Prime Front

Research documentation of the prime / zeta line: seven verified modules, one localized residual, and the complete kill list

Stefan Hamann

October 18, 2026 · v5.4 (rev dev)

Prime Front — research documents

The independent Prime Front project preserves the prime-number, zeta and Riemann-Hypothesis research line, its exact module identifiers, negative results and open assumptions.

1. `tfpt_prime_front`: complete research documentation and the kill list.
2. `note_hilbert_polya_truncations`: zeta-free finite truncations and prediction-freeze methodology.
3. `note_w1_suzuki_identification`: Galerkin identification with Suzuki's localized Weil operator.
4. `note_w3_detector_structure`: the finite detector and its structural boundary.
5. `parity_toeplitz_classification`: parity/Toeplitz classification.
6. `multilevel_toeplitz_lowerbound`: multilevel Toeplitz lower bounds.

The historical filenames and original mathematical text are retained for traceability. Historical mixed TFPT documents are preserved under `provenance/tfpt-source`; extracted research is under `documents/extracted`. No claim status is promoted by this separation. Project: https://prime-front.vercel.app.

*You are reading the **Prime Front** research record: the complete internal record of the prime / zeta line — how a discrete compiler's bookkeeping kept producing classical number-theoretic objects, which of those links became load-bearing modules, and how the remaining question was compressed, over a hundred exploratory probes, down to one strict-positivity margin input — then, once that margin was measured to be an artifact of the requirement, down to a two-object core — then down to a three-point list of which only one point is an inequality — and finally, once the boundary reformulation ran, down to that single inequality in a textbook shape (a Galerkin error with Aubin–Nitsche duality), with the prime comb named as the one structure that refuses compression — and, once the missing lemmas of that inequality were measured against their classical addresses, down to exactly one genuinely new analytic statement — and finally, with the whole chain assembled end to end on one ladder, down to a load-bearing spine that is* 96.2% *identity or completed Cholesky certificate and whose only hypothesis is a declared accounting convention. It is a **research documentation**, not a results paper. Its purpose is that a reader (or a reviewer) can see the whole route, including everything that died on the way.*

Abstract

A two-axiom discrete compiler ($c_3 = 1/(8\pi)$, $g_{\mathrm{car}} = 5$, $D_5 \oplus A_3 + \mu_4 \Rightarrow E_8$) has no business meeting the prime numbers. It does anyway: the μ_4 glue completion carries a quartic character, the signed glue census is a theta series, and the shell counts of that census turn out to be *Hecke* data. It goes further than an analogy: the E_8 shell count is literally $240\,\zeta(s)\zeta(s-3)$ as a Dirichlet series, and the lattice's self-duality gives its theta sum an exact functional equation whose fixed axis is the critical line — a framework picture, stated as such, which the document opens with and then keeps at arm's length. This companion documents the line end to end. Twenty-seven modules are load-bearing and ledger-carried (v535–v560 and v562): Hecke from geometry on the E_8 census, the Eichler trace layer, the half-integral bridge, one finite relative-trace identity of which the first three are projections, the Weil structure of the resulting family with two explicitly isolated obstructions, the amplitude route with a positive linear carrier, the matching-lemma / transport ledger package, and — out of the second phase — the margin-chain identities, the lumped M-matrix pair, the long-lag support structure, the Hardy-core

identities, the capacity-chain identities, the level-lemma identities, the Green/Szegő identities, the gauge/parity identities, the odd-sector identities, the fixed-size Ritz ceiling certificate, the four fixed-size angle instruments, the exact-form identities, the sampling/harmonics identities, the Pareto/total-variation identities, the gauge/P_{pr} identities, the cascade/vector identities, the bilinear/rank identities, the phase-2 capstone, the frame/deficit identities, the dense-limit identities of the phase-2 endgame, the Paper-II readout closure, the relative pencil one-mode structure, the separation floor, the pair-band anatomy, the Chebyshev–Loewner edge structure, the null-ray census, the two-scale kernel signs, the occupation map, the transport census, the density-dominance reduction, the prime-free closed form, the two-layer locking split, the density-fixed locking direction, the exact diagonal weight formula, the closed deterministic defect, the zero-comb identification, the rank-one pole term, the continuum determinant law, the cutoff completion, the unconditional entry certificate, the mapping completion and the 1D lock projection. Everything beyond those forty-nine is *sandbox*: an exploratory program, run as 199 preregistered probes with 4944 machine checks, which localized the residual object (the reverse-flow promotions — the v379 hardening of T133, the CP channel identities v561 of T177 and the frame-door decision v564 of T178 — live on the physics side and are counted there). That localization is the actual content of this document. The transport ledger closes exactly, leaving one coupled inequality (called **I5**) which is — *by identity, as a typing and not as progress* — equivalent to Weil positivity and hence to the Riemann Hypothesis. The subsequent arc maps I5 geographically (Connes–Consani dictionary, tightness curves, the Sonin crossing, the one-atom band), turns the positivity question into a per-zone handover induction, and then certifies its ingredients one by one until the entire remaining hardness sits in a single Loewner statement on the J-odd half of a window, reducible by Sherman–Morrison/Woodbury to one scalar ratio $r = \kappa/\varepsilon \leq 1$ (measured 0.0053–0.1814 on 16 zones), then, once ε is identified as an exact energy, to two scalars — and finally, with both scalars certified (the wing scalar unconditionally by a graded matrix cap, the boundary value by a residual certificate that carries a cancellation), to a chain that closes 16/16 conditional on exactly one strict-positivity margin input, itself two to six orders of magnitude weaker than the conclusion it buys. The penultimate part then closes the induction circle on the measured zones end to end — a certified base case, fifteen certified handover steps through a graded Loewner minorant, and the structural finding that the atom entry costs nothing — while leaving three sharp, named gaps: no reserve, no scalar step law, and no uniformity in k, with an extrapolated crossing warning at $n \approx 170$. The next part stops extrapolating and measures: on a deep ladder of 199 prime-power zones with 117 certified handovers, the crossing is located between $n = 461$ and $n = 463$ — the $n \approx 170$ figure was a fit artefact, but the wall is real, and $n^* \approx 462$ is an upper bound — and the single warning splits into three separate walls (a measured margin wall, an arithmetic ladder wall set by the twin-prime pair $521 \to 523$, and a vacuous-requirement wall at $n = 727$), while the handover mechanism itself never fails: all 117 steps certify at retention 1.000000, and the chain tears at the ratio, not at a step. The operating variable turns out to be the window depth, not the zone index. The next part acts on that reinterpretation and rebuilds the construction in a gap-coupled scaled frame (D tied to the local prime gap): two of the three walls fall *structurally* — the ladder death becomes a payable cell cost (the killer pair $461 \to 463$ now certifies spectrally with the reserve opening by a factor $\sim$1000) and the requirement wall loses its depth dependence — while the margin wall proves *frame-invariant* at exponent -0.974: it is the substance of the requirement, not geometry. The hardness is thereby traded — three arithmetic walls against two analytic demands, the positivity of one limit operator and a convergence rate below the step reserve — with the prime-gap dependence disclosed rather than hidden. The next part settles the currency question, with a fundamental reinterpretation: the wall carries the same exponent in *every* admissible currency — it is substance — but it measures the *discretisation*, not the spectrum. Under refinement at a fixed window both leading eigenvalues carry the same cell-width power (the continuum window form has no gap), the positive floor survives only as a cancellation of relative size 10^{-7}–10^{-4}, and the T109 requirement chain turns out to divide by an artifact margin; the hardness balance is restated as four parts, led by a margin-free step certificate. The next part builds that certificate and the wall dissolves: the exact Schur complement (Albert 1969, with the Moore–Penrose inverse) certifies every ladder step margin-free — eleven of them beyond the old wall, to $n = 1331$, and all seven zones where the requirement chain tore — the wall was an $O(1)$ numerator divided by an artifact floor, chains now stop only at the compute cap and never at a step, and the (R) demand itself proves grid-bound and is deleted rather than transported, leaving a two-object core: ε relative to κ via the exact Szegő identity, and

transport of an $O(0.1)$ Schur complement between grids. The next part splits that core: the transport across a real regrid certifies only mild refinement (a clean split at ratio $\rho^* \approx 1.8$, and for a principled reason — on nested ladders, where the transport error is exactly zero, the Schur floor itself falls like $\rho^{-1.7}$, so no bound can undo the drop), while a two-scale compression breaks the compute cap: a certified margin-free step at $n = 155\,921$ (117× deeper), chains of ten steps, the stopper always cost and never a step — leaving the shortest list the program has produced: three points, of which only one is an inequality (the classical Szegő–Levinson prediction-error bound for one symbol). The next part executes the reformulation point of that list, and the induction step turns out to *be* a boundary process: partial minimisation over the interior (Haynsworth) compresses the window into a state (Σ, r, σ) of $12 \times 12 + 12 + 1$ numbers that carries the global rank-one pole *exactly* (bordered elimination, no truncation) and updates by a Riccati-type recursion whose cost is independent of the window — one unbroken chain of 169 236 prepends ran to a window of 1.35 million cells (903× the old cap) at a flat 76 μs per step, declared honestly as a cost-geometry demonstration and not a Weil certificate. What refuses compression is the *prime comb* itself: the archimedean tail decays cleanly, but the reflection comb couples every incoming cell to an interior cell at $\alpha - \log n_j$ for every prime power in the window, so the full symbol never decays in the width and no sparse faithful state exists. In exchange, the one remaining inequality acquires a textbook shape: ε is the error of a piecewise-constant Galerkin method for the D-independent function $2\sinh(x/2)$, Aubin–Nitsche duality predicts the measured exponent ($\theta = 1.79 \Rightarrow s \approx 0.90$), and the target inequality is $\varepsilon(\alpha, D) \geq c_0 D^\theta \alpha^\varphi$ across the prime-power jumps. The last part then makes that inequality *theorem-shaped*: the Galerkin reading becomes four exact identities, the lower-bound direction acquires a certified two-level chain that loses no power of D ($\theta' = 1.74$ against $\theta = 1.76$), the jumps acquire exact closed forms — with the correction that prime-power entries *raise* ε, so the factor-120 falls were a sweep artefact of the earlier run — and what remains is three named analytic lemmas about one symbol, two of them constants rather than rates. The next part measures those three lemmas against their classical addresses, and two of the three stand: saturation is an *identity* here; the Schur-floor lemma is rescued by the strengthened Cauchy–Schwarz shift onto the oscillation Gram, whose arithmetic aliasing symbol suppresses the prime comb quadratically — certified positive on half the measured windows, and on a 15 680-point FFT lever the certified floor rises logarithmically and crosses zero, so the failing windows were under-resolved rather than obstructed; and the corner lemma corrects itself — a log singularity is the boundary of all powers, so no power-law corner exponent exists to cite, and the certified chain is repaired at its last step, the rate now living on the oscillation mass. The missing analytic content is thereby exactly one statement: a $D^{1.75}$ lower bound for that mass. The closing part then closes the *arithmetic* half of the remaining list as a theorem — the aliasing symbol is positive for every cell width below an explicit $D_0(\alpha) = \exp(-(\Xi(\alpha) + B))$, with Ξ the prime-power atom count and $B = -1.0474$ universal — proves that the mass bound is *not* derivable from the identities already in hand, and reduces it, through the exact identity $\kappa_{\text{end}} = 1/(1 + R)$, to a single discrete Harnack inequality with a classical address; the chain contains no zeta input anywhere. The final part then *explains* that inequality: the two increment families are the odd and the even half of one sequence, offset by a single fine cell, so given one sign on the corner cells $|R - 1| \leq 0.047$ holds *unconditionally* by a two-line pairing argument — while the per-cell version of the inequality is provably false, and the honest defect count rises from three to four, because the frame's window grows with the zone (the unconditional D_0 route covers not one handover of the real ladder) and the strengthened Cauchy–Schwarz constant decays with the width. The series then closes by *assembling* what it built: on one common resolution, thirty consecutive prime-power atoms form a run in which the output of each certified handover *is* the input of the next (the incoming atom block is bit-exactly zero, the composition residual 2.9×10^{-14}), all five stages complete on 52 of 52 ladder zones with 430 completed Cholesky certificates, and the load-bearing spine turns out to be 96.2% identity or Cholesky certificate with exactly one hypothesis — an accounting convention — so that the Harnack pair leaves the spine and survives only as a second, independent route. What remains is stated as such: not size but *uniformity* in the zone index, quantified in a six-point distance statement (no zeta input, RH unused in either direction, a finite run bounded by the smallest log-gap it contains, a measured ladder that is not all zones, uniformity as the open item, and the resulting object typed as what it is — a lower bound in numerical analysis, not a statement about zeros). A second phase opens past the finale: its first part dissolves the segment-seam question into a monotone free-resolution construction (re-gridding only at record-small gaps, all twelve affordable real seams certified), turns the

continuum step into an exact fractional-Dirichlet identity, and leaves a sixteen-point full-proof map containing exactly *two* genuinely new inequalities — a direction lemma and a zone-uniform seam floor. Its later parts dissect both inequalities (both proof-shaped; one refuted as stated and replaced by a band plus an enumeration), work the resulting four-point list cheapest first (three of the four stand at their preregistered bars; the fourth — a concentration bar — is missed honestly, by 3.6%, systematically in the re-gridding ratio), and then test the law that miss preregistered: the fitted law falls once on 331 fresh transports with the bar frozen, while the structure underneath it is solved with identities — the flat border vector sits at 1 exactly, a linear density profile at 2 exactly, everything above is curvature, and the curvature chain is a per-transport theorem holding on all 436 transports. The two pieces that break leaves are then attacked head on, and exactly one of the two stands: the graded-to-uniform *bridge* stands as an identity — the Céa/Strang defect in matrix form reproduces the directly computed uniform floor on 84 pairs with zero overshoot, explains the measured 8% false positives completely, and carries both deep seams to positive fine floors at up to 3.8× the factorisation cap — while the curvature bound honestly breaks its own frozen shape band on 13 of 545 disjoint test transports and is reduced to a zone-uniform bound on one exponent. **No claim of progress on the Riemann Hypothesis is made anywhere in this document**, and none of the sandbox findings moves any ledger marker. The document is deliberately symmetric about failure: a dedicated section lists the readings and routes that were refuted — an essential-singularity reading, a C/g proxy law, a density chain, an accumulated invariant with self-propagation, the Landau–Widom anchor, the symbol route for the odd channel, three certificate candidates for the avoidance norm plus the Combes–Thomas certificate at the boundary, and the naive margin route — because in a program of this shape the kill list is the more reliable output.

[E] exact / machine-proven [C] conditional [O] open / axiom [X] kill test

Contents

I Orientation 5

1 How to read this document 6

2 Motivation: why a compiler meets the primes 11

3 The big picture: a self-dual lattice and its own mirror 12

II The verified layer 16

4 The verified layer: forty-nine modules 16

III The localization program and the certification arc (T79–T125) 52

5 The localization program (sandbox) 53

6 The handover induction (T95–T101) 55

7 The certification arc (T102–T125) 56

IV Honest negatives: the kill list and the series balance 85

8 The kill list 85

9 The series balance (T102–T125) 87

V Toward uniformity: the second phase (T126–T199) 88

10 Phase 2: toward uniformity (T126–T199) 89

VI The closed-form theory, the surface theorems, and the detector 167

11 The closed-form theory of the parity read block: statements 167

VII The Hilbert–Pólya candidate: the operator program and the keystone 171

12 The operator program and the keystone 171

VIII Status, contracts, and open problems 292

13 Status and outlook 295

Appendices 302

A Probe index, diary parts T11–T199 303

B Symbols used in Sections 5–7 366

Part I
Orientation

This part carries orientation, not results. It states the reading rules of the document (the scope box below is the contract), gives three routes through the document for three kinds of readers, explains why a discrete compiler meets the primes at all (Section 2), and states the framework picture — a self-dual lattice and its own mirror — that motivated the whole line (Section 3). Everything said here is said again, precisely and with provenance, in the parts that follow.

1 How to read this document

Scope and status — read this before anything else

1. **No RH claim.** Nothing in this document claims progress towards, or evidence for, the Riemann Hypothesis. Where an equivalence to RH appears (Section 5) it is an *equivalence typing*: a statement about which object carries the difficulty, produced by an exact identity, and deliberately *not* a statement that the difficulty has been reduced.
2. **Two layers, never mixed.** Claims marked [ledger] are load-bearing: they have a row in `verification/status_ledger.csv`, a module in `verification/`, and a script citation here (the blue `[verification/...]` tag). Everything marked [sandbox] is exploratory work in `experiments/tfpt-discovery/`; it carries no ledger row, no marker, and no authority. Section 4 is the only section that is entirely [ledger].
3. **Status markers are quoted, never upgraded.** The markers [E] [C] [O] [X] shown in Section 4 are read verbatim from the ledger rows of v535–v563, v569–v570, v573 and v576–v596. This document introduces no new markers and moves none.
4. **Fits are declared as fits.** Every scaling law, slope, exponent and extrapolation quoted from the sandbox is a fit to finitely many measured windows and is labelled as such at the point of use. Sixteen resolved zones are sixteen zones, not all zones.
5. **Classical results are named as classical.** Kneser neighbours, Eichler/Witt, Shimura, Waldspurger, Cohen, Weil/Guinand, Bombieri, Yoshida, Connes–Consani, Gronwall/Robin, Douglas, Slepian–Landau–Pollak, Sherman–Morrison/Woodbury, Grenander–Szegő and the rest are classical and are used as such; the compiler-native part is the wiring, not the theorems.
6. **Numbers are literal.** Every numeric value in this document is transcribed from the research diary `experiments/next.txt` or from a ledger row. Figures that could not be drawn from literal data are marked *schematic* in their caption, and the caption states exactly which quantities in the figure are literal.

Reader's guide: three routes through this document

The mathematician. Enter at Part VI: the seven closed-form theorems of the parity read block, with derivation sketches (Section 11), and the one remaining arithmetic statement, posed self-contained and without TFPT vocabulary (Section 11.2). Then read the keystone — the wall in four equivalent languages, each finitely decidable per window (Section 12.6) — and, for the machine-checked backing of any single claim, the matching module subsection in Part II.
The skeptic. Read the scope box above, then Part IV: the kill list (Section 8) and the honest RH distance in six checkable statements (Section 9). Continue with the status section (Section 13) — what is certified, what is measured, what is hypothesis input — and its final fence. The off-line-zero detector line (Section 11.2) is where the program exposes itself to falsification.
The reader in a hurry. Read the motivation and the big picture (Sections 2 and 3), then the keystone round (Section 12.6), then the status section (Section 13). Everything else is reachable from those four places through the cross-references.

The diary parts are cited as **T***n* (part n of the research diary, chronological, $n = 11 \ldots 177$). The probe file, the contract name, the verdict and the check count of every part are listed in Appendix A. Section 2 explains why a physics compiler meets the primes at all and Section 3 states the framework interpretation that motivated the whole line — a self-dual lattice, its functional equation, and a mirror that returns at the far end of the program; Section 4 is the verified layer; Sections 5, 6 and 7 are the sandbox program in three acts; Section 8 is the kill list; Section 9 is the series balance (cascade, certification degree, the honest RH distance); Section 10 is the opening arc of the second phase (T126–T199: the seam architecture, the two inequalities dissected, the four-point list worked cheapest first, then the κ law tested and broken productively, then the graded-to-uniform bridge established as an identity while the curvature bound is reduced to one exponent, then the self-supply loop built one number short of closed — the secular sandwich and the Perron sign theorem — then the reverse flow: the identity block promoted to v542, a spectrum-only boundary-Dirac coupling and a certificate-floor audit that hardened v379 — then the pole-free floor closed as existence

with a sign anatomy (T134) and a bounded seam state where the Weil window provably has none (T135), and finally the M-matrix pair, which closes the a-priori radius item by Varga's identity while the exact D-bookkeeping puts the whole loss in the margin (T136, promoted as v543), and the long lags, whose support turns out to be an arithmetic stripe set and whose absolute-value envelope family is certified *dead* (T137, promoted as v544) — and finally the compensation itself, whose sign law turns out to be *interval geometry* while the m-paired Neumann certificate removes the arithmetic wall on all 77 dead blocks (T138), and the classical decay lemma, refuted at its hypothesis for an arithmetic reason while the sign law is *derived* from one exact telescoping identity and the measured core shrinks to one signed inequality at stripe distance $b \leq 16$ (T139), and finally that one inequality itself, which acquires an *exact finite core per zone* — the telescope identity lifted to the form level, the spectral radius an exact eigenvalue of a closed-geometry covering kernel times a mass-plus-Dirichlet form, the checkerboard split replacing the $O(nb)$ Weyl steps by three D-independent ones, and all the D-dependence located in the geometry (T140), and finally the Hardy ingredient itself, which *resists* but acquires an address: four exact identities put it in classical two-weight shape, the certified constant is not zone-uniform while the object it bounds is, the additive shape is dead as a shape by its own exact Weyl floor and the joint shape fails at the normalisation alone, so the rest list collapses to one closed conductance profile (T141, promoted together with T140 as v545), and at last that profile itself, *constructed* rather than guessed: the capacity decomposition exhibits the optimal Hardy weight exactly — $\Omega = 1$ by a projection identity where T141 had guessed 20.7–2724 — the failure shrinks to a constant factor $\sim$2.3 flat in D, and the rank ladder closes the whole comparison path, forcing the sharp capacity–Rayleigh route (T142), and at last that sharp route itself, which *carries*: the exact capacity–Rayleigh form is an identity on all 26 windows, Maz'ya's capacity criterion applied to the gap form lands inside its window $[1/4, 1]$ with a zone-uniform loss factor ($D^{-0.048\pm0.010}$), and D-uniformity is reduced to *one* named inequality — a non-Markovian Maz'ya capacity bound whose interval structure points at Muckenhoupt's 1972 two-weight calculus (T143), and at last that interval route itself, which *carries*: the interval class is exhausted exactly (11.4 million intervals via a Cholesky prefix-sum identity), the closed two-weight sup lands inside the Maz'ya window flat in D, the family restriction falls entirely by a max-density-subgraph bound over all 2^m sets, the Markov perturbation route is certified dead, and the certified chain has exactly *one* unproven input — the absolute Maz'ya constant c_0, its sharpest shape a Muckenhoupt-type hypothesis (T144), and at last the proof attempt itself, whose verdict is *one step missing*: the classical Maz'ya proof transcribes step by step onto the non-Markovian form, the Markov property sits in exactly *one* line (M4), that line splits with its mass half a theorem that dominates ($\sigma_{\mathrm{tot}} = 0.215$–$0.443 < 1$ everywhere), c_0 becomes explicit (2.248–4.227, flat in D) and S1′ is *certified* per window on 64/64 windows, while an explicit no-go proves that an a-priori bound on the minimiser's level profile — the level lemma L1 — is necessary and cannot be replaced by any weaker hypothesis (T145; the identity spine of the capacity cycle is promoted as v546), and at last the level lemma itself, which *stands*: L1 closes on the measurement surface as a chain of theorems and certified window inequalities with no step reading the minimiser — the proof lever is the resolvent identity $\psi = \lambda R\psi$ itself (the $\Theta(D^3)$ smallness turns from obstacle into tool), Davis–Kahan is instrumented and discarded, Maz'ya's classical dyadic 8 falls to 2 because the cake base is free, and the a-priori constant $c_0^{\mathrm{ap}} = 3.90$–4.85 lands at the size of Maz'ya's classical Dirichlet value 4 — with one genuine remainder, D-uniformity for *all* D (T146; the a-priori core is promoted as v547), and at last that asymptotics itself, which acquires an *exact shape*: the last open object becomes the identity $\Gamma = \sqrt{Q^\star}\, S_w$ — all-D uniformity split into a purely spectral and a purely geometric factor, both certified on the surface — the classical decay route is shown by computation to be asymptotically vacuous, delocalisation itself *is* the bound, the mechanism is named (a Toeplitz-minus-Hankel section whose bottom eigenvectors are Fourier modes, by Szegő/Widom theory) with the sharp prediction $Q_B \leq 2|B|$ holding, and one statement — a Szegő theorem for the diagonally reweighted section — lifts the chain to all D (T147), and at last that lifting statement itself, dissected: the second factor S_w *closes* on the surface by an $\mathrm{LDL}^{\mathsf{T}}$ layer-cake certificate, at the price of an honest negative — the Toeplitz symbol is refuted at 0, so

positive definiteness is a finite-section effect — while the lifting statement resists with its hypothesis isolated to a single named scalar, the total variation of the log-whitening weight, proved to be the roughness and not the conditioning by a controlled weight-class experiment; the first end-to-end number arrives — the certified chain delivers 8.4–15.9% of the true gap on all 48 windows (T148; the identity/certificate core of T147+T148 is promoted as v548), and at last that scalar itself, attacked through the gauge freedom: the whitening diagonal is a free choice, the constant gauge *eliminates* the total variation exactly (0.0000) at a certified sandwich price — and thereby refutes the hypothesis, since the gauges that kill the flutter move the smoothness functional by at most 0.9%: the roughness sits in the *form*, not in the multiplier, the missing input relocates to the deep modes of the pure Toeplitz-minus-Hankel section, and the Dirichlet control corrects the question to a ladder bound $\nu_k \le Ck^2$ with m-free C (T149), and at last that ladder itself, which *names the mechanism — parity*: the form is exactly the compression of the full symmetric Toeplitz section onto its antisymmetric parity sector, the sign-changing symbol's entire negative inertia sits in the *even* sector (certified 72/72) — T148's honest negative explained rather than worked around — the flutter amplitude becomes a certified form functional with a closed atom budget, and the arithmetic atoms turn out to be co-responsible for the positivity (the archimedean section alone would be indefinite), while one gate remains: the ladder constant still grows (13.81π, $x^{0.258}$ against the bar 0.25) — the gap from π exactly the arithmetic leakage of the bottom mode (T150; the identity/certificate core of T149+T150 is promoted as v549), and at last that odd-sector ladder itself, which *closes by rerouting off the ladder*: the odd grid steps over the symbol's negative window ($\theta_c/\theta_1 = 0.328$–$0.407$ on 72/72 — the positivity of the odd sector is a grid fact), the bottom spectrum is certified against the parity Laplacian ($\lambda_k \le S\mu_k^P$, $S = 1.10$–2.39), and one discrete Sobolev step yields a per-mode bound *linear* in k with a *non-growing* constant ($C_S = 11.51$–19.57, trend $x^{0.020}$) where the quadratic ladder grew — two of T150's three rest items settled (the archimedean minimum closed and attained exactly; the symbol route a computed dead end: the local model is a matrix statement, not a symbol one), one scalar remaining a fit — the bottom pencil ratio $R = K_{\mathrm{bot}}/\kappa = 3.36$–$9.71$, flat, exactly where the no-go breaks (T151; the identity/certificate core of T151 is promoted as v550), and finally that pencil ratio itself, attacked head-on: both hoped-for gifts from the smooth kernel are *refuted* — the archimedean section is itself negative in the odd sector and the atom part too, so odd-sector positivity is a *cancellation* between geometry and arithmetic, not a property of either half — while the floor nevertheless closes structurally, a Schur two-block criterion with a fixed 16-mode block certifying $\kappa \ge 0.225$–0.250 flat across all windows, leaving the ratio certified on both ends ($R \le K_{\mathrm{bot}}/t = 4.41$–$8.47$) with one term missing — an m-free ceiling on K_{bot} — and 89–92% of the dominant loss Ψ mapped onto the eight modes the certified ladder already controls (T152), and at last that mapped rebuild itself, carried out and *refuted by exactly its own factor*: Ψ is pinned between a_1/λ_1 and $1/\lambda_1$ with a_1 exactly the $8/\pi^2$ of the first parity sine — the reserve was never there — while the theorem-grade collapse closes Ψ as a term, the archimedean part turns out *positive* on the bulk parity block after an m-free peeling, and the two remaining terms lose to one and the same missing object, a Green/alignment estimate of two inverse-iteration columns (T153), and at last that estimate itself, attacked head-on — and the ceiling *closes, exactly and at fixed size*: the sixteen-column certificate $\mathrm{span}\{t_1..t_8\} + A^{-1}L_P\,\mathrm{span}\{t_1..t_8\}$ needs no residual argument at all (Ritz values are upper bounds for the eigenvalues of the same index, Courant–Fischer) and agrees with the inertia-certified true bottom ladder constant to 5.2×10^{-7} on every window, retiring the size-m factorisations from the ceiling step; the obstruction acquires a geometry — seven of eight bottom directions agree with the parity Laplacian to a fraction of a degree, *one* sits at 82.93–89.79°, and that single misalignment *is* the collapse price, recovered in full per window by one Cholesky, moving per-window end-to-end to 4.45×10^{-2}–3.13×10^{-1}, inside the target band — with both remaining terms now uniformity terms carrying certified per-window numbers, the most accessible arithmetic-free (T154; the certificate core is promoted as v551), and at last those two floors themselves, attacked head-on — the complement floor becomes an *exact* fixed-size certificate: a 12×12 problem in the exact Kac–Murdock–Szegő numbers plus one 12×8 overlap matrix reproduces the size-m eigenproblem to 0.999999783–0.999999958 on every window, the 83–90°

direction is demystified (it lives on modes 9–12, which is exactly why $K = 12$), and the collapse price is recovered 78.8–100% at fixed size — while on the block side the full symbol infimum turns out *negative* on every window against a positive section floor, so no symbol argument can ever produce that floor: the mechanism must be a finite-section Fejér cancellation (T155), and at last those two rest objects themselves, attacked head-on — and both become *single scalars*: the t_1-loss of the bottom Ritz mixing is an exact closed function $F(P, r)$ of two dimensionless numbers (verified to 2×10^{-16}), its two-line ceiling $r \le 1/(Ls) \le 1/p_1$ is tight to a factor 1.03–1.16, and the whole first term is *one* m-free lower bound on the angle $p_1 = \cos^2 \angle(t_1, e_1(A)) = 0.2010$–$0.3282$, while the block positivity is re-identified as an *alignment* fact rather than Fejér mass damping — on the minimiser an arch part and an atom part of nearly equal size cancel, the expected arch inequality ≥ 1 is refuted and the additive split is divergent (T156), and at last those two angles themselves, attacked head-on — neither falls, but both change shape: the p_1 floor becomes a *theorem* times one measured fixed-size number (the sine-block confinement, from the certified floor and ladder alone, proves $e_1(A)$ lives 97.1–98.4% inside the first sixteen parity sines, and the resolvent route then pins 97.9% of the measured angle), the whole first term is re-stated as a bound on *one* diagonal entry of a 16×16 Schur complement the chain already forms — which Cauchy–Schwarz misses by two to five orders of magnitude, because the cancellation is nearly complete — and the arch half acquires an *executed* adaptive Lipschitz certificate, while the alignment term stays a per-window domination with a shrinking margin (T157; the four instrument candidates of T155/T157 are promoted as v552), and at last the cancellation-seeing bound itself, which *exists and is a theorem*: the Thomson dual form turns the Schur entry into a Dirichlet maximum, so every trial vector bounds it from the right side — which is exactly why Cauchy–Schwarz missed by three to five orders of magnitude, a maximum evaluated at one direction — a Cholesky ladder of strictly positive terms pins the entry to within 1.13–1.27, flat, from one 16×16 factorisation the chain already builds, the entry is exactly two-dimensional once one Green column is granted, and T157's growth pointer is honestly refuted: the atom mass grows faster than the arch floor in every band, so what carries the domination is precisely the off-band coupling a dyadic argument discards, and the measured-step count falls from three to one (T158), and at last the cancellation itself, executed algebraically — and the algebra is delivered while the bound is not: seven machine-checked identities land at once, among them the gauge identity ($\sum_d w_d = 0$ exactly, because Toeplitz-minus-Hankel annihilates constant lag vectors — the form is exactly *blind* to the lag mass of the arithmetic, which is precisely where the h^2-sized halves live) and a closed 256-term Dirichlet-kernel form of the geometric weights; the honest answer to the kernel question is negative with an exponent — the h^2 does *not* telescope away, both halves grow like $h^{2.1}$ because the cancellation is the common weight, and four hoped-for routes are closed with exponents, including T158's own fixed sixteen-vector — while the genuinely new structural win is a sign law: the archimedean bulk block is exactly a symmetric Z-matrix, raw, with a closed sign-based floor comfortably above target, and the atom block obeys no sign law; the theorem count jumps from six to thirteen (T159; the theorem cores of T158/T159 are promoted as v553), and finally the pairing itself, attacked through its correlation structure — and the watershed of the phase arrives: the tail sum W^1 does not oscillate at all (three sign blocks, with a closed bound from the trigonometric zero count, so the Leibniz block device is refuted by being *worse* than the pricing it was meant to beat), three closed sign-definite moment laws evaluate the smooth archimedean half of the pairing down to the double-precision floor (a second numerical horizon, declared), and a machine-checked *sampling identity* shows that the atom half *is*, identically, a finite combination of Λ-weighted prime sums at thirty-two explicit frequencies, needed to relative depth h^{-2} and measured to cancel only to 0.00–0.37 of the trivial bound — the h^2 cancellation is the intrinsic arithmetic hardness of the problem, not an assembly artefact (T160), and finally the circularity triage itself, whose answer is the most consequential of the phase: the thirty-two frequencies are exactly the Fourier harmonics of the log-window (a theorem), the measured cancellation is fully the prime-number-theorem main term, and the required depth exceeds RH strength while sitting below the boundary term of every partial summation — the chain is *not* circular, it needs *more* than RH-strength input would supply, so the h^2 sits in the *split* and not in the primes; the analytic half

of the arch bound closes m-free with a scale-free Bernstein rate and a closed head split, the sign inequality is refuted in all four readings and replaced by a certified fraction bound, and the closed cores of T160+T161 are promoted as v554 (T161); then the third split itself, which *exists* while the exhaustion saturates — the archimedean Mellin ladder lowers the demand in closed cell moments and turns at its second rung, one Abel step makes the demand prime-free with all arithmetic in the single Chebyshev constant, and the Fejér split goes below the yardstick on every window at a price growing like $h^{2.86}$: relocated, not closed (T162); and finally the Pareto front itself, which resists *as a theorem* rather than as a failed search — the exchange law makes price and demand two coordinates of one exact identity, a four-line total-variation floor $\mathrm{TV}(x) \geq 1/\mu_1^P \sim h^2$ closes the flat-price route for every admissible trial vector in the parity sector, and the hardness is thereby identified as the spectral gap of the parity Laplacian meeting the entry normalisation — never the primes — with the theorem cores of T162+T163 promoted as v555 (T163); and finally the sector change itself, where both arms of the successor are decided — the chain spends its entry ceiling at power exactly one, yet the $O(1)$ gate turns out to be discharged window by window by a Cholesky identity, so the last analytic term collapses onto a single quantifier $\sup_m 1/g_{16}(m) < \infty$, while no sector can help, as a theorem: the entry normalisation is a gauge, the hard ratio is the same number in every sector, and floor times transfer reproduces the h^2 identically (T164); and finally the alignment question itself, which closes *by certificate* rather than by failed search — the gauge-invariant exchange identity $P_{\mathrm{pr}} = g_{16} \cdot R \cdot (\mathrm{TV}/(t_1 v)^2)/\mu_1^P$ makes demand and price one equation, so every crossing vector pays the old KMS h^2 and the alignment question was never independent (its m-uniform form *is* the quantifier), the price exponent decomposes exactly, the quarter-bar drift is settled as zone depth on a decoupled surface, and exactly one genuine open object remains — a lower bound on the sixteen-step Schur-cascade gain, a cancellation beyond absolute-value budgets — with the theorem cores of T164+T165 promoted as v556 (T165); and finally that cascade lower bound itself, dissected completely: four identities turn the fifteen ladder increments into one readable object — the whole gain is the near-collinearity $1/(1 - R_K^2)$, one rung carries most of it, and the gain is invariant under the spectral normalisation, so the h^3 is a property of the arithmetic Gram block alone — the cancellation lives in a 2×2 Gram determinant (no entry cancels, neither half is collinear alone: the same arch-against-atom mechanism as T159/T160, one level up), an anti-fitting scramble destroys the effect by a factor 4569, and the best closed route reaches gain $h^{+1.32}$ against the target $h^{+3.11}$ — an exponent gap: the one missing inequality *is* the cancellation, now stated in three equivalent dresses, with two honest contract corrections recorded (T166); and finally the most constructive dress, closed *as a construction*: at $K = 2$ the closed vector is exact — a theorem — so the third dress collapses onto the second, a machine-checked identity unifies all three dresses into *one* inequality, the threshold is mildest exactly where the vector is free, perturbation theory converges to the wrong object, the scrambled control loses positivity of the 2×2 block itself, and the whole programme rests on one scalar built from three closed lag sums, with the theorem cores of T166+T167 promoted as v557 (T167); and finally that scalar in its last three costumes — the Lagrange identity makes it a genuine sum of squares whose smallness sits entirely in a PSD kernel at one closed vector, so the hardness proves *self-similar* under exact reformulation (T168); the candidate table then closes the loop as an identity — the only threshold-meeting candidate reintroduces the determinant itself, the chain is rebuilt R4-free, and the one open object acquires standard analytic shape, a bilinear von Mangoldt sum against closed Dirichlet weights (T169); and the unconditional sieve box classifies that shape *as a theorem*: the form is written down exactly, a five-order cancellation between three explicit pieces, and it collapses back onto three linear Λ-sums by a rank-3 theorem and a Type II collapse, so R1 is finally a *near-degeneracy*, not a size — beyond every size-bounding tool — with the theorem cores of T168+T169+T170 promoted as v558 (T170); and the capstone then assembles the whole map in one connected run — all sixteen links of the reduction chain, from the I5 floor to the final R1 shape, thirteen as theorems and three as per-window certificates, the eight-route no-go battery failing exactly as classified, and the precision ledger closed — adding *zero* new uniform statements, with the capstone promoted as v559 (T171); and the frame arc then measures the map's frame dependence — the identity

tower carries unchanged beyond frame A while every carried number is frame-bound (T172), the demanded rate is itself a frame datum by the exact identity $q = 1 - s$ and the deficit between demand and delivery becomes the number of the phase (T173), and the gauge route is exhausted by theorems: everything multiplicative cancels exactly, the additive arch/comb mixture admits no factorisation, and the deficit is measured frame-free at $+0.1111 \pm 0.0222$ (5.0σ) with its driver identified as comb density per lag cell (T174) — with the theorem cores of T172+T173+T174 promoted as v560; and the placement phases are then measured directly: they are real in $\log R$ and — for the first time in the series — *causal* under an exact-rebuild intervention, yet no phase formula is certifiable because the response is provably non-smooth (the Schur floor is a near-degeneracy), the per-anchor heterogeneity dissolves substantially into the OLS error bar, and the deficit(dens) curve falls monotonically over three decades to a densest reachable bin *consistent with zero* — true zero, power-law approach and low plateau undecidable under this sieve (T175); and the reverse flow then resumes at the far end: the CP mirror turns the gauge-degree toolkit on the four CP ledger rows of the physics side — Tier-1 invariance of the $\pi/3$ structure is empty by algebra, *two* inequivalent Jarlskog-class invariants carry $\pi/3$, neither frame-coupled, four ledger readings are narrowed with no marker moves, and the exact cores are promoted as v561 (T177, landed while T176 ran); and the larger sieve then lands (T176, numbered by contract, landed after T177): the density ceiling rises $361 \to 6120$ under a $20.8\times$ sieve, both new ratio-4 bins are consistent with zero (pooled $+0.0496 \pm 0.0372$), the plateau window narrows by a factor 3.6, R1 closes as the Feynman–Hellmann identity, the $300\times$ harmonic anomaly resolves into a pole, and *the phase-2 measurement programme closes as planned* — the exact cores promoted as v562); Section 13 is the status statement.

2 Motivation: why a compiler meets the primes

TFPT is a discrete compiler with two inputs: the seam constant $c_3 = \frac{1}{8\pi}$ (P1) and the carrier rank $g_{\text{car}} = 5$ (P2). They force the lattice completion $D_5 \oplus A_3 + \mu_4 \Rightarrow E_8$, and the readouts of the Standard-Model skeleton are taken off that completion. None of this is about number theory. The prime line begins with an accident of the *glue*.

The completion is not $D_5 \oplus A_3$ but $D_5 \oplus A_3$ *plus* a glue group μ_4 — a cyclic group of order four. A cyclic group of order four carries a quartic character, and the quartic character on $\mathbb{Z}$ is χ_{-4}, the non-trivial character mod 4. Once the census of glue vectors is counted *with that sign*, three things happen at once, and all three are elementary consequences rather than choices:

1. The signed glue count is a **theta series**. Signed counting of a quadratic form is a theta series by definition; here the compiler's own count θ_3^2 is exactly the Gaussian-integer theta, $r_2(n)$, so the census counts $\mathbb{Z}[i]$-ideals (T85).
2. The character makes the count a **Dirichlet object**. χ_{-4} is the character of $\mathbb{Q}(i)$; the associated L-function is the first Dirichlet L-function, and the split law "$p = a^2 + b^2$" becomes a compiler-visible fibre property of each prime (T21, channel B: χ_4 fibre $\iff$ $p = a^2 + b^2$, verified 81/0/87/0 on $p < 1000$; χ_3 fibre $\iff$ $p = a^2 + 3b^2$, 80/0/86/0).
3. The shell counts of the census carry **Hecke eigenvalues**. That is the surprise, and it is the content of the first load-bearing module.

The first question of the series was therefore not "does this prove anything about primes" but "which of these links are *mechanism* and which are pretty coincidences". Parts T11–T13 established the signed glue theta, its L-series and the glue-character atlas (19/19, 16/16, 11/11); T14–T19 tested six candidate contracts in parallel and produced two self-corrections against the project's own earlier readings (T14 DEFLATED: the measured 2π is the universal Bisognano–Wichmann/Unruh conversion, not "the self-dual width"). The cross-finding of that batch is worth stating because it survived the entire series: the structure is exact at the census/lattice level (weight 4), and the gap is always the *same* gap — the canonical transport to the GL(1) / $\xi(s)$ world (multiplicity

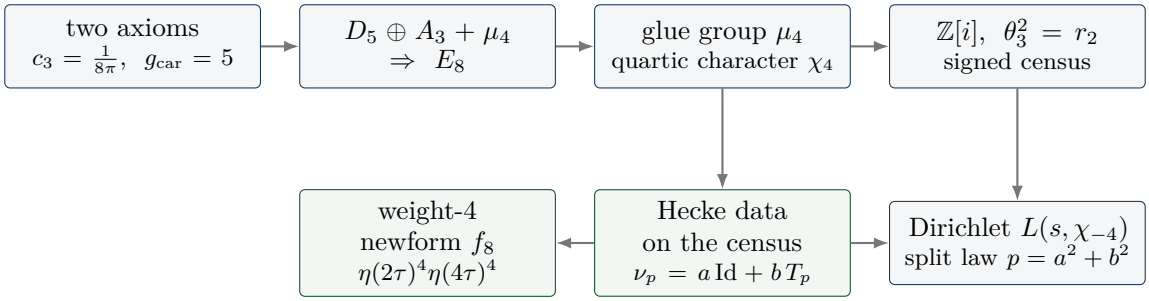


Figure 1: The chain by which the compiler acquires arithmetic. Nothing here is chosen for number theory: the glue group is forced by the completion, the quartic character is the character of a cyclic group of order four, and the signed count of a quadratic form is a theta series. The green nodes are where the line becomes load-bearing (Section 4).

one, weight 1/2, the archimedean place, the derivation of the Hecke action). One meta-contract, `ZETA.CENSUS.TO.GL1` [O].

Parts T20–T25 then killed, in four stages, the slogan "the archimedean term comes from the seam": the raw seam density of states is flat/slightly decreasing (loglog slope ≈ -0.40) while the classical digamma kernel grows logarithmically (measured 0.15917 against $1/(2\pi) = 0.159155$, rms 3×10^{-6} for $t \geq 20$) — KILLED-AS-NAIVE (T20, 25/25); the non-DOS successor located the missing logarithm in the interval cut but failed the one-constant dictionary at $2/\pi$ (T22, PARTIAL); and both dictionary closures plus the scattering-phase observable died (T23–T25, $23/23 + 25/25 + 29/29$). This matters for the rest of the document: the archimedean place was never going to come for free, and when it finally arrived (T82) it arrived from *inside* the same family, not from the seam.

3 The big picture: a self-dual lattice and its own mirror

What this section is

A **framework interpretation**, not a result. It says how the pieces fit together and why the line was pursued for a hundred parts. Every factual statement it uses is either classical (and named as such), one of the forty-nine modules of Section 4, or a sandbox probe with its part number. Nothing here is an argument about the Riemann Hypothesis, nothing here is used later as a premise, and the picture itself carries no status marker.

3.1 (i) The zeta function is literally in the geometry

Count the E_8 lattice points shell by shell. The even shells satisfy $N(2n) = 240\,\sigma_3(n)$ — [`verification/v1018_e8_directed_readout.py`] — direct lattice counting gives 240, 2160, 6720, 17 520, 30 240, 60 480 for $n = 1 \ldots 6$ (T6, one of the parts that predate the indexed range of Appendix A) — and σ_3 is multiplicative, so the counting function of the lattice is a Dirichlet series with an Euler product:

$$\sum_{n\geq 1} \frac{N(2n)}{n^s} \;=\; 240 \sum_{n\geq 1} \frac{\sigma_3(n)}{n^s} \;=\; 240\,\zeta(s)\,\zeta(s-3) \qquad (\text{at } s = 5:\ 1.705672 \text{ vs } 1.705678).$$

The Riemann zeta function stands in E_8's own counting function, with a simple pole at $s = 4$ of residue $240\,\zeta(4) = 8\pi^4/3$ (T12). This is classical lattice theory — Eisenstein series of weight 4 — and it is stated here only to fix where the arithmetic enters: not through a modelling choice, but through the object the compiler already had. The contrast is instructive: the same multiplicativity *fails* for the $D_5 \oplus A_3$ sublattice alone (52, 576, 1584, 16 128 with $576 \cdot 1584 \neq 16\,128 \cdot 52$, T6). Without the glue there is no Euler structure in the count.

3.2 (ii) The glue carries the character of the Gaussian integers

The completion is $D_5 \oplus A_3 + \mu_4$, and a cyclic group of order four carries a quartic character, which on $\mathbb{Z}$ is χ_{-4} — the character of $\mathbb{Q}(i)$. Two consequences make the primes a *fibre property* of the compiler rather than an application of it. First, the split $p \equiv 1$ versus $p \equiv 3 \pmod 4$ is visible to the census directly: the χ_4 fibre holds exactly for $p = a^2 + b^2$ (81/0/87/0 on $p < 1000$, T21). Second, the character kills a pole. Of the three character channels of the E_8 counting function, the trivial one is $240\zeta(s)\zeta(s-3)$ with its pole at $s = 4$, the spinor one is $64(1 - 2^{-s})\zeta\zeta$ and keeps it, while the μ_4-signed channel $16 \cdot 2^{-s}\eta(s)\eta(s-3) - 8L(f_8, s)$ is *entire* ($L_c(4.001) = -6.976$, finite; T12). The signed channel is the only pole-free one — exactly the ζ versus $L(\chi)$ pattern.

3.3 (iii) Hecke structure is an output of geometry, not an input

The step that made the line load-bearing is v535: the Hecke action on the census is produced by *pure lattice adjacency* — Kneser p-neighbours — with no modular input anywhere in the derivation. The marked neighbour sum is exactly an affine Hecke element, $\nu_p = a\,\mathrm{Id} + b\,T_p$, and the cusp eigenvalues come out of the glue census as $a_p = -c(p)/8$. Prime structure is what the geometry *emits*; nothing about primes was put in. The first four modules of Section 4 make this precise and end in one finite relative-trace identity of which the first three are projections.

3.4 (iv) The core point: self-duality, and a mirror that keeps coming back

E_8 is unimodular — self-dual — and self-duality is exactly the hypothesis of Poisson summation. The lattice theta sum therefore satisfies an exact functional equation, $\sum_x e^{-t|x|^2} = (\pi/t)^4 \sum_x e^{-(\pi^2/t)|x|^2}$ (verified to 25 digits, T12), whose unique self-dual width is $t = \pi$. A functional equation is a *reflection*, and the critical line is the fixed axis of that reflection. That is the geometric content of the picture: on the axis sits the symmetry, and beside it sits everything the lattice actually carries.

Two findings, at opposite ends of the program, put the same mirror on the table twice.

- **The reflection product is the transport operator** (T83, `fe_symmetrization_probe.py`, 27/27, INVARIANT-NULL). The involution algebra was computed exactly: ζ's functional equation is the reflection $J_{1/2}$ and is absorbed in evenness (on autocorrelations it acts as the identity), while the family's functional equation is J_1. Their *product* $J_1 \circ J_{1/2} = e^{\pm u}$ is the unit-line shift — the difference of the two centres *is* the transport operator, algebraically. The two reflections generate an infinite dihedral ladder; the value side is invariant under the whole ladder, the spectral cone only under its own reflection, which is what anchors it to the critical line; and $\mathrm{Fix}(J_1) \cap \{\text{Weil cone}\} = \{0\}$ exactly, so no symmetrisation can relieve I5. The verdict was a null with a structural finding: the wall is *transversal* to the functional equation, not positional.
- **The same mirror sits in the last hard residue** (T106, Section 7). The mirror parity J with $JQJ = Q$ and $JB = -BR$ splits the Weil pole exactly into $ss^{\mathsf{T}} - tt^{\mathsf{T}}$: a positive rank-one lift in the J-even channel and a negative rank-one pressure in the J-odd one. The even channel closes; everything that is still hard lives in the odd half — the anti-symmetric part of the same reflection.

Stated as a picture and nothing more: the recurring question of this line is whether the self-dual object is entirely at peace with its own mirror. That sentence is an interpretation of where the difficulty sits, in the same spirit as the classical reading of the critical line as a symmetry axis. It is *not* a reformulation of the Riemann Hypothesis, it is not used as a premise anywhere below, and it produces no claim.

3.5 (v) What the program did with the picture

It converted it, step by step, into smaller and more explicit objects. The transport ledger closes exactly (v541), which leaves the single coupled inequality I5; I5 is by identity equivalent to Weil

positivity, an equivalence *typing* and not a reduction (Section 5); the positivity question is then reorganised as a per-zone handover induction (Section 6); the induction is certified ingredient by ingredient until only one Loewner statement (R) on the J-odd half of a window remains; and (R) reduces first to one scalar ratio, then to two scalars, and finally — with both scalars certified — to one strict-positivity margin input (Section 7). The second phase continued the same conversion, and by T160 the chain localizes its residual hardness in a single classical object: by a machine-checked identity, the last open term of the pairing *is* the cancellation of $\sum_{n\le X}\Lambda(n)\,n^{-1/2}\cos(t\log n)$ at thirty-two explicit frequencies (Section 10) — a statement about a finite sum over prime powers, reported as a localization and not as progress on it. T161 then priced that object against the classical ladder, and the result relocates the hardness once more, soberly: the required cancellation depth exceeds what RH-strength input would supply and sits below the boundary term that every partial-summation bound carries, while the measured cancellation is fully accounted for by the prime-number-theorem main term — so the residual hardness is a *split-design* problem sitting above the classical prime sum, not an arithmetic statement hiding inside it (Section 10); that comparison of strengths is a triage, not a claim. The full cascade, with the type of every arrow marked, is Figure 8; the reason it is a route and not a proof sketch is stated there and in Section 13.

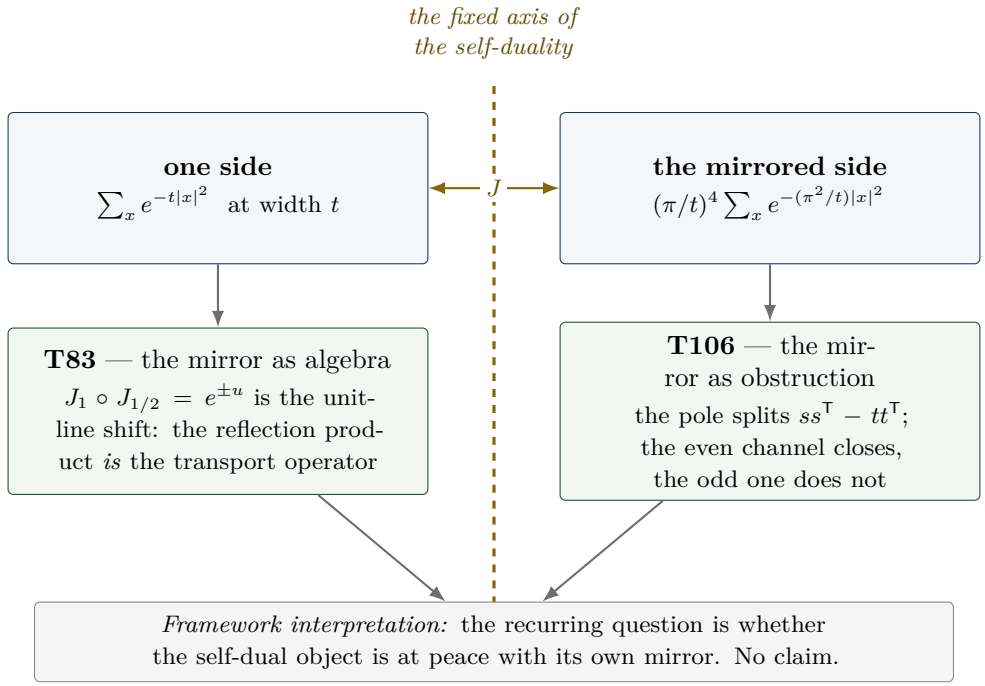


Figure 2: *Schematic.* The self-duality picture. *Literal:* the Poisson identity and its unique self-dual width $t = \pi$ (25 digits, T12); the algebraic identity $J_1 \circ J_{1/2} = e^{\pm u}$ and $\mathrm{Fix}(J_1) \cap \{\text{Weil cone}\} = \{0\}$ (T83); the exact pole splitting $ss^{\mathsf{T}} - tt^{\mathsf{T}}$ at 10^{-18} (T106). *Schematic:* the geometric arrangement, and the reading of the axis as "order" with the lattice's arithmetic content beside it. The bottom line is an interpretation and carries no status.

3.6 (vi) Two boundaries: what the geometry cannot do for the zeros

Subsections (i)–(iii) show that ζ and its Hecke structure sit literally in the E_8 geometry; this subsection records two elementary propositions fixing what that containment can and cannot deliver for the *zeros*. Both are claim-boundary statements, proved here in full; neither is a verification module, neither carries a status marker.

Proposition 1 (The Eisenstein bridge is RH-neutral). Let $\theta_{E_8}(\tau) = \sum_{v\in E_8} q^{(v,v)/2} = E_4(\tau) = 1 + 240\sum_{n\ge1}\sigma_3(n)\,q^n$ (the representation numbers $N(n) = 240\sigma_3(n)$ are the even-shell counts of subsection (i), written there as $N(2n) = 240\sigma_3(n)$; [`verification/v1018_e8_directed_readout.py`]).

Then for $\operatorname{Re} s > 4$,

$$\sum_{n\geq 1} N(n)\, n^{-s} \qquad = \qquad 240\,\zeta(s)\,\zeta(s\ -\ 3).$$

Consequently the nontrivial zeros of the L-function of the lattice theta series are exactly $\{\rho\} \cup \{\rho+3\}$, ρ ranging over the nontrivial zeros of ζ, each with the same multiplicity. The Weil explicit formula for $L(E_4, s)$ is the sum of the ζ explicit formula and its translate by 3; its positivity is equivalent to RH; no statement about the location of zeros is produced by the lattice that is not already a statement about ζ. The same holds for the character twists: the identity $D_\chi(s) = L(s,\chi)\,L(s-3,\chi)$ for the eight characters of $(\mathbb{Z}/30\mathbb{Z})^\times \cong C_2 \times C_4$ maps GRH for these Dirichlet L-functions onto itself; and the cusp form $\Delta = (E_4^3 - E_6^2)/1728$ that appears in the same geometry has an L-function whose Riemann hypothesis is a different (GL(2)) problem, not ζ's.

Proof. $\sum_{n\geq 1} \sigma_3(n)\, n^{-s} = \sum_{d,m} d^3 (dm)^{-s} = \zeta(s-3)\zeta(s)$ (absolutely convergent for $\operatorname{Re} s > 4$); the zero-set statement follows from the functional equations of both factors and the fact that $\zeta(s-3)$ has its nontrivial zeros at $\rho+3$; the explicit-formula statement is linearity of the explicit formula in the L-function's logarithmic derivative. □

The geometry contains ζ; at the level of zeros it renames, it does not inform.

A second exact bridge: the Coxeter–Euler completion. The Seifert matrix S of Bourbaki E_8 yields the Coxeter element $C = -S^{-1}S^{\mathsf{T}}$ with $\det(I - xC) = \Phi_{30}(x)$ and $\operatorname{Tr} C = -1$. The cyclotomic identity $\Phi_{30}(x) = (1-x^2)(1-x^3)(1-x^5)(1-x^{30})/[(1-x)(1-x^6)(1-x^{10})(1-x^{15})]$ is classical Möbius inversion of Euler products. Completing by a scalar, $U = 1 \oplus C$, gives $\det(I - xU) = (1-x)\Phi_{30}(x) = 1 - x^2 - x^3 + x^6 + x^7 - x^9$ and $\operatorname{Tr} U = 0$. Globally this is $Z_C(s) = \zeta(2s)\zeta(3s)\zeta(5s)\zeta(30s)/[\zeta(s)\zeta(6s)\zeta(10s)\zeta(15s)]$ and $D_{E_8}(s) = \zeta(6s)\zeta(10s)\zeta(15s)/[\zeta(2s)\zeta(3s)\zeta(5s)\zeta(30s)]$, absolutely convergent for $\operatorname{Re} s > 1/2$. The vanishing linear term is the generic consequence of $\operatorname{Tr} C = -1$ (controls: $A_1 \to 1/\zeta(2s)$; $A_2 \to 1/\zeta(3s)$; E_6 $(\Phi_3\Phi_{12}) \to \zeta(4s)\zeta(6s)/[\zeta(2s)\zeta(3s)\zeta(12s)]$; negative control B_2 $(\Phi_4, \operatorname{Tr} = 0) \to \zeta(2s)/[\zeta(s)\zeta(4s)]$, still contains $1/\zeta(s)$); E_8 only selects the divisor set of 30. The identity is Beurling-generic: it holds for any set of "primes" $Q \subset (1,\infty)$ — no prime-specific input. *Fence (verbatim):* The trace-free completion is zero-free and pole-free in $\operatorname{Re} s > 1/2$. The splitting into the scalar zeta channel and the Coxeter channel is open and RH-equivalent. No RH claim. Typed no-go `E8.COXETER.REGULARIZED_SPLIT.NO_GO.01`: any construction controlling only the $\det_2$/Hilbert–Schmidt part cannot isolate $1/\zeta(s)$; the RH content sits in the analytic continuation of $P(s) = \sum_k \mu(k)/k \cdot \log\zeta(ks)$. Like the Eisenstein bridge, the completion is RH-neutral as a finite identity. [`verification/v1019_coxeter_euler_completion.py`]

A third boundary: the Tate factor does not supply the global intersection form. For a two-dimensional carrier V, the scalar projector $T(A) = \frac{1}{2}\operatorname{Tr}(A) I$ on $\operatorname{End}(V)$ is idempotent, Hilbert–Schmidt self-adjoint and equivariant. At good places the exact splittings $V \otimes V = \operatorname{Sym}^2 V \oplus \wedge^2 V$ and $\operatorname{End}(V) = \mathbb{Q}(0) \oplus \operatorname{Ad}^0(V)$ isolate the determinant and scalar Tate zeta factors; at $p = 2$ the identity line remains inertia-fixed, although the full ramified adjoint factor is not reconstructed here. The completed scalar factor is $\Lambda_{\mathbb{Q}}(u) = 2\xi(u)/(u(u-1))$, not the entire function $\xi(u)$ itself.

A fixed finite linear carrier cannot turn this identity-side factorization into the full Weil pairing. Once more independent test directions are supplied than the carrier dimension, rank–nullity produces a nonzero kernel direction on which every pulled-back positive form vanishes. Conditional on the external strict Weil-window floor cited by the audit, this excludes every such fixed finite carrier; it does not exclude nonlinear procedures, growing finite discretizations, direct limits, or an infinite all-place realization.

The same audit corrects the historical Rankin miniature. For $b_n \geq 0$, the bound $A(X) = \sum_{n\leq X} b_n = O(X^q)$ gives only $b_n = O(n^q)$ without further input. The exact sequence $b_n = 4n^3 + \mathbf{1}_{\{n=32^m\}} n^{18/5}$ retains the X^4 leading mean and the simple pole at $s = 4$, while $b_{32^m}/(32^m)^{3+1/10} \to \infty$; hence the claimed $q \mapsto q - 1 + \varepsilon$ extraction is false. Constants defined as maxima on the same finite window are tautological there. Rankin's actual 1939 argument uses a quantitative summatory error term, not a pole and nonnegativity alone. What remains open for this exact-linear

geometric route is an infinite or compatible direct-limit all-place object carrying an independently positive global intersection form whose pullback is the complete Weil quadratic form. No RH claim. [verification/v1021_all_place_tate_rank_audit.py]

Proposition 2 (Commensurability: finite clocks cannot carry the prime orbit spectrum). **(a)** For any set $\mathcal{P}$ of primes, the real numbers $\{\log p : p \in \mathcal{P}\}$ are linearly independent over $\mathbb{Q}$.

Proof. Suppose $\sum_{p\in\mathcal{P}_0} a_p \log p = 0$ with $\mathcal{P}_0$ finite, $a_p \in \mathbb{Q}$ not all zero; clear denominators to integers n_p; exponentiate: $\prod_{n_p>0} p^{n_p} = \prod_{n_p<0} p^{-n_p}$. The two sides are positive integers with disjoint sets of prime factors; by uniqueness of prime factorization both equal 1, so all $n_p = 0$, a contradiction. □

(b) Corollary. Let a dynamical system have all its closed-orbit periods in $T \cdot \mathbb{Z}_{>0}$ for a single $T > 0$. This covers every action of a finite cyclic group — in particular the Coxeter element of E_8 (order $h = 30$, characteristic polynomial Φ_{30}, the Seifert readout of the compiler) — and more generally any finite-order symmetry, whose closed orbits have pairwise commensurable periods. If two closed orbits had periods $c \cdot \log p$ and $c \cdot \log q$ for distinct primes $p \neq q$ and a common scale $c > 0$, then $\log p / \log q \in \mathbb{Q}$, contradicting (a). Hence such a system carries at most one prime length: **no finite-order clock realizes the prime orbit spectrum** $\{\log p\}$ (nor $\{\log p^k\}$). A dynamical realization in the Selberg–Deninger sense (closed orbits $\leftrightarrow$ prime powers, lengths $\leftrightarrow \log p^k$) requires a flow with infinitely many $\mathbb{Q}$-linearly independent primitive periods — a non-periodic (Anosov-type) dynamics, which the finite E_8 data do not provide.

Numerical shadows — experiments only, no RH claim

[sandbox] Rounds r607 (`event_lindblad_twokey_probe.py`, verdict KILLED(RECOORDINATIZATION): the ordered event Lindbladian reproduces finite Weil positivity to 10^{-15} in true, scrambled and permuted worlds alike, labels decorative) and r613 (`prime_e8_ordered_herglotz_probe.py`, verdict NO_IDENTIFICATION: at zero free evolution all ordered E_8 jumps commute, so the monodromy depends only on their sum and the order is decorative to 10^{-14}; with free evolution the order becomes visible but carries no prime-specific information, residuals at Euler points indistinguishable from scrambled controls) are the numerical shadows of Proposition 2: the finite structure alone cannot see the primes, and the free evolution that breaks commensurability is not arithmetic. Both rounds live in `experiments/tfpt-discovery/` and are not part of the verified layer.

The prime front therefore rests on the verified identities of Section 4, not on any zero-location claim; RH-relevant work is carried in the RH program (`rh/`) with its own claim boundary.

Part II
The verified layer

The single section of this part is the only place in the document that is entirely load-bearing: every claim has a module in `verification/`, a row in the status ledger, and an explicit script citation at the point of use. The subsections follow the module numbers and can be read independently; each states what its module checks and what it deliberately does not. Nothing in the sandbox parts that follow upgrades anything here.

4 The verified layer: forty-nine modules

This section is entirely [ledger]. Each subsection states the claim as the ledger states it, cites the module, and quotes the ledger's own status markers. Nothing here depends on Sections 5–7.

The forty-nine modules and their ledger markers

Module	Claim id	Content	Markers
v535	HECKE.GEOM.01	Hecke from geometry on the E_8 census channels	[E] + [C] + [O]
v536	HECKE.GEOM.EICHLER.01	Eichler trace layer at the E_8 lattice	[E] + [C] + [O]
v537	HECKE.GEOM.HALFINT.01	half-integral bridge	[E] + [C]
v538	HECKE.GEOM.RTF.01	compiler relative trace identity	[E] + [C]
v539	RTF.GNS.WEIL.01	Weil structure of the compiler family	[E] + [C]
v540	RTF.GNS.AMP.01	amplitude route and positive linear carrier	[E] + [C]
v541	RTF.GNS.LEDGER.01	matching lemma and transport ledger package	[E] + [C]
v542	PRIME.MARGIN.IDENT.01	margin-chain identities of phase 2 (per instance)	[E]
v543	PRIME.MMATRIX.IDENT.01	lumped M-matrix pair of phase 2 (per instance)	[E]
v544	PRIME.LONGLAG.SUPP.01	long-lag support structure of phase 2 (per instance)	[E]
v545	PRIME.HARDY.IDENT.01	Hardy-core identities of phase 2 (per instance)	[E]
v546	PRIME.CAPCHAIN.IDENT.01	capacity-chain identities of phase 2 (per instance)	[E]
v547	PRIME.LEVEL.LEMMA.01	level-lemma identities of phase 2 (per instance)	[E]
v548	PRIME.GREEN.SZEGO.IDENT.01	Green/Szegő identities of phase 2 (per instance)	[E]
v549	PRIME.GAUGE.PARITY.IDENT.01	gauge/parity identities of phase 2 (per instance)	[E]
v550	PRIME.ODD.SECTOR.IDENT.01	odd-sector identities of phase 2 (per instance)	[E]
v551	PRIME.RITZ.CEIL.01	fixed-size Ritz ceiling certificate of phase 2 (per instance)	[E]
v552	PRIME.ANGLE.INSTR.01	four fixed-size angle instruments of phase 2 (per instance)	[E]
v553	PRIME.EXACT.FORM.IDENT.01	exact-form identities of phase 2 (per instance)	[E]
v554	PRIME.SAMPLING.HARM.01	sampling/harmonics identities of phase 2 (per instance)	[E]
v555	PRIME.PARETO.TV.01	Pareto/total-variation identities of phase 2 (per instance)	[E]
v556	PRIME.GAUGE.PPR.01	gauge/P_{pr} identities of phase 2 (per instance)	[E]
v557	PRIME.CASCADE.VECT.01	cascade/vector identities of phase 2 (per instance)	[E]
v558	PRIME.BILINEAR.RANK.01	bilinear/rank identities of phase 2 (per instance)	[E]
v559	PRIME.PHASE2.CAPSTONE.01	phase-2 capstone: the sixteen-link chain (per instance)	[E]
v560	PRIME.FRAME.DEFICIT.01	frame/deficit identities of phase 2 (per instance)	[E]
v562	PRIME.DENSE.LIMIT.01	dense-limit identities of the phase-2 endgame (per instance)	[E]
v563	PRIME.PAPER2.READOUT.01	Paper-II readout closure (per instance)	[E]
v569	PRIME.PENCIL.ONEMODE.01	relative pencil one-mode (declared surface; measured readouts)	[E]
v570	PRIME.PENCIL.SEPFLOOR.01	separation floor certified (closed scalars; declared surface)	[E]
v573	PRIME.PAIRBAND.01	pair-band anatomy: the dominance is long-range	[E]
v576	PRIME.CHEBLOEWNER.01	Chebyshev–Loewner edge structure (cross formula; null cone; N^{-4})	[E]
v577	PRIME.NULLRAY.01	null-ray locking census (partial carrier; measured)	[E]
v579	PRIME.MACROKERNEL.01	two-scale kernel signs (geometric half	[E]

Marker key: ***[E]*** *exact / machine-proven ·* ***[C]*** *conditional (holds under named hypotheses) ·* ***[O]*** *open / axiom ·* ***[X]*** *falsifiable kill test. Per-claim fine type in* `verification/status_ledger.csv`.

Every row carries `NO marker moves` in the ledger. The [O] on v535 and v536 is the same open gate throughout: *weight-4* $\to$ GL(1) *transport.*

4.1 Hecke from geometry (v535)

The Hecke structure of the μ_4-glue census is lattice-native: it comes out of Kneser p-neighbour geometry, not out of modular input [verification/v535_hecke_from_geometry.py] (25 checks, ∼11 s; consolidated promotion of T27/T31/T32).

- **Kneser carries Hecke** [E]: #iso_pts $= p^7 + p^4 - p^3$ and #iso_lines $= \sigma_3(p) \cdot \#\mathbb{P}^3(\mathbb{F}_p)$ identically and by full enumeration at $p = 2, 3, 5, 7$ (135/1120/19656/137600 lines). The Eisenstein eigenvalue $\sigma_3(p) = 1 + p^3$ is an exact factor of a lattice-native correspondence count.
- **The marked neighbour sum is an affine Hecke element** [E]: $\nu_p = a\,\mathrm{Id} + b\,T_p$ exactly (overdetermined, residual 0), with $b = \sigma_3 + a_p$; the cusp rule $a_p = b - \sigma_3$ recovers $a_3 = -4$ and $a_5 = -2$.
- **Census redundancy is 2-adic oldform structure** [E]: $\dim V = 7 = 5{+}2$, $\pi_{\mathrm{cusp}} = (28{-}T_3)/32$, only 2-adic levels occur.
- Fences carried *inside* the claim: the $p = 5$ geometric $S[1]$ block is a T31-validated *freeze* [C]; the weight-4 $\to$ GL(1) transport is [O].

4.2 The Eichler trace layer (v536)

The smooth (Eisenstein) background and the coherent (cusp) interference separate exactly [verification/v536_eichler_trace_layer.py] (23 checks, ∼30 s; promotion of T33/T36/T37/T42). The Witt layer gives $\lambda_{\mathrm{Eis}} = \sigma_3(\#\mathbb{P}^3 - \sigma_3) = L - \sigma_3^2$ identically (anchors 336, 3780, 19264 at $p = 3, 5, 7$) [E], and the Eichler identity $\lambda_{\mathrm{geom}} = \lambda_{\mathrm{Eis}} + a_p^2$ holds two-sidedly (352/3784/19840). The signed cusp reading is $a_p = -c(p)/8 \Rightarrow (-4, -2, +24)$, and the assembler $R(p) = a_p^2$ is exact for $p \le 31$. The $p = 5$ type-(ii) block is again a validated freeze [C]; the transport gate stays [O].

4.3 The half-integral bridge (v537)

In the 70-element compiler weight-5/2 theta monoid there is *exactly one* object g with $\mathrm{Sh}_{t=2}(g) = -8f_8$ coefficientwise [E], it is an exact $T(p^2)$-eigenform with the weight-4 eigenvalues $(-4, -2, 24, -44, 22)$ [E], and $U_4(g) = 0$ places it outside the Kohnen plus space — a scope fence, not a defect [E] [verification/v537_halfintegral_bridge.py] (20 checks, ∼90 s; promotion of T38/T41/T44/T45). The Waldspurger/Baruch–Mao constant $R = 23.1873585645\ldots$ is constant on ten discriminants $d \equiv 1 \bmod 8$ (spread $\le 10^{-12}$), typed [C] because it is an AFE-numeric measurement.

4.4 One finite relative trace identity (v538)

The strongest structural statement of the verified layer: v535, v536 and v537 are three *projections* of one finite relative-trace identity of Jacquet type [verification/v538_relative_trace_identity.py] (18 checks, ∼14 s; promotion of T53). $\mathrm{Tr}_V(\nu_p \circ \pi)$ is computed twice, independently — geometrically from the glue census with no modular input, and spectrally from $\eta(2\tau)^4\eta(4\tau)^4$ with no geometry — and the two agree exactly for all $(p, \pi) \in \{3, 5, 7\} \times \{\mathrm{Id}, \pi_{\mathrm{Eis}}, \pi_{\mathrm{cusp}}, \pi_{E_4}, \pi_{f_8}\}$ (anchors, e.g. $p = 7$: full trace 727680, f_8 channel 19840) [E]. The Eichler split *is* the orbit decomposition: principal orbit $a\,\mathrm{Id} \to \lambda_{\mathrm{Eis}}$, elliptic orbit $b\,T_p \to a_p^2$, cross terms cancel exactly. And the period side is literally compiler geometry: g is the quaternary lattice form $n = (x^2 + y^2)/2 + 2z^2 + u^2 + 2w^2$ with sign $(-1)^{|u|+|w|}$, giving [signed lattice count]$^2 = R\,|d|^{3/2} L(f_8 \times \chi_d, 2)$ at relative 10^{-12} [C]. The identity is *finite*; the infinite RTF is explicitly open.

4.5 The Weil structure of the family (v539)

The family carries a fully identified Weil structure *up to two explicitly isolated obstructions*, and the obstructions are promoted content, not a footnote [verification/v539_weil_structure_family.py] (25 checks, ∼6 s; promotion of T55/T61/T62/T63). The canonical GNS pairing is a direct integral over the Gelfand spectrum with fibre mixing $\leq 5.04 \times 10^{-16}$; the trivial Sato–Tate isotype is algebraically a GL(1) core, $G_0 = (1+Y)/(1-Y)$; the prime side is an exact linear relation to shifted ζ-Weil forms at relative $\leq 9 \times 10^{-16}$. The two obstructions are (i) a sign/doubling residue (metaplectic, sym^2) that survives every preregistered canonical projection, and (ii) a Plancherel correction which is irreducibly non-automorphic. Finite-class positivity of Q_{fam} is measured [C] ([4.369, 11.486]); dense-class positivity is open. The ledger row says it in one line: *not almost-RH.*

4.6 The amplitude route and the positive linear carrier (v540)

The route out of the square plane [verification/v540_amplitude_linear_carrier.py] (34 checks, ∼3 s; promotion of T67–T72). An amplitude Dirac operator $D = \left(\begin{smallmatrix} 0 & V \\ V^{\mathsf{T}} & 0 \end{smallmatrix}\right)$ with $V[d,m] = \sqrt{w_d}\, b(d)\chi_d(m)$ exists and is Hecke-equivariant [E]; the geometric polarisation $b = N_+ - N_-$, $\Theta = N_+ + N_-$ is lattice-exact with $N_\pm \geq 0$ for all $n \leq 50\,000$ [E]; the Cohen seed $\Theta(d) = -48L(-1,\chi_d)$ is exact-rational on 159 live d [E]; the prime side is a *pure plus* combination $Q = Q_\zeta(g_-) + Q_\zeta(g_+)$ at relative 2×10^{-15} [E]; the functional equation of the linear family holds at relative 1.4×10^{-40} [E]. The open boundary is *inside* the claim: the gap functional λ^* on $n \equiv 6 \bmod 8$, with an exact Farkas witness (0.0511), is a named limit, and positivity is asserted only in the Euler region.

4.7 The matching lemma and the transport ledger (v541)

The largest module of the line [verification/v541_matching_lemma_ledger.py] (33 checks, ∼10 s; promotion of T78–T82 and T85; every core check recomputed, not cited).

- **Window proof** [E]: the matching lemma is proved exact-integer on $[4, 10^6]$ — $7S(j) < 40A(j)$ at every atom, 0 violations over 939 870 clash atoms, no-overflow proven in exact integers, 268 big-integer rechecks; exact margin $X = 22242575693/270725400000 = 0.082159$ at $j^* = 554400$; $\rho_{\mathrm{crit}} = 1.143989 > 21/20$. A maximal-greedy identity makes the certificate uniform in the target set and the greedy weights.[1]
- **Transport ledger** [E]: $Q_{\mathrm{Weil}} = Q_{\mathrm{cert}} + \Delta_{\mathrm{arch}} + \Delta_2$ closes with residual 4.4×10^{-16} on a 24-row battery, and $\Delta_{\mathrm{pole}} \equiv \Delta_{\mathrm{conv}} \equiv 0$ are *proven*, not measured (kernel collapse in `sympy`, exact scale invariance).
- **Character-exact envelope** [E]: $-\psi = (\chi_{-4} + \frac{1}{4}\chi_8 + \frac{1}{4}\chi_{-8})\Theta$ coefficient-exact for all odd $n \leq 50\,000$; confinement holds as a *set equality* on 10^6.
- **Archimedean term internal** [E]: the Legendre duplication bridge $(2\pi)^{-s}\Gamma(s) = \frac{1}{2}\Gamma_{\mathbb{R}}(s)\Gamma_{\mathbb{R}}(s{+}1)$ and the kernel identity $k_\zeta = K_{\mathrm{fam}} - K_{\mathrm{shift}}$ at 7.9×10^{-31} pointwise.
- **Frontier facts** [C]: unlifted coherent chain crossing at $k^* = 14$ ($\log_{10} N^* = 23.1$); Robin tail factor 6.16 at $J = 10^6$ and divergent — the constant route stays open.
- **Named limits as content.** Two limits are part of the claim rather than hidden by it: one classically-shaped open lemma (correlated cancellation on credit-rich non-coherent tail atoms), and the I5 equivalence typing discussed next. The row states explicitly: *NOT "almost RH"*; `ZETA.HP.CARRIER` untouched.

[1] Prior-art note (literature pass, 2026-07-27): the coefficient laws L1–L4 that T77/T78 use to shape the lemma are specialisations of Cohen 1975 (Math. Ann. 217: the Hurwitz class-number series $H(r,n)$ and the Cohen–Eisenstein series of weight $r + \frac{1}{2}$) and of Pei–Wang 2003 — corollaries of known theory, not discoveries. The restricted divisor inequality itself remains the independent part.

4.8 The margin-chain identities of phase 2 (v542)

The first promotion *out of* the second phase — and the narrowest module of the line [verification/v542_margin_chain_identities.py] (44 checks, ∼0.2 s; promotion of the identity/theorem block of T128–T131, recomputed on small frame-A windows, nothing cited from the sandbox). What is promoted is exactly the algebra: nine identities and per-instance theorems, each a numerical residual against a *preregistered* tolerance plus at least one mutation control that must fail by $\geq 10^{-3}$ relative. What is *not* promoted is everything the sandbox still owes: no fit, no graded floor, no Lanczos value, and no statement uniform in the zone index. The battery is 24 seam instances ($n = 3 \ldots 97$, $\rho = 1.25 \ldots 4.00$, $h_c = 24 \ldots 214$, $h_f = 36 \ldots 299$; every inverted or diagonalised matrix ≤ 300), 43 graded/fine grid pairs ($m \leq 172$) and 400 randomised profiles.

- **Border geometry and the (T) identity** [E]: $\sum_i (1 - g_i) = t_l + t_r$ exactly (5.5×10^{-13}), so the κ weights are a probability vector and the flat border vector gives $\kappa_{\text{flat}} = 1$; and $\tau_{\text{dn}} = \|y\|^2 - m_{\text{prot}} + m_{\text{fill}} - V_{\text{bord}}$ at 2.2×10^{-16}, with τ recomputed independently from the *full* odd overlap matrix (1.1×10^{-13}).
- **The profile identity and the curvature chain** [E]: $p_{N-1} = 2 - p_0 + 2E$ at 2.3×10^{-16} (instances) and 1.4×10^{-15} (400 random profiles), hence the equivalence $2E > p_0 \iff p_{N-1} > 2$ with 0 exceptions and both branches realised (60/400); the Abel/Hölder chain holds per instance with 0 violations, and the *same* chain without the curvature term is violated on 88/400 profiles — which is the honest statement of why the curvature term is in it.
- **Assembly and the compression defect** [E]: the matrix-free two-scale assembly equals $J^{\mathsf{T}} Q J$ (5.2×10^{-16}), the graded overlap operator equals J (5.2×10^{-14}), and $S_{\text{graded}} - S_{\text{uniform}} = R^{\mathsf{T}} X^{-1} R$ holds as a *matrix* identity on all 43 grids (2.1×10^{-14}) with a PSD defect — the compression error is one-sided *by* the identity, which is what T130 needed and all it gives.
- **Sandwich, sign, counting** [E]: the secular sandwich holds on all 48 pole-free sections at which $A > 0$ *and* $\varepsilon > 0$ are *checked* (relative width ≤ 1.4448), with Albert's sign equivalence verified in both directions (6 rescaled controls with $\varepsilon < 0$ all give $\lambda_{\min}(Q) < 0$); $S^{-1} > 0$ entrywise (measured on 48/48 blocks) implies a sign-constant and simple ground vector, with a control matrix where hypothesis and conclusion fail together; and $n_{\text{run}} \leq n_{\text{cross}} + 1 \leq s_2 + 2$ together with $\sum_j |\Delta p_j - \overline{\Delta p}| = \text{TV}(P)$ hold without exception on a battery that is not vacuous (s_2 up to 31, n_{run} up to 33).
- **Named limits as content.** Every statement is per instance on small windows; nothing is uniform in the zone index. The κ *law* is *not* claimed — T129 found it false in general (Section 10) — only the chain underneath it. The sandwich carries no floor value: positivity of the pole-free section at depth (M25d) stays open. The Perron step is an implication with a *measured* hypothesis. And the defect identity says nothing about the *size* of the defect, which is the open exponent of T130.

4.9 The lumped M-matrix pair (v543)

The second promotion out of the second phase: the identity block of the M-matrix question that T134 posed and T136 answered [verification/v543_lumped_pair_identities.py] (35 checks, ∼1 s; promotion of T136, recomputed on small frame-A windows). The battery is 20 seam windows ($n = 4 \ldots 59$, $\rho = 1.25 \ldots 4.00$, $h = 25 \ldots 299$; every factorised or diagonalised matrix ≤ 300) giving 40 symmetric matrices — pole-free odd sections and border Schur blocks — of which 39 carry positive off-diagonals at all, plus 312 shifted ladder rungs and 8 whitened two-grid rungs. Every statement is a residual against a *preregistered* tolerance with at least one mutation control that must fail by $\geq 10^{-3}$ relative.

- **The lumped pair and the congruence** [E]: with Δ the positive off-diagonal part and $L_\Delta = \text{diag}(\Delta \mathbf{1}) - \Delta$, the pair $A_B = A + L_\Delta$ is Stieltjes, L_Δ is a graph Laplacian, so $A_B \mathbf{1} = A \mathbf{1}$

(1.6×10^{-15}) and $A_B \succeq A$ — an *upper* bound on $\lambda_{\min}(A)$ and a floor never. The congruence $A = A_B^{1/2}(I - W)A_B^{1/2}$ with $W = A_B^{-1/2} L_\Delta A_B^{-1/2} \succeq 0$ holds as a *matrix* identity (4.4×10^{-15}), $\lambda_{\max}(W) = \rho(L_\Delta A_B^{-1})$ (4.4×10^{-15}), and the equivalence $\lambda_{\max}(W) < 1 \iff A \succ 0$ is checked in *both* directions (39 shifted controls in the indefinite range all give $\lambda_{\max}(W) \geq 1$) — the Loewner sandwich is circular by itself, and that declared circularity is the content.

- **The anchor and the empty shift ladder [E]**: with $x = A_B^{-1}\mathbf{1} \geq 0$ and $A_B^{-1} \geq 0$ entrywise (Ostrowski 1937, *checked*), $\lambda_{\min}(A_B) \geq 1/\max_r x_r$ on all 39 rows (min slack 3.0×10^{-3}, sharpest ratio 0.9070) — one solve, one sign check, no eigenvalue. Lumping commutes with diagonal shifts, $(A - sI)_B = A_B - sI$ (2.5×10^{-16} on 312 rungs), and $(A_B - sI)^{-1} \geq A_B^{-1} \geq 0$ entrywise (Berman–Plemmons 1979), so $\max_r x_r(s)$ and the a-priori margin are non-decreasing along the whole ladder: the shifted ladder is *structurally* empty — a theorem, not the measurement T136 first made.
- **Varga's identity and the a-priori Jacobi radius [E]**: for the regular splitting $A_B = D_0 - N_0$ (all three hypotheses checked), $\rho(J) = \tau/(1+\tau)$ with $\tau = \rho(A_B^{-1}N_0)$ at 4.8×10^{-15} ($\rho(J) = 0.7347$–0.9860), and the Collatz–Wielandt bracket at the anchor vector gives an *a-priori* $\rho(J) \leq 0.9861 < 1$ with no eigenvalue anywhere, sharp on the measured gap by 1.0062–1.0531. Controls: a non-regular splitting breaks the identity (1.1×10^{-1}); at a sign-changing test vector the quotient bound is false on 39/39.
- **The k-scanned M18 majorant — a negative result [X]**: the whitened mass identity $\|L^{-1}\hat{s}\|^2 = \hat{s}^{\mathsf{T}}U^{-1}\hat{s}$ holds at 7.2×10^{-16} and the triangle chain is a *valid* majorant at every k separately (39 (rung, k) pairs over $k = 2 \ldots 64$, 0 violations), so minimising over the ladder is legitimate — and it does not reach the preregistered bar 1/2 on any rung (best 3.080) while the *measured* bad mass is ≤ 0.0120. The localisation term alone already exceeds the bar (0.542): the loss is the bound, and the bar is not moved.
- **Named limits as content.** Per instance on small windows; nothing uniform in the zone index, in D or in the gap. The T136 D-exponents (anchor $D^{-0.56}$, gap $D^{2.72}$, slack $D^{0.12}$) are fits and stay in the sandbox. The congruence carries no floor; the anchor and Varga rows are implications with hypotheses *checked at the instance*; the ladder row kills a route and says nothing about the true $\lambda_{\max}(W)$.

4.10 The long-lag support structure (v544)

The companion module: what the positive off-diagonal part *is*, and which bounding families are dead `[verification/v544_long_lag_support.py]` (24 checks, ∼0.2 s; promotion of T137 on the same 20 windows, 3–25 prime-power atoms, 20555 positive off-diagonal edges, 455 loaded stripes, 39 bracketed lumped comparisons).

- **The support is a comb stripe set [E]**: the kernel splits exactly as $c = c_{\text{arch}} - c_{\text{comb}}$ with $c_{\text{comb}} \geq 0$, and the archimedean section alone has *all* off-diagonals ≤ 0 on 20/20 windows, so every positive off-diagonal of A is comb-generated (control: the full section carries $+2.0 \times 10^{-1}$). All 20555 positive edges have their Hankel index $M - 1 - r - t$ inside the atom-hat index set — per-window fraction exactly 1.000000 — so $\operatorname{supp}(\Delta)$ is a union of anti-diagonal stripes at prime-power atom positions (the converse is reported, not claimed), and each stripe is a perfect matching (0 endpoint collisions), so its edge vectors are *exactly* orthogonal and L_Δ restricted to a stripe is a direct sum of 2×2 blocks.
- **The amplitude certificate [E]**: $A_{rs} = A_{rs}^{\text{arch}} - c_{\text{comb}}[|r-s|] + c_{\text{comb}}[\text{anti}]$ gives $\Delta_{rt} \leq c_{\text{comb}}[M - 1 - r - t]$ on all 20555 edges (excess -1.9×10^{-6}, sharpest ratio 0.99998532) — no eigenvalue, no norm; the Toeplitz lag in place of the Hankel index breaks it on 20415/20555. Honestly undecided and *not* promoted: at $h \leq 300$ the atom hats never overlap, so the tighter one-atom refinement (T137: worst ratio 2.268 at depth) coincides with the certificate here.

- **The Green function, bracketed two-sidedly** [E]: for the lumped comparison $B = D_0 - N_0$, $J = D_0^{-1}N_0 \geq 0$, the anchor identity $q_J = \max_r (Jx)_r/x_r = 1 - \min_r 1/(d_r x_r) < 1$ (8.2×10^{-16}) needs no measurement; the Neumann partial sum to $K = 8$ is then an *entrywise* lower bound for B^{-1} (-2.1×10^{-4}) *because the discarded terms are nonnegative*, and the anchor-weighted ceiling is an entrywise upper bound (-1.4×10^{-1}) on 39 rows. This delivers item (2) of the T136 rest list; dropping the tail makes the upper bound false.
- **The absolute-value envelope family is certified dead** [X]: with $E = B^{-1}L_\Delta$, Gershgorin, every row-sum norm and every Collatz–Wielandt quotient at every *positive* weight are upper bounds for $\rho(|E|)$, so one rigorous *lower* bound decides the family at once — $\rho(|E|) \geq 1.3141 \geq 1$ on 20/20 rows (the cheapest four members all ≥ 1.5937). No member can ever certify $\lambda_{\max}(W) < 1$. The *signed* radius on the same rows is $\rho(E) = \lambda_{\max}(W) \leq 0.99995781$: the absolute value alone destroys the cancellation, and what survives is a sign-preserving bound that does not exist yet.
- **Named limits as content.** Per instance on small windows, no decay profile and no amplitude law (T137's decay measurements are fits). The support row is *one* inclusion; the amplitude row bounds the amplitude and *not* $\lambda_{\max}(W)$, and the structure bound assembled from it does not beat the margin (T137: 0 of 35 windows) and is not promoted; the bracket is for the *lumped comparison*, not for the target, whose entrywise inverse positivity at depth stays open; the envelope row closes a route, not the question, and moves no marker. Demko–Moss–Smith 1984 is cited as a model and *not* applied — the comparison is dense.

4.11 The Hardy-core identities (v545)

The identity block of the finite core: what the signed inequality *is*, in closed form, and which two bound shapes are *valid* on it [`verification/v545_hardy_core_identities.py`] (36 checks, ∼1 s; promotion of T140 and T141 on 11 frame-A windows, $n = 8 \ldots 139$, $h = 58 \ldots 285 \leq 300$, core $m = 57 \ldots 284$, 193–5102 positive off-diagonal edges, 10–87 loaded stripes, $D = 7.578 \times 10^{-3} \ldots 2.388 \times 10^{-2}$ — a factor 3.15, so the last row is a comparison and not a single point — exact $\rho(W) = 0.997759$–0.999981; the explicit $n_e \times n_e$ signed Gram is formed only where $n_e \leq 900$).

- **The form lifting** [E]: with M the edge–interval incidence matrix and $C = \mathrm{diag}(\sqrt{\Delta})M$, the T139 telescope lifts from the *entries* of the signed Gram to its *form*, $\mathrm{Gram} = CHC^{\mathsf{T}}$ at 3.0×10^{-15}, hence $\mathrm{rank}(\mathrm{Gram}) \leq h - 1$ (measured 57–183 against 193–882 edges); the unweighted incidence breaks it (1.6).
- **The exact finite-core reduction** [E]: the covering kernel in closed geometric form, $K_{rs} = W([r \wedge s,\, r \vee s])$ — entrywise nonnegative and monotone by inclusion on 11/11 — equals $M^{\mathsf{T}}\Delta M$ (1.3×10^{-15}), and $\rho(W) = \lambda_{\max}(K^{1/2}HK^{1/2}) = 1 - \lambda_{\min}(A, A_B)$ $(5.4 \times 10^{-14}) = \lambda_{\max}(\mathrm{Gram})$ (7.8×10^{-16}): three independent routes to the same number, no truncation anywhere. The factorwise product $\lambda_{\max}(K)\lambda_{\max}(H)$ misses by 7.8×10^2, which is why the joint shape below is needed. Energy reordering $H = \mathrm{diag}(s) + L_N$ at 4.7×10^{-16}; the Dirichlet weights are positive on 0.676–0.872 of the off-diagonal area (a measurement, no law).
- **The checkerboard split** [E]: at group length exactly b, $\mathrm{Band}_b = D + A_{\mathrm{even}} + A_{\mathrm{odd}}$ *entrywise* (0.0) on all 9 (window, b) pairs over $b = 4, 8, 16$, and the three-step Weyl sequence dominates $\lambda_{\max}(\mathrm{Band}_b)$ with 0 exceptions (worst ratio 1.6940) at a step count independent of the up to 9 groups; group length $b/2$ loses entries (3.5×10^{-1}).
- **The four Hardy identities** [E]: (i) $\rho(W) = u^{\mathsf{T}}Hu/u^{\mathsf{T}}K^{+}u$ at $u = K^{1/2}v$ (6.6×10^{-13}); (ii) $L_N = D^{\mathsf{T}}BD$ with $B_{kl} = \sum_{r \leq k \wedge l,\, k \vee l < s} N_{rs}$, the same closed covering formula one grid in (4.9×10^{-16}); (iii) $Q = B_+\mathbf{1}$ (1.4×10^{-15}) — the Cauchy–Schwarz weight of T140 *is* a Gershgorin row sum; (iv) $DKD^{\mathsf{T}} = L_\Delta$ on the interior nodes (2.7×10^{-14}) with the endpoint edge mass 0.4886–15.15 as diagonal, a *local* quantity. The crossing kernel of $|N|$ does not reproduce L_N (2.6×10^{-1}): the signs are load-bearing.

- **The closed Muckenhoupt quantity** [E]: $A_{M0} = \max_k Q_k(\Delta\mathbf{1})_{k+1}$ by two closed routes agreeing to 8.3×10^{-15}; its value (65.5–4244) is *reported* and neither its size nor its zone behaviour is promoted. Replacing the local endpoint mass by $\lambda_{\max}(K)$ inflates it by ≥ 358.
- **Two certified shapes, as *valid* bounds** [E]: every Loewner step is certified by a completed Cholesky (-1.5×10^{-14}, -2.3×10^{-10}). The additive Hardy chain dominates $\rho(W)$ on 11/11 (factor 3.074–10.758); its own exact two-term Weyl split dominates too (1.691–2.503) and lies *below* the chain everywhere, so it is the floor of the additive family; the joint shape $\Lambda(H, Y) \cdot \Omega(Y)$ with a positive definite Jacobi Hardy form Y dominates on 11/11 with the closed-form geometric profile (8.815–74.872 at $\Omega = 8.33$–36.2) and on 11/11 with the per-zone tridiagonal read-off of K^+. Promoted is the *dominance*; the factors are not. Controls: the same Y without its mass term is singular — the constant vector lies in its kernel (5.7×10^{-17}) while H does not vanish there — so the certificate *refuses* on 11/11; and dropping the *positive* part instead of the negative one falls below $\rho(W)$ on 11/11.
- **The sign-versus-absolute separation, as a comparison** [X]: over the measured D range the exactly bounded object spreads by 1.36× while every absolute variant spreads by at least 54.94× (Loewner drop 62.03, Cauchy–Schwarz 54.94, product 56.92), and dropping the positive part of H costs 9.96–464.63×. The power laws are *fits*, labelled as fits: $D^{-0.138\pm0.094}$ (signed) against $D^{-0.776\pm0.352}$ (certified Muckenhoupt) and $D^{-2.748}$, $D^{-2.820}$, $D^{-2.918}$ (absolute).
- **Named limits as content.** Per instance on small windows; nothing uniform in the zone index or in D, and every exponent above is a fit that stays in the sandbox. The bound rows promote *validity* and not size: neither shape beats the target, the additive shape is dead as a *shape* by its own Weyl floor, and the joint shape is decided at the normalisation Ω. The finite-core row is an *identity* and not a bound — the zone-uniform discrete Hardy inequality it reduces the question to stays open, and no marker moves toward it.

4.12 The capacity-chain identities (v546)

The identity spine of the capacity cycle T142–T145 of Section 10: what the certified chain *is*, per instance and machine-checked, with the one open lemma fenced out [verification/v546_capacity_chain_identities.py] (26 checks, ∼0.5 s; promotion of the identity blocks of T142–T145 on 12 frame-A windows, $n = 4 \ldots 139$, $h = 26 \ldots 285 \leq 300$, $D = 7.578 \times 10^{-3} \ldots 2.789 \times 10^{-2}$, 32–5102 positive off-diagonal edges; the covering kernel is positive definite on 11 of them, where the first two items run; each statement carries a mutation control that must fail by $\geq 10^{-3}$).

- **The capacity decomposition (T142)** [E]: $K^{-1} = D^{\mathsf{T}} J^{-1} D + xx^{\mathsf{T}}/\mathrm{cap}$ with $J = DKD^{\mathsf{T}}$ (assembled entrywise, 3.8×10^{-15}), $x = K^{-1}\mathbf{1}$ the equilibrium charge and $\mathrm{cap} = \mathbf{1}^{\mathsf{T}} K^{-1} \mathbf{1}$, as a matrix identity ($1.4 \times 10^{-13}$); under the congruence $u = K^{1/2} v$ the Dirichlet half is an *orthogonal projection* ($P^2 = P$ at 1.9×10^{-12}, $\Omega = \lambda_{\max}(P) = 1$ to 5.1×10^{-12} — the T142 normalisation, where T141 had guessed 20.7–2724), its complement *is* the capacity rank-one, and it annihilates the equilibrium direction $K^{-1/2}\mathbf{1}$. Controls: dropping the rank-one (3.4×10^{-1}), a 1% cap perturbation (3.4×10^{-3}).
- **The exact capacity–Rayleigh identity (T143)** [E]: with $H = \mathrm{diag}(s) + L_N$ (4.7×10^{-16}), $L_N = D^{\mathsf{T}} B D$ for the signed crossing kernel (4.9×10^{-16}) and $\rho(W) = 1 - \lambda_{\min}(A, A_B)$ (5.4×10^{-14}), the assembled quotient $[\, d^{\mathsf{T}}(J^{-1} - B)d + (x^{\mathsf{T}}v)^2/\mathrm{cap} - \sum_k s_k v_k^2 \,]/[\, d^{\mathsf{T}} J^{-1} d + (x^{\mathsf{T}}v)^2/\mathrm{cap} \,]$ equals $v^{\mathsf{T}}(K^{-1} - H)v / v^{\mathsf{T}} K^{-1} v$ for *every* v (9.5×10^{-14} on 24 random vectors per window), reproduces $1 - \rho(W)$ at the exact top eigenvector (1.7×10^{-9} under the *declared* 1/gap conditioning bar 5.4×10^{-8}), and every random quotient stays $\geq 1 - \rho(W)$ (the inf property); on the constant vector the whole denominator *is* the capacity term. Controls: dropping the cap term (rel = 1), the $|N|$ crossing kernel (2.6×10^{-1}).

- **Cholesky nestedness (T144)** [E]: $\mathrm{cap}_E([a, a+j))$ equals the prefix sum of squared forward-substitution entries of *one* triangular solve — 3404 interval capacities from 36 solves at 2.0×10^{-14} against independent Schur solves, exhaustive on the median window (every prefix, 2.8×10^{-14}), monotone in b by construction. Control: the reversed ordering computes *suffix* capacities and deviates by $\geq 4.6 \times 10^{-1}$.
- **The whitening certificate (T144)** [E]: $\lambda_{\max}(W) \leq \kappa_{\mathrm{up}}$ (completed Cholesky) $\leq$ the Gershgorin ceiling on every window ($\kappa_{\mathrm{up}} = 1.2245$–$1.4236$ against ≤ 2.1213); the licensed chain direction $\lambda_{\min}(E)/\kappa_{\mathrm{up}} \leq \lambda_{\min}(A, A_B)$ holds on every window — a *denominator* comparison, untouched by T142's numerator obstruction — and the Cholesky refuses at the shrunken shift $0.995\,\lambda_{\max}(W)$.
- **The M4 split and the certified chain (T145)** [E]: the mass half of Maz'ya's step M4 — $\sum_k f_k^2 \leq \psi^2$ *pointwise* (a theorem for nonnegative truncations, exact slack ≥ 0), the row-wise implication on rows with $(E\mathbf{1})_i \geq 0$, and the full step $\sigma_{\mathrm{tot}} = 0.2197$–$0.4425 < 1$ on every window although the Markov hypothesis is absent. Licence 4 is stated with $|R|$ and *not* R^+, with the witness $R = \left[\begin{smallmatrix} 1 & -1/2 \\ -1/2 & 1 \end{smallmatrix}\right]$, $x = (1, -1)$ ($3.0 > 2.0$) built in as a counterexample check. The sign-free chain is ordered on every window, and S1′ is *certified per window on both routes*: cert_lam_max$(R) \leq 12\,\sigma_{\mathrm{tot}}\Psi_{\mathrm{pos}}$ (energy, $c_0 = 2.6369$–5.3099) and $\leq G_{\mathrm{dy}}\Psi_{\mathrm{abs}}$ (sign-free, $c_0 = 2.5154$–6.1449), better route $c_0 = 2.5154$–4.2265 against the classical Dirichlet value 4; the all-sets density bound carries Charikar's cited constant, bracketed by exhaustive enumeration of all 4095 subsets of a small block. Control: $c_0 \times 10^{-3}$ breaks the certificate on every window.
- **Named limits as content.** Per instance on small windows; nothing uniform in the zone index or in D, and every T142–T145 exponent is a fit that stays in the sandbox. The c_0 of the last row is *measured* at the minimiser's own layer cake — the a-priori *level lemma* (T145's L1) stays open, unclaimed and unapproached, and no marker moves toward it. Maz'ya's strong-type half is *not* used as a theorem anywhere: it is the thing the cycle transcribes.

4.13 The level-lemma identities (v547)

The a-priori core of T146 (Section 10): the same level constant that v546 certified with a *measured* input, now supplied as a functional of the form alone [verification/v547_level_lemma_identities.py] (20 checks, ∼0.4 s; promotion of the four a-priori pieces of T146 on 12 frame-A windows, $n = 4 \ldots 139$, $m = 26 \ldots 285 \leq 300$, $D = 7.578 \times 10^{-3} \ldots 2.789 \times 10^{-2}$, plus 9 random positive definite forms for the form-independent items and a stress ladder $m = 64 \ldots 288$; each statement carries a mutation control that must fail by $\geq 10^{-3}$).

- **The union identity for nested cakes (T146)** [E]: for a nested decreasing chain with positive coefficients, $\sum_{j,l} c_j c_l\,|S_j \cup S_l| = 2T\|\psi_t\|_1 - \|\psi_t\|^2$ with $T = \sum_j c_j$ and $\psi_t = \sum_j c_j \mathbf{1}_{S_j}$ — form-independent and $O(m)$, the reason the cake base is movable at all; checked against T145's $K \times K$ double sum on all 12 window minimisers and 9 random vectors (max rel 0.0) *and* against the union sizes computed from the sets themselves with no nestedness used. Controls: shuffling one proper level set breaks it (3.2×10^{-2}), a 1% perturbation of the heaviest coefficient breaks the double sum (4.8×10^{-3}).
- **The layer-cake counting lemma at a general base (T146)** [E]: $G(b) \leq 2b^2\,\|v\|_\infty\|v\|_1/\|v\|^2 + \varepsilon(b)$ for *any* vector v at all 6 preregistered bases (margin ≥ 1.08 on the battery and on every window minimiser); at $b = 2$ the leading constant is the classical dyadic 8 *exactly*, and the base is *free* because the chain consumes only the lower domination $|v| \leq \psi_t$: the a-priori constant gains $4/b_*^2 = 3.7612$ over base 2 on every window. Controls: the bound $\times 10^{-3}$ fails everywhere, and the one-level-down cake (ψ_t/b) breaks the domination on every draw — the +1 in the cake exponent is load-bearing.
- **The resolvent delocalisation bound (T146)** [E]: $\psi = \lambda R\psi$ to 9.0×10^{-14} — an *identity*, so no Davis–Kahan transfer step appears anywhere — and per coordinate (Cauchy–Schwarz), with

$\lambda_{\mathrm{up}} = \min_j \mathrm{Rayleigh}(Re_j) \geq \lambda_{\min}$ (Rayleigh–Ritz; window ratio 1.0592–1.2883) and the column norms certified from above with the linear-solve residual paid for,

$$\|\psi\|_\infty \;\leq\; \lambda_{\mathrm{up}} \max_j \|Re_j\|\,\|\psi\|$$

on every window, within a factor 1.061–1.337 of the truth. *The lever*: the trivial ceiling $\max_j \|Re_j\| \leq 1/\lambda_{\min}$ would give $\Gamma \leq \sqrt{m} = 5.1$–$16.9$ (useless); the certified columns sit a factor 2.85–10.17 *below* it (preregistered bar 2.0), giving $\Gamma = 1.8356$–2.2797 — read off the form, never off the minimiser. Controls: a 1% λ perturbation breaks the identity (1.0×10^{-2}), the sup bound $\times 10^{-3}$ fails on every window.

- **The composite level lemma (T146)** [E]: $c_0^{\mathrm{ap}} = 2b^2\,\Gamma \min(1, \Gamma_1) + \varepsilon = 3.9042$–$4.8488$ dominates the measured level constant on all 12 windows (slack 1.986–2.418) and stays under the preregistered ceiling $16 = 4\times$ Maz'ya's Dirichlet value 4; the certified window inequality $\mathrm{cert\_lam\_max}(R) \leq c_0^{\mathrm{ap}} \Psi_{\mathrm{abs}}$ holds on 12/12 (both sides by completed Cholesky, the density the $|R|$ one with Charikar's cited constant), closing the chain at loss factor 0.0423–0.1272 of the exact gap — v546's certified chain with its one measured input replaced by a functional of the form. *The built-in discriminator*: on T145's no-go form $R = aa^{\mathsf{T}} + \varepsilon I$, $a_i = i^{-1/2}$, Γ grows $3.676 \to 6.798$ over $m = 64 \ldots 288$ (ratio 1.85 $\geq$ the preregistered 1.5) and $c_0^{\mathrm{ap}} = 11.10$ *exceeds* every window value — the bound must be worse there, since a candidate that did not fail on the no-go would prove a false statement — while the theorem half survives on every no-go size and the Dirichlet path-Laplacian control stays flat (max/min 1.0063). Control: $c_0^{\mathrm{ap}} \times 10^{-3}$ breaks the certificate on every window.
- **Named limits as content.** Per instance on small windows — a *finite list* of certified window inequalities with an explicit maximum; nothing uniform in the zone index or in D, and explicitly *no* statement for all D: the asymptotic delocalisation of the Green columns (T146's remainder) stays open and typed open. Only the sign-free / Green constant is bounded a priori — the energy-route σ_{tot} stays *measured* — and every T146 exponent is a fit that stays in the sandbox. Davis–Kahan 1970 is cited as the transfer step this route deliberately avoids; Maz'ya's strong-type half is *not* used as a theorem anywhere.

4.14 The Green/Szegő identities (v548)

The identity/certificate core of T147 and T148 (Section 10): the two scalars the level constant of v547 is made of, certified on the same class of windows [`verification/v548_green_szego_identities.py`] (21 checks, ∼0.4 s; promotion of the identity/certificate pieces of T147+T148 on 12 frame-A windows, $n = 4 \ldots 139$, $m = 26 \ldots 285 \leq 300$, $D = 7.578 \times 10^{-3} \ldots 2.789 \times 10^{-2}$, plus 7 random positive definite forms for the form-independent items and a weight-class model ladder $m = 64 \ldots 288$; each statement carries a mutation control that must fail by $\geq 10^{-3}$).

- **The factorisation identity (T147)** [E]: for any symmetric positive definite form with Green matrix R,

$$\Gamma = \sqrt{m}\,\lambda_{\mathrm{up}} \max_j \|Re_j\| \;=\; \sqrt{Q^\star} \cdot S_w\,, \qquad S_w = \lambda_{\mathrm{up}} \|R\|_F\,, \qquad Q^\star = m \max_j (R^2)_{jj} / \operatorname{tr}(R^2)\,,$$

exact at 2.2×10^{-16} on all 12 windows and 7 random forms — the all-D question splits into a purely spectral and a purely geometric scalar and into nothing else; the certified factorisation (residual-paid column norms, Frobenius bracket) dominates the true Γ everywhere, with $S_w = 1.1186$–1.5777 and $Q^\star = 2.0879$–2.8634 on this list. Controls: a 1% $Q^\star$ perturbation breaks the identity (5.0×10^{-3}), perturbing the largest Green column breaks the factorisation.

- **The bottom-block equidistribution constant (T147)** [E]: with Π_B the spectral projector onto $B = \{k : \lambda_k \leq 10\hat{\lambda}\}$ (preregistered rule, $|B| = 3 \ldots 5$), $\sum_j (\Pi_B)_{jj} = |B|$ *exactly* (3.3×10^{-16}), hence $Q_B = m \max_j (\Pi_B)_{jj} \geq |B|$ is a theorem (max $\geq$ mean) — and the *measured* $Q_B/|B| =$

1.5091–1.7980 lies in the preregistered band $[1, 2.2]$ against the Szegő/Widom sharp reference $Q_B \le 2|B|$ (a measurement on a finite list, typed as such; T148's honest negative stands beside it: $f(0) < 0$ on every surface window, so the mechanism enters nowhere as a theorem). Controls: the rank-$|B|$ coordinate projector gives $Q_B = m$ and overshoots the band by ≥ 3.9; a 1% basis scaling breaks the trace identity.

- **The second-difference ℓ^1 ceiling (T148, the lever)** [E]: from the exact Dirichlet eigenpair $L_0 s_k = \mu_k s_k$ (KMS 1953) and $\mu_k \ge 4k^2/(m+1)^2$ ($\sin t \ge 2t/\pi$), the coefficient bound $|a_k| \le \min(1, \|s_k\|_\infty \|L_0^p \psi\|_1 / \mu_k^p)$ for $p = 1, 2$ holds against the *true* coefficients on the bottom mode of every window and on random unit vectors (worst excess ≤ 0); the chain $\|a\|_1 \le \mathrm{ceil}_\mu$, $\mathrm{ceil}_\nu \le 2\sqrt{\nu}+1$ with $\nu = (m+1)^{3/2}\|L_0\psi\|_1/(2\sqrt{2})$ — every power of m cancelled, ν alone carries the m-dependence — and $m\|\psi\|_\infty^2 \le 2\|a\|_1^2$ holds everywhere. *Calibration*: for the Dirichlet bottom mode $\|a\|_1 = 1$ exactly and $\nu = 3.1415 \to \pi$ at every size. Controls: the ceiling $\times 10^{-3}$ fails on every window, a one-index μ shift breaks the coefficient identity (1.5×10^{-1}).
- **The layer-cake S_w certificate (T148)** [E]: the completed LDL^T inertia count at every rung of the preregistered ladder $\tau_i = \rho_i \lambda_{\mathrm{lo}}$ ($\rho = 1.5 \ldots 1024$, λ_{lo} a certified positive floor) equals the direct spectral count *exactly* on all 96 (window, rung) pairs (Sylvester 1852; Bunch–Kaufman 1977 — a certificate, never a sorted list), and the layer cake $\sum_k \lambda_k^{-2} \le N_1/\lambda_{\mathrm{lo}}^2 + \sum_i (N_{i+1}-N_i)/\tau_i^2 + (m-N_L)/\tau_L^2$ certifies $S_w \le 1.3288$–1.8220 per window (true 1.1186–1.5777, slack ≤ 1.216); nothing here says m-free — that reading lives in the sandbox as a labelled trend. Control: *dropping the bottom layer* breaks the bound on every window — the bottom eigenvalue is the weight.
- **The Liouville transform and the weight-class discriminator (T148)** [E]: the chain

$$m\|\psi\|_\infty^2 \;\le\; \kappa_\Lambda\, m\|\hat{\varphi}\|_\infty^2 \;\le\; 2\kappa_\Lambda\, \|a(\hat{\varphi})\|_1^2 \;\le\; 2\kappa_\Lambda\, \mathrm{ceil}(\hat{\varphi})^2\,, \qquad \varphi = \Lambda^{-1/2}\psi\,,$$

  holds stepwise on every bottom-block mode of every window — the exact pencil equivalence $E\psi = \lambda\psi \Leftrightarrow A\varphi = \lambda\Lambda\varphi$ is a change of variables, not a perturbation (Davis–Kahan 1970 cited as the route T148 computed to be vacuous and *not* used) — with $\kappa_\Lambda = \max\Lambda/\min\Lambda = 1.3868$–$2.5805$ *a priori* (a ratio of two diagonal entries of the form). *The discriminator*: on a model section with a genuine order-2 zero ($f(0) = 0$, $f''(0)/2 = 1$, verified exactly), over the ladder $m = 64 \ldots 288$ at matched κ_Λ (mismatch 0.0013), the bounded-variation multiplier keeps ν flat (max/min 1.003) while the rough multiplier ($\mathrm{TV}(\log\Lambda)$ larger by 162.6) grows by 19.9 — the rough class *must* be worse, and it is: the operative hypothesis is the roughness of the multiplier, not its condition number. Control: the first Liouville step $\times 10^{-3}$ fails on every mode, provably (the step slack is bounded by κ_Λ^2).
- **Named limits as content.** Per instance on small windows — a *finite list* of certified window inequalities with an explicit maximum; nothing uniform in the zone index or in D, and explicitly *no* statement for all D: the one open lifting input ($\nu_L \le F(\mathrm{form})$ with F m-free; equivalently bounded $\mathrm{TV}(\log\Lambda)$ or any weaker smoothness of the multiplier) stays open and typed open. The symbol-level order-2 hypothesis is *refuted* (T148: $f(0) < 0$ on every surface window) and enters nowhere as a theorem; σ_{tot} stays retired as a route; every T147/T148 exponent is a fit that stays in the sandbox.

4.15 The gauge/parity identities (v549)

The identity/certificate core of T149 and T150 (Section 10): the gauge *freedom* around the machinery `v548` certified in the Jacobi gauge, and the parity *structure* underneath it `[verification/v549_gauge_parity_identities.py]` (21 checks, ∼0.3 s; promotion of the identity/certificate pieces of T149+T150 on 12 frame-A windows, $n = 4 \ldots 139$, $m = 26 \ldots 285 \le 300$, $D = 7.578 \times 10^{-3} \ldots 2.789 \times 10^{-2}$, with 48 gauge assemblies, the full-section parity items on the 6 windows with $M \le 300$, and a parity control model at $m = 2001$; each statement carries a mutation control that must fail by $\ge 10^{-3}$).

- **The gauge freedom (T149)** [E]: for *any* positive diagonal $\tilde{\Lambda}$ the whitening $\tilde{E} = \tilde{\Lambda}^{-1/2} A \tilde{\Lambda}^{-1/2}$ is an exact congruence, so $\lambda_{\min}(A) \geq \text{cert\_lam\_min}(\tilde{E}) \cdot \min_j \tilde{\Lambda}_j$ is a *valid* certified lower bound for every gauge — an identity plus one Rayleigh step — and the family maximum is valid too (all 48 assemblies: Jacobi, const, moving-geometric-mean, and the arithmetic-free archimedean diagonal); the factorisation identity $\Gamma = \sqrt{Q^\star} S_w$ holds *gauge-invariantly* (max rel 2.4×10^{-16}); the const gauge (the geometric mean of the Jacobi diagonal) has $\mathrm{TV}(\log \tilde{\Lambda}) = 0$ *exactly* on every window — T149's elimination, not reduction, of T148's blocking scalar — with the certified entrywise sandwich $\sigma = c_2/c_1 = 1.3868$–$2.5805$. Controls: a 1% $Q^\star$ perturbation breaks the identity (4.99×10^{-3}); on the shifted indefinite form $A - 1.05\lambda_{\min} I$ the completed-Cholesky floor *refuses* on 12/12.
- **The two-regime discriminator (T149, the refutation as a check)** [E]: the flutter-smoothing gauge (moving geometric mean, preregistered half-width $m/16$) removes $\mathrm{TV}(\log \Lambda)$ by $\geq 2.88\times$ on every window while moving the Liouville-*transported* bottom-block smoothness functional $\nu_{\tilde{L}}$ by at most 0.0088 — and even the matched-amplitude *rough* re-gauge (lattice-scale oscillation) moves it by at most 0.0046 (bar 0.01 for both): the roughness $\nu_{\tilde{L}}$ responds to is *not* in the multiplier — T149's withdrawal of the TV hypothesis, wired as a per-instance check. Control: the same functional read *without* the transport (on the whitened modes ψ, which carry the multiplier) jumps by 0.41–9.76 under the same rough re-gauge — the invariance is carried by the transport, not by numbness of the functional.
- **The explicit zone diagonal (T150)** [E]:

$$\Lambda_r \;=\; c_0 - c_{M-1-2r} + \sum_{s \neq r} \max\bigl(c_{|r-s|} - c_{M-1-r-s},\, 0\bigr)\,,$$

  assembled lagwise with no matrix anywhere, equals the lumped-pair diagonal on every window (max rel 5.9×10^{-16}) — the whitening diagonal is an *explicit* functional of the zone's lag arithmetic and of nothing else; the lag vector splits *exactly* as $c = c^{\mathrm{arch}} + c^{\mathrm{atom}}$ with $c^{\mathrm{atom}} \leq 0$ entrywise, and the arithmetic mass carries the closed budget $\|c^{\mathrm{atom}}\|_1 \leq \sum_j \mu_j \leq 4B\sqrt{N}$ with $B = 1.038821$ the *verified* maximum of $\psi(n)/n$ over the whole atom table (Chebyshev 1852, computed and never assumed; Abel summation carries the closed form). *The honest negative as content*: the archimedean section alone is *indefinite* on 12/12 windows (completed $\mathrm{LDL}^{\mathsf{T}}$, up to 5 negative eigenvalues) while the full section has zero — the atoms are *co-responsible* for the positivity, so the additive perturbation reading (Weyl 1912 / Bauer–Fike 1960 / Davis–Kahan 1970, cited, computed dead by T150 at 2.4–5.8 orders, *not* used) is structurally empty and only the multiplicative / Loewner gauge step transfers. Control: dropping the positive part in the zone formula breaks the identity (4.25×10^{-1}).
- **The parity compression (T150, the named mechanism)** [E]: with U_-, U_+ the antisymmetric / symmetric parity isometries of the full window,

$$U_-^{\mathsf{T}}\, T_M(c)\, U_- \;=\; T - H\,, \qquad U_+^{\mathsf{T}}\, T_M(c)\, U_+ \;=\; T + H\,, \qquad U_+^{\mathsf{T}}\, T_M(c)\, U_- \;=\; 0\,,$$

  exactly (max rel 3.9×10^{-16}, cross block 1.4×10^{-16}) on all 6 windows with $M \leq 300$: the reflection-odd section used since T106 *is* the compression of the full symmetric Toeplitz section onto its antisymmetric parity sector (Basor–Ehrhardt 2009: the parity sectors are the natural objects of the Toeplitz+Hankel algebra, cited as the address and never as an authority); the completed $\mathrm{LDL}^{\mathsf{T}}$ counts *localise* the negative inertia — the full section's negative eigenvalues (forced by the sign-changing symbol, $f(0) = -29.2\ldots - 6.3 < 0$; T148's honest negative *re-read*, never re-derived) sit entirely in the *even* sector while the odd sector counts zero: $f(0) < 0$ is explained rather than worked around. Control: the sign-flipped basis does not reproduce $T - H$ (3.99×10^{-1}).
- **The parity-sine calibration and the ladder certificate (T150)** [E]: the parity sines $t_k(r) = \frac{2}{\sqrt{N}} \sin(2\pi k(r{+}1)/N)$, $N = 2m{+}1$, are the exact eigenpairs of the parity Laplacian

(reflecting corner 3, forced by antisymmetry, not chosen) at $\mu_k^P = 4\sin^2(\pi k/N)$ — eigen-residual 2.3×10^{-13} — and the smoothness ladder *calibrates* on the exact model: $\nu_k^P = \pi k^2$ for $k = 1\ldots12$ at $m = 2001$ (max deviation 3.0×10^{-5}), so the ladder form $\nu_k \le Ck^2$ is the right question and π its exact floor; on every window, in the const gauge ($\varphi = \psi$ exactly — the ladder of the *pure* Toeplitz-minus-Hankel section, no multiplier, no arithmetic weight), the parity coefficient licence holds, $\nu_k^P \le Ck^2$ with the per-window certified constant $C = 11.70$–17.28, and the derived per-mode ceiling $m\|\psi_k\|_\infty^2 \le 2(2\sqrt{C}k+1)^2$ is a certified instance inequality — an m-free C is exactly the one open input and is *not* claimed. Controls: the ladder $\times10^{-3}$ fails everywhere, a one-index calibration shift misses by 0.75.

- **Named limits as content.** Per instance on small windows — a *finite list* of certified window inequalities with an explicit maximum; nothing uniform in the zone index or in D, and explicitly *no* statement for all D: the one open input after T150 (an m-free constant C in the odd-parity-sector ladder $\nu_k \le Ck^2$ for a sign-changing-symbol Toeplitz section) stays open and typed open; TV($\log\Lambda$) stays withdrawn as a hypothesis (T149), σ_{tot} stays retired as a route (T148); every T149/T150 exponent is a fit that stays in the sandbox.

4.16 The odd-sector identities (v550)

The identity/certificate core of T151 (Section 10): the *reroute* off the quadratic ν ladder that v549 certified — through the pencil against the parity Laplacian and one elementary Sobolev step, to a per-mode bound *linear* in k [verification/v550_odd_sector_identities.py] (19 checks, ∼0.2 s; promotion of the identity/certificate pieces of T151 on 12 frame-A windows, $n = 4\ldots139$, $m = 26\ldots285 \le 300$, $D = 7.578\times10^{-3}\ldots2.789\times10^{-2}$, with parity control models at $m = 241$ and $m = 2001$ and a no-go size ladder $m = 64\ldots256$; each statement carries a mutation control that must fail by $\ge 10^{-3}$).

- **The grid step-over and the honest negative (T151)** [E]: the parity sines live on the *shifted* grid $\theta_k = 2\pi k/N$, $N = 2m{+}1$, which never contains $\theta = 0$, with $\mu_k^P = 2 - 2\cos\theta_k$ *exactly* (2.2×10^{-16}; eigen-residual 2.3×10^{-13}); the symbol has $f(0) = -47.6\ldots-6.3 < 0$ (T148's honest negative, re-read and never re-derived) with a nonempty negative window $[0,\theta_c)$, and $\theta_c/\theta_1 = 0.371$–$0.485 \le 0.9$ on *every* window: the odd grid *steps over* the negative window — odd-sector positivity is a grid fact, stepped over rather than cancelled. *The honest negative beside it, promoted as content* (T150's rest item 3 answered negatively): the symbol-only pencil floor $\kappa_{\mathrm{sym}}(1-\rho)$ is vacuous ($\rho \ge 1$) on 12/12 windows and the symbol moves by 0.45–3.27 relative across *one* mode spacing — no pointwise symbol statement is admissible at the bottom mode's resolution: the local model is a *matrix* statement, not a symbol statement. Controls: the Dirichlet corner breaks the eigenpairs (1.0), a one-index shift misses by 6.5×10^{-3}.
- **The certified bottom ladder (T151)** [E]: $\lambda_k(A) \le S\mu_k^P$ for $k \le 8$, certified per window by 8 completed LDL$^\mathsf{T}$ inertia counts (Sylvester 1852; Bunch–Kaufman 1977 — a certificate, never a sorted list), $S = 1.1019$–6.4725: the odd sector's bottom spectrum *is* the parity-Laplacian ladder up to a printed $O(1)$ factor — order 2 in k, with no trace of $f(0) < 0$ in it (the Rayleigh floor $L_P \ge \mu_1^P I$ absorbs it); the count equals the direct spectral count on all 96 (window, k) pairs. Control: $S\times10^{-3}$ certifies no eigenvalue below it anywhere.
- **The discrete Sobolev step and the π price (T151)** [E]: $\|\nabla\psi\|_2^2 = \langle\psi, L_0\psi\rangle - \psi_{\mathrm{last}}^2$ exactly (7.6×10^{-15}), $\|\nabla\psi\|_2^2 \le \langle\psi, L_P\psi\rangle$, and $\|\psi\|_\infty^2 \le 2\|\psi\|_2\|\nabla\psi\|_2$ on the true window modes, the parity sines and random unit vectors — elementary, licensed by the odd sector's *virtual node* $\psi_{-1} = 0$ (Hardy–Littlewood–Pólya 1934 / Agmon 1965 the continuum address); calibration: on the exact parity model the pencil is $(1,1)$ (certified $|\kappa - 1| = 1.1\times10^{-12}$ at $m = 241$) and the Sobolev per-mode price is π to 5.1×10^{-8} at $m = 2001$ — the honest, m-free cost of leaving the ℓ^2 world. Controls: dropping the boundary term breaks the identity (4.8×10^{-2}), the Neumann energy undercounts a first-node vector by 0.50.

- **The closed archimedean minimum (T151; T150's rest item 2 closed and attained)** [E]: the archimedean lag kernel has two shape properties, checked per window and never assumed — $c_i^{\mathrm{arch}} < 0$ for $i \geq 1$ and c_i^{arch} non-decreasing on $i \geq 1$ — and given those, $\min_r \Lambda_r^{\mathrm{arch}} = c_0^{\mathrm{arch}} - c_{M-1}^{\mathrm{arch}}$ *exactly* (max rel 0.0; value 2.1719–3.4674), attained at $r = 0$: a closed two-term functional of the smooth kernel, no matrix, no eigenvector, no loss. Control: the full atom-including kernel violates the monotone shape on every window and the closed formula misses its minimum by $\geq 8.7 \times 10^{-2}$.
- **The linear per-mode bound and the no-go discriminator (T151)** [E]: with $\kappa = 0.1939$–0.3599 the certified pencil floor (*one* completed Cholesky of $A - \kappa L_P$ per window, the floating-point floor carried as fl/μ_1^P; the direction re-read on the actual bottom eigenvalue: $\kappa\mu_1^P \leq \lambda_1(A) \leq S\mu_1^P$) and $K_{\mathrm{bot}} = S$, the chain

$$\langle \psi_k, L_P \psi_k \rangle \;\leq\; \frac{\lambda_k(A)}{\kappa} \;\leq\; \frac{K_{\mathrm{bot}}\,\mu_k^P}{\kappa} \qquad \Longrightarrow \qquad b_k := m\|\psi_k\|_\infty^2 \;\leq\; 2m\sqrt{\frac{K_{\mathrm{bot}}\,\mu_k^P}{\kappa}} \;\leq\; C_S\, k$$

  holds with $C_S = 12.081$–$26.953 \leq 2\pi\sqrt{K_{\mathrm{bot}}/\kappa}$ on the bottom 8 modes of every window — *linear* in k where the quadratic ν ladder grew (v549), every m-power cancelled, nothing additive anywhere (the $\Theta(D^3)$ bottom that killed the Weyl / Bauer–Fike / Davis–Kahan route never enters) — and the bound dominates the truth $m\|\psi_k\|_\infty^2$ and the parity Dirichlet energy on every read mode; the bottom pencil ratio $R = K_{\mathrm{bot}}/\kappa = 3.7362$–$18.6607$ is the *one* scalar the chain still owes, printed per window and typed *open* in m. *The discriminator*: on the T145 no-go form the ratio *explodes* on the size ladder ($1.686 \times 10^3 \to 2.667 \times 10^4$, factor 15.8 over $m = 64 \ldots 256$) while the exact parity model holds $R = 1$ to 1.1×10^{-12} — a route that did not fail there would prove a false statement. Controls: the ladder $\times 10^{-3}$ fails against the true sups, the inflated floor 1.5κ makes the Cholesky *refuse* on 12/12.
- **Named limits as content.** Per instance on small windows — a *finite list* of certified window inequalities with an explicit maximum; nothing uniform in the zone index or in D, and explicitly *no* statement for all D: the one open input after T151 (the m-freeness of the bottom pencil ratio $R = K_{\mathrm{bot}}/\kappa$ for the odd parity sector of a sign-changing-symbol Toeplitz section) stays open and typed open — its flat trend ($x^{0.037 \pm 0.015}$) is a fit that stays in the sandbox; the symbol-only route stays certified vacuous; TV($\log \Lambda$) stays withdrawn as a hypothesis (T149), σ_{tot} stays retired as a route (T148).

4.17 The fixed-size Ritz ceiling certificate (v551)

The certificate core of T154 (Section 10): the *fixed-size* replacement for the ceiling step that v550 certified by per-window LDL^{T} counts of size m — the sixteen-column subspace $\mathrm{span}\{t_1..t_8\} + A^{-1}L_P\,\mathrm{span}\{t_1..t_8\}$ carries a Courant–Fischer ceiling that *attains* the inertia-certified true bottom ladder constant, so the ceiling step needs no factorisation of size m and, by direction, no residual argument at all [verification/v551_ritz_ceiling_certificate.py] (16 checks, ∼0.4 s; promotion of the certificate core of T154 on the 12 *deepest* prime-power zones admitting a frame-A window inside the cap, $n = 29 \ldots 139$, $m = 96 \ldots 291 \leq 300$, $D = 7.248 \times 10^{-3} \ldots 1.815 \times 10^{-2}$ — the deep end of the accessible surface, mirroring T154's own surface selection and declared rather than tuned: on the shallowest zones $n \leq 11$ more than one bottom direction leaves the sine block, and the fixed-depth certificate is *not* claimed there; a no-go size ladder $m = 48 \ldots 288$, parity controls at $m = 64, 128, 256$, and 3 Dirichlet control windows; each statement carries a mutation control that must fail by $\geq 10^{-3}$).

- **The direction lemma (T154)** [E]: for orthonormal Q and $W = Q^{\mathsf{T}}AQ$, $\lambda_k(A) \leq \theta_k$ for *every* $k \leq \dim$ — Ritz values are *upper* bounds for the eigenvalues of the *same index* (Courant–Fischer 1920 / Cauchy 1829), verified against the exact spectrum on the certificate subspace of every window and on 24 random subspaces (one-sided excess 0.0) — so the fixed-size ceiling needs *no* residual argument; Temple 1928 / Kato 1949 is a floor device and is used nowhere in the

module. Control: the *reversed* reading (Ritz values as lower bounds) is recognised as false on 12/12 windows, θ_k overshooting λ_k by a factor ≥ 18.6 somewhere on every window.

- **The sixteen-column certificate (T154)** [E]: the subspace has *fixed* dimension 16, independent of m (one Green step $A^{-1}L_P t_k$ by Cholesky back-solves, no inverse formed), and its ceiling $K^F = \max_k \theta_k/\mu_k^P$ (8 completed Choleskys of matrices $\leq 8 \times 8$) equals $K^F = 1.1019$–1.5845 with $K^F \geq \max_k \lambda_k/\mu_k^P$ on every window *and* $|K^F/K_{\text{bot}} - 1| \leq 9.9 \times 10^{-7}$ against the declared cap 10^{-4} on 12/12, where K_{bot} is the inertia-certified true ladder constant (8 completed LDL^T counts of size m per window; count = direct spectral count on all 96 pairs): the fixed-size ceiling *attains* the truth, and the size-m counts are retired from the ceiling step. Controls: the parity sines alone overshoot by 41.6–739 (failing the cap everywhere — T152/T153's $O(m^2)$ overshoot reproduced at fixed size), and corrupting the Green columns ($L_P t_k$ instead of $A^{-1}L_P t_k$) breaks the attainment on 12/12 (Kaniel 1966 / Paige 1971 quoted as the mechanism, never used as a bound).
- **The one-direction anatomy (T154)** [E] (angles measured): the median of the seven aligned principal angles between the bottom eight of A and the bottom eight of L_P is 0.03–0.14° while the largest is 83.79–89.70° on every window (Björck–Golub 1973) — the misalignment is carried by *one* nearly orthogonal direction, and that single direction *is* the collapse price $\lambda_1(A)/(t\,\mu_1^P) = 4.408$–$7.042$ ($t = 0.2250$–0.2500 the certified Schur two-block floor, $k_b = 16$), recovered *in full* per window by one completed Cholesky of $A - \gamma I$ (size m, certified, not fixed size, labelled as such): the price is a uniformity gap, not a size gap. Control: $A = L_P$ through the same instrument returns $K_{\text{bot}} = K^F = 1$ and zero misalignment (deviations $< 10^{-5}$, the inertia ladder's backoff step), with the price exactly $1/t$.
- **The no-go discriminator (T154)** [E]: on the T145 no-go reconstruction $c(l) = 1/(1+l)$ the odd section is positive definite and the Schur floor survives ($t = 0.05$ on all 4 sizes), but K_{bot} explodes ($\times 35.4$) and the sixteen-column ceiling explodes *with* it ($\times 57.5$) over $m = 48 \ldots 288$ — the instrument does not report flatness where flatness is false and manufactures no closure; the Courant–Fischer direction is violated nowhere (16/16 positive-definite cases); and the *Dirichlet* control (the Hankel half removed, arithmetic kept bit for bit) stops being positive definite — $\lambda_1 = -5.60 \ldots -3.55$ certified on 3/3 — so no ceiling exists there: the reflection half is load-bearing for positivity (T154's second refutation of the boundary-reflection reading).
- **Named limits as content.** Per instance on small windows — a *finite list* of certified window inequalities with an explicit maximum; nothing uniform in the zone index or in D, and explicitly *no* statement for all D: the attainment $K^F = K_{\text{bot}}$ is exhibited per window and *not* promoted to a theorem (T154's no-go stress shows the certificate overshoots there by 1.12–2.19 — exact attainment is a property of the real section, not of the instrument); the two open inputs after T154 (R1′, an m-free floor for $\lambda_{\min}(B_{HH})$ on the bulk parity block; R2′, an m-free lower bound for the L_P floor on the complement of the eight bottom Ritz directions — arithmetic-free, worth 91–100% of the collapse price) stay open and typed open; every floor consumed is certified per window at size m; `v550`'s ladder stays as stated, and nothing is re-opened.

4.18 The four fixed-size angle instruments (v552)

The instrument cores of T155 and T157 (Section 10): the four fixed-size instruments the angle campaign produced — none of them closes an angle, and none is promoted as anything but a per-instance instrument [`verification/v552_angle_instruments.py`] (18 checks, ∼0.4 s; everything recomputed on `v551`'s declared surface, the 12 deepest prime-power zones admitting a frame-A window inside the cap, $n = 29 \ldots 139$, $m = 96 \ldots 291 \leq 300$; a no-go size ladder $m = 48 \ldots 288$; parity and Dirichlet controls at $m = 64, 128, 256$; each statement carries at least one mutation control that must fail loudly).

- **The complement-floor certificate (T155)** [E]: $\min_{v \perp W} v^\mathsf{T} L_P v \geq \mu_{13}^P - \lambda_{\max}(M^{1/2}(I - GG^\mathsf{T})M^{1/2})$ with $M = \text{diag}(\mu_{13}^P - \mu_k^P)$ and $G = T_{12}Q_W$ — an identity plus *one* certified

Rayleigh bound on the high block, a theorem for every m and every subspace, direction checked: the certificate *never* exceeds the exact size-m complement bottom (12/12) and agrees with it to $1 - \text{ratio} \le 1.01 \times 10^{-3}$ against the declared cap 2×10^{-3} on this small surface (the agreement tightens with depth: T155 measured $\le 2.2 \times 10^{-7}$ at $h = 142 \dots 1293$, sandbox, not consumed here). Exactness controls in *both* known configurations: $W = \text{span}\{t_1..t_8\}$ returns μ_9^P and the *worthless* configuration $W = \text{span}\{t_2..t_{13}\}$ returns μ_1^P, both to 2.0×10^{-11} — the instrument reports its own worthlessness. Mutation: one corrupted overlap row moves the certificate by ≥ 0.549, only downward.

- **The sine-block confinement (T157)** [E]: from the certified pencil floor $A \succeq tL_P$ (Schur two-block, $k_b = 16$) and the certified ladder ceiling $\lambda_1 \le S\mu_1^P$ (v551's sixteen-column certificate) *alone*, with $\gamma = Te_1(A)$ the parity coordinates of the bottom eigenvector, $\|\gamma_H\|^2 \le \lambda_1/(t\,\mu_{17}^P) \le (S/t)/\rho_{17} = 0.0153$–$0.0245 < 1$ on 12/12: $e_1(A)$ lives 97.5–98.5% inside the first 16 parity sines by a *per-instance theorem* (measured tail 6.7×10^{-6}–3.0×10^{-5}; T146's measured "98 per cent" replaced by one line). Negative control: substituting the pencil ceiling κ for the ladder S makes the bound *vacuous* (17.6–$477 > 1$ on 12/12, at least 997× above the S form) — κ carries the whole atom mass, and the choice of ceiling is the whole content. Mutation: the inflated floor κL_P is refuted at $e_1(A)$ itself by 3.3×10^3–1.0×10^5.

- **The Schur-entry identity (T157)** [E]: $s = \mu_1^P\, t_1^{\mathsf{T}} A^{-1} t_1 = (B^{-1})_{11}$ exactly (worst relative 1.1×10^{-11}) and $(B^{-1})_{LL} S_L = I$ to 5.8×10^{-11} with $S_L = B_{LL} - B_{LH} B_{HH}^{-1} B_{HL}$ the *same* fixed-size 16×16 Schur complement the T152..T157 chain already forms, with the literal-entry direction $1/s \le (S_L)_{11}$ on 12/12: $1/s = 4.0460$–5.8962 is an $O(1)$ number carried by *one* fixed-size object, so T156's R2″ loss ceiling $r \le 1/(Ls)$ is a statement about one entry. Negative control (T157's refutation wired as a check): the inversion-free Cauchy–Schwarz ceiling $\hat{a} - \|b\|^4/(b^{\mathsf{T}} B_{HH} b)$ is valid and *useless*, off by 250–7.27×10^3 ($\hat{a} = 1.13 \times 10^3$–$3.11 \times 10^4$ is the same entry *without* the Schur subtraction) — the cancellation in $b^{\mathsf{T}} B_{HH}^{-1} b$ is nearly complete and Cauchy–Schwarz cannot see it. Mutation: a one-index shift breaks the identity by 1.55–2.15.

- **The adaptive Lipschitz ceiling (T157)** [E]: interval bisection with the *local* Lipschitz constant $\text{lip}(a) = d/g(a) + 2f/g(a)^2$ certifies $\inf f^{\text{arch}}/(4\sin^2(\theta/2)) \ge t = 0.25$ on 12/12 windows in 519–1201 symbol evaluations (certified floor 0.2505–0.3062, never above the fine-grid minimum — direction checked): the arch one-variable statement carries an *executed* certificate per window, not a grid observation. The *global* θ_{lo} constant is documented insufficient (a uniform grid would need 2.14×10^3–7.33×10^4 points, up to 61.4× the adaptive cost). Mutation: fed a target above the true grid minimum the certificate *refuses* on 12/12 — it cannot be talked into certifying a false floor.

- **The no-go discriminator and the controls** [E]: on the T145 family $c(l) = 1/(1+l)$ the certified Schur floor *survives* ($t = 0.05$ on all 4 sizes) while both new instruments break — the confinement bound goes vacuous (7.93–$413 > 1$, ×52.0 over $m = 48 \dots 288$) and $(S_L)_{11}$ explodes ($1185 \to 7.11 \times 10^4$, ×60.0) against the flat 4.05–5.90 real band: the instruments do not report confinement or $O(1)$ entries where both are false. Parity control $A = L_P$ exact to 4.9×10^{-14} ($B = I$, $s = 1$, zero tail); the Dirichlet residual hits the predicted $2/\sqrt{N}$ to 7.4×10^{-5}.

- **Named limits as content.** Per instance on small windows — a *finite list* with an explicit maximum; nothing uniform in the zone index or in D, and explicitly *no* statement for all D: *nothing about the angles is closed* — the two open terms after T157 (an m-free lower bound on $\hat{g}_1^2$, equivalently an m-free upper bound on the one Schur diagonal entry $1/s$, since the resolvent route is a theorem times that one measured fixed-size number; and an m-free R1″ domination with a non-shrinking margin, which T157 certifies per window with a 7.3×10^{-4} margin at the largest window and a *shrinking* trend) stay open and typed open; every floor t and ceiling S consumed is certified per window and its m-freeness is exactly what remains open; v551's ceiling and v550's ladder stay as stated, and nothing is re-opened.

4.19 The exact-form identities (v553)

The theorem cores of T158 and T159 (Section 10): the algebra of the sixteen-form itself — the dual variational structure, the exact lag sum with its gauge identity, the two closed scalars, and the one exact sign law the campaign found — and it closes *neither* open term [verification/v553_exact_form_identities.py] (25 checks, ∼0.2 s; everything recomputed on v551/v552's declared surface, the 12 deepest prime-power zones admitting a frame-A window inside the cap, $n = 29 \dots 139$, $m = 96 \dots 291 \le 300$; no-go ladders $m = 48 \dots 288$ on *both* T145 forms; parity and Dirichlet controls at $m = 64, 128, 256$; each statement carries at least one mutation control that must fail loudly).

- **The Thomson dual form and the positive Cholesky ladder (T158, P1)** [E]: $s = (B^{-1})_{11} = \max_x(2x_1 - x^{\mathsf{T}}Bx)$ (Maz'ya 1985 / Miclo 1999) with the direction *executed*, not asserted — 8 random trial vectors per window never exceed s, so every trial is an upper bound on the entry $1/s$: exactly the variational structure Cauchy–Schwarz lacks. The ladder $g_K = \sum_{j \le K} y_j^2$, $y = L^{-1}e_1$ from *one* Cholesky of the leading 16×16 block (nested Schur complements, Schur 1917 / Haynsworth 1968): all 16 terms strictly positive, partial sums strictly monotone, every $g_K \le s$, and $1/s \le 1/g_{16} \le 1/g_1 = B_{11}$ literally on 12/12; tight to $(1/g_{16})/(1/s) = 1.0642$–$1.1522$ against the declared cap 1.5 (T158 measured ≤ 1.2738 at $h = 254 \dots 1393$, sandbox, not consumed here). Mutation: a 5% corruption of one off-diagonal entry destroys positive definiteness — the Cholesky *refuses* on 12/12.
- **The two-dimensionality (T158, P2)** [E]: $\mathrm{span}\{t_1, A^{-1}L_P t_1\}$ attains the Dirichlet maximum s exactly ($g/s = 1$ to 9.0×10^{-13} on 12/12) for the theorem reason $L_P t_1 = \mu_1^P t_1$ (KMS 1953; eigenpair residual 8.1×10^{-13}) — the entry is a *two-dimensional* quantity the moment one Green column is granted. Negative controls: $\mathrm{span}\{t_1, t_2\}$ (no Green column) misses s by a factor 1.42–4.8, and the corrupted Green column $L_P t_1$ (no solve) misses by 255–7.7×10^3.
- **The gauge identity and the TmH signature (T159, P4/P7)** [E]: the section map annihilates constant lag vectors *entrywise with zero tolerance* (odd_toeplitz($\mathbf{1}$) = 0 exactly — the mechanism in one line), hence the gauge identity $\sum_d w_d = 0$ holds to 1.1×10^{-14} of $\sup|w|$ for the per-window optimiser and 3 random sixteen-vectors: the form is *blind* to the lag mass of c, and the lag mass is exactly where the arch and atom halves of the sixteen-form are of size h^2 and cancel. The premise — the Toeplitz-minus-Hankel signature $A_{r,s} - A_{r+1,s+1} = f(r+s)$ — is constant along every anti-diagonal to 1.4×10^{-16}. Mutations: a one-index Hankel shift breaks the gauge identity by 2.7×10^{-7}–4.2×10^{-5}; the T145 form $R = aa^{\mathsf{T}} + \varepsilon I$ has signature defect 7.4×10^{-2} — not a parity section, the gauge identity is not even *defined* for it.
- **The exact lag sum and the two closed scalars (T159, P2/P5)** [E]: $x^{\mathsf{T}}B_{LL}x = \sum_d c_d\, w_d$ ($w_d = T_d - H_d$, T the autocorrelation and H the convolution of $y = \sum_k (x_k/\sqrt{\mu_k^P})\, t_k$) against the direct assembly to 2.7×10^{-16} of the absolute scale $\sum_d |c_d w_d|$ — and the honest price *measured*: relative to the cancelled total the same identity holds only to 1.4×10^{-11} on this small surface (the sandbox at $h \le 1293$ loses 4.0–8.1 digits; that depth *is* the h^2 obstruction). The two closed scalars: $w_0 = \sum_k x_k^2/\mu_k^P$ (the $N^2/4\pi^2$ factor itself) to 4.2×10^{-16} and $2w_0 - w_1 = \|x\|^2$ to 7.4×10^{-16}, because $L_P = \mathrm{odd\_toeplitz}(2, -1, 0, \dots)$ and the t_k are its exact eigenvectors (KMS 1953) — the huge scale and the $O(1)$ scale sit in the *same* two weights. Mutations: a 1% perturbation of the largest lag breaks the lag sum by 2.2×10^{-3}; a one-index μ shift breaks w_0 by 0.60.
- **The Z-matrix law and the Collatz–Wielandt floor (T159, U1/U2)** [E]: B_{HH}^{arch} *is* a symmetric Z-matrix, *raw* — every off-diagonal entry ≤ 0 with no tolerance on 12/12. The closed sign-based floor $\min_k (B_{HH}^{\mathrm{arch}}\mathbf{1})_k = 0.9093$–$1.1284 \ge t = 0.25$ with the theorem direction verified per instance: floor $\le \lambda_{\min}(B_{HH}^{\mathrm{arch}}) = 0.9138$–$1.1432$ exactly (the symmetric M-matrix criterion, Perron 1907 / Frobenius 1912 / Collatz 1942 / Wielandt 1950), beating the sign-free norm route by 0.69–8.8 on every window. The honest limit as content: the atom block obeys

no sign law (0.44–0.53, a coin toss), the full block is not a Z-matrix and its row-sum floor is *negative* ($-48.3\ldots-0.79$) against a positive exact bottom — admissible and vacuous, R1″ stays open. Mutation: the checkerboard similarity $S=\mathrm{diag}((-1)^k)$ — an isometry, $\lambda_{\min}$ moves by 0 exactly — destroys the raw law (fraction $1.0000\to 0.494$–0.498): the Z law is an entrywise fact, not a spectral one.

- **The no-go discriminator and the controls** [E]: the T145 no-go breaks on three axes of this module's structure (Z law the exact opposite at fraction 0.000, signature broken, diagonal varies by 0.94) while on the $c(l)=1/(1+l)$ reconstruction the ladder *theorem* survives and the size explodes ($1/g_{16}=1187\to 7.11\times10^4$, $\times59.9$, tracking the true $1/s$ — the instrument reports the collapse instead of hiding it). Parity control: $A=L_P$ *is* the section of the lag vector $(2,-1,0,\ldots)$ entrywise, and the same machinery returns $B=I$, $s=1$, every ladder partial sum $g_K=1$ exactly constant in K, a zero gauge sum and lag sum $=\|x\|^2=1$ (worst 4.9×10^{-14}); the Dirichlet residual hits the predicted $2/\sqrt{N}$ to 7.4×10^{-5}.
- **Named limits as content.** Per instance on small windows — a *finite list* with an explicit maximum; nothing uniform in the zone index or in D, and explicitly *no* statement for all D: *neither open term is closed* — R2″ (one explicit pairing inequality: a bound on $\sum_d(\Delta c)_d W_d^1$ that uses the *correlation* between Δc and W^1 rather than their sizes, since $\ell^1\times$ sup pricing is refuted at every Abel rung) and R1″ (a device for the atom off-band block that is neither a norm nor a sign) stay open and typed open; T159's refutations (the fixed sixteen-vector, the one-sine rung, the power-law ansatz family, the Abel ℓ^1-pricing, the checkerboard law) are sandbox negatives and *not* promoted; the numerical horizon T159 declared ($\mathrm{cond}(B_{LL})>10^{12}$ past $h\approx1292$) does not reach this surface and is not consumed; v552's instruments and v551's ceiling stay as stated, and nothing is re-opened.

4.20 The sampling/harmonics identities (v554)

The closed theorem cores of T160 and T161 (Section 10): what the sixteen-form *pairs against* — the atom half as a Λ-weighted sampling at the Fourier harmonics of the log-window, the closed moment laws and the total-variation theorem of the smooth half, the closed head split of the archimedean kernel with its scale-free Bernstein rate, and the certified off-diagonal fraction bound that replaces the refuted sign law — and it closes *no* open term [verification/v554_sampling_harmonics_identities.py] (21 checks, ∼0.5 s; everything recomputed on v551–v553's declared surface, the 12 deepest prime-power zones admitting a frame-A window inside the cap, $n=29\ldots139$, $m=96\ldots291\le300$, plus a declared Λ-battery of six log-spaced cut-offs $X=10^3\ldots3.2\times10^5$ for the PNT item — finite von-Mangoldt sums only, no matrix anywhere; each statement carries at least one mutation control that must fail loudly).

- **The sampling identity (T160, P2)** [E]: $\sum_d c_d^{\mathrm{atom}}\,w_d=-\frac12\sum_{n\le X}(2\Lambda(n)/\sqrt{n})\,\widehat{w}(\log n)$ (plus one reflected term below the grid), $\widehat{w}$ the linear interpolant of the lag weights — exact to 5.6×10^{-15} of the absolute atom scale on 12/12 windows: the atom half of the pairing *is* a Λ-weighted sampling at the prime-power positions, hence identically a finite combination of sums $\sum_n\Lambda(n)\,n^{-1/2}\cos(t\log n)$ at 32 explicit frequencies. Mutation: a one-index shift of the sampling grid breaks by 5.1×10^{-5}–3.1×10^{-3}.
- **The moment laws and the TV theorem (T160, P1/P4)** [E]: $m_0=\sum_d w_d=0$ (1.0×10^{-16} of $\|w\|_1$), $m_1=\sum_d d\,w_d=-[S_0^2+2\sum_j P_j^2]$ (9.2×10^{-16}) and $m_2=\sum_d d^2w_d=-[2S_1-(M-1)S_0]^2$ (3.4×10^{-14}, a perfect square), both sign-definite forms strictly negative on every window; and U3 as a *theorem with checked hypotheses* ($c^{\mathrm{arch}}\le0$ and monotone on $d\ge1$, checked on 12/12, never assumed): $\mathrm{TV}(c^{\mathrm{arch}})=3.9817$–$4.8991\le|c_0^{\mathrm{arch}}|+2\sup_{d\ge1}|c_d^{\mathrm{arch}}|<6$, a closed window-independent constant. Mutations: a one-index prefix shift breaks m_1 by $\ge1.25\times10^{-2}$, an $(M-2)$ mis-normalisation breaks m_2 by $\ge6.1\times10^{-2}$, and the full atom-including kernel violates the monotone hypothesis on 12/12.

- **The harmonic-frequency theorem and the PNT main term (T161, P3)** [E]: $t_j\,(2\alpha) = 2\pi j$ exactly (2.8×10^{-14}) on every window and every battery cut-off — the 32 frequencies are precisely the Fourier harmonics of the log-window $[0, 2\alpha]$, so the atom half is the vector of the first 32 Fourier coefficients of the measure $\sum_n (2\Lambda(n)/\sqrt{n})\,\delta_{\log n}$; at those frequencies $\theta = 2\pi j$ collapses the partial-summation main term to the closed, parameter-free prediction $(\sqrt{X} - 1)/(\frac{1}{4} + t^2)$ (de la Vallée Poussin 1896; the Mellin factor of $x^{-1/2}$), and on the battery the finite sums agree with it to 0.0007–0.097 of the trivial mass against the declared cap 0.12, the subtraction reducing the aggregate residual to 0.008–0.466 ≤ 0.5 of $\sum |S|$ on every cut-off. Mutations: a wrong Mellin pole worsens the aggregate residual by ×1.8–×95.8, and a mass-matched *uniform* fake measure misses by 0.23–0.51 $\ge 1.5\times$ the cap — the agreement is a property of the von-Mangoldt measure, not of any measure with the same mass.
- **The head split and the scale-free Bernstein rate (T161, P1)** [E]: $c_d^{\mathrm{arch}} = \Psi_d + D\,\widehat{G}(dD, D)$ with $\Psi_d = -\frac{1}{2}[(d+1)\log(d+1) - 2d\log d + (d-1)\log(d-1)]$ closed, D-free and m-free (the triangle-smeared simple pole; the $\log D$ cancels identically) and $\widehat{G}$ the smear of the symbolically regularised $g_{\mathrm{reg}} = g - 1/(2w)$ — exact to 3.2×10^{-16} of $\sup|c^{\mathrm{arch}}|$ at every lag of every window, the two Ψ representations agreeing to 2.9×10^{-14}; and $\rho^* = (3 + \sqrt{5})/2 = 2.618034$: the root of $x^2 - 3x + 1$ (0.0), returned exactly by the $D \to 0$ limit of the peeled interval $[0.4\alpha, 2\alpha]$, invariant under $\alpha \times 10$ (it depends only on the interval ratio 5), every window's $\rho(D) = 2.588$–2.608 below it (Bernstein 1912). Mutations: dropping Ψ misses c^{arch} by 0.99–1.00; the wrong interval ratio moves the limit to 3.0.
- **The off-diagonal fraction bound and the sign refutation (T161, P2)** [E]: the gauge identity licenses one boundary-free Abel step and $Q^{\mathrm{arch}} = \sum_{k,l} a_k a_l \sum_{d\ge 1} (\Delta c^{\mathrm{arch}})_d\, R^1_{kl}(d)$ exactly (2.0×10^{-13}); the certified per-window inequality is |off-diagonal block| $\le \frac{1}{4}\,|Q^{\mathrm{arch}}|$ (measured 0.171–0.212 on 12/12; the sandbox measures 0.1035–0.1713 over $h = 50$–1218, flat — a fit, not consumed) — and the *refuted* sign law is wired as the negative control: the arch half is negative while its off-diagonal block is positive on 12/12 (the harmful direction for the Thomson dual) and 176 of 240 off-diagonal pairs violate the per-pair form on every window, so a one-sign statement was never the right object and nothing single-signed is promoted. Mutation: a one-index R-kernel shift breaks the decomposition by $\ge 6.8 \times 10^{-5}$.
- **The no-go discriminator and the controls** [E]: the head split is a statement about *the* T115 kernel — on the T145 no-go reconstruction $c(l) = 1/(1 + l)$ the same closed formula misses by 2.39 of $\sup|c|$; the closed Dirichlet cosine-sum identity agrees with the brute-force sum to 1.8×10^{-14} on 64 random triples including the degenerate branch; $L_P t_k = \mu_k^P t_k$ to 1.4×10^{-14} with the basis orthonormal to 1.0×10^{-14} (KMS 1953); odd_toeplitz$(c^L) = L_P$ at zero tolerance.
- **Named limits as content.** Per instance on small windows — a *finite list* with an explicit maximum; nothing uniform in the zone index or in D; finite Λ-sums are allowed and used, *no* RH statement is made, and the δ triage of T161 (required depth against RH strength) is a documentation item, *not* a claim of this module; the open terms after T161 — R-A′ (the prime-free log-moment $\sum_d \Psi_d w_d$, outside the polynomial ladder), R-B′ (the m-free 16×16 Gram fraction bound, Fejér 1915 / Schur 1917), R-C′ (*one* split of the pairing with $\delta_{\mathrm{eff}} < \frac{1}{2}$; T161 refutes the arch/atom and arch+smooth/fluctuation splits with numbers), R-D (a fifth R1″ device) — stay open and typed open; v553's algebra stays as stated, and nothing is re-opened.

4.21 The Pareto/total-variation identities (v555)

The closed theorem cores of T162 and T163 (Section 10): the *price structure* of the pairing — the exchange law that makes demand and price two coordinates of one object, the total-variation floor that closes the flat-price route in the parity sector, the chain-derived price axis of the Fejér knob, the Mellin cell moments and the boundary-free Abel step with its closed loss reason, and the Lerch/Frullani log-moment on three independent routes — and it closes no open term beyond the

negative closure of R-C‴ that the floor inequality itself is [`verification/v555_pareto_tv_identities.py`] (23 checks, ∼0.4 s; everything recomputed on v551–v554's declared surface, the 12 deepest prime-power zones admitting a frame-A window inside the cap, $n = 29 \dots 139$, $m = 96 \dots 291 \le 300$; the one arithmetic input $\kappa = 0.038821$ *verified* at every jump point of $\psi(x)/x$ on the von-Mangoldt table in $[100, 3.2 \times 10^5]$, reproduced to 4.2×10^{-7}; each statement carries at least one mutation control that must fail loudly).

- **The exchange law and the crossing criterion (T163, R1.2)** [E]: $\delta_{\mathrm{bnd}}(x) = \frac{1}{2} + \log(2\kappa\, g_{16}\, \mathrm{TV}(x)/P(x))/\log X$ is an identity at all 612 grid points of the declared Fejér knob (50 geometric σ on $[1, 4096]$ plus ∞, 12 windows; 5.1×10^{-16} relative, admissibility $Q(x) > 0$ everywhere), and the crossing criterion $\delta_{\mathrm{bnd}}(x) < \frac{1}{2} \iff P(x) > 2\kappa\, g_{16}\, \mathrm{TV}(x)$ holds as an exact equivalence at every point: price and demand are two coordinates on *one* object, and the crossing price is a closed formula. Mutation: the wrong constant (κ for 2κ) moves the law by 6.5×10^{-2}–3.4 at every grid point.

- **The total-variation floor (T163, R2.2 — the theorem that carries the verdict)** [E]: (T1) $w_0(x) = \|a(x)\|^2$ to 4.9×10^{-16} (the parity rows are orthonormal); (T2) $\mathrm{TV}(x) \ge |w_0(x)|$ by telescoping with $w_M = 0$; (T3) the normalisation $x_1 = 1$ forces $\|a\|^2 \ge 1/\mu_1^P$, hence $\mu_1^P\, \mathrm{TV}(x) = 5.00$–$28.84 \ge 1$ on all 708 trial vectors built in the module (51 knob settings plus 8 random admissible sixteen-vectors per window), against the closed floor $1/\mu_1^P = 1/(4\sin^2(\pi/N)) = 943.6$–$8609.6 \sim h^2$; (T4) with the crossing criterion, every sub-$\frac{1}{2}$ grid point pays $P > 2\kappa g_{16}/\mu_1^P = 15.4$–$142.8$ (all 54 sub-$\frac{1}{2}$ points respect it) — a flat-price sub-$\frac{1}{2}$ demand is impossible in the parity sector on this surface, the inequality that closes R-C‴ negatively. Negative control: the *inadmissible* vector $0.01\, x^*$ ($x_1 \ne 1$) does undercut the floor ($\mu_1^P \mathrm{TV} = 2.5$–$2.9 \times 10^{-3}$) and is recognised as inadmissible — the floor is a property of the normalisation meeting the smallest parity eigenvalue (Kac–Murdock–Szegő 1953), not of the machinery.

- **The chain-derived price axis (T163, R1.0)** [E]: $\sigma = 1$ gives $x = e_1$ exactly with $P = g_{16}/g_1$ (the $K = 1$ rung of the T158 Cholesky ladder, the T157 route-(0) certificate; 4.1×10^{-16}) and $\sigma = \infty$ gives $g_{16}\, Q(x^*) = 1$ (the $K = 16$ top rung; 1.2×10^{-12}) — the Fejér sweep is a path between two certificates the chain already owns; the ladder carries all 16 terms strictly positive with strictly monotone partial sums, and the knob is monotone in both coordinates on every window (the knob curve *is* the Pareto front). Mutation: a corrupted off-diagonal sixteen-block entry makes the Cholesky *refuse* on 12/12.

- **The Mellin cell moments and the Abel step (T162, C2/C5)** [E]: the k-th rung of the archimedean Mellin ladder is the closed sum $\sum_d w_d\, C_d(-(2k + \frac{1}{2}))$ with $C_d(s)$ the exact tent integral (Mellin 1896), reproduced by an independent per-cell Gauss–Legendre quadrature to 2.8×10^{-13} on every rung $k = 1 \dots 4$ (shifted tent centres break by $\ge 1.3 \times 10^{-2}$); the boundary-free Abel identities hold at levels $k = 1 \dots 3$ to 8.3×10^{-17} (Abel 1826), the dropped-boundary level-1 form — the one the chain uses — agrees because the gauge identity kills the boundary term exactly, the optimal level is *one* for T162's closed reason (net factor $32\pi/\alpha = 40.6$–$59.4 > 1$, confirmed by the measured level-2/level-1 bound ratio 30.8–$69.1 > 1$), and a gauge violation breaks the dropped form by its closed prediction $v_0 c_0 M$ to 1.2×10^{-15} — T162's positive control.

- **The Lerch/Frullani log-moment (T162, R-A′)** [E]: $L = \sum_d \Psi_d w_d$ agrees on three independent routes on every window — the direct sum with Ψ_d closed; the double-Abel form *with* its Wronskian boundary term $-\frac{1}{2} M \log M\, w_{M-1}$ (2.8×10^{-14}; dropping the term breaks by 2.2×10^{-4}–1.3×10^{-2}); and the Lerch/Frullani integral with the $d = 1$ term peeled in closed form (1.8×10^{-15}; 44 declared panels) — the peel constant $\int_0^\infty ((1 - e^{-y})/y)^2 dy = 2\log 2$ reproduced to 2.2×10^{-16} and $\Psi_1 = -\log 2$ exactly (Frullani 1828 / Lerch 1887 / Clausen 1832).

- **The no-go staircase and the controls** [E]: coarsening the lag grid over $\nu = 4/2/1/0.5$ degrades the arch/atom cancellation *monotonically* in the median (damage $1.00 \to 3.49 \to 10.81 \to 24.98$, the $\nu = 4$ column an exact null control) — the T145 resolution no-go breaks monotonically in

the collision rate; the closed Dirichlet cosine-sum identity to 2.4×10^{-14} including the degenerate branch; KMS eigenpairs to 1.4×10^{-14}; odd_toeplitz$(c^L) = L_P$ at zero tolerance.

- **Named limits as content.** Per instance on small windows — a *finite list* with an explicit maximum; nothing uniform in the zone index or in D; the T163 h-exponents ($P_{\text{aff}} \sim h^{3.05}$, $P_{\text{cross}} \sim h^{1.91}$, the TV exponent $\sim h^{2.00}$) are sandbox fits and are *not* consumed — what is consumed is the closed per-window floor; finite Λ-sums are allowed and used, *no* RH statement is made ($\frac{1}{2}$ appears in exactly one role: as the strength of a classical statement a required depth is compared against); the open terms after T163 — R-E (the successor, both arms prime-free: a growing $1/s$ ceiling for the downstream chain, or a sector whose gap does not vanish like h^{-2}), R-B‴ (an h-uniform positivity margin), R-D (a fifth R1″ device) — stay open and typed open; v554's identities stay as stated, and nothing is re-opened.

4.22 The gauge/P_{pr} identities (v556)

The closed theorem cores of T164 and T165 (Section 10): the *gauge structure* of the pairing — the theorem that the entry normalisation is a gauge, the exact conservation law that no sector shift can remove the h^2, the linearity that makes the chain spend its entry ceiling at power exactly one, the gauge-invariant P_{pr} identity that closes the successor question R-F by certificate, and the Schur-cascade direction with its exact exponent closure — and it does not close the one remaining quantifier [verification/v556_gauge_ppr_identities.py] (23 checks, ∼0.3 s; everything recomputed on v551–v555's declared surface, the 12 deepest prime-power zones admitting a frame-A window inside the cap, $n = 29\ldots139$, $m = 96\ldots291 \le 300$; the one arithmetic input $\kappa = 0.038821$ *verified* at every jump point of $\psi(x)/x$ on the von-Mangoldt table in $[100, 3.2\times10^5]$; each statement carries at least one mutation control that must fail loudly).

- **The gauge theorem (T164, S2.1)** [E]: $x_1 = 1$ fixes $a_1 = 1/\sqrt{\mu_1}$, i.e. only the *scale* of a; Q and TV are homogeneous of degree two in a; hence the demand ratio Q/TV of the same a-direction — and with it δ_{bnd} — is the *same number in every sector*: unchanged to 2.9×10^{-11} on 2520 sector-by-trial-vector combinations (5 weight families × 7 shifts × 6 Fejér taper widths on 12 windows), with $\text{TV} \ge |w_0|$ in every sector and the pure shift scaling the certified value by exactly $\theta = \mu_1^P/(\mu_1^P + \kappa_s)$ (2.2×10^{-11} on 432 points) — the crossing criterion $Q/\text{TV} > 2\kappa$ contains no normalisation and cannot be moved by *any* sector change; candidate (i) is settled in closed form (the unprojected spectrum has $\mu_0 = 0$ exactly, the lowest remaining eigenvalue is $\le \mu_1^P$ on 12/12 — the full space is strictly worse). Mutation: the wrong transport map (the x-coordinates carried unchanged instead of the a-direction) moves the ratio by 2.0×10^2–5.4×10^3.
- **The h^2 conservation law (T164, S2.2)** [E]: floor × transfer $= 1/(\mu_1^P+\kappa_s)\cdot(1+\kappa_s/\mu_1^P) = 1/\mu_1^P$ *identically* at every shift on the declared grid (72 window-by-shift points, 2.2×10^{-16} of $1/\mu_1^P$): a sector shift can flatten the T163 floor but pays the identical factor back as a transfer — the $h^2 = 1/(4\sin^2(\pi/N))$ is conserved, never removed; it lives in the *ratio*, not in the floor. Mutation: the wrong transfer misses the product by 0.9995 at the largest shift on every window.
- **The power-one spend as an identity (T164, S1)** [E]: the r-ceiling of the downstream chain is $r \le U/L$ with $L = \lambda_1(A)/\mu_1^P = 1.1019$–$1.5845$ computed per instance from the full window — *linear* in the entry ceiling U, so $d\log r/d\log U = +1$ exactly (central difference at $U = 1/g_{16}$, within 1.1×10^{-14} of 1 on 12/12): an h^ε ceiling costs $h^{-\varepsilon}$ of r-margin for every $\varepsilon > 0$ — no free tolerance in the consumption step (the measured power of the *full* T156 loss margin, $-1.0005\ldots-1.0000$, is a sandbox number and is not consumed). Mutation: the quadratic consumption $r = U^2/L$ reads slope 2.0000 and is recognised.
- **The P_{pr} identity and the predicate equivalence (T165, T1 — the spine of the R-F closure)** [E]: the gauge-invariant dashboard satisfies $P_{\text{pr}} = g_{16}\cdot R\cdot(\text{TV}/(t_1 v)^2)/\mu_1^P$ with no inequality anywhere — four factors, each a named object of the chain: the R-E-A quantifier $g_{16} = 0.1559$–0.2227, the demand $R = Q/\text{TV}$, the T163 floor $\text{tvf} = \text{TV}/(t_1 v)^2 \ge 1$, and the

Kac–Murdock–Szegő 1953 scale $1/\mu_1^P = 943.6$–$8609.6 \sim h^2$ — exact to 4.3×10^{-16} on all 192 battery vectors (6 knob settings + 6 random sixteen-vectors + 4 random *full-window* vectors per window), gauge-invariant under $v \to tv$ to 1.5×10^{-11} against the declared cancellation horizon; the predicates $R > 2\kappa$ and $P_{\rm pr} > 2\kappa g_{16}\,\mathrm{tvf}/\mu_1^P$ agree as booleans on all 192 (no search is run — T165's ascent stays sandbox): the demand half of R2″ and the gauge-invariant crossing condition are *one* predicate, so R-F is strictly stronger than the R2″ demand and its m-uniform form *is* R-E-A; tvf ≥ 1 pointwise, hence every crossing vector pays $P_{\rm pr} > 2\kappa g_{16}/\mu_1^P = 15.4$–$142.8$ — above the preregistered tolerance $\Theta_{\rm TOL} = 10$ on 12/12 windows: the R-F conjunction is *empty* on this surface by an identity plus a floor. Mutation: the price with $(t_1 v)$ *unsquared* is degree one over degree two, scales like t, and moves by 1.0–1.0×10^4 under the same round trips.

- **The Schur-cascade direction and the exponent closure (T164/T165)** [E]: $s = \mu_1^P\, t_1^{\mathsf T} A^{-1} t_1 = 0.1696$–$0.2472 \geq g_{16} \geq g_1$ window by window, i.e. $1/s \leq 1/g_{16} \leq 1/g_1 = B_{11}$ — the T158 ladder is a chain of upper bounds on the entry ceiling with the T157 route-(0) rung the loosest (Schur 1917 / Haynsworth 1968), and $P_K = \hat{a}\, s = 254.8$–$7.7 \times 10^3 \geq 1$ is the Kantorovich 1948 admissibility the T156 kernel consumes; the exponent closure is an *identity of the lists* — per instance $\log(1/g_1) = \log(g_{16}/g_1) + \log(1/g_{16})$ to 1.6×10^{-15}, the least-squares fits closing to 1.3×10^{-15} by linearity (T165's $+3.110 - 3.049 = +0.061$ is this identity on the sandbox surface; the exponent *values* are sandbox fits and are not consumed — what is promoted is that the whole certification lives in the cascade gain g_{16}/g_1, the object of the one open quantifier). Mutations: the closure with g_8 in place of g_{16} breaks by 0.10–0.24; a corrupted off-diagonal sixteen-block entry makes the Cholesky *refuse* on 12/12.
- **The no-go staircase and the controls** [E]: coarsening the lag grid over $\nu = 4/2/1/0.5$ degrades the arch/atom cancellation *monotonically* in the median (damage $1.00 \to 3.49 \to 10.81 \to 24.98$, the $\nu = 4$ column an exact null control); the closed Dirichlet cosine-sum identity to 2.9×10^{-14} including the degenerate branch; KMS eigenpairs to 1.4×10^{-14}; odd_toeplitz$(c^L) = L_P$ at zero tolerance.
- **Named limits as content.** Per instance on small windows — a *finite list* with an explicit maximum; nothing uniform in the zone index or in D; the T164/T165 h-exponents and the measured anatomy (the free ascent, the η(Cap) profile, the decoupled ν surface with its zone-depth finding, the retired constant $U_{\rm ref} = 4.90 \to 5.7327$) stay in the diary and Section 10; finite Λ-sums are allowed and used, *no* RH statement is made; **the one open object after T165 stays open and typed open**: $\inf_m g_{16}(m) > 0$, equivalently a lower bound on the sixteen-step Schur-cascade gain g_{16}/g_1 uniform in m — a quantifier over a certified flat list; R-B‴ (an independent α/h surface) stays open as narrowed by T164; v555's identities stay as stated, and nothing is re-opened.

4.23 The cascade/vector identities (v557)

The closed theorem cores of T166 and T167 (Section 10): the *anatomy of the cascade* and the *exactness of the closed vector at $K = 2$* — the prefix / minor / regression identities that make the sixteen rungs closed objects of the raw arithmetic Gram block, the diagonal invariance that makes the gain blind to the KMS normalisation, the exact $K = 2$ identity that makes the fast-null-vector free (the third dress collapses onto the second), the pivot identity that makes every candidate excess an exact number, the unification that makes the three dresses *one* inequality, and the trial theorem with the L_P no-go and the scramble type-change as must-breaks — and it does not close the one remaining scalar [verification/v557_cascade_vector_identities.py] (23 checks, ∼0.1 s; everything recomputed on v551–v556's declared surface, the 12 deepest prime-power zones admitting a frame-A window inside the cap, $n = 29 \ldots 139$, $m = 96 \ldots 291 \leq 300$; the one arithmetic input $\kappa = 0.038821$ *verified* at every jump point of $\psi(x)/x$ on the von-Mangoldt table in $[100, 3.2 \times 10^5]$; each statement carries at least one mutation control that must fail loudly).

- **The cascade identities (T166, U1)** [E]: with Q_K the leading $K \times K$ block of the normalised

sixteen-block and $G = TAT^{\mathsf{T}}$ the raw arithmetic Gram block — the *prefix* property ($g_K = \sum_{j\le K} y_j^2$ from *one* Cholesky equals the per-K direct solves $e_1^{\mathsf{T}} Q_K^{-1} e_1$ at every $K = 1 \dots 16$; 192 comparisons, 1.1×10^{-12}), the *minor* form $1/g_K = \det G_{[1..K]}/(\mu_1^P \det G_{[2..K]})$ in log space to 2.7×10^{-12} on 36 (window, K) pairs, $K \in \{2, 6, 16\}$ (the rungs are ratios of consecutive Gram minors of the *raw* block — closed objects, not factorisation outputs; Schur 1917 / Haynsworth 1968), and the *regression* form $g_K/g_1 = 1/(1 - R_K^2)$ with R_K^2 the squared multiple correlation of mode 1 on modes $2 \dots K$ (8.8×10^{-13}) — the open object *is* a near-collinearity statement. Mutation: the minor with the wrong row/column deleted misses by 0.99–1.45.

- **The diagonal invariance (T166, I4)** [E]: the gain $g_K/g_1 = B_{11}(Q_K^{-1})_{11}$ is unchanged under $B \to DBD$ on 144 random positive diagonals across $K \in \{2, 6, 16\}$ (2.7×10^{-12}): the cascade does *not* see the KMS $1/\sqrt{\mu}$ normalisation — the gain, and with it the whole h^3 of the sandbox surface, is a property of the arithmetic Gram block alone (Kac–Murdock–Szegő 1953 enters only through the spectrum). Mutation: a non-diagonal congruence moves the gain by 2.7–3.8.

- **The exact $K = 2$ identity (T167, V1 — the central positive result)** [E]: the *closed* vector $u = (1, -Q_{21}/Q_{22})$ *attains* the constrained minimum on every window — $u^{\mathsf{T}} Q_2 u = 1/g_2 = Q_{11}(1 - r_{12}^2)$ to 1.1×10^{-12} with $r_{12}^2 = Q_{12}^2/(Q_{11}Q_{22})$, and $B_{11} g_2 = 1/(1 - r_{12}^2)$ *identically* to 1.2×10^{-12} (measured $1 - r_{12}^2 = 2.8 \times 10^{-4}$–$5.9 \times 10^{-3}$ on this small surface; the h-trend is a sandbox fit and is not consumed): at $K = 2$ the fast-null-vector needs *no* approximation — the vector is free, dress (c) collapses onto dress (b), and everything the gain certifies at $K = 2$ is carried by *one* scalar built from three closed lag sums. Mutation: the wrong-sign vector pays the relative excess $4r_{12}^2/(1 - r_{12}^2) = 6.7 \times 10^2$–$1.5 \times 10^4$.

- **The pivot identity and the unification (T167, V1/V4)** [E]: for $u = x_K^* + \delta$ with $\delta_1 = 0$, $u^{\mathsf{T}} Q_K u = 1/g_K + \delta^{\mathsf{T}} Q_K \delta$ *exactly* (2.0×10^{-12} on 252 evaluations across three δ scales) — every relative excess $\rho = g_K\, \delta^{\mathsf{T}} Q_K \delta$ is an exact number, never an estimate; the Rayleigh envelope holds on the battery, its $\delta_1 = 0$ ceiling $\lambda_{\max}(Q_{HH})$ is attained exactly (1.1×10^{-15}) and Cauchy-interlaced, turning ρ into the vector threshold $\mathrm{eps}_{\mathrm{vec}} = \sqrt{\rho/(g_K \lambda_{\max})}/\|x^*\|$ with the square law $\mathrm{eps}_{\mathrm{ent}} \sim \mathrm{eps}_{\mathrm{vec}}^2$; the sign-aligned entrywise perturbation attains $|x^{*\mathsf{T}} E x^*| = \varepsilon S_K$ exactly (3.8×10^{-16}, $S_K = |x^*|^{\mathsf{T}} |Q| |x^*|$), so the entry threshold $\mathrm{eps}_{\mathrm{ent}}(K) = \rho_{\mathrm{carry}}(1/g_K)/S_K$ is exact — and the *unification*: $(1/g_K)/S_K$ is the one cancellation ratio of all three dresses — T166's determinant ratio over the absolute-value mass (dress a), at $K = 2$ literally $(1/g_2)/S_2 = (1 - r_{12}^2)/(1 + 3r_{12}^2)$ verified to 1.2×10^{-12} (dress b, up to the closed factor $1 + 3r_{12}^2 \to 4$ — T166's independently derived quarter), and the entry threshold itself (dress c): *one* inequality, not three. Mutation: $\delta_1 = 0.1$ breaks the pivot identity by exactly the closed cross term $2\delta_1 = 0.2$ (2.0×10^{-12}).

- **The trial theorem and the two must-breaks (T166/T167)** [E]: x_K^* attains $1/g_K$ (2.1×10^{-12}) and all 288 random trials with $u_1 = 1$ satisfy $u^{\mathsf{T}} Q_K u \ge 1/g_K$ (min ratio 15.14) — every closed candidate is a certified *lower* bound on g_K, never an upper one (Schur 1917); the L_P no-go: for $A = L_P$ the normalised sixteen-block is the identity (1.6×10^{-15}) and the cascade gain is exactly 1 (deviation 0.0) at $m = 64/128/256$ — a candidate blind to the off-diagonal block entries cannot carry a single power of h; the scramble type-change: uniform random atom positions at the *same* total mass make B_{11} itself lose positivity on 49/60 declared draws and on a majority of draws on 12/12 windows (range $-1.05 \times 10^5 \dots 2.4 \times 10^4$ against the true $B_{11} = 1134.9$–$31117.9 > 0$), so g_2 does not even exist there — at $K = 2$ the collapse under scrambling is a change of *type*, not a factor. Plus exact Dirichlet/parity controls (2.3×10^{-14} / 1.4×10^{-14} / zero tolerance).

- **Named limits as content.** Per instance on small windows — a *finite list* with an explicit maximum; nothing uniform in the zone index or in D; the T166/T167 h-exponents ($1 - r_{12}^2 \sim h^{-2.921}$ on frame A, the certified gain exponent $h^{+2.921}$, the threshold monotonicity in K, the $K = 6$ separation $h^{+1.138}$) and the measured anatomy (the rung profile, the polarisation split, the Kato radius, the headroom) stay in the diary and Section 10; finite Λ-sums are allowed and used,

no RH statement is made; **the one open object after T167 stays open and typed open**: R1, the single scalar — an m-free upper bound on $1 - r_{12}^2 = 1 - Q_{12}^2/(Q_{11}Q_{22}) \le C\,h^{-3+\varepsilon}$, three closed lag sums and nothing else; the routes T167 closed off stay closed as sandbox negatives (perturbation theory of every order around the KMS bottom mode — the Kato series converges to the *wrong* object; position-blind surrogates) and are not promoted as theorems; R-B‴ stays open as narrowed; v556's identities stay as stated, and nothing is re-opened.

4.24 The bilinear/rank identities (v558)

The closed theorem cores of T168, T169 and T170 (Section 10): the *exact shape of the one remaining scalar* — the Lagrange/Wronskian anatomy that makes the near-parallelism a property of the arithmetic metric alone, the exponent account with the unconditional Gershgorin certificate that makes ν_1 cheap, the T169-TH7 det-collapse that closes the loop T167 scalar → T168 factor → T169 candidate as an identity, the exact bilinear von Mangoldt form with its polarisation expansion and explicit kernel double sum, the rank-3 theorem and the Type II collapse that collapse the bilinear form back onto three *linear* Λ-sums, and the R4-free chain with the L_P no-go and the scramble discriminator as must-breaks — and it does not close that scalar [verification/v558_bilinear_rank_identities.py] (27 checks, ∼0.2 s; everything recomputed on v551–v557's declared surface, the 12 deepest prime-power zones admitting a frame-A window inside the cap, $n = 29\ldots139$, $m = 96\ldots291 \le 300$; the one arithmetic input $\kappa = 0.038821$ *verified* at every jump point of $\psi(x)/x$ on the von-Mangoldt table in $[100, 3.2\times10^5]$; each statement carries at least one mutation control that must fail loudly).

- **The Lagrange/Wronskian anatomy (T168, W0/W1)** [E]: the *lag-sum* identity $\hat a_{ij} = t_i^{\mathsf T} A_h t_j = \sum_d c_d W_d^{ij}$ with the closed T163 correlation weights, the $1/\sqrt{\mu}$-normalised block equal to $X^{\mathsf T}AX$ (1.6×10^{-15}; T169-TH3 — no conditioning ladder enters); *Euclidean orthogonality* $t_1 \cdot t_2 = 0$ to 4.4×10^{-16} — the near-parallelism $1 - r_{12}^2 \to 0$ of the whole programme is created *entirely* by the arithmetic metric A_h and not at all by the vectors; the *Wronskian telescope* $(\mu_1-\mu_2)u_r v_r = W_{r-1} - W_r$ with $W_r = u_{r+1}v_r - u_r v_{r+1}$ and $W_{-1} = W_{h-1} = 0$ (the telescope closes at the window edge; 1.9×10^{-17}), and the maximal-minor identity $\frac12\sum M^2 = \|u\|^2\|v\|^2 - \langle u,v\rangle^2 = 1/(\mu_1\mu_2)$ exactly (4.6×10^{-16}; Lagrange 1773) — the Wronskians carry *zero* arithmetic. Mutation: the telescope with the wrong eigenvalue pair misses by 1.66–1.67 relative (the closed miss is $(\mu_2-\mu_3)/(\mu_1-\mu_2) \to 5/3$).
- **The exponent account and the Gershgorin certificate (T168 W2 / T169 X2)** [E]: $1 - r_{12}^2 = \nu_1\nu_2/(\hat a_{11}\hat a_{22})$ *identically* (2.5×10^{-13}; measured $1 - r_{12}^2 = 2.8\times10^{-4}$–$5.9\times10^{-3}$ on this small surface, the h-trend a sandbox fit); the large eigenvalue carries the unconditional closed bound $\nu_1 \le \max(\hat a_{11}, \hat a_{22}) + |\hat a_{12}|$ (Gershgorin 1931 — T169's R3) on 12/12 windows with loss factor 1.1885–1.1947: ν_1 is cheap, the whole remaining difficulty sits in ν_2. Mutation: dropping the $|\hat a_{12}|$ term fails on every window ($\nu_1 - \max(\hat a_{11}, \hat a_{22}) = 0.30$–$1.17 > 0$).
- **The det-collapse theorem (T169-TH7)** [E]: on the Rayleigh family $u(t) = (t, -1)$ the one threshold-meeting candidate $K7 = \sqrt{\hat a_{22}/\hat a_{11}}$ collapses in closed form, $R(K7) = 2\sqrt{\hat a_{11}\hat a_{22}}\,\det\hat A/[(\hat a_{11}+\hat a_{22})(\sqrt{\hat a_{11}\hat a_{22}} + \hat a_{12})]$ (8.7×10^{-13}) — it meets the threshold precisely *because* it reintroduces $\det\hat A$: the t^*-language is provably exactly as hard as the determinant, and the loop closes as an identity; the usable form $\nu_2 \le 2\det\hat A/(\hat a_{11}+\hat a_{22}) = 2\nu_1\nu_2/(\nu_1+\nu_2)$ holds on 12/12, tight to 1.9980–1.9999 < 2 exactly (Rayleigh 1877). Mutation: the closed constant $1/\sqrt{2}$ overshoots ν_2 by 17.1–2.3×10^2.
- **The exact bilinear von Mangoldt form (T170, TH1–TH3)** [E]: the exact split $\hat A = B - S$ with $S_{ij} = \sum_{n\le X}(\Lambda(n)/\sqrt{n})\,X_n[ij]$, $X_n[ij] = \phi_n \cdot W^{ij}$ the two-point spline read of the closed weights, against the direct block (1.7×10^{-14}); the polarisation $\det\hat A = \det B - D(B,S) + \det S$ with $D(P,Q) = P_{11}Q_{22} + P_{22}Q_{11} - 2P_{12}Q_{12}$ (1.0×10^{-10} against the declared near-null horizon) — arch-arch, arch-Λ (linear), Λ-Λ (bilinear); the double sum $\det S = \sum_{n,m\le X}\Lambda(n)\Lambda(m)K(n,m)/\sqrt{nm}$ with $K(n,m) = \frac12 D(X_n, X_m)$, verified through the *explicit* kernel matrix and never through the factorisation (7.5×10^{-16}; Cauchy–Binet 1812/1815) —

R1's bilinear shape is a theorem; the wedge reading holds to the measured rank-one defect 4.6×10^{-3}–3.1×10^{-2} (an honest certificate, not exact). Mutation: the wrong cross-coefficient breaks the expansion by 64–430 relative.

- **The rank-3 theorem and the Type II collapse (T170, TH4–TH5)** [E]: the polarisation matrix on (X_{11}, X_{22}, X_{12}) is $\left[\begin{smallmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -2 \end{smallmatrix}\right]$ with eigenvalues $\{-2, -1, +1\}$ — rank 3, signature $(1, 2)$, exact integers — so the kernel $K = PMP^{\mathsf{T}}$ has $\sigma_4/\sigma_1 \le 1.6 \times 10^{-16}$, effective rank 1.24–1.36 ≤ 3 and nonzero spectrum equal to $\mathrm{spec}(M\,P^{\mathsf{T}}P)$ (1.0×10^{-15}) on every window: the bilinear form *is* the rank-3 polynomial $S_{11}S_{22} - S_{12}^2$ in three linear Λ-sums; Vaughan's identity $\Lambda = a_1 + a_2 + a_3 + a_4$ is verified coefficient by coefficient for every $U = V \in \{3, 4, 5, 6\}$ (residual 0.0; Vaughan 1977) and the Type II kernel $g(\log m + \log d)/\sqrt{md}$ has 99.9%-energy rank $[2, 2, 2, 2]$ across the whole U scan (log-additivity of the closed weights) against $[20, 15, 11, 9]$ for same-size generic Gaussian kernels (4.5×–10× more) — the large sieve rewards the generic case and returns the trivial operator norm here (Montgomery–Vaughan 1973, cited as the *priced* route; the sandbox route-table deltas stay in Section 10).
- **The R4-free chain and the two discriminators (T170-TH6 / T169 X3)** [E]: $1 - r_{12}^2 = \det \hat{A}/(\hat{a}_{11}\hat{a}_{22})$ is an identity (0.0) — a bound on $\det \hat{A}$ *is* a bound on the target with no positivity of A_h, no Rayleigh detour, no R4: the Weil fence is never approached; the T169-TH9 chain $[\max(\hat{a}_{11}, \hat{a}_{22}) + |\hat{a}_{12}|]\,\nu_2/(\hat{a}_{11}\hat{a}_{22})$ is a valid upper bound on 12/12 with loss exactly the Gershgorin loss; the L_P no-go: $\hat{a}_{12} = 1.1 \times 10^{-17}$ identically and $1 - r_{12}^2 = 1$ with no decay at $m = 64/128/256$; the scramble discriminator: uniform random atom positions at the *same* masses leave the rank-3 collapse *untouched* ($\sigma_4/\sigma_1 \le 2.1 \times 10^{-16}$ on all 60 draws — it is algebra, blind to where the atoms sit) while the value of $1 - r_{12}^2$ moves by a median factor 834.8 (range 69–1.8×10^4): the arithmetic content sits *entirely* in the joint values of (S_{11}, S_{22}, S_{12}) against B — which is exactly why no size-bounding tool reaches R1. Plus exact Dirichlet/parity controls (1.2×10^{-14} / 1.4×10^{-14} / zero tolerance).
- **Named limits as content.** Per instance on small windows — a *finite list* with an explicit maximum; nothing uniform in the zone index or in D; the T168–T170 h-exponents ($1 - r_{12}^2 \sim h^{-2.92}$ on the deep frame-A surface, the route-table deltas, the five-order cancellation of the reference window, the $h^{-2.711}$ row-angle trend) and the measured anatomy stay in the diary and Section 10; finite Λ-sums are allowed and used, *no* RH statement is made; **the one open object after T170 stays open and typed open**: R1, now finally *classified* but not closed — an m-free unconditional certificate that the two rows of $\hat{A}$ become collinear at rate $h^{-3+\varepsilon}$, equally a joint near-degeneracy of the three explicit linear Λ-sums (S_{11}, S_{22}, S_{12}) against the archimedean block — a rate statement the module neither makes nor approaches; the T168/T169/T170 verdicts (MINORS-RESIST / RATIO-RESISTS / SIEVE-RESISTS) are sandbox verdicts; v557's identities stay as stated, and nothing is re-opened.

4.25 The phase-2 capstone (v559)

The capstone *verifies the map and claims nothing new* (T171, contract `FINAL.MAP`, Section 10): T126–T170 reduced the I5 uniformity question through sixteen exact reformulations to *one* object (R1); T171 reproduced every link of that reduction in one connected sandbox run; this module is the load-bearing version of that run — the whole sixteen-link chain as one machine-checked pass (13 links THEOREM: algebra for every h, checked on instances; 3 links CERT: per window, no quantifier in m), the eight-route no-go battery (each classified dead route exhibited failing on an instance exactly as classified), and the precision ledger with both self-similarity identities — and it adds *zero* new uniform-in-m statements, which is precisely why it is promotable [`verification/v559_phase2_capstone.py`] (35 checks, ∼0.3 s; everything recomputed on v551–v558's declared surface, the 12 deepest prime-power zones admitting a frame-A window inside the cap, $n = 29 \ldots 139$, $m = 96 \ldots 291 \le 300$; the one arithmetic input $\kappa = 0.038821$ *verified* at every jump

point of $\psi(x)/x$ on the von-Mangoldt table; the 16×16 low block positive definite on 12/12 — a numerical fact about finite matrices, never routed through the Weil criterion).

- **The chain, sixteen links in one pass** [E]: (1) the T152 Schur two-block criterion — $TL_PT^{\mathsf{T}} = \mathrm{diag}(\mu^P)$ to 1.3×10^{-15} (KMS 1953), the identity pair $x^{\mathsf{T}}Gx = v^{\mathsf{T}}A_hv$ / $x^{\mathsf{T}}x = v^{\mathsf{T}}L_Pv$ to 1.8×10^{-14}, the criterion *holds* at $0.5\lambda_{\min}$ and *fails* at $1.02\lambda_{\min}$ (sharp, the direction check), Haynsworth product 3.8×10^{-15} (Schur 1917 / Haynsworth 1968); (2) the T151 Sobolev bound $b_k \le C_S k$ with $C_S = (2\pi + 2)/\sqrt{N}$, worst ratio 0.4816; (3) the T153 Ψ-collapse $\Psi(x) = \sum_d c_d w_d = x^{\mathsf{T}}Gx$ to 6.3×10^{-16} with $\sum_d w_d = 0$ to 9.0×10^{-17} (the Z-law) and cancellation headroom 2.1×10^{15} (the declared horizon); (4) the T154 Ritz ceiling — Cauchy interlacing and the dual chain $g_{12} \le g_{16} \le g_{32} \le s$ with $g_{32}/s = 0.9520$–0.9849 from *below* (Cauchy 1829; Courant–Fischer 1920); (5) the T155 12×12 floor [CERT] — holds at $(1 - 10^{-9})\lambda$, fails at $(1 + 10^{-4})\lambda$, certified $t = 0.2295$–0.3719; (6) the T156 $F(P, r)$ chain [CERT] — $P_K \ge 1$, $r_K \ge 0$ at $K = 2, 4, 8, 16$ with certified L increasing, $L(16) = 6.6$–15.6; (7) the T158 Thomson/ladder duality — the dual maximum equals the Cholesky rung to 3.1×10^{-12}, every increment strictly positive, 36/36 random admissible trials below; (8) the T160 sampling identity — the four-Dirichlet-kernel closed weights equal the correlation weights to 1.1×10^{-15} (Dirichlet 1829) and the atom half of Ψ equals the sampled Λ-mass to 9.9×10^{-15}; (9) the T163 TV floor / Abel swap to 1.8×10^{-11} with $\mathrm{TV}(w) \max|C_d|$ overshooting $|\Psi|$ by 5.2×10^4–1.1×10^6 (Abel 1826); (10) the T164 gauge/quantifier collapse — $R = \Psi/\mathrm{TV}$ scale-invariant to 2.2×10^{-11}, $\Psi(x^*) = 1/g_{16}$ to 7.0×10^{-12}, 48/48 perturbed admissible vectors larger; (11) the T165 P_{pr} identity to 1.1×10^{-16} with $P_{\mathrm{pr}} = 1$ at x^*; (12) the T166 cascade $g_{16}/g_1 = 1/(1 - R_{16}^2)$ to 1.8×10^{-12}, diagonal-invariant to 2.3×10^{-12}; (13) the T167 $K = 2$ exactness — $G_{11}g_2 = 1/(1 - r_{12}^2)$ to 6.8×10^{-13} with the closed minimiser, $K = 2$ *milder* than $K = 16$ by 1.62–5.20; (14) the T169 det collapse, R4-*free* — $1 - r_{12}^2 = \det\hat{A}/(\hat{a}_{11}\hat{a}_{22}) = \nu_1\nu_2/(\hat{a}_{11}\hat{a}_{22}) = \det G_2/(G_{11}G_{22})$ and $1/g_2 = G_{11}(1 - r_{12}^2)$, all $\le 4.2 \times 10^{-12}$, consuming *no* positivity of A_h (T170-TH6; the Weil fence is never approached); (15) the T170 rank-3 theorem — J-eigenvalues $\{-2, -1, +1\}$ exact, the split to 1.7×10^{-14}, the polarisation to 1.0×10^{-10}, $\det S = \frac{1}{2}y^{\mathsf{T}}Jy$ to 3.9×10^{-16}, $\sigma_4/\sigma_1 \le 1.6 \times 10^{-16}$ (Lagrange 1773; Cauchy–Binet 1812/1815); (16) the T170 R1 shape [CERT] — $\sin\angle(\mathrm{row}_1, \mathrm{row}_2) = |\det\hat{A}|/(|r_1||r_2|)$ via the 2×2 Lagrange identity (3.6×10^{-16} at the natural scale; 3.7×10^{-15} as algebra on well-conditioned controls), measured $\sin\angle = 1.169 \times 10^{-4}$–$2.467 \times 10^{-3} \le 10^{-2}$ per window — *the rate and the quantifier in m stay open: that statement is R1.*
- **The no-go battery, eight routes failing as classified** [E]: the size budget (the ℓ^1 majorant overshoots $|\det\hat{A}|$ by 2.7×10^3–1.6×10^4); the symbol infimum (-47.6 – $-21.3 < 0$ on 12/12 — no symbol-positivity argument can deliver the floor; Fejér 1915 cited); the sector/gauge change (an invariance, moves $\le 4.2 \times 10^{-12}$); perturbation theory (ν_2 sits 9.8×10^2–2.0×10^4 below $\|\hat{A}\|$; Kato 1949 the closed-off route); band-local domination (the atom half dominates $\times 1.71$–2.83); the sieve box (the operator-norm bound on $\det S$ alone overshoots $\times 8.8 \times 10^3$–7.5×10^4 — by the rank-3 theorem a bound on $\det S$ alone cannot see the three-term cancellation; Montgomery–Vaughan 1973 / Vaughan 1977 the *priced* route); the scramble (the value moves $\times 679$ median, range 68–2.3×10^6, while the rank-3 collapse is *untouched* on all 60 draws); and the L_P control (the Gram exactly diagonal, cascade gain = 1 to 0.0).
- **The precision ledger** [E]: the needed joint relative precision on (S_{11}, S_{22}, S_{12}) against the archimedean block — the value $1 - r_{12}^2 = 2.752 \times 10^{-4}$ at the deepest window ($m = 285$) — is finer than the RH-equivalent yardstick $h^{-1/2}$ by $215\times$ and finer than the best unconditional $h^{-0.996}$ by $13\times$, *on this surface*; both self-similarity identities check (6.8×10^{-13} / 3.1×10^{-13}, the t^* overshoot ≥ 0 everywhere, sign-agnostic; Rayleigh 1877). Mutations fail loudly (the wrong regression row, the wrong Thomson entry, the off-by-one swap, the wrong polarisation coefficient — the 10^{-2} bar missed by orders).
- **Named limits as content.** Per instance on small windows — a *finite list* with an explicit maximum; nothing uniform in the zone index or in D; the T171 trend fits (the pooled sin-

angle $h^{-2.33}$, the frame-split rates $\nu = 4$: $h^{-1.86}$ / $\nu = 6$: $h^{-2.87}$, the no-go growth exponents, $\delta = +1.044$ against 3.0, the full-surface precision numbers 2.284×10^{-7} / $1.2 \times 10^{5}\times$ / $3.1 \times 10^{3}\times$ at $h = 1444$) are sandbox numbers on T171's two-frame surface and stay in Section 10; finite Λ-sums are allowed and used, *no* RH statement is made; **the one open phase-2 object stays open and typed open**: R1 — an m-free unconditional certificate that the two rows of $\hat{A}$ become collinear at rate $h^{-3+\varepsilon}$, a joint near-degeneracy of the three explicit linear Λ-sums against the archimedean block; R2/R3 booked (T169), R4 off the path (T170-TH6), nothing else open in phase 2; the T171 verdict MAP-COMPLETE is a verdict about the *map*, not about R1, and stays a sandbox verdict; v558's identities stay as stated, and nothing is re-opened.

4.26 The frame/deficit identities (v560)

The gauge route of phase 2, closed as algebra (T172 `FRAME.BEYOND` + T173 `FRAME.RATE` + T174 `CANCEL.IDENTITY`, Section 10): the companion to v559, which certified the sixteen-link map on the small frame-A surface — this module certifies what T172/T173/T174 established *about* that map's frame dependence, and it makes no rate claim and no uniformity claim [`verification/v560_frame_deficit_identities.py`] (24 checks, ∼0.5 s; a small declared surface: a 4×3 (anchor, h) rectangle of gap-blind frame-B cells, prime-power anchors $n = 41 \ldots 557$ crossed with the declared ladder $h = (160, 226, 320)$, comb *complete* on every cell, positive definite low block on 12/12; one non-prime-power instance $n = 210$ and one truncated-comb stress instance $n = 1009$; the one arithmetic input $\kappa = 0.038821$ *verified* at every jump point of $\psi(x)/x$ on the table; the builder's signature is $(n_{\text{zone}}, \alpha, M)$ — no frame label is an argument of any observable).

- **The identity transfer + the sieve-horizon discriminator** [E] (T172): four representative chain links — the KMS/Schur pair $TL_P T^{\mathsf{T}} = \text{diag}(\mu^P)$ (3.2×10^{-16}; quadratic-form pair 5.2×10^{-15}), the lag-sum / correlation identity $v^{\mathsf{T}} A_h v = \sum_d c_d w_d$ (1.4×10^{-16} of the larger half; Abel 1826), the R4-free det collapse $1 - r_{12}^2 = \det \hat{A}_2/(\hat{a}_{11}\hat{a}_{22}) = \nu_1\nu_2/(\hat{a}_{11}\hat{a}_{22}) = \det G_2/(G_{11}G_{22})$ (at 0.020 of the conditioned bar $64\varepsilon/|1 - r_{12}^2|$ — the 2×2 determinant is a near-null number and the bar says so), and the additive split $\hat{A} = \hat{A}_{\text{arch}} + \hat{A}_{\text{comb}}$ (7.6×10^{-16}) — re-derived *exactly* on a gap-blind frame-B instance *and* a non-prime-power-anchor instance: the links use no property of the frame rule or the anchor arithmetic, which is why they transfer. The discriminator: the truncated-comb instance turns *indefinite* ($\lambda_{\min} = -9.6 \times 10^4$) against 12/12 positive definite complete-comb cells — indefiniteness sits at the declared sieve horizon, not at a frame or zone — and the identity links keep holding there, because none of them uses positivity.
- **The $q = 1 - s$ identity + the calibration** [E] (T173): $hD = \alpha$ per cell (8.9×10^{-16}) forces $\log D = \log \alpha - \log h$ pointwise, hence the OLS slopes obey $q = 1 - s$ to 2.9×10^{-14} (OLS is linear in its response — the identity lifts from cells to slopes): T171's h^{-3} is the $q = 1$ idealisation of a $\Theta(D^3)$ law *as an identity*; the gauge-degree account comes out as integers — $G2 = t/\lambda_{\max}(B_{LL})$ and the delivery $1 - r_{12}^2$ both at $(d_{\text{amp}}, d_\mu) = (0, 0)$, so $G2$ is the *unique* doubly gauge-invariant arithmetic member of T173's family; $G3 = \mu_1^P G2$ identically (2.2×10^{-16}, the offset-2 explained: $G2$ is μ_1^P-blind); and the $G6$ tautology $(D/\alpha)^3 h^3 = 1$ exactly.
- **The blindness theorems + the additive limit** [E] (T174): the amplitude $c \to \kappa c$ is a gauge of GAP, of $1 - r_{12}^2$ and of R over twelve decades (worst 0.026 of the per-cell conditioned bar; rebuilt from scratch 0.098); the scalar $\mu_1^P = 4\sin^2(\pi/(2h+1))$ — *the entire h^{-2} channel of the demand* — cancels exactly on both sides (0.001 of bar); the delivery is invariant under the full positive diagonal group while a non-scalar diagonal moves the demand by 0.23–1.40 (the asymmetry is the anatomy); and the additive limit: neither the arch term nor the comb term alone has a positive definite low block on a single cell (0/12 + 0/12 against 12/12 for the sum) — no factorisation $\hat{A} = (\text{frame}) \times (\text{arithmetic})$ exists, so T173's P6 is *not* promoted to a theorem: only its multiplicative half is.
- **The KMS shape band (unconditional) + the placement discriminator** [E] (T174/T172): with $x = \pi/(2h+1)$ and $K = 16$, $|\sqrt{\hat{S}_k/k^2} - 1| \le \varepsilon_{\text{cert}} = (Kx)^2/6$ on 12/12 and the k^2-swap

moves $\log \mathrm{GAP}$ by at most 0.312 of the certified band $2\log((1+\varepsilon)/(1-\varepsilon))$ (Rayleigh 1877; Schur 1917) while the delivery does not move at all; the value-multiset scramble destroys the positive low block on 24/24 declared draws and moves $|R|$ by a median factor 10^6 — the demand side is a statement about *where* the prime powers are. Every statement carries a mutation control that fails loudly.

- **Named limits as content.** Per instance on a small declared surface — a finite list with an explicit maximum; nothing uniform in the zone index or in D; **the frame-free deficit is a measurement, not a claim of this module**: T174's $+0.1111 \pm 0.0222$ (5.0σ, 151 anchor clusters, cluster-robust; 0.4σ from T173's $+0.155 \pm 0.102$) stays a sandbox MEASURED number in Section 10, as do all T172/T173/T174 h-exponents, the density anatomy and the ν/density collinearity; finite Λ-sums are allowed and used, *no* RH statement is made; **the one open phase-2 object stays open and typed open**: R1, now stated frame-free; the residual per-anchor scatter ($\chi^2/\mathrm{dof} = 3.02$) and the sparse corner were the sandbox items tested in T175 (Section 10.65: the scatter is substantially an error-bar artefact, the sparse corner is not an assembly artefact); the T172/T173/T174 verdicts PARTIALLY-PORTABLE / DEFICIT-VARIES / PARTIAL-CANCEL are sandbox verdicts; v559's chain stays as stated, and nothing is re-opened.

4.27 The dense-limit identities (v562)

The exact cores of the phase-2 endgame (T176 `DENSE.LIMIT`, Section 10.66) — the probe that raised the sieve twentyfold, extended the deficit(dens) curve to a densest bin consistent with zero, and closed the phase-2 measurement programme as planned. This module carries the identities, not the measurement: everything is recomputed per instance on a small declared surface, and the big sieve is *not* rebuilt — the identities are sieve-size-free, which is their point [`verification/v562_dense_limit_identities.py`] (16 checks, ∼0.5 s, deterministic; 18 prime-power anchors in $[40, 565]$ crossed with the declared $2^{1/4}$ ladder $h = (128, 152, 181, 215, 256, 304) = 108$ cells, comb complete, positive definite on 108/108; the one arithmetic input $\kappa = 0.038821$ verified at every jump point of $\psi(x)/x$ on the table).

- **R1 closed: the Feynman–Hellmann identity** [E] (T176 P2, the strongest): $d\log R/d\delta = v^{\mathsf{T}} d\tilde{A} v/\lambda_{\min} - w^{\mathsf{T}} d\tilde{A} w/\lambda_{\max} - d\mathrm{DEL}/\mathrm{DEL}$, with $d\tilde{A}$ the odd-Toeplitz image of the *exact* lag derivative $dc/d\delta$ (the assembly moves $0.5\mu_p$ per unit phase between two adjacent lag cells and nothing else; piecewise linearity checked at 7.0×10^{-12} of the increment), reproduces the full nonlinear rebuild on 6/6 declared instances at the minimum of the declared step ladder (2.3×10^{-7} at $\delta = 10^{-6}$; T176's own optimum 3.37×10^{-6}) *with the U-shape exhibited* — round-off limited below, truncation limited above, both $\geq 3\times$ the minimum: a wrong formula has a floor, not a minimum (mutations — swapped eigenvectors, dropped $d\mathrm{DEL}/\mathrm{DEL}$ — miss by a factor 4×10^1 while the true form sits at 5×10^{-8}). T175's measured intervention slope (879) is retired as a separate claim, and the decomposition corrects the expected story: the near-degeneracy channel $d\mathrm{DEL}/\mathrm{DEL}$ carries the largest share on every instance (0.54–0.98; T176: 84% median).
- **The phase pole** [E] (T176 P3): on the densest declared instances, positive definiteness survives only 2% of the phase period (declared 64-point grid, bisection to 40 levels), the loss point satisfies $\delta_{\mathrm{PD}} \cdot |d\log R/d\delta| = 7.2$ median — inside the declared $O(1)$ band, a pole at the natural scale — and the two lowest modes of the normalised low block sit within 0.02–0.17 of each other already at $\delta = 0$: the intervention walks along a crossing, it does not create one. The ∼300× harmonic anomaly of T175 is *retired*: a first-harmonic regression over the period estimates a Fourier coefficient of a function with a pole in the interval.
- **Sieve consistency at construction level** [E] (T176 P4): a bigger sieve adds atoms above the old cap and nothing else, so on anchors whose comb is complete under the old cap the lag vectors agree *bit for bit*, R exactly, and the cluster-robust bin deficit identically; the must-break moves exactly where it must (the deepest declared anchor past the old cap). T176's cell-by-cell audit

— all six T175 bins at 0.0σ under the 20.8× sieve — is this construction fact plus statistics, and it is what lets any later probe raise the cap without re-deriving the chain.

- **The estimator + the anti-promotion as a check** [E] (T176 P1): the cluster-robust bin statistic is deterministic and exactly reproducible under a permuted cell order — and it is *ladder-dependent*, exhibited per instance (fine $2^{1/4}$ vs coarse $\sqrt{2}$ ladder shift 0.036 on the module surface; T176 saw up to 0.20): **any ledger row quoting a bin deficit must quote the rung ladder with it**, and `PRIME.DENSE.LIMIT.01` does exactly that. The occupancy-preserving phase scramble destroys positive definiteness on 6/6 draws (T176: 194/194) — a one-sided destruction control, never a calibrated null.
- **Named limits as content.** T176's SITS-AT-ZERO verdict — pooled $+0.0496 \pm 0.0372$ over the two new bins, plateau window 0.074 at 2σ, a 3.6× narrowing — stays a sandbox MEASURED number on T176's 2.5×10^7 surface *with its rung ladder* (the ladder caveat is part of the quote); the limit stays open and typed open (true zero at finite density, power-law approach and low plateau remain indistinguishable under any resource-reachable sieve — one more decade of window would cost `ATOM_MAX` $\sim 2.5 \times 10^9$); the phase-2 closure is a closure of the *measurement programme*, not of the question; no RH statement anywhere, and the open quantifier at link 16 is untouched.

4.28 The Paper-II readout closure (v563)

The check closure of the classification paper (Paper II, `parity_toeplitz_classification`): every number that paper quoted from an exploration probe without a recomputing module is recomputed on the identical declared surface, and the one mechanism it declared as a caveat without exhibiting it — the rung-ladder shift of the binned exponent — is exhibited as a *structure fact* [`verification/v563_paper2_readouts.py`] (24 checks, ∼0.5 s; pure readout/identity checks, no new claim, no rate, no uniformity).

- **The reference window** [E]: the declared T170 frame-A scan, bit for bit, lands on $h = 540$ (node cap $X = 2.465 \times 10^4 = 157^2$), and the four quoted numbers are recomputed — $\det B = 6.18$, $-D(B,S) = -198.8$, $\det S = 192.6$, $\det \hat{A} = 4.22 \times 10^{-3}$: a 4.7-order cancellation between three pieces of size ∼200, a further factor 383 above the absolute target — with the split and the polarisation identity at round-off (mutations: the wrong cross-coefficient misses by 2.7×10^3; the position scramble moves $|1 - r_{12}^2|$ by 4×10^5 while every identity keeps holding).
- **The ℓ^1-to-signed ratio** [E]: 40.6× on the declared 900-node subsample (the price of any triangle-inequality route); the kernel double sum equals $\det S$ at round-off (Cauchy–Binet), the kernel diagonal carries 5.9%.
- **The tightness range where cheap** [E]: the declared Fejér trial family reproduces the quoted envelope $\mu_1^P\mathrm{TV} \in [5.00, 23.20]$ with both ends attained on 22 windows; the floor ≥ 1 holds on all 1122 trial vectors, and the inadmissible vector undercuts by exactly λ^2 (degree-2 homogeneity, 1.9×10^{-16}).
- **The rung-ladder shift is anchor selection** [E] (the Paper-II caveat, exhibited): the fine $2^{1/4}$ and coarse $\sqrt{2}$ ladders select *different anchor sets* in the same declared bin (13 vs 12; an anchor needs three in-bin rungs), and removing the extra anchors reproduces the whole shift (+0.0490 anchor part against −0.0135 rung part at fixed anchors); curvature transport with the paper's measured $a_2 = -0.0842$ moves the shift by only −0.0111 (a wrong curvature is 11.9× louder): the shift lives in *which* anchors enter the bin, not in how the rungs sample the curvature.
- **Named limits as content.** Readouts of measured sandbox quantities stay typed MEASURED with their ladders — the v562 anti-promotion applies to every number here, this module included; no bound, rate or uniformity statement anywhere.

4.29 The relative pencil one-mode structure (v569)

The same declared surface, read through a new exact lens [verification/v569_lambda_pencil_onemode.py] (6 checks, ∼2 s; construction base: the T170 frame-A scan of v563, imported read-only, bit for bit). The rank-3 polarisation of Paper II is exactly (2+1)-Lorentz geometry, and the low 2×2 block carries the relative pencil $Z = B^{-1}S$ with the exact identities $\det(B - S) = \det B\,(1 - \lambda_1)(1 - \lambda_2)$, $D(B, S) = \det B\,(\lambda_1 + \lambda_2)$, $\det S = \det B\,\lambda_1\lambda_2$ (proved symbolically in the parabolic self-code module; here executed on the real surface at worst residual 2×10^{-14} across all 70 windows).

- **The labels, corrected honestly** (MEASURED): the *locking* mode is λ_2, hugging 1 (median 1.000021, range $[-0.85, 1.0005]$), while the spectator λ_1 is large — $[4.53, 180.64]$, every window above 1. (The review that motivated the question guessed the separated mode *below* 1; on the real surface it sits far above.)
- **The one-mode answer** (MEASURED): $\min|1 - \lambda_1| = 3.53$ across all 70 windows, so $\det(I - Z)/(1 - \lambda_2) = 1 - \lambda_1$ is uniformly bounded away from zero on the reachable surface and the two-dimensional cancellation reduces to the *single scalar* $1 - \lambda_2$: verdict ONE-MODE.
- **The reference window [E]**: on $h = 540$ the locking factor is $1 - \lambda_2 = -2.27 \times 10^{-5}$ and the pencil reproduces $\det \hat{A} = 4.22 \times 10^{-3}$ — the 4.7-order cancellation of Paper II *is* the single factor $1 - \lambda_2$.
- **Must-break [E]**: the position scramble at the same masses explodes $|1 - \lambda_2|$ by $\times 1.9 \times 10^6$ and destroys the pencil geometry (both eigenvalues negative) while every identity keeps holding: the locking is arithmetic placement, not an identity artefact.
- **Named limits as content.** Declared finite surface only — Problem 7.1 of Paper II (the uniform quantifier) is *untouched*: its structure is typed (one scalar mode), not bounded; whether λ_1 stays separated as h grows is a new named open question, strictly weaker than Problem 7.1; MEASURED stays MEASURED, no rate, no bound, no RH statement.

4.30 The separation floor, certified (v570)

The open question v569 left behind, reduced and certified on the surface [verification/v570_separation_floor.py] (8 checks, ∼13 s; construction base: the same T170 frame-A scan, read-only). Two one-line theorems — $P = \det Z = \det S/\det B > 0$ and $T = \operatorname{tr} Z > 0$ give $\lambda_1 \geq \sqrt{P}$ (since $\lambda_1^2 - P = \lambda_1(\lambda_1 - \lambda_2)$), and $P \leq 0$ gives $\lambda_1 = T - \lambda_2 \geq T$ — turn the spectator separation into a *closed per-window floor*.

- **The certification [E]**: on all 70 declared windows the floor is sound and clears the margin, $\min F = 2.130 \geq 1.05$ — the v569 separation is no longer only a measured eigenvalue fact, it follows from $\det S/\det B$ and $\operatorname{tr} Z$ alone (conservative against the measured $\min \lambda_1 = 4.534$, as a floor should be).
- **The dominance** (MEASURED): $P = \det S/\det B$ lies in $[4.5, 120.6]$ (median 30.1) and *grows* with depth (log-log slope +0.900 in h, with its surface ladder); the exact link $1 - T + P = \det(B - S)/\det B$ makes trace and determinant dominance one question.
- **The cheap route is dead [E]** (honest negative): $\det S$ is itself a pair-level cancellation object — Cauchy–Binet exact, negative pair mass up to $13.7 \times \det S$ (median 8.0×), best single-pair witness only 0.13–$0.34 \times \det B$ against the ≥ 1.10 it would need: no two-atom witness certifies the dominance.
- **Must-break [E]**: the position scramble flips the trace sign ($T = -78.28$ against $+32.15$ real) — the $T > 0$ gate detects the destroyed geometry loudly; the dominance is arithmetic placement.
- **Named limits as content.** The h-uniform dominance $\det S \geq (1 + c)^2 \det B$ is the *same* open question v569 named, now in sharper closed-scalar form — typed as pair-level size-vs-cancellation (an 8:9 cancellation, not a 4.7-order one, but not free); Problem 7.1 untouched; no rate, no bound, no RH statement.

4.31 The pair-band anatomy: the dominance is long-range (v573)

Where the certified dominance lives [verification/v573_pair_band_structure.py] (6 checks, ∼6 s; construction base: the same declared scan, read-only, with a declared kernel subset — every twelfth window with ≤ 16000 atoms plus the $h = 540$ reference).

- **The near region drags** (MEASURED): *every* band below 16 lag cells *and* the diagonal of the Cauchy–Binet pair kernel are negative on every selected window — a near-diagonal mass budget aims at the wrong sign.
- **The far band carries everything** (MEASURED): the net far-band share of $\det S$ sits in the stable band [1.55, 1.91] while its gross positive mass grows $4.1 \to 13.0 \times \det S$: the dominance is *long-range coherence*, not local mass.
- **The sharper anatomy** (MEASURED): the excess is *scale-locked* — pairs $\geq h/2$ apart carry a large growing net while the bands 16..256 stay negative — and *weight-concentrated*: the top-10%-weight pairs alone net 0.85–$1.00 \times \det S$ on every selected window, essentially $\det S$ itself (everything below the top decile cancels to ∼0); the lag-grid phase plays no role (flat profile across the declared bins, spread shrinking with h — the phase-mechanism candidate is dead).
- **Must-break, both directions** [E]: the position scramble *explodes* $\det S$ by $\sim \times 49$ — the real placement holds $\det S$ fifty-fold *below* random while organising its far-band coherence — and collapses the far share below 1.
- **Named limits as content.** A *typing* of the v570 dominance question: its certificate must control *heavy-pair coherence at window-scale separation*; declared surface and subset only, MEASURED with ladders, no bound, no rate, no RH statement.

4.32 The Chebyshev–Loewner edge structure (v576)

The second external review's Toeplitz strand, audited and exact [verification/v576_cheb_loewner_edge.py] (9 checks, ∼6 s; the corpus definitions read-only).

- **The cross formula** [E]: the polarized lag weight of two parity modes is closed, $w_d^{(ij)} = \frac{2}{N}\,\frac{\sin\omega_j \sin(d\omega_i) - \sin\omega_i \sin(d\omega_j)}{\cos\omega_i - \cos\omega_j}$ — exact symbolically at $N = 7, 9, 11$ (all pairs, all lags) and at 10^{-11} on the surface: the whole $K = 2$ read block is analytically closed.
- **The edge theorem** [E] (exact, not asymptotic): $\sin((N-s)\omega) = -\sin(s\omega)$ turns the edge read into a scaled *Loewner matrix* of U_{s-1}: rank $\leq s - 1$; the first edge read ($s = 2$) is rank *one* by theorem (sv-ratio 10^{-16} on the real objects).
- **The Pythagorean null bridge** [E]/[C] (typed compression): the leading modes-$(1, 2)$ profile $[[1, 2], [2, 4]]$ has Lorentz image $(5, -3, 4)$ — null exactly, the same triple as $(g_{\mathrm{car}}, N_{\mathrm{fam}}, |\mu_4|)$ via Euclid's $(i^2{+}j^2,\ i^2{-}j^2,\ 2ij)$; one quadratic generator fed by the anchor values $(1, 2)$ and the parity modes $(1, 2)$: a bridge, *not* new independent evidence.
- **The defect coefficient** [E] (symbolic series): $\det W^{(s)}/(W_{11}W_{22}) = -\frac{3\pi^4}{175}(s^2{-}4)(11s^2{+}6)N^{-4} + O(N^{-6})$ — vanishing at the rank-one read, and locally one order *better* than the h^{-3} demand of Problem 7.1: the open hardness is the weighted summation across scales, not any single edge read.
- **The pair-kernel polynomial** [E]: $P(s, z) = 5s^4 - 21s^2z^2 + 19s^2 + 5z^4 + 19z^2 + 24$ with $P(2, 3) = 0$ exact and sign transition $s/z \approx 0.503/1.987$ — near scales couple negative, far scales positive: the analytic local prototype of the v573 measured pattern; the macro kernels $W^{\infty}(\sigma)$ are verified at 5%.

- **Named limits as content.** The three-regime split refines Problem 7.1 without touching its quantifier: edge *closed* at N^{-4}, macro (the weighted two-scale summation) *open*, transition *open*; no uniformity, no RH statement.

4.33 The null-ray locking census (v577)

The review's Priority-2 test, executed verbatim [verification/v577_nullray_census.py] (5 checks, ∼2 s).

- **The alignment is real** (MEASURED): the pencil locking mode aligns with the exact null ray $(2, -1)$ at 23.5–33.8° (raw; median 28.1°) on all 70 windows — everywhere below the 45° isotropic mean.
- **Honest**: the decline is real but *slow* ($h^{-0.10}$ raw, $h^{-0.12}$ B-metric) — the review's clean success bar (uniform convergence to zero) is *not* met on this surface.
- **Must-break** [E]: the position scramble destroys the alignment (28.1 → 49.4° raw): arithmetic structure; the scramble-blind kill is excluded.
- **Verdict**: a real *partial* carrier — consistent with the three-regime split (the edge regime, whose null ray this is, closed; the macro regime bends the direction); the named follow-up is the macro-regime direction from $K_\infty(\sigma, \tau)$, not more angle fits.

4.34 The two-scale kernel signs (v579)

Regime B gets its explicit object [verification/v579_macro_kernel_signs.py] (6 checks, <1 s).

- **The sign map is exact** [E]: $\operatorname{sign} K_\infty(\sigma, \tau)$ matches the sign of the real two-lag kernel $D(W(d_1), W(d_2))$ on *every* resolved grid cell at $h = 300$ (332/332; 330/332 at $h = 150$, two finite-size cells).
- **Same-scale coupling is negative, analytically** [E]: the diagonal $\det W^\infty(\sigma)$ is negative on ∼72% of $(0, 1)$ (positive islands only at $\sigma \geq 0.66$) — the macro origin of the v573 near-diagonal drag; the near/far asymmetry is kernel geometry (mean −0.18 near-diagonal vs −0.01 far).
- **Must-break** [E]: the wrong polarisation (cross term added instead of subtracted) breaks the sign match loudly.
- **Named limits as content.** The *geometric* half of Regime B closes; the *arithmetic* half — the Λ-weighted occupation of the (σ, τ) cells by actual prime pairs, the summation that must beat the 8:9 cancellation — is exactly the remaining open content; no uniformity, no RH statement.

4.35 The arithmetic occupation map (v580)

The arithmetic half gets its measured object [verification/v580_occupation_map.py] (5 checks, ∼3 s).

- **The lag-side identity is exact** [E]: $\det S = \frac{1}{2} \sum_{d_1, d_2} c_{\mathrm{at}}(d_1) c_{\mathrm{at}}(d_2)\, D(W(d_1), W(d_2))$ — the atom-lag double sum against the polarized weight kernel (from the T163 correlation theorem), verified at 10^{-9} with the binned map resumming exactly.
- **The signs follow the kernel** (MEASURED, central): *every* significant occupation cell sign-matches $K_\infty(\sigma_1, \sigma_2)$ on every tested window (50/50, 67/67, 65/65, 65/65): the primes *populate* the geometric sign map — the open Regime-B content is the *amounts*, not the signs.
- **Structure** (MEASURED): mid-range diagonal block net negative, cross-scale cells net positive — the v573 anatomy on the lag side; scramble control: the identity resums exactly while $\det S$ explodes ×49.

4.36 The transport census: the multilevel lemma dies honestly (v581)

The review's Priority-4 lemma, measured [verification/v581_locking_transport.py] (4 checks, ∼2 s).

- **The census [E]**: 40 doubling pairs (h' within 15% of $2h$) on the declared surface; mode coordinates are scale-free, so the comparison needs no prolongation operator at the 2×2 level.
- **Small but non-decaying** (MEASURED): the per-octave drift of the locking direction is 0.3–6.7° (median 2.35°) — but it *grows* ($h^{+0.36}$) against the proposed bar $\sin \leq Ch^{-1-\delta}$: the increments are not summable, and the review's own kill criterion fires — the route is ended without romantic extension.
- **Typing**: consistent with v577 and v579 — the macro regime re-bends the direction per window; a scalar-defect transport along a *fixed* direction is not available.
- **Round-22 sharpening (2026-08-06)**: in the spinor coordinates of [verification/v807_lorentz_nullselector.py] the apparent h-oscillation of the locking direction is revealed as α-jitter (window-width order, not height order): the spinor slope is *strictly monotone* in α (66/66 pairs, Kendall 1.000); the kill above stands — the h-order bars remain failed — but the drift is a coordinate artifact, not noise (see sec:k5-edge-machine).

4.37 The density-dominance reduction (v582)

The quantitative occupation question, answered at its first level [verification/v582_density_dominance.py] (5 checks, ∼2 s).

- **Density-driven** (MEASURED, central): the smoothed comb (moving average; declared windows 2–20% of the lag range) reproduces $\det S$ through the exact lag-side identity on every regular window — 0.998–0.999 at 2%, 0.987–0.993 at 5%, degrading smoothly to 0.81–0.89 at 20%: the dominance is carried by the deterministic density *profile*; the prime fluctuations contribute ∼1%.
- **The decisive control** (MEASURED): on the scrambled window the smooth model captures only 0.70 — the real comb is anomalously *smooth* at these scales; the arithmetic-placement content (v573: $\det S$ suppressed ∼50× below random) is the profile, not fine noise.
- **Honest boundary**: the anomalous window $h = 1292$ (the $P \leq 0$ window of v570) deviates (1.24) — at the sieve horizon the fluctuations are not negligible.
- **The relocation.** The v570 h-uniform dominance question reduces — up to a measured ∼1% fluctuation correction needing only a crude bound — to a positivity/growth statement about the *smooth density functional*: PNT-level regularity, not prime fluctuations; the named next step is a closed-form bound on that functional.

4.38 The prime-free closed form (v583)

The named next step of v582, executed [verification/v583_pnt_model.py] (8 checks, ∼4 s).

- **The two-term law** (MEASURED): on the atom table, $\text{mass}(u) = 4e^{u/2} - 2(\zeta'/\zeta)(1/2)$ — the pole term plus the $s = \frac{1}{2}$ constant, *no zeros* — holds with an oscillation band < 1.5.
- **Prime-free closed form** (MEASURED, central): the grid model with density $2e^{u/2}$ and the *parameter-free* theoretical constant, run through the untouched v563 pipeline, reproduces $\det S$ on all 69 regular windows to 1.016–1.143 (mean 1.039), the S entries to 0.995–1.001, and the dominance growth with correlation 0.998.
- **Honesty**: the determinant is a fine cancellation — constant sensitivity ∼0.38/unit; the 2–14% residual band is the zero-oscillation of the constant term.

- **The boundary typed**: at $h = 1292$ the prime-free model gives $+1844$ where reality gives -1465 — the sign flip at the sieve horizon *is* genuine prime-fluctuation content; the $P \leq 0$ anomaly of v570 is now typed.
- **The relocation, final form.** The h-uniform dominance splits into (a) a deterministic *prime-free* two-integral inequality $\det S_{\mathrm{PNT}}(h) \geq (1+c)^2 \det B(h)$ and (b) a zero-oscillation bound entering through the constant — explicit-formula territory, named honestly.

4.39 The two-layer locking split (v585)

The complementary side of v583 [verification/v585_pnt_locking_split.py] (6 checks, ∼3 s).

- **The locking is arithmetic** (MEASURED, central): the same prime-free model that reproduces $\det S$ to 1.02–1.14 *misses* the Paper-II defect $\delta = \det(B-S)/((B-S)_{11}(B-S)_{22})$ by 2–4 orders of magnitude (median ratio 1350, growing $\sim h^{1.08}$): the dominance is density, the *depth* of the lock is the primes.
- **Geometry vs depth**: the prime-free pencil also locks its eigenvalue near 1 (a coarse lock at the 10^{-2}–10^{-4} level) — the v569 one-mode geometry needs no primes; only the fine depth does.
- **The two decay layers** (MEASURED): density $\delta_{\mathrm{PNT}} \sim h^{-1.43}$ (always positive, log-scatter 0.70) $\times$ arithmetic $\sim h^{-1.08}$ extra = real $\sim h^{-2.51}$; the real defect goes *negative* exactly at the deep windows $h = 1219, 1445$ (plus the typed 1292 anomaly), the deterministic model never does.
- **The budget relocation.** Problem 7.1 asks $\delta \leq Ch^{-3+\varepsilon}$: the density layer supplies ~ -1.4 deterministically (classical analysis of the closed model), the arithmetic layer the remaining ~ -1.1 (zero-oscillation territory).

4.40 The density-fixed locking direction (v586)

The direction side of the split [verification/v586_pnt_lock_direction.py] (7 checks, ∼3 s).

- **Direction-density-fixed** (MEASURED, central): the real locking eigenvector of $B^{-1}S$ agrees with the prime-free model's to max 0.331°, median 0.030°, decaying $\sim h^{-1.16}$ on all 69 regular windows — while the depth differs by 1350× (v585): *the primes are a pure depth amplifier along a density-fixed direction.*
- **The witness**: the deterministic direction certifies the real defect — the quadratic form of the real $B - S$ along it reproduces the minimal eigenvalue within $[0.98, 2.9]$ (median 1.42): the arithmetic content of Problem 7.1 is one quadratic form along one deterministic vector.
- **The v577 puzzle resolved**: the deterministic direction itself drifts (v_2/v_1 from -1.73 to -1.19) — the slow angle decay against the fixed null ray $(2,-1)$ was a comparator artifact; the drift limit under $1/\log h$ is -0.551 (R^2 0.67), consistent with -0.5, *not settled. Round-22 update (2026-08-06): in spinor coordinates the drift is strictly monotone in the window width α (66/66, Kendall 1.000) — ordered approach, not oscillation; the limit identification stays open (the approach $2 - r \sim \alpha^{-0.23}$ is glacial) ([verification/v807_lorentz_nullselector.py]).*
- **Must-break**: scrambled combs misalign by 6–25° (vs 0.026° real).

4.41 The exact diagonal weight formula (v587)

The diagonal companion of v576 [verification/v587_w_closed_form.py] (6 checks, ∼2 s).

- **The new exact formula**: the corpus diagonal lag weight is the *single* closed expression $W_{kk}(d) = \frac{2}{N}\left[(N-1-d)\cos(\omega_k d) + \sin(\omega_k(d+1))/\sin\omega_k\right]$ for $d \geq 1$ ($W(0) = 1$) — machine-exact (1.9×10^{-15}) for $h = 7$–1000, modes $k = 1, 2, 3$, all lags.
- **Structure**: the building blocks are closed Dirichlet sums; the apparent piecewise break at $d = m$ *cancels* on assembly; the reflection-edge suppression is an exact two-term cancellation.

- **The payoff**: the exact cell-sum $S_{ij} = \sum_d \lambda(\text{cell})\, W_{ij}(d)$ reproduces the v583 prime-free model — entries to 1.0011–1.0036, determinant to 0.9999–1.0003: the S-side of the deterministic layer is closed-form.
- **The macro dictionary**: the limit is $g_k(\sigma) = 2(1-\sigma)\cos(2\pi k\sigma) + \sin(2\pi k\sigma)/(\pi k)$, and the v579 kernel is its edge-anchored version identically ($W^{\infty}(\sigma) = g(1-\sigma)$).

4.42 The closed deterministic defect (v588)

Both sides of the defect are one function [verification/v588_closed_delta.py] (6 checks, ∼5 s).

- **One function**: $G_{ij}(\beta) = \sum_{d\geq 1} e^{-\beta d} W_{ij}(d)$ — exact finite geometric-trig series over the v587 formulas, machine-exact.
- **Pole vs the trivial zeros**: the S side is the pole exponent ($\beta = -D/2$); the B side is the *trivial-zero ladder* — $c_{\text{ar}}(d) = -Df(dD)$ with $f(w) = e^{-w/2}/(1-e^{-2w}) = \sum_n e^{-(2n+1/2)w}$ (verified 1.000002; near field $d \leq 32$ as explicit quadrature constants).
- **The census** (MEASURED): closed B to 2×10^{-5} (det to 6×10^{-5}) and the closed defect $\tilde{\delta}$ to 0.9999–1.0023 against the v585 deterministic layer on *all* 70 windows; census decay $h^{-1.40}$ (the v585 slope -1.43).
- **The picture**: the deterministic layer of Problem 7.1 is the explicit formula's skeleton in closed form — pole against Γ-factor; the named remaining step for a theorem is elementary asymptotics of G and the ladder; the arithmetic layer stays with the primes.

4.43 The zero-comb identification (v589)

The arithmetic layer gets its object [verification/v589_zero_comb.py] (4 checks, ∼16 s).

- **The identification**: the residual mass oscillation of the atom table *is* the explicit-formula zero comb $-4\sum_{\gamma>0} \sin(\gamma u)/\gamma$ — correlation 0.82 (slope 1.28, the truncation deficit) with the first 200 damped zeros; pointwise 1.00 at $u = 3$.
- **Partial depth recovery** (MEASURED): at the smallest window ($h = 184$, $\pi/D = 208$) the comb improves the lock ratio 176 → 31 (a factor 5.7), non-monotone in the truncation — honest; the deep windows stay out of reach ($h = 540$: 2646×).
- **The calibration**: the zero-oscillation bound is a quantified target — control $\sum_\gamma \sin(\gamma u)/\gamma$ at window resolution π/D with amplitude accuracy set by the lock depth; no bound claimed.

4.44 The rank-one pole term (v591)

The direction law derived [verification/v591_pole_rank_one.py] (5 checks, ∼2 s).

- **The identity** (EXACT): the Laplace denominators factor completely — $D_{11} = (a^2+4\pi^2)^2$, $D_{22} = (a^2+16\pi^2)^2$, $D_{12} = (a^2+4\pi^2)(a^2+16\pi^2)$ — and the pole part is exactly separable rank-one, $S^{\text{pole}} = -32\pi^2 a e^a\, g g^T$ with $g_k = k/(a^2+4\pi^2k^2)$: $\det S^{\text{pole}} \equiv 0$.
- **The direction law** (EXACT): $v_2/v_1 = -(a^2+16\pi^2)/(2(a^2+4\pi^2))$, limit exactly $-\frac{1}{2}$ (the Pythagorean null ray), approach $-6\pi^2/(a^2+4\pi^2)$.
- **The match** (MEASURED): the law reproduces the measured locking directions to 0.04–1.4% across the ladder ($h = 1445$: -1.2248 vs -1.2253); via v586 it describes the *real* corpus locking direction.
- **Resolutions**: the v577 null-ray conjecture is *derived*; the v586 drift is the explicit closed approach; the density layer is the correction theory of the rank-one pole term.

4.45 The continuum determinant law (v592)

The correction theory begins [verification/v592_continuum_det_law.py] (6 checks, ∼2 s).

- **The determinant law** (EXACT): $\det S(a)$ is closed; the e^{2a} coefficient is identically zero (the rank-one cancellation) and the exact leading law is $\det S \sim 640\pi^2 e^a/a^3$ — one full exponential order suppressed, derived not measured.
- **The dominance growth, derived** (MEASURED): the real $P = \det S/\det B$ tracks the closed formula at log-log correlation 0.9985 — the closed-form origin of the v570 growth; the finite-N gap is typed ($0.19 \to 0.50$, the named remaining expansion layer).
- **The entry certificate** (conditional): the arithmetic deviation of every tested entry is bounded by $\sup|\mathrm{Osc}| \times \mathrm{TV}$ with margin 47.6× — rigorous given the measured oscillation sup; the unconditional sup *is* the zero-oscillation theorem (v589).

4.46 The cutoff completion (v593)

The density layer closes end-to-end [verification/v593_cutoff_completion.py] (4 checks, ∼4 s). The finite-N gap of the continuum determinant (v592: 0.19–0.50) was the integration *cutoff*: with the σ_0-corrected closed integrals (still elementary) the entries match the model to 0.7–1% and the determinant to 0.2–1%; the residual is second-order and shrinks monotonically (the $O(1/N)$ kernel terms cancel exactly in the v587 formula).

4.47 The unconditional entry certificate (v594)

The first unconditional statement [verification/v594_unconditional_cert.py] (5 checks, ∼2 s). Partial summation (machine-verified as an identity on the atom table) plus the published bound $|\psi(x)-x| < 0.94\sqrt{x}$ ($11 < x \le 10^{19}$, Büthe 2018, via verified zeros; the surface sits sixteen orders inside the range) give the unconditional sup $|\mathrm{Osc}(u)| \le 17.4$ on the declared surface (measured: 1.474); the certificate $|S^{\mathrm{real}} - S^{\mathrm{model}}| \le \sup \times \mathrm{TV}$ holds with margin 565× on every tested window — the entry-level arithmetic layer needs no conjecture and no runtime zero input. The determinant-level lock stays with the zero-oscillation theorem.

4.48 The mapping completion (v595)

Identity-level closed forms [verification/v595_mapping_completion.py] (4 checks, ∼4 s). The v593 residual was the σ *mapping*: the lag variable is $\sigma = d/N$, so the continuum cell sum carries $a_{\mathrm{eff}} = ND/2$ with limits $[u_0/(ND),\ 2\alpha/(ND)]$ ($ND = 2\alpha + O(D)$, which is why the α-form carried a first-order residual). With the exact dictionary the closed entries *and* determinant match the model to 1.00000–1.00017 across the ladder (1.00000 at $h = 1445$); the kernel correction enters at 9×10^{-6} (the genuine $O(1/N^2)$ piece). Honest amendment: v593's attribution of its residual to kernel corrections is superseded — the numbers stand, the interpretation was wrong.

4.49 The 1D lock projection (v596)

The near-identity along the closed direction [verification/v596_lock_projection.py] (6 checks, ∼35 s). Projecting the defect onto the *closed* v591 direction (parameter-free), the zero-side functional cancels the closed density value in a near-identity: $|q_{\mathrm{real}}/q_{\mathrm{model}}|$ has median 7.7×10^{-4}, decaying $\sim h^{-1.01}$; the coupling spectrum is *collective* (max single-zero share 5.9%, 56 zeros for half the mass) — equidistribution-flavored, killing single-resonance explanations; scrambled combs break the identity by six orders of magnitude. After the closed density layer, the arithmetic content of Problem 7.1 is *one approximate identity per window* with explicit couplings.

Part III
The localization program and the certification arc (T79–T125)

From here through Part V the document is a research diary: preregistered sandbox probes, chronological, none of them load-bearing (promoted cores are cited where they landed in Part II). This part contains the first three acts: the localization of the residual difficulty into one coupled inequality, I5, equivalent by identity to Weil positivity (Section 5); the reorganization of that inequality as a per-zone handover induction (Section 6); and the certification arc T102–T125, which converts the ingredients of the induction into identities and certificates until the remaining hardness sits in named, checkable statements (Section 7).

5 The localization program (sandbox)

Provenance

Everything from here to Section 13 is [sandbox]: exploratory probes in `experiments/tfpt-discovery/`, no ledger row, no marker, no authority. The findings are reported because the *shape* of the residual is the useful output, not because any of them is established.

5.1 I5: the ledger closes and one inequality is left (T79)

The transport wall — the thing that had blocked the line since T20 — was fully decomposed in T79 (`transport_ledger_probe.py`, 22/22). The key observation is that the odd-prime side of the Weil functional agrees *exactly* with the certified plus combination (relative 2.6×10^{-16}): the prime side was never the wall. What remains after the ledger closes is a minimal transport set of two exact identities (proved), two classically-provable bounds, and exactly one coupled inequality:

> **(I5)** $\quad Q_{\mathrm{cert}}(h) + \Delta_2(h) + A_{\mathrm{fam}}(h) - A_{\mathrm{shift}}(h) \;\geq\; 0$ for all autocorrelations h.

I5 is the prime $\leftrightarrow$ archimedean coupling. By the closed ledger it is, as an identity, equivalent to Weil positivity and hence to RH. The diary is explicit about what that means: *the built-in safeguard fired.* RH does not follow from finite bookkeeping; the hidden infinite step is exactly I5 — and the value of T79 is that the step was *found and named* rather than concealed. This is an equivalence typing, not progress.

T82 then changed I5's *type* (`heat_arch_probe.py`, 22/22, ARCH-INTERNAL): Δ_{arch} is exactly the internal Γ difference $A_{\mathrm{fam}} - A_{\mathrm{shift}}$ via Legendre duplication, verified on 18/18 battery rows at max relative 6.6×10^{-15}. I5 stopped being a coupling between two worlds and became a *self-consistency statement about one heat family.* Again: a type change is not a proof.

T83 (`fe_symmetrization_probe.py`, 27/27, INVARIANT-NULL) then asked the obvious question of the self-duality picture (Section 3): can the functional equation itself do the work? It cannot, and the way it fails is informative. The ζ reflection $J_{1/2}$ acts as the identity on autocorrelations — the symmetry is already absorbed in evenness, and $Q_{\mathrm{Weil}}(Jh) = Q_{\mathrm{Weil}}(h)$ holds floating-point-exactly on 15 test functions — and the family reflection J_1 moves nothing in the value geography (5/24 overlap invariant, λ^* covariant-exact) while pushing $m_+ h$ out of the Weil cone in closed form, with $\mathrm{Fix}(J_1) \cap \{\text{Weil cone}\} = \{0\}$ exactly. The structural finding underneath the null is the one quoted in the big picture: the product of the two reflections is the unit-line shift, i.e. the transport operator, and the wall is transversal to the functional equation rather than positioned by it. Symmetrisation is therefore not a route; the same mirror returns, as an obstruction, in T106.

5.2 The Connes dictionary (T87) and where positivity is anchored

T87 (`i5_connes_dictionary_probe.py`, 22/22, DICTIONARY-EXACT-CORE) identified the I5 one-family form as a compiler instance of the semi-local Connes structure. Four exact matches: the two-shell orbit route equals Connes' expression at relative 6.3×10^{-16}; the orbit kernel of Connes–Consani (2021) is literally the compiler's Poisson ladder at $\rho = e^u$ (`sympy` plus 1.7×10^{-50}); h_+ is by definition the compiler kernel $k_\zeta = K_{\text{fam}} - K_{\text{shift}}$; and $k_\zeta = 2\theta'_{\text{RS}}$ — the derivative of the *Riemann–Siegel phase*, at relative 0.0. The dihedral shift is the scaling group $\mathbb{R}^*_+$.

The most useful part of T87 is not a match but a *boundary*. The two positivity programs are complementary: Connes–Consani anchor positivity *before* the primes (the prime-free Sonin window, support $(\frac{1}{2}, 2)$, i.e. $u \in (-\log 2, \log 2)$), the compiler anchors it *after* the primes (atom certificates, all supports). The window boundary is exactly the first prime atom $u = \log 2$ (`sympy`-exact; on narrow rows the prime side, Q_{cert} and Δ_2 all vanish identically). The sectors *touch* and do not overlap; the I5 coupling lives in the crossing region between them, outside both proved sectors.

5.3 The tightness map (T88) and the crossing (T89–T91)

T88 (`i5_tightness_probe.py`, 27/27) mapped where I5 is actually tight: a band-limited zero plateau (classical), safe zones, and *eight* near-vertical tight curves whose spacing follows the Γ-side density law (ratio 1.01 ± 0.08, computed zero-free — no spectral identification is claimed or made). There are no negatives on genuine autocorrelations; the earlier T76 negatives were the shadow of missing archimedean/$p = 2$ bookkeeping, exactly as the ledger predicted.

T89 (`sonin_crossing_probe.py`, 19/19) established that the window compression of the Weil form *is* Bombieri's classical object, and settled the sharpness question: the boundary at $\log 2$ is *not* sharp. The margin falls smoothly from $a \approx 0.5$ towards the floor with no structure at $\log 2$; the prime-free sector still has margin 0.557 at its own boundary and keeps ≥ 0.25 of the reference comfort to $a \approx 0.75$ (+0.057 beyond the proved boundary). The residual subspace grows softly $0 \to 1 \to 2 \to 3 \to 4$ (out of 32) and carries the global minimum. T90 (`residual_dissection_probe.py`, 17/17) made those residual vectors explicit — n-stable to $\leq 2.1°$, closed Gaussian$\times$cos/Gaussian$\times$sin fits with 99+% capture — and decided the core question: the residual is *not* just pole coupling (3/10 vectors are controlled by no structure at all; beyond $a \approx 0.85$ genuinely pole-free atom $\leftrightarrow$ arch content remains).

T91 (`thin_band_analytic_probe.py`, 19/19) identified the band as the *one-atom zone* and found the mechanism that dominates the rest of the document: beyond an inner edge the prime atom $u = \log 2$ becomes *load-bearing* — the prime *rescues* the form where the archimedean margin is exhausted (rescue term $+0.22 \ldots + 0.41$, purely odd sector). Two uncertainty constants were decided in closed form: $a \cdot t_{\text{rms}} \to \pi$ (Wirtinger, 0.3%) and $a \cdot t_{\text{cent}} \to 2\,\text{Si}(\pi) - 4/\pi = 2.4306$ (two independent routes, 10^{-25}).

The band that these three parts map out is narrow and its landmarks are explicit. Reading outwards from the classical side, the map is: classical edge 0.34657, prime-free crossing 0.3723–0.3763, band top $\alpha^* = 0.46265$, second atom $\log 3/2 = 0.54931$. Everything in Sections 6 and 7 happens inside that interval, between the first and the second prime atom, and the four numbers are the coordinates used throughout. The map was re-derived by a third independent implementation (T93, `band_reconciliation_probe.py`, 41/41) before it was used further.

An attempt to convert the band into a proof was made and did not succeed. T92 (`zone_extension_proof_probe.py`, 36/36, T-SKELETON) certified something small and real — $Q \geq 6.7 \times 10^{-12} > 0$ on an 8-dimensional sine-window subspace across the whole band, with an error certificate and a 1057-point covering — but not the lattice zone extension it was aiming at, and it named two independent walls: $\lambda_{\min}$ collapses geometrically (factor 11.4 per mode), and the high-mode complement would need $N^* \gtrsim 5069$ modes. The finding, and the reason it is quoted here rather than buried, is a statement about instruments: *the finite-block route is structurally the wrong instrument for this target, not an under-resourced one.* That is what motivated the change of approach in Section 6.

5.4 The blind prime demonstration (T94)

A separate, deliberately modest exercise. T94 (`prime_blind_demo_probe.py`, 16/16, BLIND-100) used the exact geometric criterion "odd n is prime $\iff r_4(n) = 8(n+1)$" (Jacobi 1834, on the $\mathbb{Z}^4 = \text{rank-}|\mu_4|$ theta tower) to predict all 753 primes of the never-seen window $[1\,000\,001,\ 1\,010\,000]$ — count and MD5 declared *before* any ground truth, then evaluated: exactly 753, zero false positives, zero false negatives, 100.00% on all 10^4 integers, with an AST-enforced prediction path containing no divisibility test (no `isprime`, no sieve, no `%`, `//` or `/`). The cost is part of the result: a sieve wins by $\sim$820$\times$ (1 ms against 0.83 s). This is structure completeness, not an algorithmic advance — and the *spectral* prediction (zeros $\to$ primes) remains bound to I5/RH and is not claimed.

6 The handover induction (T95–T101)

The band work left a picture: each prime atom rescues its own zone and hands over to the next. T95–T101 turned that picture into an induction, and then measured whether the induction can run.

6.1 The relay is real (T95, T96)

T95 (`continuum_extension_probe.py`, 28/28) proved the C1 chain unconditionally ($|h_f(\log 2)| \le 1/2$ by disjoint supports with margin 0.2305; the windowed symmetrised shift has $\|S\| = 1/2$ exactly, 1.1×10^{-16}) and found that the binding mechanism is *not* the two-bump extremal (share 0.046) but the atom rescue: above $\alpha_{\text{free}} = 0.371654$ the prime-free minimiser has $h(\log 2) < 0$ down to -0.328, so the atom adds positivity exactly in the losing direction.

T96 (`relay_test_probe.py`, 21/21) is the model of a self-correcting run. The apparent "edge" of T95 at α^* was a map artefact: the value $+7.8 \times 10^{-6}$ reproduced exactly, but it sits on a smooth, strictly positive curve with no feature at all ($\lambda_{\min} > 0$ on the whole of $[0.38, 0.86]$, decay law $\lambda_\infty \sim \exp(-49.2\alpha)$). The *relay*, however, is real — and it lives entirely in the counterfactual: remove the $\log 3$ atom and $\lambda_{\min}$ crashes to -0.445, with the loser being exactly the anti-double-bump at distance $\log 3$ (alignment -0.99) and a rescue identity at 5×10^{-15}. All four handover windows are positive: each atom arrives strictly *before* it is needed ($+0.025/ + 0.009/ + 0.011/ + 0.007$ for $k = 1..4$). Three consequences were recorded: it is a margin problem, not an edge problem; numerics as a witness is exhausted beyond $\alpha \approx 0.55$; and the counterfactual is the proof target, because it works on $O(0.1)$-sized quantities instead of 10^{-6}.

6.2 The induction step and its one missing lemma (T97, T98)

T97 (`relay_induction_probe.py`, 105/105) gave the induction step proof shape. Alignment is true and sharp with a constant law; the $t = 0$ killer loss is proved by three structure identities (old atoms vanish identically on E_-; the pole rescue flips with factor $1 - \cosh(u/2)$; $|\hat{f}|^2 = 2(1 - \cos(tu))|\hat{g}|^2$), giving $k_{\text{eff}} = (1 - \cos(tu))k(t)$ with infimum $-1.11\ldots - 2.59$ instead of $k(0) = -5.37$.[2] The structural pearl: E_0 is *literally* the same form at a smaller window (7×10^{-14}) — the induction hypothesis, by self-similarity. What remained was a single cross-block lemma about a single vector.

T98 (`cross_lemma_probe.py`, 44/44) attacked it and produced the most instructive negative result of the series: the measured $\sqrt{\lambda_0}$ law is Douglas (1966) range inclusion, i.e. a *tautology* of the positivity one is trying to prove — the identification *dissolves* the lemma rather than proving it. Three T97 premises were refuted in the same run. The replacement target is an exact scalar inequality with no constants and no near-null vector:

$$D_k(\alpha) \quad := \quad -\lambda_{\min}\Big(Q|_{E_-} - Q_{-0}(Q|_{E_0})^{-1}Q_{0-}\Big) \quad \le \quad \tfrac{\mu_k}{2}.$$

[2] In closed form (a T92 side result) $k(0) = -\gamma - 3\log 2 - \pi/2 - \log\pi$. Prior-art note (literature pass, 2026-07-27): in substance this is the classical digamma value $\psi(1/4) = -\pi/2 - 3\log 2 - \gamma$, shifted by $-\log\pi$, as it appears in Suzuki 2023 (J. London Math. Soc. 107) — a good consistency check, not a find.

T99 (`dk_recursion_probe.py`, 23/23) then found the a-priori structure: an exact mirror-parity selection rule ($J_- Q_{-0} J_0 = -Q_{-0}$ at 2×10^{-15}) giving $D_k = \max(D_{\text{even}}, D_{\text{odd}})$, with the binding even channel coupling only to the *odd* spectrum of the smaller window — so the fragile near-null mode is identically absent from the binding channel. T100 (`bulk_remainder_probe.py`, 27/27) closed the bulk remainder from 11/24 to 24/24 samples and showed the 10.8% drift was a grid artefact.

6.3 The race (T101)

T101 (`k_asymptotics_probe.py`, 31/31, CROWDING-TRENDS) resolved 16 zones ($n = 2 \ldots 29$) and separated two questions that had been fused. The *mathematics* trends self-supportingly: the primitive quantities are flat over 16 zones (w/g slope $+0.002 \pm 0.178$; the relative D_k margin does not trend; $D_k \le \mu_k/2$ holds 64/64 and never fails). Atom *strengths* do not fall ($\mu \sim n^{-0.087}$), only the *spacings* do ($n^{-0.615}$, exact) — pure arithmetic. What loses the race is the *closure instrument*: the demand ratio r_k decays like $\exp(-0.1622k)$ (fit, ± 0.0562). The collapsed one-parameter law $w_k = 0.0838 \cdot (\text{atom spacing})/\mu_k$ reduced the scatter from 8.85× to 2.66× with zero residual trend.

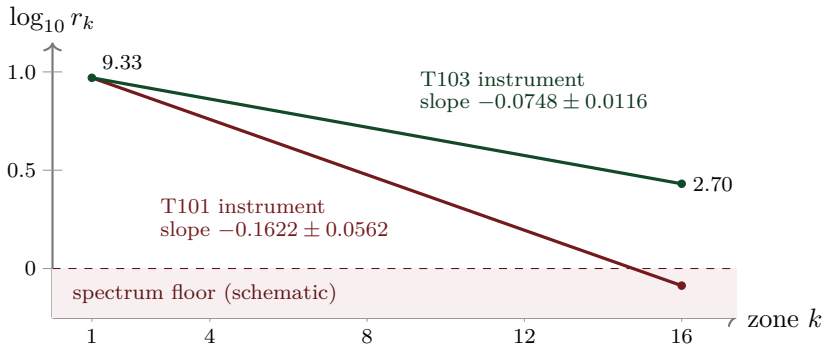


Figure 3: *Schematic.* The instrument race over the 16 resolved zones. *Literal:* the two decay slopes (-0.1622 ± 0.0562 measured in T101; -0.0748 ± 0.0116 measured in T103, a factor 2.2 flatter), and the T103 endpoints $r_1 = 9.33 \to r_{16} = 2.70$ with the statement that the improved instrument never leaves the spectrum. *Schematic:* the straight-line interpolation in $\log_{10}$, the common anchor of the two lines, and the position of the "spectrum floor". Both slopes are declared fits in the diary. What loses the race is the instrument, not the inequality: the true inequality $D_k \le \mu_k/2$ holds 64/64 and its relative margin does not trend.

T101 also stated what an asymptotic theorem would need, and that list has not changed since: (A) the collapsed lower bound $w_k \ge c\, g_k/\mu_k$ with a fixed $c > 0$ — a sharp, non-perturbative, *arithmetic* lower bound at a Weil-positivity edge, with Taylor arguments forbidden by the $\exp(-c/\delta')$ onset; (B) a uniform relative margin; (C) a replacement for the bulk remainder; (D) a finite check of the low zones. If crowding ever wins, the hardness sits entirely in (A).

7 The certification arc (T102–T125)

The last twenty-four parts stopped improving measurements and started converting them into certificates — and, where that failed, into explicit refutations. The arc closes with T125, which assembles the certificates into one chain and states what that chain is and is not (Section 9).

7.1 The mechanism: a crossing, not a singularity (T102)

T102 (`arithmetic_bound_probe.py`, 42/42, MECHANISM-IDENTIFIED) computed the mechanism exactly. The k-th atom acts on $E_-/E_0/E_+$ as $\text{diag}(-\frac{1}{2}, 0, +\frac{1}{2})$, so $Q_{k-1} = Q_{\text{full}} - \frac{\mu_k}{2} P_- + \frac{\mu_k}{2} P_+$: the atom strength μ_k enters exactly once, linearly. That gives a two-sided sandwich through the

Schur profile $\sigma_k(\delta)$, with $\mu_k/2 \le \sigma_k$ implying that the handover holds, and $2w_k$ lying between the crossings in 16/16 zones.

Two corrections came out of the same run. First, the onset is *anchored* at $\delta_c = 2w_k$ — a crossing of two finite quantities ($R^2 = 0.968$) — and exponential-singular versus power behaviour is not separable; T96's essential-singularity reading is compatible but no longer distinguished. Second, and more consequentially, the binding constraint *flips*: no concentration condition binds. The bare E_- form is strongly positive ($2.65 \ldots 3.52$, i.e. 4–14× $\mu_k/2$), the pole flip and band mass saturate their classical ceilings (96.9–97.1% of Cauchy–Schwarz and Landau–Pollak/prolate respectively), and the onset is manufactured *entirely* by the Schur dressing against $E_0 \oplus E_+$, which eats 35.7%–97.3% of the bare eigenvalue. The loser does not live in E_-.

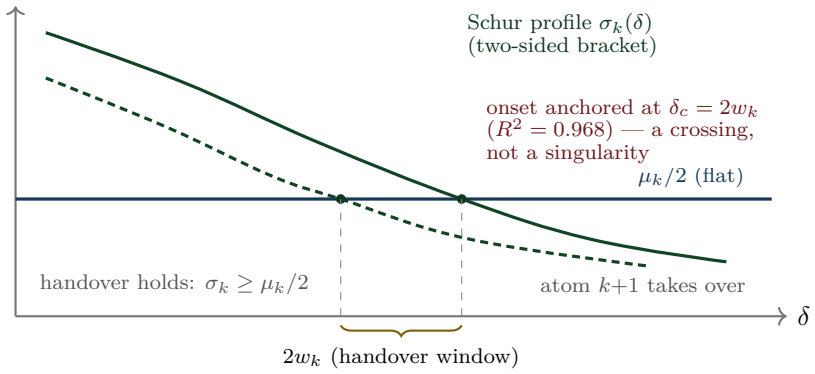


Figure 4: *Schematic.* The crossing mechanism of T102. *Literal:* μ_k enters exactly once and linearly, so the condition is a two-sided sandwich through the Schur profile $\sigma_k(\delta)$ against the flat level $\mu_k/2$; the window width $2w_k$ lies between the crossings in 16/16 zones; the onset is anchored at $\delta_c = 2w_k$ with $R^2 = 0.968$; the candidate bound $w_k \ge A_k \mu_k^{1/q_k}$ holds 16/16 with ratio 0.749–0.940. *Schematic:* the curve shapes and all absolute scales.

The same run performed the most useful kill of the arc: a triple decomposition test showed that the T101 law in C/g was a *proxy*. g_k is causally impossible (the form Q_{k-1} is blind to the next atom), statistically dispensable (a causal law $C \cdot u$ has CV 23.3% against 27.6%), and arithmetically only a ceiling ($C_k \le g_k \mu_k$ exactly; the 16-zone extrapolation violates the ceiling from $k = 69$; checked over 18 120 prime-power atoms to $n = 200\,000$).

7.2 The instrument, rebuilt (T103), and the two doors

T103 (`instrument_probe.py`, 29/29, INSTRUMENT-IMPROVED) rebuilt the closure instrument with certified weights $w_j \ge 1/\lambda_j$ and a θ-weighted band sum, reproducing both anchors exactly. With *one* fixed, k-uniform instrument ($\Lambda_0 = 3$, $r = 2$, no per-zone tuning) the closure map jumps from 7/64 to 44/64 and the race slope halves (Figure 3). The price is explicit: the number of explicit modes grows $2 \to 232$. And the verdict relocated the remainder: it is no longer the bulk (the bulk is measurably not low-rank — effective rank up to $0.579 \dim E_-$) but the *wing slack* $S = 1 - \rho$, which falls $0.2091 \to 0.0392$.

The convergence recorded at the end of T102 is the pivot of the whole arc: T103's wing slack $S = 1 - \rho$ and T102's Schur dressing are *the same object seen from two sides*. Both doors lead to the wing near-null direction.

7.3 Inductive chains, and the death of the margin route (T104)

T104 ran as *two independent arms* after a file collision between two parallel workers was resolved by keeping both implementations of the same contract — `schur_profile_bound_probe.py` (21/21) and `schur_profile_chain_probe.py` (47/47). All core findings converged, which makes it a de-facto independent replication.

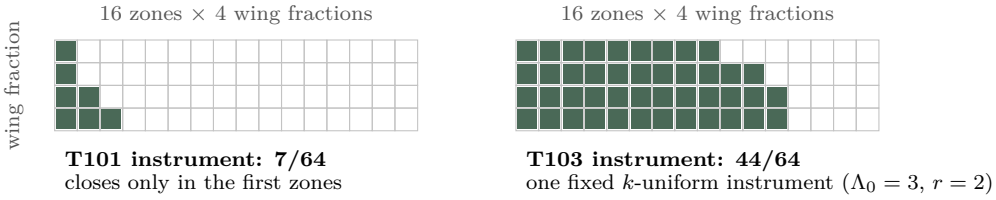


Figure 5: *Schematic.* The closure map. *Literal:* the grid is 16 zones × 4 wing fractions = 64 cells; the old instrument closes 7 of them, the rebuilt one closes 44 with a single fixed k-uniform setting ($\Lambda_0 = 3$, $r = 2$) and no per-zone tuning. *Schematic: which* cells are shaded — the diary records the totals, not the pattern; the shading is drawn as a monotone wedge purely to visualise the counts.

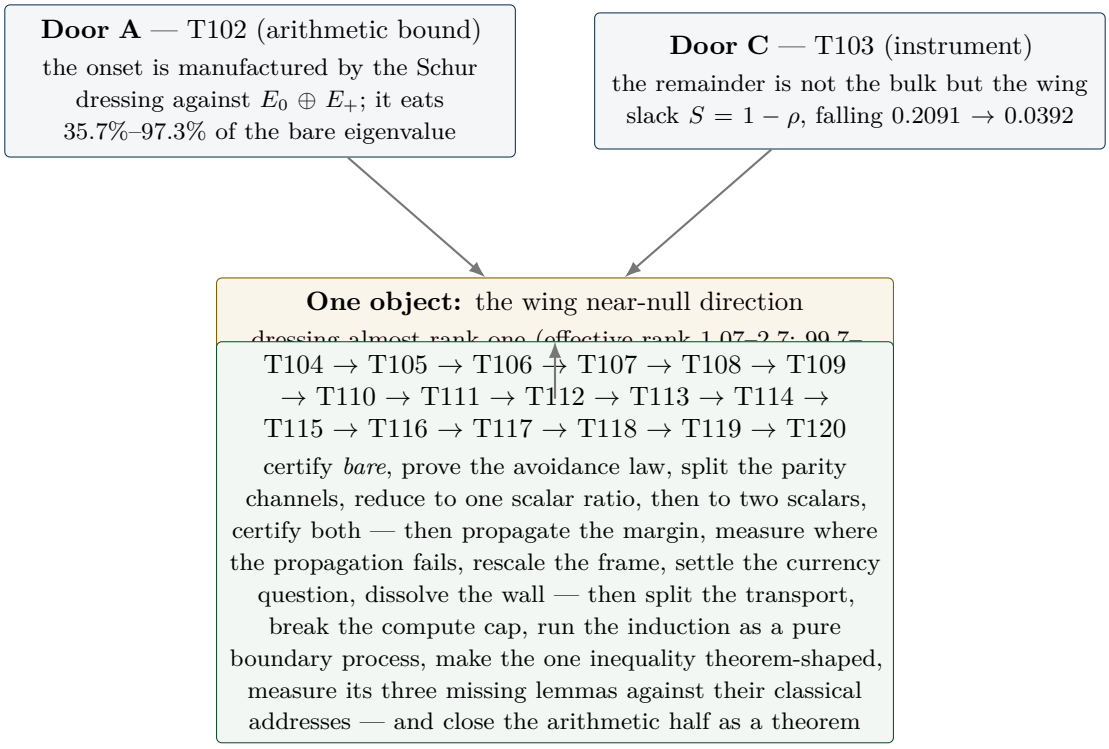


Figure 6: Two doors, one object. Both entries are literal diary findings; the convergence statement ("the same object from two sides") is quoted from the T102 closing note.

- **The naive margin route is dead**: $M \geq m \cdot \mathrm{Id}$ holds in 0/16 zones; the coupling mass to margin ratio is 10^3–10^6; m vanishes in the continuum like $M^{-1.7}$ (fit).
- **The mechanism is avoidance**: the coupling B_- decouples from the soft bulk modes — the softest mode carries 0.00% of the coupling mass for $n \geq 3$, and the mass sits in mid/high bands (lowest band only 2–12%). Positivity is not saved by a margin but by structural avoidance.
- **Exact spectral-split chains close** 16/16 with finite data: arm A via the exact block-inverse identity with matrix caps in PSD order (headroom 1.15–1.44, reach $\gamma_{\max} = 0.50$–0.90); arm B via a threshold split with an $O(1)$, resolution-stable threshold $w = 2.47$–5.90 — but requiring $r_{\min} = 64 \ldots 1024$ soft modes $\propto M$, which makes the continuum statement a spectral-density condition.
- A recommendation from T96/T103 was tested and *rejected*: the prolate wing basis loses against the raw basis in both arms.

Consequence: the hard core migrated from σ_k as a whole to four named ingredients — a lower bound on $\mathrm{bare}_k = \lambda_{\min}(Q_{\mathrm{full}}|_{E_-})$, the soft dressing scalar L, finite induction data, and a one-sided

edge estimate instead of a fit. The chain around them is classical (Schur complement, exact block inverse, Parseval/Bessel, Weyl) and fit-free.

7.4 The certified bare bound and the avoidance theorem (T105)

T105 (`bare_wing_bound_probe.py`, 28/28, ONE-OF-TWO) delivered one of the two missing ingredients in closed form. First it resolved the T104 arm discrepancy: *bare* depends only on (u_k, δ) — the window centre cancels ($\le 0.74\%$ drift under $8\times$ refinement) — with the exact split bare $= \mu_k/2 + b_0$. Then three classical steps (Bessel; a Legendre/level-set optimisation at $k(\pi/\delta)$; Cauchy–Schwarz at the pole pair) collapse to a closed lower bound:

$$b_0 \quad \ge \quad \frac{\delta}{\pi}\int_0^{\pi/\delta} k(t)\,dt \quad - \quad \delta\,K_{\max} \quad - \quad 4\left(\cosh\tfrac{u}{2} - 1\right)\sinh\tfrac{\delta}{2}.$$

This is the band average of the archimedean kernel over the band conjugate to the wing, of size $\sim \log(1/2\delta) - 1$. It is positive in 16/16 zones (1.27–3.54) at 81.1–92.7% of the measured value (median 92%), it sharpens as the wing flattens, and it needs no eigenvalue or induction data.

Second, the *avoidance law* became a theorem rather than an observation: $Q_{\text{full}} \succeq 0 \Rightarrow \|B^{\mathsf T} v_i\|^2 \le w_i \Lambda_-$ (a semi-inner-product argument; coupling proportional to the mode eigenvalue), verified on every mode of every zone with worst ratio 0.377. That explains the measured 0.00% avoidance exactly. Alongside it, an exact parity superselection: $JQJ = Q$ (5.7×10^{-15}) and $JB = -BR$ (3.4×10^{-15}) — the two channels do not mix, and the softest mode is J-even in 16/16 zones. Notably the *Euclidean* angles are not small (cosine up to 1.0): the avoidance lives in the form metric, as a Friedrichs angle.

What T105 left is one Loewner statement: $Q_{\text{full}}(\alpha) \succeq (\mu_k/2) P_-^{(k)} \iff \rho \le 1 - (\mu_k/2)/\text{bare}_k$ — a Friedrichs-angle / spectral-density condition, explicitly *not* a margin condition. The free cap $L \le \Lambda_- = 5.63$–7.52 diverges like $\log(1/D)$, so the certificate has to come from the spectral density near the soft end.

7.5 Parity superselection: the pole splits (T106)

T106 (`friedrichs_angle_probe.py`, 32/32, DENSITY-MAPPED) killed two routes and found the structural result of the arc. The kills are in Section 8. The finding:

$$ab^{\mathsf T} \;+\; ba^{\mathsf T} \qquad = \qquad ss^{\mathsf T} \;-\; tt^{\mathsf T} \qquad (10^{-18};\; Ja \;=\; b)$$

Here J is the mirror parity of Section 3, arriving for the second time and now as the obstruction itself. The rank-2 Weil pole splits *exactly* into a positive rank-one lift in the J-even channel and a negative rank-one pressure in the J-odd channel. The Toeplitz part has exactly one negative eigendirection; it is J-even, and $ss^{\mathsf T}$ removes it precisely there. The odd channel never needed the pole at all.

The channel-wise angle chain closes 16/16 in the even channel and 9/16 in the odd channel, against only 3/16 unsplit; all 16 handovers close at operating depth in both channels under the exact Schur criterion, with certified b_0 at 89–93%. Certified after T106: bare, the parity split, the pole splitting, the channel inertia, the avoidance law, the growth inequality, the accumulation bounds. Measured: ρ_-.

7.6 One scalar ratio (T107)

T107 (`odd_channel_closure_probe.py`, 30/30, SCALAR-TRACTABLE) reduced the matrix statement (R) to a scalar one. Sherman–Morrison gives exactly (R) $\iff s^* \le 1$ (identity at 3.4×10^{-11}; equivalence confirmed 16/16 against Rayleigh–Ritz; precondition $G \succ 0$ via Cholesky 16/16) — but s^* is *saturated* ($0.99885 \ldots 0.99999$, with only $n = 2$ at 0.2569), so no uniform s^* chain can work. The Woodbury decomposition rescues it: $s^* = \tau + \kappa$ and

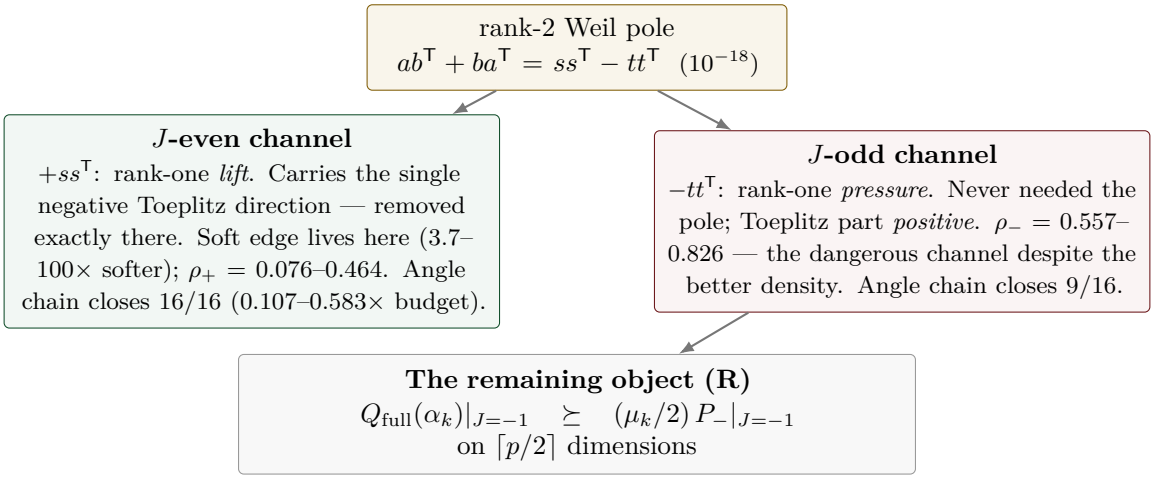


Figure 7: Parity superselection (T106). All numbers are literal; the layout is illustrative. The practical consequence: it is not the density that decides the angle but the *avoidance* — the dangerous channel is the one with the better density. T105's certified bare bound sits 16/16 exactly in the hard channel, and ρ_- is resolution-stable ($\leq 17.6\%$).

$$\text{(R)} \iff r := \frac{\kappa}{\varepsilon} \leq 1, \qquad \varepsilon := 1 - \tau, \qquad \text{measured } r = 0.0053 \ldots 0.1814.$$

Two orders of magnitude of headroom instead of five decimal places, and s^* is resolution-stable (spread $\leq 2.60\%$ over $M = 600/900/1200$). The synthesis is stated with the usual asymmetry: certified-closed 0/16, measured 16/16 (with $q = 1.31$–4.16 at $M = 1200$, replacing T106's 0.747–1.156 — a factor 1.7–2.5 gain). Exactly one object is missing: a certified positive lower bound for $\varepsilon = 1 - \tilde{t}^\mathsf{T} T_{\text{odd}}^{-1} \tilde{t}$ — how far the pole vector is from exhausting the odd channel — or, since ε and κ both fall with M ($\varepsilon \sim M^{-1.99\ldots-1.69}$, fit), a certificate for the *ratio* r itself. The named route is via a wing statement of the T105 type plus an avoidance norm, explicitly not via Szegő.

7.7 The ε identity, and two scalars (T108)

T108 (`ratio_certificate_probe.py`, 44/44, EPSILON-IDENTITY) turned the missing positivity of ε from a bound to be won into an *identity*. Everything hangs on the pole response $x := T_{\text{odd}}^{-1}\tilde{t}$, and three exact identities (verified at 10^{-11}–10^{-13}) pin it down: $\varepsilon = x^\mathsf{T} Q|_{\text{odd}} x/\tau$ — so ε *is* a $Q|_{\text{odd}}$-energy, not an accident of subtraction — $1/\varepsilon = 1 + \tilde{t}^\mathsf{T} Q|_{\text{odd}}^{-1} \tilde{t}$, and the overlap of ε and κ is exactly one rank-one term $hh^\mathsf{T}/\varepsilon$, so that (R) $\iff \hat{r} = (\mu/2)\lambda_{\max}(\Gamma + hh^\mathsf{T}/\varepsilon) \leq 1$. The third route supplies the classical reading: $\tilde{t}$ is exactly two-term geometry, τ a 2×2 Christoffel–Darboux form, and ε exactly the square of the last Cholesky pivot — the Szegő/Levinson prediction error. *The positivity of ε coincides with the induction positivity in the odd channel*; it is not a second, independent thing to certify.

Two consequences, and one correction of the previous picture. The consequence: (R) reduces from an $M/2$-dimensional Loewner statement to **two scalars** — $\omega < 1$ (measured 0.2366–0.7531, 16/16; the T105 structure survives the parity split verbatim, $V^\mathsf{T} S|_{\text{odd}} V = -\frac{1}{2} I$ at 4.9×10^{-14}) and one statement at the vector x. The chain (R) $\Leftarrow [\omega < 1] \wedge [\lambda_{\min}(Q|_{\text{odd}}) \geq (\mu/2)\tau\theta/(1-\omega)]$ has $C1$ closing 16/16 (margin 5.5–178; ≥ 1 at all 48 ladder points, 2.7–343), $C2$ 15/16, $C3$ 0/16 — the first time a margin chain in this arc is *not vacuous*: the induction margin it needs is explicitly 9.3×10^{-8}–3.7×10^{-3}, i.e. two to six orders of magnitude *below* $\mu/2$. The correction: the soft-edge picture of T107 was wrong in its mechanism — τ sits to 99.66% on modes $\lambda \geq 1$, and $\|h\|^2$ is small by sign cancellation ($\sum |k| = 401$ versus $\sum k = 1$), not by softness. T107's Cauchy–Schwarz chain turns out to be exactly the Weyl bound (slack 1.0000).

What is exactly open after T108 is one boundary value. The remaining object is a certified upper bound for the avoidance norm $\|V^\mathsf{T} T_{\text{odd}}^{-1} \tilde{t}\|^2$, and on 8 zones that is literally the single edge

entry x_0 of an explicit vector (suppressed at $|x_0|/\max|x| = 2.9\times10^{-3}$–$5.7\times10^{-3}$, with a linear edge profile $r^{1.01}$ quoted as a fit) — a boundary-decay statement about an explicit vector, the same object class as the T105 carrier separation. $\hat{r}$ is flat in M (6.1× more stable than ε) and $r, \hat{r} < 1$ at all 92 (zone, M) points up to $M = 3000$, tightest at 0.9648: measured, on resolved zones, as always. That boundary value, and the still-uncertified ω, are exactly what the next part attacked.

7.8 Boundary certificates: cancellation, not decay (T109)

T109 (`boundary_decay_probe.py`, 29/29, BOUNDARY-CERTIFIED) closed the arc: both remaining scalars are now certified, and the whole chain for (R) closes 16/16 conditional on exactly one explicit input. Three findings and one honest remainder.

First, the mechanism. The edge value x_0 is not small because anything *decays* — it is small because a sum *cancels*. Carrier separation is excluded geometrically: $\tilde{t}$ has its maximum at $r = 0$; the source sits on the boundary itself. The Combes–Thomas hypothesis is satisfied (T_{odd} is of Jaffard class, with local rate $\rho(s) = D(\frac{1}{2} + 2/(e^{2s}-1))$, matched at a factor 0.85–1.00, quoted as a fit) — but its conclusion is empty: the Green sum for x_0 cancels by a factor 44–4.6×10^4 on 15/16 zones (the known outlier $n = 2$ does not cancel), and the edge profile is a nearly linear zero, $r^{0.77\ldots1.28}$ as a fit. A boundary condition, not a decay length — and no absolute-value bound can survive a cancellation.

Second, the x_0 certificate that works. Four candidates were run certified-against-measured: Combes–Thomas, evaluated with the *exact* Green row so that no constant could rescue it, is *refuted* (1/16; an upper bound on its whole class); a T-metric Cauchy–Schwarz at the edge entry closes 1/16; the exact Szegő/Levinson two-point evaluation exhibits the cancellation but bounds nothing. The fourth is the new object: a residual / goal-oriented certificate (Prager–Synge; second order in the two residuals) which *carries* the cancellation by construction instead of bounding it away. Sharpness 1.0000–2.4937 against the measured norm, 16/16 within budget.

Third, ω is cracked — unconditionally. The certificate is a graded matrix cap in PSD order: one trial level per soft direction, validity by a single Cholesky factorisation (Sylvester inertia) plus Loewner antitonicity of the inverse, $n_{\text{top}} = 17$–512. It gives $\omega_{\text{cert}} = 0.2561$–$0.7569$ against the measured 0.2366–0.7531 — a factor 1.004–1.124 — with *no* hypothesis input. The compression Schur distance that blocked T108 shrinks from 0.18–0.52 to 0.0024–0.039, and the ungraded cap (one level for the whole soft block, which is what T108's global route amounted to) is vacuous everywhere: the grading is the whole difference. The trial data are numerically generated; their *validity* is trial-independent (identity + Cholesky + Loewner), only their sharpness is not.

The synthesis feeds both certificates from one object ($\Gamma_{\text{cert}} = S_{\text{cert}}^{-1}$): the chain closes 16/16 with margin 1.3–10.4 and is M-stable (44/48 ladder points to $M = 3000$; the certification price 1.5–28.6 does not grow). What remains open is exactly **one input**: the *strict* positivity margin of the odd channel, $\lambda_{\min}(Q|_{\text{odd}}) \geq 1.7\times10^{-6}$–$7.5\times10^{-3}$ against a measured 1.4×10^{-5}–5.1×10^{-2} — two to six orders of magnitude *below* $\mu/2$, i.e. strictly weaker than the conclusion it buys; the induction hypothesis as it stands supplies only the non-strict $Q|_{\text{odd}} \succeq 0$. Declared caveats: floating point is not audited, and $\lambda_{\min}$ is read through Rayleigh–Ritz. Whether that one number propagates itself through the induction is exactly what the next part measured.

7.9 The margin propagates: the circle closes on the measured zones (T110)

T110 (`margin_propagation_probe.py`, contract `MARGIN.PROPAGATION`, 28/28, MARGIN-PROPAGATES — with a precision statement) asked the only question T109 left open: does the strict scalar margin $m_k = \lambda_{\min}(Q|_{\text{odd}})$ propagate itself along the zone ladder, against the explicit demand need109(k) of the T109 chain? Three findings close the circle on the measured zones; three gaps state precisely what a theorem would still need.

The margin map. On a common grid (16 zones, $\delta_0 = 0.00284$) the margin falls monotonically, 6.95×10^{-2} ($n = 2$) to 1.66×10^{-5} ($n = 29$), against need109 = 3.09×10^{-7}–3.87×10^{-4}: $m_k \geq$ need109 on 16/16 zones, with ratio 2.09–179.6. Dilution during pure window growth is weak

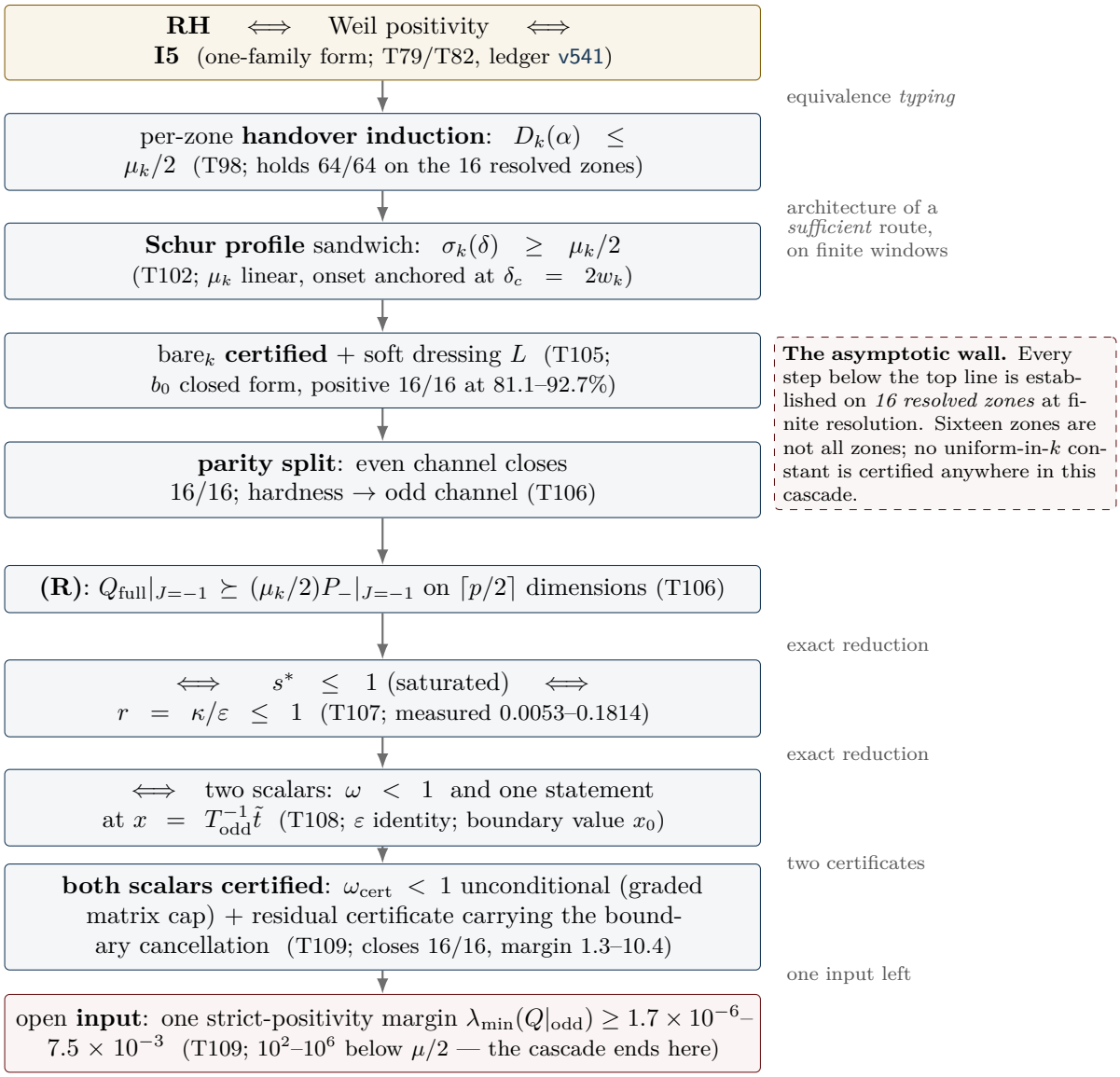


Figure 8: The reduction cascade, with its arrow types made explicit. The top line is an *equivalence typing* produced by an exact identity (Section 5); the arrows below are the architecture of a would-be sufficient route, verified on finitely many windows only. Since T109 the cascade ends at one strict-positivity margin input; T110 shows that this input propagates itself along the measured ladder (certified base case, graded Loewner minorant), leaving three gaps — which T111 then drives deep and measures as three separate walls (the two subsections below). The dashed box is the reason this figure is not a proof sketch.

— a factor 1.02–2.05 per atom gap, i.e. 0.993–0.997 per cell, a $1 - O(1/M)$ loss; the scaling fits are honestly poor (the power law wins 7/16). The structural finding of the part is the atom entry: it costs *nothing*. The naive expectation was a $-(\mu/2)$ pressure on the odd channel; measured, the entry *raises* $\lambda_{\min}$ on 15/15 handovers (the first-order Kato term has helping sign), and at $n = 29$ the atom-free window is not even positive (-9.2×10^{-3}): the atom makes the window co-positive. The T106 killer argument — transport destroys a demand that *points* somewhere — has no analogue for the scalar floor.

The step law. The growth step splits exactly (embedding error $\leq 2.6 \times 10^{-14}$), and $\max |\mu N| = 0.0$ on 15/15: the new atom lies outside the old lag reach (gap 0.0606 against $\delta_0 = 0.00284$), so its restriction to the old window is the exact zero matrix — everything is decided by the bordering. Scalar routes fail decisively: bordered Weyl closes 0/15 and the Schur/Friedrichs scalar cap is vacuous 15/15 (both recorded in the kill list). What passes certified is the **graded Loewner minorant** — its validity condition is exactly the induction hypothesis, and the level is then found by bisection through a Cholesky/Sylvester factorisation: with one soft direction ($n_{\text{soft}} = 1$) it holds 15/15 with retention 1.0000; the strictly scalar variant ($n_{\text{soft}} = \dim$) closes 0/15.

Base case and circle test. The base case $n = 2$ is *certified* by an explicit Cholesky of $Q|_{\text{odd}} - \lambda I$ at three grid resolutions ($\lambda_{\min} \geq 6.93 \times 10^{-2}$, a factor 88–180 over need). The circle test then runs the whole chain from that certified base value: the strictly scalar induction breaks at $k = 3$ (its Schur factor compounds to 2.7×10^{-36}), but the graded chain runs through completely — 16/16 zones above need109, each step one Cholesky, with the propagated floor as the only input. On the measured zones, the induction circle closes end to end.

The three gaps — the asymptotic mountain, made precise

1. **No reserve.** f_{crit} at the first handover is 1.00; the chain lives on a retention of $1 - 2 \times 10^{-7}$, and a factor-2 loss anywhere would break it within two steps. What a theorem needs is a step law *without* loss.
2. **No scalar step law.** Structurally excluded, not merely missed: the new cells attach at the boundary layer where the pole source sits, so any one-number summary of the bordering is blind to exactly the direction that decides it.
3. **No uniformity in k.** Every certificate above is one finite Cholesky; flat-in-k is a measurement, not a theorem. And the measured trend warns rather than reassures: $m_k \sim n^{-1.93}$ against need109 $\sim n^{-0.98}$, ratio $\sim n^{-0.96}$ (fit, rms 0.735) — extrapolated crossing at $n \approx 170$. Propagation gets *harder* with k, not easier. (T111 subsequently *measured* that crossing: it sits at $n^* \approx 462$, not 170 — the extrapolation was a fit artefact of the 16-zone window, but the wall itself is real. See the next subsection.)

7.10 The three walls (T111)

T111 (`deep_zone_stress_probe.py`, contract `DEEP.ZONE.STRESS`, 23/23, CROSSING-CONFIRMED) stopped extrapolating and measured. On exactly the T110 cell grid — the first 16 zones reproduce T110 bit-exactly — the ladder was extended to *every* prime-power zone $n \leq 1000$ plus a thin tail to 3001: 199 zones in the margin map, with the chain ladder running $n = 2 \ldots 521$ through 117 handovers.

The margin wall, measured. The ratio m_k/need109 falls 179.59 → 0.0174 across the deep ladder, and the crossing is now a measurement, not a fit: the last zone with ratio ≥ 1 is $n = 461$, the first below is $n = 463$ (ratio 0.935). The $n \approx 170$ figure of T110 was a fit artefact of the 16-zone window — the wall sits much further out, but it is real. The failure follows the *prime branch*: prime-power zones (512, 529, 625, 729), whose atoms are weaker, survive longer than the primes around them. The fit families were again compared honestly (best by AIC: a broken-scale law with $n^* = 501$; the jackknife band of the pure power law is $[298, 352]$), and a depth-preserving resolution study shows that refinement pushes the ratio *down*: $n^* \approx 462$ is an *upper* bound, not an estimate to be rounded up.

The ladder wall, arithmetic. The uniformity object is no longer flat: n^*_{soft} runs $1 \ldots 8$ across the

117 handovers. But retention stays 1.000000 at every step (largest single loss 5.96×10^{-8}), and the atom entry stays constructively free on 117/117 — the restriction of the new atom to the old window is the exact zero matrix, deep into the ladder. The new and harder finding is the *ladder wall*: the nested step needs a lag gap exceeding twice the window depth, and the twin-prime pair $521 \to 523$ (gap $0.003831 < 2D = 0.005617$) kills the ladder *arithmetically* — at frozen cell width, $n = 521$ is the deepest reachable zone, and the bound is set by twin primes, not by spectra. An arithmetic scan to 40 000 finds 4052/4284 consecutive prime-power pairs endangered; letting D shrink like $1/n$ instead forces the cell count to grow like $n \log n/4$ (horizon $n \approx 690$ at $h \le 1500$).

The circle tears at the ratio, not at a step. The no-reserve finding of T110 softens: $f_{\mathrm{crit}} \sim n^{-0.39}$ (fit) — the reserve *opens* with depth, and the bottleneck stays at the first step $2 \to 3$. The circle test runs from the certified base case (factor 179.6) and breaks first at $k = 109$, $n = 463$ — exactly where the margin map measures the crossing. All 117 handovers certify at retention 1.000000, including beyond the break: what fails is the demand ratio, never the handover mechanism.

The three walls — and the reinterpretation

The hardness balance names the driver: the growth of $1/\kappa$ (exponent +0.86) against the margin (exponent −0.68). Three separate walls, in order of appearance:

1. **The margin wall**, $n^* \approx 462$: measured, an upper bound (refinement pushes it down); the T109-chain demand ratio crosses 1 between $n = 461$ and $n = 463$.
2. **The ladder wall**, $521 \to 523$: purely arithmetic — at frozen cell width the nested handover dies at the first twin-prime gap below $2D$, independently of any spectral quantity.
3. **The requirement wall**, $n = 727$: $\omega_{\mathrm{cert}} \ge 1$ makes need109 vacuous on 46 deep zones — beyond it the T109 chain no longer states a demand at all.

The reinterpretation that comes with the measurement: the operating variable is the window *depth*, not the zone index n — doubling the cheeks moves the margin wall from 463 to 47. A proof along this route must beat the exponent gap $-0.974 = -0.681 - 0.293$ and couple the depth δ_0 to the local prime gap.

T112 subsequently ran the construction in exactly that coupled frame (next subsection): walls (2) and (3) fall *structurally*, wall (1) proves frame-invariant.

7.11 The scaled frame: two walls fall, the margin wall is frame-invariant (T112)

T112 (`adaptive_scaling_probe.py`, contract `ADAPTIVE.SCALING`, 20/20, SCALING-PARTIAL) stopped freezing the cell width and coupled it to the local scale. Two couplings were built: frame A, gap-coupled, $D_k = g_k/(2\nu)$ with g_k the local prime-power gap and ν the resolution constant of the frame; and frame B, mean-field, $D_k = \log n/(2\nu n)$ — the smooth PNT version of the same idea. The first finding is a structural asymmetry: *frame B has no ladder* (only 38% of consecutive pairs nest; the first failure is already at $n = 3$) — the nested handover lives exclusively in the gap coupling, not in any smooth $1/n$ law. T105 admissibility, re-measured in the scaled frame, sets a resolution floor $\nu \ge 4$ (at $\nu = 1$, 3/10 test zones are inadmissible). The flatness verdict is honest and double-edged: the ratio is *not* flat, but the exponent is *frame-invariant* — the components shift massively (m_k: $n^{-1.93} \to n^{-0.95}$; need109: $n^{-0.98} \to n^{+0.03}$, i.e. practically n-independent in the scaled frame), yet the difference stays −0.974.

The three walls, scaled. The **ladder wall is structurally gone**: with $D_k = g_k/(2\nu)$ the nested step has exactly ν cells per end *by construction* — twin pairs included — verified as an exact arithmetic lemma over all zones; the atom entry is the exact zero matrix by construction rather than by arithmetic accident; and the T111 killer pair $461 \to 463$ now certifies *spectrally* ($h = 1418$, retention 1.000000), with the reserve *opening*: $f_{\mathrm{crit}} = 8.1 \times 10^{-4}$–$4.2 \times 10^{-3}$ instead of 1.00 — roughly a factor 1000. (The pair $521 \to 523$ would need $h = 1634$, above the dimension cap: arithmetically co-certified, spectrally declared honestly as not run.) The ω **wall is no longer a depth wall**: the depth coefficient is $n^{-0.079 \pm 0.121}$, compatible with zero — ω_{cert} is driven by the resolution and the local gap, not by the depth of the ladder; a residual requirement-vacuum tail of 5 zones on the deepest ladder remains and is not booked as "gone". The **margin wall stays**,

practically unmoved: the first sub-1 zone sits at $n = 449$ against the frozen frame's 461–463 (a factor 1.07) — it is not geometry but the substance of the T109 requirement itself.

The limit object. Overlaying the scaled windows, the archimedean part (0.657) and the pole part (0.558) contract Cauchy-like with amplitude laws $D^{-0.26}$ and $D^{+0.37}$; the atom part does *not* contract (0.935) — it is the prime-gap statistics itself, written in scaled coordinates. The spectral shape does not drift: $\lambda_2/\lambda_1 \sim n^{-0.029\pm0.052}$. The resulting formulation is a split object: *[a deterministic limit shape: arch + rank-one pole] + [the atoms as a controlled, non-convergent perturbation].* New and uncomfortable is the **regrid object**: the zone frames are not nested, so Rayleigh–Ritz transfers nothing between them; the regrid cost is measured at rate $(D'/D)^{+2.93}$ — an object that simply does not exist in the frozen frame, disclosed here because any limit argument has to pay it.

The trade — and the disclosed prime-gap dependence

Three arithmetic walls are exchanged for two analytic demands:

[P1] **Positivity of the limit operator** — *one* operator instead of a sequence of matrices; the deterministic part (arch + rank-one pole) contracts, the atom part is the perturbation to be controlled.

[P2] **A convergence rate below the step reserve** — the Cauchy rates and the regrid cost against the opening reserve f_{crit}.

The prime-gap inputs are disclosed rather than hidden. Upward, Bertrand–Chebyshev 1852 ($g_k \le \log 2$; used and verified in-run) and the trivial even-gap bound suffice. The only remaining arithmetic parameter is the normalised gap $\theta_k = g_k\, n/\log n = 0.18$–$4.12$ — and downward, θ is *provably* unboundedly small: by Zhang 2014 / Maynard 2015 (infinitely many bounded gaps), $\theta \to 0$ on subsequences, so any gap-coupled construction must be uniform in $\theta \to 0$, at the price of the $1/\theta$ cost explosion.

T113 subsequently answered the currency question (next subsection): the wall is currency-invariant — and what it measures is the discretisation, not the spectrum.

7.12 The currency question: the wall is substance, and it measures the discretisation (T113)

T113 (`limit_operator_probe.py`, contract `LIMIT.OPERATOR`, 27/27, SUBSTANCE-CONFIRMED) answered the question T112 left open — is the margin fall in the scaled frame a normalisation artefact? — and the answer reinterprets the program's hardest number.

The currency table. The floor/requirement ratio carries the *same* exponent, $n^{-1.168\pm0.259}$, in all five currencies tested (raw, per $\lambda_{\max}$, per trace density, per D, per D^2) — and the invariance is structural, not accidental: the piecewise-constant basis is L^2-orthonormal (Gram matrix exactly I, so there is no free D-power to spend), and the zone matrices are restrictions of *one* quadratic form to subspaces of the same L^2 space (cross-grid form identity at relative 1.2×10^{-14}). The floor's homogeneity degree in the cell width is exactly 1.0; the requirement's is 1.0 under every legitimate rescaling. No flat currency exists. The wall is currency-invariant: substance, not bookkeeping.

Two surprises about what that substance is. First, the limit object is *not* "arch + pole with the atoms as a perturbation": the atom-free window form is deeply negative ($\lambda_{\min} = -3.08$ to -19.36, growing like $n^{+0.60}$), the atom block is equally large (norm 3.21–19.44), and the positive floor survives only as a *cancellation* of relative size 1.3×10^{-7}–9.7×10^{-5} — on the floor vector, -11.64 against $+11.64$. The Weyl bound is saturated: norm perturbation theory is five orders of magnitude too coarse for this object. Second, there is *no plateau under refinement*: at a fixed window, on nested grids over 8 levels (a factor 128 in resolution), $\lambda_1 \sim D^{1.83}$ and $\lambda_2 \sim D^{1.76}$ — the *same* power, with λ_2/λ_1 flat. **The continuum window form has no gap; m_k measures the gap of the discretisation.** This resolves the T112 tension exactly: λ_2/λ_1 constant while the floor falls is what it looks like when both eigenvalues carry the same D-power.

The atom part is almost invisible at the floor. The entering (deepest) atom contributes almost nothing to the coupling (8.3×10^{-11}–1.3×10^{-6}) — which is why handover steps certify at retention 1.000000 while the margin tears. The floor is almost pure geometry: $\log m =$

const $+ 1.87 \log D - 2.07 \log \alpha$, with an arithmetic residual of a factor 1.13 and no $\Lambda(n)$ dependence. The $\theta \to 0$ stress test separates cost from substance: the operator norm does *not* diverge ($\theta^{-0.005}$); an extra atom at the window edge shifts the floor by only 0.2–0.4%, while the same atom in the interior shifts it 10^7 times more — the $1/\theta$ explosion of T112 is purely a *cost* statement ($h \sim \nu n/\theta$ cells), not an operator instability.

Regrid, re-read. The raw regrid rate $(D'/D)^{3.85}$ is mostly the floor's own D-power — a normalisation jump, not an instability; the reserve certifies $f_{\text{crit}} = 9.5 \times 10^{-2}$ (better than the T112 estimate), and after subtracting the D-power the reserve buys 15 steps (against 0 on the raw jump).

The new hardness balance — replacing [P1]/[P2]

The T112 trade is restated by measurement; this balance supersedes the earlier two-demand form.

[P1] **Semidefiniteness of the balanced form without a gap** — the continuum window form is semidefinite at best; strict limit-floor formulations demand what refinement demonstrably refuses.

[P2] **Certify the D-power itself** — a Grenander–Szegő-type consistency statement for $\lambda_{\min}$ of the scaled form; the power, not an abstract rate, is the object.

[P3] **New, and now leading: the requirement homogeneity** — need109 carries $D^{0.63}$ against the floor's $D^{2.38}$: the T109 chain divides by an *artifact* margin. The repair the measurements point at is a *margin-free step certificate* — a Schur/Loewner formulation certified from semidefiniteness alone, never dividing by a margin.

[P4] $\theta \to 0$ **is a cost-uniformity demand only** — the frame needs $h \sim \nu n/\theta$ cells, but the operator itself is θ-stable.

Disposition after T114 (next subsection): [P3] is **discharged** — the margin-free step certificate exists and runs past the wall; [P4] stands as a pure cost statement; a new **[P5]** appears (the (R) demand is grid-bound and is deleted, not transported); and [P1]/[P2] are restated as the two-object core — ε relative to κ, and Schur transport between grids.

T114 (`margin_free_step_probe.py`, contract `MARGIN.FREE.STEP`) then rebuilt the chain without the margin division — and the wall dissolved.

7.13 The wall dissolves: the margin-free step (T114)

T114 (`margin_free_step_probe.py`, contract `MARGIN.FREE.STEP`, 22/22, WALL-DISSOLVES) built the certificate that [P3] demanded, and the answer to the preregistered question — does the margin wall disappear as a requirement artefact? — is yes.

The margin-free step. The structural fact that makes it possible was already on the table (T110/T111): the restriction of the grown window's form to the old window is bit-exactly Q_{old} — the entering atom's block is the exact zero matrix — so margin-freeness *is* that property, once the step stops asking for more. Route (i) is Albert 1969: the generalised Schur criterion with the Moore–Penrose inverse,

$$Q' = \begin{pmatrix} A & C \\ C^{\mathsf{T}} & X \end{pmatrix} \succeq 0 \iff X \succeq 0, \quad \operatorname{ran}(C^{\mathsf{T}}) \subseteq \operatorname{ran}(X), \quad A - CX^{+}C^{\mathsf{T}} \succeq 0$$

(the range condition is Douglas 1966) — no margin appears anywhere. It certifies 27/27 ladder steps, *eleven of them beyond the old wall* (467, 479, 512, 529, 547, 577, 661, 773, 887, 1129, 1331; deepest $h = 1495$). The exact 4×4 Schur complement is an $O(0.1)$ object with *no* cancellation: $\lambda_{\min}(S) = 0.068$–$0.154$, i.e. 42–67% of the block scale. Route (ii), the T110 graded minorant in its degenerate limit $x_{\text{in}} = 0$, dies structurally on 27/27 — which retypes T110's margin: it was not paying for positivity but for an *abandoned direction*. Route (iii), unshifted Cholesky, passes 27/27 against the declared backward-error floor. Negative controls are sharp (5/5), and the certificate is frame-invariant at $\nu = 8$ (3/3). All seven zones at which the T109 chain tears — including the wall zone $n = 449$ itself, where need109$/m = 1.241$ — are certified by the margin-free step.

The wall in one line. The same quantity, bounded through norms (Weyl: $\lambda_{\min}(A) - \|C\|^2/\lambda_{\min}(X)$), is *negative* by a factor of 2.4×10^5–9.6×10^7: an $O(1)$ numerator divided by

the 10^{-6} artifact floor. Every norm route had to fail — and the exact Schur complement passes through none of it.

The favourable power direction. On nested refinement chains at a fixed window, $\varepsilon \sim D^{1.77\text{–}2.01}$ and $\kappa \sim D^{3.64\text{–}4.84}$ (fits) — the preregistered "same power" expectation is refuted *in the favourable direction*: κ falls faster, so the scale-free ratio $r = \kappa/\varepsilon \sim D^{1.6\text{–}3.1}$ shrinks under refinement. The ratio closure $r = 2 \times 10^{-4}$–0.103 holds on 27/27 steps (11/11 beyond 463). The circle [base semidefiniteness (a declared hypothesis input) + margin-free step + ratio closure] closes 27/27; ten multi-step chains run (21 certified steps, the longest of length 4); and *every* chain ends at the cap $h \leq 1500$ — a window *cost* — never at a step. Regrid honesty: 0/464 gap ratios are dyadic, so the non-nested transport object of T112/T113 remains real.

[P5]: the (R) demand is grid-bound. Holding the physical demand $(\mu/2)P_-$ fixed and refining the grid, ω grows monotonically (5/5) and crosses 1 (4/5): the (R) formulation is *false in the continuum* — it is to be *deleted*, not transported. The exact Schur complement needs no demand surrogate in its place.

Cancellation robustness. The 10^{-7}-relative cancellation of T113 has 5.3–8.8 decimal digits of headroom above the Cholesky backward-error floor (Wilkinson/Higham), and it sits almost entirely in the rank-one pole — whose positivity is exactly the T108 Szegő identity ε.

The new core — two objects

The four-part balance collapses. [P3] is discharged (the margin-free step certificate exists and runs past the wall), [P4] was only ever a cost statement, and [P5] deletes (R). What is left:

[P1] $\varepsilon > 0$ **with controlled size relative to** κ — the exact Szegő/Levinson identity of T108, now the single positivity object of the program.

[P2] **Transport of the exact Schur complement between non-nested grids** — an $O(0.1)$-sized object with no cancellation, sharpened from the earlier abstract D-power demand.

Disposition after T115 (next subsection): [P2] **splits** — the transport across a real regrid certifies only mild refinement ($\rho \lesssim 1.8$), for a principled reason (the Schur floor itself falls under refinement), while a two-scale compression breaks the compute cap; [P1] **sharpens** to one scalar inequality, the classical Szegő–Levinson prediction-error bound. The core is restated as the three-point list at the end of the next subsection — only one point of which is an inequality.

T115 (`schur_transport_probe.py`, contract `SCHUR.TRANSPORT`) then ran the two last bricks — and split them.

7.14 Transport blocked, cap broken: the two-scale compression (T115)

T115 (`schur_transport_probe.py`, contract `SCHUR.TRANSPORT`, 26/26, TRANSPORT-BLOCKED — with a large compression gain) took the two-object core apart: transport of the exact Schur complement across the ladder's non-nested grids, and a multi-resolution compression of the already-certified interior.

The transport splits at $\rho^* \approx 1.8$ — for a principled reason. Haynsworth's partial minimisation $y^{\mathsf{T}} S y = \min_z [y; z]^{\mathsf{T}} Q' [y; z]$ makes $\lambda_{\min}(S)$ available without any inverse and without a margin; the two-sided transport bracket built from it is an *identity* (bookkeeping), with the evidence sitting in the consistency defect η and in the lower end. On the eleven real regrid pairs of the ladder ($\rho = 1.023$–3.425) the lower end is positive on 7/11, with a clean split: every pair with $\rho \leq 2.061$ certifies and none with $\rho \geq 2.291$ does; a synthetic scan puts the threshold at $\rho^* = 1.83$, against a median real refinement ratio of 2.001 — 39% of the ladder's 7686 refinement pairs are covered. The blocker is *not* the transport machinery: the exact cell-overlap projection beats three deliberately wrong transfer maps on 9/9 (by factors up to 270), and on *nested* ladders — where the transport error is exactly zero — $\lambda_{\min}(S)$ itself falls like $\rho^{-1.73}$ to $\rho^{-1.65}$. The drop is real, so no bound can undo it. Two sharpenings came free: non-nestability is a matter of *integrality*, not dyadicity (0/14 807 gap ratios are integer, the closest miss 3.4×10^{-5}), and the measured defect $\eta = 2.7 \times 10^{-2}$–$8.1 \times 10^{-2}$ is 10^3–10^6 times smaller than its a-priori Céa/Strang norm surrogate —

which is exactly where the margin division would have re-entered.

The compression breaks the cap. The two-scale window (interior merged into blocks, the boundary strip kept fine, the merge anchored at the centre) preserves $X_{\text{mixed}} = Q_{\text{old,mixed}}$ *exactly* (rel 0.0) — the compressed step is still margin-free — and Albert certifies 66/66 (zone, q) combinations up to $q = 64$, with the compression error certified *one-sided and second order* in the projection defect (Rayleigh–Ritz plus stationarity of the fine minimiser; Céa/Strang; 66/66) and savings down to $m/h = 0.043$. The deep run then breaks the T114 cap: the margin-free step certifies at $n = 155\,921$ (117× T114's 1331), on a fine lattice of $h = 93\,470$ cells (62× the hard cap $h \le 1500$) compressed to $m = 1490$; re-coarsening inside a chain is free (Rayleigh–Ritz, rel $\le 2.1 \times 10^{-12}$); the longest chain is now *ten* steps (T114: four), with 33 certified steps over four chains; the stopper is the cost cap on 3/4 runs and a failing step on 0; and every certified quantity sits 10^5–10^{11} times above the declared Cholesky backward-error floor. Stated honestly: $\lambda_{\min}(S_{\text{mixed}}) \approx 5.1$ is 52× the uniform value because the coarser Galerkin space sees fewer soft directions — a weaker statement about the continuum, declared as such, and flat across the whole deep band. The contiguous reach an induction needs moves from $n \le 125$ to $n \le 5437$ (factor 43.5): the $n \log n$ cost barrier is *divided*, not broken.

The remaining list — three points, one inequality

The shortest list the program has produced. From T114's balance, the margin (dissolved), the (R) demand (grid-bound, deleted), the floor/ε transport (the object to transport is $\lambda_{\min}(S)$, the slowest-falling of the three) and the compression risk (it does not break the step) are all *struck*. What is left:

(1) [P1] as one scalar inequality — given $T = Q + \tilde{t}\tilde{t}^{\mathsf{T}} \succ 0$, positivity of $Q|_{\text{odd}}$ is *equivalent* to $\varepsilon = 1 - \tilde{t}^{\mathsf{T}} T^{-1} \tilde{t} \ge c(D) > 0$: the classical Szegő–Levinson prediction-error bound for one explicit symbol (the Haynsworth double-Albert device: $T \succ 0 \wedge \varepsilon \ge 0 \iff Q \succeq 0$). This is the target-sentence candidate. Measured: $\varepsilon \sim \rho^{-1.71}$, falling slightly faster than $\lambda_{\min}(S) \sim \rho^{-1.69}$ (fits).

(2) An a-priori bound on the transport defect η — a regularity statement about the minimiser z^*; it would turn the measured transport into a certificate for $\rho \le 1.8$.

(3) A boundary formulation (suggested by the exact-zero lemma) — never represent the old interior at all; it would remove the compute cap *and* the regrid at once.

Only (1) is an inequality; (2) is a regularity statement, (3) is a reformulation.

Disposition after T116 (next subsection): (3) is **run and splits** — the boundary recursion itself is exact (the pole rides in the state with no truncation, and the march cost is flat), but the state cannot be *bounded*: the prime comb refuses compression. (2) is **devalued** — a single boundary march needs no regrid, so the transport defect no longer sits on the critical path. (1) **sharpens** into a two-variable law with a named classical proof technique: ε is a Galerkin error, Aubin–Nitsche duality predicts the measured D-exponent, and the jumps at prime-power entries become an explicit side-condition.

T116 (`boundary_formulation_probe.py`, contract `BOUNDARY.FORMULATION`) then ran the reformulation — and it held exactly where the contract feared it would break, and broke exactly where it seemed free.

7.15 The boundary recursion: exact pole, flat cost, and the prime comb (T116)

T116 (`boundary_formulation_probe.py`, contract `BOUNDARY.FORMULATION`, 33/33, RICCATI-PARTIAL) executed point (3) of the list: never represent the old interior at all — run the induction as a pure boundary process.

The recursion stands, and the pole is free. The state is the triple (Σ, r, σ) obtained by partially minimising the pole-free form over the interior (Haynsworth 1968): the identity $t^{\mathsf{T}} T^{-1} t = r^{\mathsf{T}} \Sigma^{-1} r + \sigma$ holds on six zones at rel 6.7×10^{-16}, the Haynsworth double-Albert equivalence certifies 6/6 with sharp negative controls, and the Riccati-type chaining of two eliminations is *exact* (synthetic 1.4×10^{-16}; bit-exact on real windows). The first feared obstruction — pole bookkeeping — dissolves for free: the rank-one pole is genuinely *global* (its boundary value at half depth is 38.6%, against 0.2% for the archimedean part), but the pair (r, σ) carries it *exactly* through bordered

elimination, with no truncation anywhere.

The tail bookkeeping breaks — on the prime comb. The second feared obstruction was the archimedean kernel tail, and it is innocent: the arch part decays cleanly, $\exp(-1.73\,\omega)$. What does not decay is the *full* symbol — it stays at $\geq 93\%$ of its maximum beyond every tested width, and 100% of the off-band mass sits in the stripes of the two *prime combs*, carried by the reflection comb: every incoming cell couples through a Hankel term to the interior cell at $\alpha - \log n_j$, for *every* prime power $\sqrt{n} < n_j < n$. A truncation of the state would have to discard 1.3×10^2–1.1×10^6 times the margin ε it must respect, a budget that grows like $n^{2.17}$ ($\mu_j \sim n^{-1/4}$ against $\varepsilon \sim n^{-1.8}$; fits); 11/18 real truncations break outright; a comb-aware state that keeps the stripes still spans 83.6% of the window and reaches only rel 20; and the ceiling of the whole idea is a $\Theta(\log^2 n)$ gain. There is no sparse faithful state — the stripes drift with fill-in.

The deep boundary run — a cost-geometry demonstration, declared as such. One unbroken chain of 169 236 prepends ran to a window of $h = 1\,354\,088$ cells — 903× the old cap — with the entire state $12 \times 12 + 12 + 1$ numbers and the cost *flat*: 76 μs per step, first-to-last-decile ratio 1.00. Stated honestly, this is *not* a Weil certificate: at the fixed cell width $D = 1.9 \times 10^{-6}$ the band contains no atom, the stopper is $\varepsilon < 0$ at $h = 1.35 \times 10^6$ — the *band model* loses positivity there, not the machine — and the atom reach stays at $n = 173$. What the march certifies is the cost geometry of the boundary formulation.

The one inequality, sharpened into a law with a proof technique. The measured law is $\varepsilon \approx 8.34\, D^{1.790}\alpha^{-6.04}$ (jackknife errors ≤ 0.041, rms ≤ 0.097; φ from a fixed-D sweep, ± 0.08; all fits). ε is *not* smooth in α: it jumps by factors up to 120 at prime-power entries, so any smooth ansatz is wrong from the start. (T117 subsequently resolved the same sweep cell by cell and *corrected* this reading: the entries themselves move ε *up*, and the factor-120 falls are the accumulated smooth bordering decay between sweep points — see the T117 subsection. The criticality and D-law statements of this paragraph are unaffected.) The continuum form is *exactly critical* — $\tilde{t}^{\mathsf{T}} T^{-1} \tilde{t}$ reaches 0.999975745, and Szegő–Kolmogorov says the pole is representable in the limit — with every measurement $\geq 1.1 \times 10^5$ above the declared Cholesky floor. The classical hit: in the Galerkin reading, $\tilde{t}$ represents the D-independent function $2\sinh(x/2)$, ε is the error of a piecewise-constant Galerkin method for it, and Aubin–Nitsche duality predicts $\varepsilon \sim D^{2s}$: the measured $\theta = 1.79$ gives $s \approx 0.90$, exactly the regularity a log singularity plus window edges would produce. The target inequality is now formulated: $\varepsilon(\alpha, D) \geq c_0\, D^{\theta}\alpha^{\varphi}$, uniformly across the jumps.

The remaining list after T116 — one inequality, one obstruction

Won: the exact pole (no truncation), the exact Riccati chaining, and flat cost per step. Broken: **[SUPPORT]** — the prime comb; there is no sparse faithful state, and the stripes drift with fill-in. Devalued: **[P2]** — a single boundary march needs no regrid, so the transport defect leaves the critical path. The basis is uncritical. What remains on the critical path:

(1) **[P1] as a two-variable law** — $\varepsilon(\alpha, D) \geq c_0\, D^{\theta}\alpha^{\varphi}$ with $\theta = 1.790$ and $\varphi = -6.04$ (fits), to be proved *across* the factor-120 jumps at prime-power entries; the proof technique is named (piecewise-constant Galerkin error for $2\sinh(x/2)$, Aubin–Nitsche duality, $s \approx 0.90$).

(2) **The comb as the honest support obstruction** — any bounded-state formulation must either carry the reflection stripes ($\Theta(\log^2 n)$ ceiling) or find a representation in which the comb is diagonal.

Disposition after T117 (next subsection): (1) is **theorem-shaped** — the Galerkin reading becomes four exact identities, the lower-bound direction acquires a certified two-level chain that loses no power of D, and the jumps acquire exact closed forms with the *opposite* sign: entries *raise* ε, the factor-120 falls were a sweep artefact, and the jump side-condition is *removed* from the ansatz. What is left of (1) is three named analytic lemmas about one symbol, two of them constants. (2) is untouched.

T117 (`epsilon_theorem_probe.py`, contract `EPSILON.THEOREM`) then took the inequality apart — keeping the one honest asymmetry visible everywhere: the classical Galerkin machinery produces *upper* bounds for approximation errors, and the target needs a *lower* bound.

7.16 The one inequality becomes theorem-shaped (T117)

T117 (`epsilon_theorem_probe.py`, contract `EPSILON.THEOREM`, 23/23, THEOREM-SHAPED) converted the T116 reading into identities and a certified chain. The direction problem is stated once and kept visible: Céa 1964, Aubin 1967/Nitsche 1968 and Bramble–Hilbert 1970 all bound a Galerkin error from *above* by an interpolation error; the missing lower-bound ingredient carries a classical name — the *saturation assumption* (Bank–Smith 1993; Dörfler–Nochetto 2002) — and is known to be an assumption, not a theorem. Every classical anchor below is therefore cited with the direction it actually gives.

The Galerkin identity, exact. Four identities, verified on six windows to 10^{-10} relative or better, none of them asymptotic: (i) $\tilde{t}$ is the cell functional of the *D-independent* function $f(x) = 2\sinh(x/2)$ (independent quadrature, rel 5.2×10^{-16}); (ii) the family is *exactly nested* — with P the block-averaging prolongation, $T_c = P^{\mathsf{T}} T_f P$ and $\tilde{t}_c = P^{\mathsf{T}} \tilde{t}_f$ (rel 2×10^{-14}) — so ε is a Galerkin best-approximation error of *one* bilinear form on a nested family, and its monotonicity under refinement is a *theorem*, not a fit; (iii) $\varepsilon = \min_v [\,1 - 2\tilde{t}^{\mathsf{T}} v + v^{\mathsf{T}} T v\,]$ with the two-level Pythagoras $\varepsilon_c - \varepsilon_f = \|u_f - P u_c\|^2_{T_f}$ (rel 6.2×10^{-11}); (iv) the dual-norm residual form $\varepsilon_c - \varepsilon_f = \|\tilde{t}_f - T_f P u_c\|^2_{T_f^{-1}}$, whence $\varepsilon > 0 \iff \tilde{t} \notin T(V_c)$: positivity is a *non-membership* statement — which is exactly why the quantitative version is hard, since a non-membership statement carries no rate by itself.

The lower bound, honestly. The crude psd-minorant route ($\varepsilon \geq 1 - \tilde{t}^{\mathsf{T}} G^{-1} \tilde{t}$ for $0 < G \leq T$) is measured dead: criticality forces any useful minorant to be sharp to relative 8.3×10^{-4} in the one direction $u = T^{-1}\tilde{t}$ — which is the same as knowing u. What survives is the *two-level chain*

$$\varepsilon_c \;\geq\; \varepsilon_c - \varepsilon_f \;=\; y^{\mathsf{T}} S y \;\geq\; \lambda_{\min}(S)\,\|y\|^2 \;\geq\; \lambda_{\min}(S)\,D_f^3 \max_j g_j^2/2 \;>\; 0,$$

with y the oscillation of the fine dual solution, S the T-Schur complement onto the oscillation space (Haynsworth 1968), and the last step the one-cell Payne–Weinberger 1960 cell-variance inequality used in its *lower* form — the only place where a classical approximation statement points the right way. Every link is an identity or a completed Cholesky: certified on all 19 nested pairs, bound/$\varepsilon \in [0.111, 0.185]$, everything at least 6.7×10^5 above the declared backward-error floor. The rate: $\theta'(\text{sum}) = 1.74$ against $\theta = 1.76$ (jackknifed fits) — a loss of 0.04 powers of D, i.e. *none*, because $\lambda_{\min}(S)$ *grows* under refinement (+0.20 per halving of D; log-versus-small-power undecidable on an 8× lever, and nothing in the chain rests on deciding it): the chain costs a constant, not a power (bound/ε spread 1.195 over the whole lever). The one-cell version costs a further 0.93 powers — the classical price of replacing a sum over cells by its largest term.

The jumps, exactly — and a correction to T116. Growing the window by one cell per end at fixed D is a pure prepend in frame A, so the bordered inverse gives the *exact* drop $\varepsilon(h{+}1) = \varepsilon(h) - r_0^2/s_0$ (Levinson 1947; rel 3.7×10^{-12} over consecutive steps). A prime-power entry at fixed window is a *corner* update supported on the two outermost Hankel anti-diagonals, of rank exactly $k_0 + 1$ (k_0 the corner distance), and Woodbury/Sherman–Morrison reproduces its effect on ε in closed form on all 23 measured entries (rel 2.1×10^{-11}). The direction *corrects* T116: all 23 entries move ε *up* (factors 1.0003–1.0506; for $k_0 = 0$ the update is $+\beta e_0 e_0^{\mathsf{T}}$ with $\beta > 0$, so an entry can never by itself lower ε at that corner distance). The T116 factor-120 falls were a *sweep artefact*: resolved cell by cell over 400 steps, the largest single-cell drop is a factor 0.9555, the entry share of the total log-drop is −0.3% *with the opposite sign*, and the falls are the accumulated smooth Levinson product between sweep points spaced 150–300 cells apart. The jump side-condition disappears from the ansatz; what a bound must survive is a smooth $\alpha^{-6.07 \pm 0.03}$ decay (fit; the α lever is short and bound by the compute cap $h \leq 1400$, declared as such).

The theorem candidate — and the three-lemma missing list

Candidate. Let $V_c \subset V_f$ be the reflection-odd piecewise-constant spaces on $(-\alpha, \alpha)$ with M and $2M$ cells, T the pole-free odd Weil form, $\tilde{t}$ the pole functional of $f(x) = 2\sinh(x/2)$. Assume (H1) $T_f \succ 0$ [certified per window]; (H2) $\lambda_{\min}(S) \geq \kappa > 0$ on the oscillation space [certified per window; measured to grow under refinement]; (H3) one coarse cell carries a non-vanishing discrete slope of the dual solution

[measured]. Then the four-line chain above gives $\varepsilon_c \geq \kappa\, D_f^3 G^2/2 > 0$; at fixed D the α-decrease per added cell pair is exactly r_0^2/s_0; and prime-power entries only raise ε — so the certified bound carries the full measured rate. Line-by-line attribution: the identities are exact algebra, (H1)/(H2) are completed Choleskys, the cell step is classical, the exponents are jackknifed fits.

What is still missing — three named analytic lemmas about one symbol, each with a classical address:

(1) a uniform lower bound on *one* local slope of the dual solution $u_f = T_f^{-1}\tilde{t}_f$ (H3 as analysis): the corner asymptotics of T^{-1} for a symbol with a log singularity — the Kac–Murdock–Szegő 1953 / Widom 1974 circle of results;

(2) a lower bound for $\lambda_{\min}(S)$ at *one* level (H2 as analysis): S is the form seen by oscillations, i.e. the symbol at the Nyquist frequency π/D, where the archimedean weight is largest — the measured monotonicity then carries it;

(3) the saturation constant: dropping ε_f costs the factor $1 - \varepsilon_f/\varepsilon_c$, measured in $[0.675, 0.744]$ — literally the Bank–Smith / Dörfler–Nochetto saturation assumption, bounded away from 0 by a constant, so it costs no rate.

Two of the three are *constants*, not rates. None of this touches the T116 **[SUPPORT]** comb obstruction — that is a separate question about *state compression*, not about ε.

Status after T118 (next subsection): (3) is **classically done** — saturation is an *identity* here, $\varepsilon_c - \varepsilon_f = y^{\mathsf{T}} S y$, plus the one hypothesis Bank–Smith themselves assume; (2) is **certified on windows and arithmetically reduced** — not through the symbol of S itself, which is a weighted *harmonic* mean of the aliasing pair and vacuous on the comb, but through the strengthened Cauchy–Schwarz shift onto the oscillation Gram; (1) is **corrected and open** — the corner exponent does not settle, because a log singularity is the boundary of all powers, and (H3) must be upgraded to a mean-square statement, which also repairs the chain's last step.

T118 (`symbol_lemmas_probe.py`, contract `SYMBOL.LEMMAS`) then measured the three lemmas against their classical addresses, one by one — and two of the three stand.

7.17 Two of the three lemmas stand (T118)

T118 (`symbol_lemmas_probe.py`, contract `SYMBOL.LEMMAS`, 36/36, TWO-OF-THREE) put each missing lemma against its classical address. The outcome, stated before the detail: the saturation lemma (3) is an *identity* here; the Schur-floor lemma (2) acquires a fully certified classical route on half the measured windows and an arithmetic reduction on all of them; the corner lemma (1) does *not* contradict its address — the address simply does not cover this symbol — and its failure *corrects* both an old exponent estimate and the last step of the T117 chain. What the theorem still needs is exactly one genuinely new analytic statement.

The harmonic versus the arithmetic aliasing mean. The two-level oscillation Schur complement of a Toeplitz form has an *exact* two-grid symbol (re-derived in closed form; local Fourier analysis, Brandt 1977/Hackbusch 1985; multigrid for Toeplitz, Fiorentino–Serra 1991), and it is the weighted *harmonic* mean of the symbol at a low frequency and at its Nyquist partner: $1/\sigma_S = \cos^2(\theta/2)/f(\theta+\pi) + \sin^2(\theta/2)/f(\theta)$. A harmonic mean needs *both* entries positive, and f is negative on a comb of dips at the window's own resolution scale: $\inf \sigma_S = 0$ on 14/14 (zone, level) rows — every pointwise route through σ_S is *vacuous* on the real symbol, not merely weak (a perturbative split recovers at most 2.6×10^{-5} of $\lambda_{\min}(S)$; the comb is non-perturbative). The classical reason is computed rather than asserted: $\lambda_{\min}$ at finite M is a Fejér–Riesz *moment* problem, not a pointwise infimum — the ground state of S *concentrates* on the negative set (3.3–43× the uniform share), and positivity is a cancellation of relative size 10^2–10^4 between the positive and negative parts of the symbol integral. Coarse-graining shows where the mass sits: the one-cell-averaged symbol is positive on 14/14 windows (a heuristic, declared as such). The growth is a *logarithm*, not a power: on an FFT-only lever of up to 128× in D the log model beats the power model (at $b = 0.44$–0.93 per $\log(1/D)$), exactly what the log-singular kernel demands (Kac–Murdock–Szegő 1953; Widom 1974). And the pole mass in the oscillation space is $\leq 2.1 \times 10^{-5}$ — an identity via the DTFT factor $|\sin(\theta/2)|$ — which is why the finite-section effect is absent there.

Saturation is an identity here — and the CBS shift rescues Lemma B. For a nested piecewise-constant pair, saturation is not an assumption but the *identity* $\varepsilon_c - \varepsilon_f = y^{\mathsf{T}} S y$ (rel 4.9×10^{-11}), so β is *computed*: $\beta \in [0.252, 0.336]$ over 20 triples, with the monotonicity in the favourable direction; what remains of Lemma C is the one hypothesis Bank–Smith 1993 themselves assume. Their strengthened Cauchy–Schwarz constant γ (Yserentant 1986) is then the lever that rescues Lemma B: the identity $\lambda_{\min}(L_z^{-1} S L_z^{-\mathsf{T}}) = 1 - \gamma^2$ (rel 2.1×10^{-15}) moves the symbol question off S and onto the oscillation Gram, whose symbol $\sigma_z(\phi) = \sin^2(\theta/2) f(\theta) + \cos^2(\theta/2) f(\theta + \pi)$ is the *arithmetic* mean of the same aliasing pair — the low-frequency negativity that poisoned the harmonic mean is suppressed *quadratically* by $\sin^2(\theta/2)$. The route is then certified: $\inf \sigma_z > 0$ on 7/14 windows (systematically: every window with $\alpha \le 1.30$ and $h \ge 200$), with the certified bound recovering up to 52% of the measured $\lambda_{\min}(S)$; on *all* 14 windows the reduction $\lambda_{\min}(S) \ge (1 - \gamma^2)\, \lambda_{\min}(T_h(\sigma_z))$ holds at 32–76% sharpness — Lemma B is reduced to the positivity of a plain Toeplitz section of an explicit symbol; and on the long FFT lever (M up to 15 680, no matrix ever formed) the certified floor $\inf \sigma_z$ *rises* logarithmically and *crosses zero* on 3/5 zones — the failing windows were *under-resolved*, not obstructed.

The corner lemma corrects itself — and repairs the chain. The classical statement is an edge exponent: for a symbol vanishing like $|\theta|^{2s}$ the finite-section solution vanishes like $\mathrm{dist}(\cdot, \mathrm{edge})^s$. The estimator is calibrated on a model class where this is exact ($|2\sin(\theta/2)|^{2s}$, lags in closed form; the Kac–Murdock–Szegő 1953 closed-form inverse as the $s = 0$ control). On the *real* symbol the corner exponent *drifts*: $+0.238$ per halving of D (3–7.4σ) — $\log(1/|\theta|)$ is the boundary of all powers, so there is no asymptotic (s, κ) to cite. (The old T108/T109 24-cell exponent was doubly mis-estimated; its apparent stability was the cancellation of two errors.) The (H3) correction: the single-cell form is mis-dimensioned — the concentration factor grows like $h^{0.99}$, the slope mass sits on a fixed *fraction* of cells — and the repair is at the T117 chain's last step: the rate lives on $\|y\|^2$ (the stage-two bound carries $D^{1.751}$ against $\varepsilon \sim D^{1.770}$ — lossless; the single-cell stage costs $D^{2.74}$), so (H3) must be a mean-square statement.

The theorem after T118 — five unconditional links, two hypotheses

Candidate (V2). Five unconditional links — the saturation identity $\varepsilon_c - \varepsilon_f = y^{\mathsf{T}} S y$, the CBS identity $\lambda_{\min}(L_z^{-1} S L_z^{-\mathsf{T}}) = 1 - \gamma^2$, the isometry ordering onto $T_h(\sigma_z)$, Grenander–Szegő in the lower direction, and the certified Cholesky floors — give, for every window the probe touched,

$$\varepsilon(\alpha, D) \;\ge\; (1 - \gamma^2)\, \lambda_{\min}\big(T_h(\sigma_z)\big)\, \|y\|^2 .$$

Two hypotheses close it: **(H1)** γ bounded away from 1 uniformly in (α, D) — the standard CBS hypothesis of the two-level literature, measured $1 - \gamma^2 \ge 0.181$ with the favourable trend; **(H2)** $\|y\|^2 \ge c\, D^{1.75}$ uniformly — (H3) upgraded to mean square, and *the* one genuinely new analytic statement the theorem needs, not importable from the Kac–Murdock–Szegő/Widom power-law corner theory because this symbol is log-singular with atoms.

The per-lemma missing lists. **L-B** (two points, both *arithmetic*): $\inf \sigma_z > 0$ for all $D < D_0(\alpha)$ — D_0 set by the comb depth against the Nyquist-band mass, an atom-counting question — and narrow-dip positivity of finite sections. **L-C** (one point): γ-uniformity, i.e. (H1). **L-A** (two points): the mean-square form (H3′), and the Fisher–Hartwig extension of the corner theory to log-singular symbols with atoms.

Disposition after T119 (next subsection): the first L-B point is now a **theorem** with an explicit $D_0(\alpha)$ and a universal constant; the second is diagnosed as a moment problem, with one window newly certified by the large sieve; (H2) is proven *not derivable* from the links above — genuinely new content, as suspected — and reduced, through the mean-square form (H3′), to a single scalar that is itself an exact identity, $\kappa_{\mathrm{end}} = 1/(1 + R)$; what remains of the whole list is one discrete Harnack inequality plus two constants.

T119 (`oscillation_mass_probe.py`, contract `OSCILLATION.MASS`) then took the two halves that remained — the arithmetic half of Lemma B and the oscillation-mass half (H2)/(H3′) — and closed the first as a theorem.

7.18 The arithmetic half closes; one Harnack inequality remains (T119)

T119 (`oscillation_mass_probe.py`, contract `OSCILLATION.MASS`, 27/27, ARITHMETIC-DONE) is the part where the comb, the standing obstruction since T116, becomes an explicit and *bounded* object. The outcome, stated before the detail: the arithmetic half of Lemma B is a theorem with an explicit $D_0(\alpha)$; the narrow-dip point is diagnosed as a moment problem, with one window newly certified by the large sieve; (H2) is proven *not derivable* from the existing identities — and the sharpest identity of the run reduces the whole oscillation-mass half to one discrete Harnack inequality with a classical address.

The D_0 theorem, with a universal constant. The lag sequence splits exactly as $c = c_{\mathrm{arch}} + c_{\mathrm{comb}}$, and the comb half has a *closed-form* symbol: every prime-power atom (u_j, μ_j) contributes exactly one cosine at the frequency of its own lag, split over the two nearest integer lags by linear interpolation — and under the aliasing average σ_z the *parity* of the lag index decides everything (an even lag survives at full strength, an odd one is multiplied by $-\cos\theta$). Two consequences, both elementary and both new: the whole comb is bounded *uniformly in D* by the atom count, $|\sigma_z^{\mathrm{comb}}| \le \Xi(\alpha) = \sum_{n \le e^{2\alpha}} 2\Lambda(n)/\sqrt{n} \sim 4e^{\alpha}$ (Chebyshev); and the archimedean growth is an *identity*, $\inf \sigma_z^{\mathrm{arch}} = \log(1/D) + B$ with slope exactly one and a *universal* $B = -1.0474$ (drift $\le 3.1 \times 10^{-4}$ over $\alpha = 0.98$–4.70; measured slope 1.0030 ± 0.0012). Their conjunction is the theorem:

$$\inf \sigma_z > 0 \quad \text{for all} \quad D < D_0(\alpha) = \exp\big(-(\Xi(\alpha) + B)\big),$$

verified deep on a 1024× FFT-only lever (M up to 65 536, sixteen zones to $n = 11\,981$, no matrix ever formed).

The two-regime correction, and the large sieve's one win. The deep run corrects T118's reading: there are two regimes. Narrow zones ($\alpha \le 2.00$, 5/16) cross zero *certified* at the frame's own resolution; wide zones ($\alpha \ge 2.22$, 11/16) also cross — the theorem is not in doubt — but at $M \sim 10^{13}$–10^{187}: a defect of the *constant*, not of the theorem. The narrow-dip point (L-B.2) becomes a certifiable criterion through the large sieve (Montgomery–Vaughan 1974, used in the honest direction: an upper bound on the L^2 mass a polynomial of degree $< h$ can place on the dips, which is exactly what a lower bound on $\lambda_{\min}$ needs). It wins *once* — a certified positive section floor $+0.396$ on a window where $\inf \sigma_z = -0.349$, the series' first certificate of section positivity *without* pointwise positivity — and it dies structurally for $\alpha \ge 2.5$, where a single Fejér bump fits inside one dip: the obstruction is a moment problem, not a localisation problem. A by-product is proved exactly: coarse-graining can only raise the section floor, $\lambda_{\min}(T_h(\sigma * K)) \ge \lambda_{\min}(T_h(\sigma))$ by unitary averaging — the direction *opposite* to T118's heuristic reading, which is thereby retired.

(H2) is genuinely new — proven, not suspected. $\|y\|^2$ is the squared L^2 distance of the fine piecewise-constant solution from the coarse space, so (H2) is an *inverse* approximation statement. Its structure is dissected: the mass is distributed (corner share 0.18–0.42, middle 0.19–0.38, inner half 0.36–0.48 — no single-cell carrier), the exponent bookkeeping closes ($\varepsilon \sim D^{1.761}$, $\|y\|^2 \sim D^{1.961}$, $\lambda_{\min}(S)\|y\|^2 \sim D^{1.757}$ — the second stage loses no power), and the Levinson structure of the solution is verified in full (the corner identity $u_0 = -\sqrt{2}\,\rho/E$ to 2.8×10^{-8}, the mass budget $1 - \varepsilon = \sum_n \rho_n^2/E_n$ to 2.5×10^{-13}, exactly one negative pivot). The derivability question is answered *no*: the energy route is valid and empty — it returns $\varepsilon \ge r\,\varepsilon$ with $r < 1$, a tautology — so (H2) is genuinely new analytic content, exactly as T118 suspected.

The constant becomes an identity. The sharpest result of the run removes the constant from (H3′): Cauchy–Schwarz is nearly sharp, the corner increments are monotone on 100% of the cells, and the end-cell share $\kappa_{\mathrm{end}} = 0.4906$–$0.5093$ is flat over 50 rows and 15 ladder pairs and *exponent-blind* — it drifts by ≤ 0.0019 per level while the corner exponent itself creeps by $+0.046$–0.242 per level, a factor 128 less. And it is not a measured constant at all: for a monotone corner profile,

$$\kappa_{\mathrm{end}} = \frac{1}{1+R}, \qquad R = \frac{\sum_j |a_j|}{\sum_j |w_j|},$$

exactly (verified to 1.1×10^{-16}; a telescoping sum), with w_j the within-cell and a_j the across-cell increment of u and R measured 0.9634–1.0349. The one hypothesis left inside the chain is therefore

a *discrete Harnack inequality* for the increments of a log-singular Toeplitz inverse — the profile does not oscillate at the mesh scale — and not a bare constant.

The theorem after T119 — fourteen links, three defects

Candidate (V3). Fourteen links close the chain $\varepsilon_c \geq (1-\gamma^2)\,\lambda_{\min}(T_h(\sigma_z))\,\|y\|^2 > 0$ for every window the probe touched: eight identities (among them, new in T119, the closed-form comb symbol, the parity rule, the Levinson corner value with its mass budget, and $\kappa_{\text{end}} = 1/(1+R)$), one arithmetic bound (the atom count $\Xi(\alpha)$, uniform in D), one certified growth law (the archimedean identity with universal B), one theorem (the explicit D_0), the isometry, Cauchy–Schwarz, and the chain itself — five of the fourteen new in T119.

The defect list, shortest form — three items:

(D1) the discrete Harnack inequality $\sum_j |a_j| \leq R_{\max} \sum_j |w_j|$ on the corner cells — the one genuinely new analytic statement, with a classical address (Widom 1974; Trench 1964 for the explicit inverse; Böttcher–Silbermann); *any* finite $R_{\max}$ gives $\kappa_{\text{end}} \geq 1/(1+R_{\max}) \approx 0.49$;

(D2) the D_0 constant for wide α — the theorem is unconditional, but at the frame's own resolution the pointwise floor is negative for $\alpha \gtrsim 2$, and the section route that would cover it is a Fejér–Riesz moment problem;

(D3) γ-uniformity — (H1), inherited from T118 unchanged.

Distance to a paper, honestly: as a *conditional* lemma paper — under (D1) and (H1), $\varepsilon \geq c\,D^{\theta}$ with an explicit c — the material is essentially complete, and the missing work is writing, not discovery; an unconditional version needs exactly the Harnack statement. The chain is classical numerical analysis end to end and contains no zeta input anywhere.

Disposition after T120 (next subsection): (D1) is *explained* — $R \approx 1$ is parity symmetry, and the summed inequality follows unconditionally from one sign of the corner increments by a two-line pairing argument, while the per-cell version is provably false — so the one hard statement is replaced by a strictly weaker and far more classical hypothesis; but (D2) is enlarged (the frame consumes broad windows by construction, and the unconditional D_0 route covers not one handover of the real ladder) and (D3) loses ground as a uniformity claim ($1-\gamma^2$ drifts down with α): the defect count goes three → four.

T120 (`harnack_probe.py`, contract `HARNACK`) then attacked that defect list head on — and explained the Harnack core while sharpening the side defects.

7.19 The Harnack core is proof-shaped; the wide-window defects sharpen (T120)

T120 (`harnack_probe.py`, contract `HARNACK`, 21/21, HARNACK-EXPLAINED) is the part where the one hard statement of V3 stops being an inequality about two families and becomes a two-line consequence of a symmetry. The outcome, stated before the detail: $R \approx 1$ is *parity symmetry* — the two increment families are one sequence offset by a single fine cell — with an unconditional $|R-1| \leq 0.047$ certificate; two structural routes to the remaining hypothesis are refuted, and the per-cell version of the inequality is provably *false*; and the honest defect count rises from three to four, because the α-audit shows the unconditional D_0 route covers not one handover of the real ladder and the CBS constant decays with the width.

The parity symmetry, and the certificate. The two families of (D1) are not two objects: with $v_i = u[i+1] - u[i]$ the single increment sequence of the *same* solution $u = T^{-1}\tilde{t}$, the within-cell and across-cell increments are the even and the odd half, $w_j = v_{2j}$ and $a_j = v_{2j+1}$ — offset by exactly one fine cell. If v has one sign on the corner cells (measured at 100% of them on every one of 3915 rows), the difference of the two parity sums is a sum over *disjoint* adjacent pairs, so

$$|R-1| \;\leq\; \frac{\sum_j |v_{2j+1} - v_{2j}|}{\sum_j |v_{2j}|} \tag{C1}$$

unconditionally — no monotonicity, no smoothness. Measured over the whole sweep, (C1) gives $|R-1| \leq 0.04745$ against a measured 0.03485, and it dominates the measurement on all 3112 sign-pure rows, as it must. If v is moreover monotone the left side telescopes to a boundary term over a bulk sum, $|R-1| \leq |v_{2n_C} - v_0| / \sum_j |v_{2j}| \leq (C-1)/n_C$ (C2) — recorded as *conditional* (the

monotone share is only 0.50–0.96) and kept as the explanation of the rate: R sits at 1 because a shift by one fine cell is a boundary perturbation, and R is resolution-blind because refining raises n_C and shrinks the boundary term at the same rate. The certificate itself confirms the rate: (C1) decays like $n_C^{-0.58\pm0.05}$ (a fit). The brute-force sweep (155 windows × resolutions × corner fractions, 3915 rows) keeps $R \in [0.9098, 1.1011]$ in the corner regime with sign purity exactly 1.0000; the single excursion above 1.1 sits at the clamped minimal corner $n_C = 4$, exactly where the boundary-term mechanism predicts it, and for $n_C \geq 8$ the window certificate is $\kappa_{\text{end}} \geq 0.480827$. One negative result belongs to this positive one: the *per-cell* Harnack inequality is *false* — $|a_j/w_j|$ reaches 4.8×10^3 — so no cellwise comparison can be true; only the summed form holds, which is exactly the form (C1) proves.

Two structural routes refuted. Both obvious routes to the one remaining sign hypothesis are now closed, each by a measurement. First, the 40×40 corner block of T^{-1} is entrywise positive on 18/18 (zone, level) rows without exception — and the checkerboard alternative is refuted exactly (0.5000, a coin flip) — but it is *not* TP2 (the share of non-negative 2×2 minors is only 0.51–0.88), so the Gantmacher–Krein oscillation-matrix route is closed; nor is T an M-matrix. Second, differencing $Tu = \tilde{t}$ once gives an *exact* discrete boundary-value problem for the increments alone, $Kv = s - u_0 G\mathbf{1}$ with $K = D_1 T L$ (relative residual 2.4×10^{-12}), with a smooth, positive right-hand side — the correct classical shape for a maximum-principle argument — but K^{-1} is *not* entrywise positive (positive share 0.44–0.51, minimum entry -333): the discrete maximum principle is not unproved, it is *false*. The reason is visible in the solution itself: u is monotone at the corner and oscillatory in the bulk — that is the prime-power comb. The corner block of K^{-1} *is* entrywise positive (0.9961–1.0000), so a localised statement is not excluded — but the corner components of v contract full rows of K^{-1} carrying a genuine cancellation (no-cancellation ratio 10^{-7}–10^{-3}), so what the sign hypothesis needs is a corner-localised Fisher–Hartwig decay estimate (Widom 1974; Böttcher–Silbermann), not a sign pattern.

The α-audit: (D2) is not vacuous but total. The frame's window is not a free parameter: in frame A the handover at the atom $u_k = \log n_k$ uses $D = g_k/(2\nu)$ and $M_o = \lceil u_k/D \rceil + 1$, hence $\alpha_o = u_k/2 + O(D)$ *exactly* (maximum deviation 1.5×10^{-2} over all 732 admissible handovers) — the consumed half-width *is* half the log of the zone, and only 1.4% of the handovers have $\alpha \leq 2$. Because $B = -1.0474$ is a property of the archimedean kernel alone and hence universal (spread 1.2×10^{-6}), the criterion $\log(1/D_{\text{frame}}) \geq \Xi(\alpha) + B$ can be evaluated on the whole zone table: it holds on exactly 3 of 1492 zones ($n = 2, 3, 4$), i.e. for $\alpha \leq \alpha^* = 0.693$ — *below* the first admissible handover $n = 7$. Not one step of the induction is covered by the unconditional D_0 route (the resolution it would need explodes to $M \sim 10^2$–10^{188}). The fallbacks, measured and labelled: the comb supremum is only 0.038–0.603 of its certified cap Ξ — an incoherence gain of up to 1.7×, but a grid supremum, hence a measurement and never a certificate (a certified version would be an exponential-sum estimate over prime powers) — and the large-sieve narrow-dip floor (Montgomery–Vaughan 1974) is positive on 2/6 surveyed zones, at $n = 32$ where $\inf \sigma_z = -0.45$ — the dip route genuinely buys something — and dies beyond $\alpha \approx 2$.

The γ formula, and the wrong direction. The CBS constant has an exact local symbol identity: with $f_1 = f(\theta)$, $f_2 = f(\theta + \pi)$, $\sigma_c\sigma_z - \tau^2 = f_1 f_2$ (relative residual 3.9×10^{-16}), hence $1 - \gamma(\theta)^2 = f_1 f_2/(\sigma_c\sigma_z)$ — (D3) is no longer a bare measured constant but a symbol infimum, the same class of object as $\inf \sigma_z$. Two honest caveats, both measured: the naive symbol infimum is 25–210× *below* the matrix constant (the finite odd section never sees the worst aliasing pair — a finite-section transfer is needed, not the infimum), and the measurement itself moved: $1 - \gamma^2$ *falls* with α (fitted slope -0.297 ± 0.013, range 0.079–0.283, flat in the resolution at a fixed zone) — T119's ≥ 0.181 is reproduced on its own α-range but is *not* uniform.

The theorem after T120 — eighteen links, four defects

Candidate (V4). Eighteen links (7 identities — among them, new in T120, the increment system $Kv = s - u_0 G\mathbf{1}$ and the symbol identity $\sigma_c\sigma_z - \tau^2 = f_1 f_2$ — 4 certified, 2 classical, 2 measured, and the chain), and the one hard statement of V3 is *replaced*: (D1) is no longer a bare inequality but a proved

two-line implication — the pairing certificate (C1), unconditional given one sign of the corner increments. What a lemma paper could claim today: *let T be the finite odd Weil form on a window, $u = T^{-1}\tilde{t}$, and suppose the increments of u have one sign on the outer n_C coarse cells; then $|R-1| \le \delta(n_C)$ with the explicit pairing ratio δ, hence $\kappa_{\text{end}} \ge 1/(2+\delta)$ and $\|y\|^2 \ge (u_{\text{end}} - u_0)^2/(2n_C(2+\delta)^2)$* — everything after "suppose" is proved; measured, $\delta \le 0.0964$ for $n_C \ge 8$, falling like $n_C^{-0.58}$.

The defect list — four items (was three):

(E1) *one sign of the corner increments* — the single hypothesis of (C1)/(C2), measured on 100% of the corner cells on all 3915 rows; the two obvious structural routes are refuted (the corner of T^{-1} is positive but not TP2; the global discrete maximum principle is false), so what remains is a corner-localised Fisher–Hartwig estimate (Widom 1974; Böttcher–Silbermann);

(E2) *a uniform bound on the pairing ratio δ* — a discrete gradient estimate for the increments of $T^{-1}\tilde{t}$, strictly weaker than the old Harnack statement (any finite δ suffices, $\kappa_{\text{end}} \ge 1/(2+\delta)$);

(E3) *D_0 for the whole ladder* — (D2), enlarged: the frame consumes $\alpha = u_k/2$ by construction and the criterion fails at $\alpha^* = 0.693$, below the first admissible handover; the open routes are a certified comb-incoherence bound or the large-sieve dip geometry, which works on the first zones and dies beyond $\alpha \approx 2$;

(E4) *the α-dependence of γ* — (D3) with the direction now known: $1-\gamma^2$ shrinks like $e^{-0.30\alpha}$ (a fit), so uniformity is not merely unproved but questionable in the direction that matters.

Honest movement: the one hard statement got much easier — a two-line implication with one classical hypothesis — and the side defects got sharper and both got worse; the count went three → four, and that is the honest direction.

Disposition after T121: (E3) is retired *as stated* — the chain link is a section eigenvalue, not a symbol infimum, and the section stayed positive on the whole real ladder — and is replaced by the section lemma (E3*) with the Hankel term; (E4) is replaced by one named poly-log, (E4*); (E1) and (E2) keep their kind, with the corner certificates restated at $n_C = h/16$ (T121 found the $h/8$ convention used here too wide); and the symbol-to-section transfer this list implicitly relied on is refuted (the odd sector's Hankel term). The next subsection reports the measurements.

7.20 Section positivity survives the real ladder; the net balance names one poly-log (T121)

T121 (`wide_window_probe.py`, contract `WIDE.WINDOW`, 21/21, WIDE-RESTRUCTURED) asks the question the T120 defect list left sharpest: does the σ_z route survive the *real* ladder — the one whose window grows with the zone, $\alpha_o = u_k/2$? It does, but not as the statement T120 wrote down. The route survives as a *section* statement and dies as a *symbol* statement; the net α-balance of the whole chain loses only a single poly-log to a demand that itself falls like α^{-6}, and that poly-log splits without residue into two repairable steps; one link of the chain, filed by T120 as classical, is honestly refuted on the way.

Section positivity instead of symbol positivity. The chain never needed $\inf \sigma_z > 0$; it needs $\lambda_{\min}$ of the *section* $T_h(\sigma_z)$ at the frame's own resolution. Over 16 frame windows of the real ladder ($n = 7 \dots 283\,303$, $\alpha = 0.985 \dots 6.277$, $M_{\text{frame}} = 118 \dots 431\,162$, $h = 29 \dots 107\,790$) the section is *positive on* 16/16 while the symbol infimum is negative on 8 of them — and where it is negative, the section eigenvalue sits at 0.64–3.23 $\times\, |\inf \sigma_z|$ on the *positive* side. Direction kept honest: the dense rows ($h \le 1700$) carry a shifted-Cholesky certificate with the Wilkinson–Higham floor; the large rows are matrix-free FFT-Lanczos *measurements* (a Ritz value is an upper bound for $\lambda_{\min}$, so it can refute positivity but never prove it). The mechanism is quantified at its classical address — the Christoffel function (Szegő 1939; Grenander–Szegő 1958, Ch. 5): a degree-h polynomial cannot concentrate below the Fejér bandwidth $2\pi/h$, and every comb dip is only 0.156–0.720 of one Fejér width. The certified per-dip moment budget $\lambda_{\min} \ge \text{thr} - \sum_j (\text{thr} + \delta_j)\, w_j\, (h+1)$ is positive on 8/16 rows (up to $\alpha = 3.446$, sharpness 0.20–0.76) and tears above, for a computed reason: the dip *count* grows like $e^{2.41\alpha}$ (a fit) against the budget's factor h. So (E3) is reduced from a symbol-infimum requirement that no ladder step satisfies to a *section lemma* the measurement satisfies everywhere: a Christoffel lower bound surviving $O(e^{\alpha})$ Fejér-subresolved dips — a Szegő/Fisher–Hartwig question

about the comb, no longer a symbol question.

The net α-balance — the core number of the part. Every factor of the chain is measured on one two-dimensional (D, α) lever (14 zones × 3 resolutions, 42 rows). The individual links do fall with α, exactly as T120's (E4) recorded: $1-\gamma^2 \sim \alpha^{-0.79}$, $\lambda_{\min}(A_z) \sim \alpha^{-1.07}$, $\|y\|^2 \sim \alpha^{-5.63}$, so the assembled supply falls like $\alpha^{-7.49}$. But the *demand* falls almost as fast ($\alpha^{-5.93}$ on this lever; T116 quoted -6.04): 79% of the decay (E4) complained about is the demand's own. What is left is the only α deficit of the chain,

$$\mathrm{SUPPLY}/\varepsilon_c \;\sim\; D^{-0.177\pm0.096}\,\alpha^{-1.565\pm0.135}$$

(fits; the measured ratio is 4.9×10^{-3}–0.15 over $\alpha = 0.98$–6.31) — net-negative in α by one to two powers (the refit per resolution lands in $-1.88\ldots-1.62$ — a range, not a decimal) but *uniform in* D: the D exponent is zero within its own band, so refining the resolution neither helps nor hurts. The deficit is then decomposed *exactly*: the identity $y^{\mathsf T}Sy/\mathrm{SUPPLY} = r_{\mathrm{ray}}\, r_{\mathrm{cbs}}/(1-\gamma^2)$ holds to 2.8×10^{-14}, so $\alpha^{-1.57}$ splits without residue into $\alpha^{-1.02}$ from the *Rayleigh step* (using $\lambda_{\min}(A_z)$ on a y that is nowhere near the minimal eigenvector; measured slack 1.29–13.19), $\alpha^{-0.51}$ from the CBS step (measured slack 3.42–16.67) and $\alpha^{-0.04}$ from discarding ε_f — *nothing* of it sits in the value of γ. The named rebuild is therefore structural, not a new constant: bounding $y^{\mathsf T}A_z y$ directly (legitimate, because y is the oscillation part of the solution, not an arbitrary vector) recovers 65% of the deficit by itself, and the CBS step's own slack covers the rest.

Link (5) refuted: the Hankel term. The section comparison $\lambda_{\min}(A_z) \geq \lambda_{\min}(T_{h_c}(\sigma_z))$ — which T120's assembly used as a classical Grenander–Szegő transfer — is *false* on 42/42 rows (ratio 0.357–0.916, drifting with the window). The reason is structural and was hiding in the odd coordinates: $A_z = Z^{\mathsf T}(\mathrm{Toeplitz} - \mathrm{Hankel})Z$, and Grenander–Szegő carries the even Toeplitz part only; the reflection Hankel term is not sign-definite. A defect made visible rather than created: the section lemma (E3*) must be about A_z, not about $T_{h_c}(\sigma_z)$.

The corner sign, and the inconclusive Fisher–Hartwig comparison. The one-sign run of the increments covers the corner region the (C1)/(C2) certificates need on 24/24 (zone, resolution) rows at $n_C = h/16$ (run share 0.21–0.42 of the section against the required 0.125 — a safety factor of 1.68–3.38, the sign uniformly negative); at the $h/8$ convention T119/T120 used it holds on only 21/24, so the certificates are restated at $h/16$ — which costs nothing, because δ grows like $n_C^{+0.29}$ (a fit): halving n_C *improves* κ_{end}. The α-slope of the run share is -0.0258 ± 0.0042 (a fit); extrapolated linearly, the $h/16$ margin would be exhausted only near $\alpha \approx 11.7$ (an extrapolation, not a prediction). The Fisher–Hartwig comparison itself is *inconclusive, and that is the result*: the local exponent of the corner row of T^{-1} drifts by $-0.008\ldots+2.934$ between dyadic windows where the log-corrected FH shape would allow only -0.120 — the corner is in no asymptotic power regime at any resolution a factorisation reaches, and an FH exponent quoted from these numbers would be a fit masquerading as a structure. What the block does deliver: the no-cancellation ratio 4.6×10^{-8}–7.1×10^{-5} settles that (E1) can *never* be a sign-pattern argument, and the minimal sufficient statement (E1′) is written out as an explicit head-versus-tail inequality (head share $1/(1+1.63)$ to $1/(1+11.73)$ of the mass at $J = h/16$), with the tail exponent as the one unknown a Widom-type estimate must supply.

δ over the full ladder. The pairing quotient stays in 0.0126–0.1331 over 360 rows spanning $n = 7\ldots299\,983$ and three corner fractions — always below 1, no outlier — so $\kappa_{\mathrm{end}} \geq 1/(2+\delta) \geq 0.4688$ everywhere measured, and $\delta \sim n_C^{+0.29}$ confirms the $h/16$ restatement is free.

The theorem after T121 — the count stays four, every entry changes kind

Candidate (V5). Nineteen links: the identities (Schur/Haynsworth, CBS/Yserentant, the T120 symbol identity, the aliasing form of σ_z, $\kappa_{\mathrm{end}} = 1/(1+R)$), the certified pieces (the pairing certificate (C1); the shifted-Cholesky section floors; the Christoffel/Fejér per-dip budget up to $\alpha = 3.446$), the classical steps (CBS + Rayleigh, lower; Cauchy–Schwarz, lower), the measured/fitted entries (section positivity on the ten Lanczos rows; the one-sign run; the net balance) — and one link *refuted*: (5), the symbol-to-section transfer, killed by the odd sector's Hankel term.

The defect list — four items, each with a changed kind:

(E1) *one sign of the corner increments* — unchanged in kind, sharpened twice: the certificates move to $n_C = h/16$ (clean on all rows there, only 21/24 at $h/8$; the move is free since $\delta \sim n_C^{+0.29}$), and (E1′) is now an explicit head-versus-tail inequality whose tail exponent a corner-localised Fisher–Hartwig/Widom estimate must supply — the FH comparison is inconclusive at reachable h, and no sign-pattern argument can ever do the job (no-cancellation ratio 4.6×10^{-8}–7.1×10^{-5});

(E2) *a uniform bound on the pairing ratio* δ — unchanged: no outlier over 360 rows to $n = 299\,983$, $\delta = 0.0126$–0.1331, any finite δ suffices;

(E3*) *the section lemma, with the Hankel term* (replaces (E3)) — $\inf \sigma_z > 0$ at frame resolution is *retired as a requirement*: the chain link is a section eigenvalue, positive on 16/16 frame windows up to $\alpha = 6.277$, $h = 118\,115$, including 8 windows with a negative symbol infimum. What replaces it is strictly smaller but has two parts: (a) a Christoffel-function lower bound surviving $O(e^{\alpha})$ Fejér-subresolved dips (the certified budget delivers it for $\alpha \le 3.446$ and tears above); (b) the odd sector's Hankel term, which refutes link (5) as stated — the lemma must be about $A_z = Z^{\mathsf{T}}(\text{Toeplitz} - \text{Hankel})Z$, not about $T_{h_c}(\sigma_z)$;

(E4*) *one residual poly-log in* α (replaces (E4)) — the net balance carries $\alpha^{-1.565\pm0.135}$ and $D^{-0.177\pm0.096}$: net-negative by one to two powers of α (per-resolution refits $-1.88 \ldots -1.62$), *uniform in* D. Strictly better than (E4) as T120 wrote it: 79% of the supply's α-decay is the demand's own, and the deficit sits in the Rayleigh step ($\alpha^{1.02}$) and the CBS step ($\alpha^{0.51}$), none of it in γ — the address is a structural lower bound for $y^{\mathsf{T}} A_z y$, which alone recovers 65%.

Honest movement: the counter stays at four — the target of getting back below three was not met, because the residual α deficit is real — but every entry changed kind, and one link recorded as classical was found refuted, a defect made visible rather than created.

Disposition after T122: both named repairs landed. Link (5) is retired by an *identity* — $A_z = W^{\mathsf{T}}\text{Toeplitz}_M(c)\,W$ with $W = BZ$ isometric, so the Hankel term is the reflection half of an isometry and not an error term — and the certified band bound that replaces it is non-vacuous to $\alpha = 6.31$, where the Christoffel budget tore at 3.45; the Rayleigh step is made structural and is *sharp* (residual slack 1.00–1.03, flat in α); the certified deficit halves to $\alpha^{-0.73}$, exactly uniform in D, and with the measured CBS coupling the balance closes to within the chain's own ceiling. (E3*)/(E4*) are thereby absorbed into one band-margin estimate (F3) plus one structural CBS estimate (F4). The next subsection reports the measurements.

7.21 The Hankel term is an isometry half; the certified deficit halves (T122)

T122 (`net_balance_repair_probe.py`, contract `NET.BALANCE.REPAIR`, 20/20, NET-IMPROVED) attacks the two worst-case steps the T121 decomposition named — the Rayleigh step and the link-(5) transfer — and both repairs land. The headline is structural: the odd sector's Hankel term, the object that refuted link (5), was never an error term to be estimated.

The Hankel repair is an identity, not an estimate. The compressed Hankel term has a closed form, $(Z^{\mathsf{T}}HZ)_{jk} = b_{j+k}$ with $b_s = (c_{K-2s} - 2c_{K-2s-1} + c_{K-2s-2})/2$ — half a second difference of the lag sequence, exactly like the Toeplitz-side lags — verified to 7.0×10^{-16} on all 36 (zone, resolution) rows. So the oscillation injection Z suppresses *both* halves by the same DTFT factor $|2\sin(\theta/2)|^2$: the measured norm ratio is 0.123–0.781, i.e. $O(1)$ — the T118 mechanism does not make the Hankel term small. The honest anatomy (the archimedean half numerically of rank 10–14, the Hartman/Peller smoothing; the comb half corner-localised on few anti-diagonals) is structure, not a bound: the valid Weyl floor $\lambda_{\min}(T_{h_c}) - \|Z^{\mathsf{T}}HZ\|$ is positive on only 12/36 rows (up to $\alpha = 2.44$) and nowhere near sharp. The valid replacement for link (5) is an *identity*: with $W = BZ$,

$$A_z \;=\; W^{\mathsf{T}}\,\text{Toeplitz}_M(c)\,W, \qquad W^{\mathsf{T}}W = I \;\; \text{(exact to 0.0)}$$

— the Hankel term *is* the reflection half of an isometry. From this and a certified cell envelope ($g_{\text{thr}} = (\text{thr} - \ell)_+ \ge 0$; Parseval is exact because $|Wv|^2$ has degree $< M$) follows the certified band bound

$$\lambda_{\min}(A_z) \;\ge\; \text{thr} - \lambda_{\max}(G), \qquad G = W^{\mathsf{T}}\text{Toeplitz}_M(g_{\text{thr}})\,W \succeq 0,$$

non-vacuous on 36/36 rows up to $\alpha = 6.31$ — where the T121 Christoffel budget tore at $\alpha = 3.45$ — recovering 0.429–0.995 of the true $\lambda_{\min}(A_z)$; the $e^{2.41\alpha}$ dip count no longer enters anywhere (measured gain over the per-dip Nikolskii budget: $8.4\text{–}1.1 \times 10^2$). The honest caveat is where the threshold sweep puts its optimum: $\mathrm{supp}(g) = 0.95\text{–}1.00$ of the circle — one certified end of a one-parameter family whose other end (small thr) is the dip budget that tore.

The structural Rayleigh step is sharp. "y is high-band" is a statement about Wy at the window resolution (99.3–100.0% of its mass above $|\theta| = \pi/2$, with $\|Wy\|^2 = \|y\|^2$ exact), not about y itself (0.1–1.7% high-band in the coarse coordinates — the oscillation lives in the injection, not in the coefficients). The same certified band identity, evaluated on y instead of over the whole oscillation space — $y^{\mathsf{T}} A_z y \geq \mathrm{thr}\|y\|^2 - y^{\mathsf{T}} G y$ — is positive on 36/36 rows, never exceeds the truth, and beats the worst-case product $\lambda_{\min}(A_z)\|y\|^2$ by a factor 1.28–18.90: the slack it removes is exactly T121's r_{ray}, growing like $\alpha^{+0.83}$, and the residual slack of the structural bound is $q_{\mathrm{str}} = 1.00\text{–}1.03$ with drift $\alpha^{-0.002\pm0.001}$ — flat, i.e. *sharp*: no α-dependence of the chain passes through the Rayleigh step any more. The CBS step is equally pessimistic: the actual coupling is $\kappa_y = 0.0067\text{–}0.5311$ against the worst case $\gamma^2 = 0.7173\text{–}0.9647$ (a factor of up to 100+, growing like $\alpha^{+1.52}$), and $1 - \kappa_y = r_{\mathrm{cbs}}$ exactly — T121's slack, now with a name. One lever limit declared: at $M \leq 2048$ the coarse envelope of σ_z is nonnegative on 13/36 rows — the deep dips T118/T121 chased appear only at the frame's own resolution.

The new net balance. Five supplies on one (D, α) lever (12 zones × 3 resolutions, 36 rows), all sound (36/36 below the truth $S_4 = y^{\mathsf{T}} S y$). The T121 baseline reproduces exactly ($S_0/\varepsilon_c \sim \alpha^{-1.562\pm0.163}$); repairing link (5) alone does not change the exponent at all ($\alpha^{-1.565}$ — a uniform floor cannot beat $\lambda_{\min}(A_z)$; V1's contribution is that the floor is certified over the whole ladder instead of tearing at $\alpha \approx 3.45$); with both repairs the *certified* balance is

$$S_2/\varepsilon_c \;\sim\; D^{-0.003\pm0.075}\, \alpha^{-0.729\pm0.090}$$

(fits; measured 0.033–0.207): the deficit *halves*, and it is exactly uniform in D. Adding the *measured* CBS coupling gives $\alpha^{-0.113\pm0.063}$ — statistically indistinguishable from the chain's own ceiling $S_4/\varepsilon_c \sim \alpha^{-0.116\pm0.062}$ (the cost of discarding ε_f, untouched by both repairs). The residual decomposition closes to 2.9×10^{-16}: $-0.729 - 0.002 + 0.615 = -0.116$. In one sentence: V1 buys the ladder, V2 buys the exponent, and the chain closes *up to its own ceiling* as soon as the CBS step is no longer a worst case.

The theorem after T122 — four defects, consolidated

Candidate (V6). The chain renumbered: the Haynsworth identity; the exact CBS split $y^{\mathsf{T}} S y = (1 - \kappa_y)\, y^{\mathsf{T}} A_z y$ with $\kappa_y \leq \gamma^2$ classical (Yserentant); the new two-grid identity $A_z = W^{\mathsf{T}} \mathrm{Toeplitz}_M(c) W$ ($W = BZ$, $W^{\mathsf{T}} W = I$) — the line that retires link (5); the certified cell envelope; the certified band bound (5R) with its uniform, y-free variant (Gershgorin); and the quoted demand fit.

The defect list — four items (T119 = 3, T120 = T121 = V6 = 4; the target < 4 not met, but the kinds changed twice over):

(F1) *one sign of the corner increments* — unchanged from T120/T121 and untouched by this part: 24/24 clean at $n_C = h/16$, no sign-pattern argument possible, the minimal statement is the head-versus-tail inequality (E1′) with an open tail exponent;

(F2) *a uniform bound on the pairing ratio δ* — unchanged and untouched: 0.0126–0.1331 over 360 rows, no outlier, a discrete gradient estimate still missing;

(F3) *an analytic (D, α) bound for the band margin* $1 - y^{\mathsf{T}} G y/(\mathrm{thr}\|y\|^2)$, measured 0.024–0.363 over $\alpha = 0.98\text{–}6.31$ with fitted drift $D^{-0.042}\alpha^{-0.525}$ — the one *new* item, and it absorbs link (5), the Nikolskii budget and both readings of (E3)/(E4): a Fejér/Christoffel question about how the comb dips of the window symbol overlap the high-band mass of the two-level residual, not an eigenvector question. An α-flat band margin would carry the certified chain to within the CBS step of the truth;

(F4) *a structural CBS estimate* — $\kappa_y \leq \gamma^2$ is the last worst-case step in the chain (worth $\alpha^{+0.615}$ in the balance), and the actual coupling is only 0.009–0.551 of it.

Honest movement: link (5) is not repaired but *retired* — it was a false comparison between an isometric compression and one of its two halves — and the per-dip Nikolskii budget went with it; what is left of

the old (E3)/(E4) is a single scalar estimate about dips versus high-band mass, plus the CBS item.

Disposition after T123: the two scalar items were run against the same band machinery and they *split*. (F3) closes almost for free — the reduction to the band profile of Wy costs under 1%, and the oscillation block gains uniform certified positivity, (5R*) — while (F4) resists structurally: every certified route to κ_y needs the *coarse* form from below, whose near-null direction is smooth and lives on a cancellation that no pointwise symbol minorant can see. The remaining $\alpha^{+0.500}$ gap to the ceiling is thereby identified with the ε_f lid itself — the recursion base case. The next subsection reports the measurements.

7.22 The two-isometry completion: the margin closes, the coupling resists (T123)

T123 (`structural_cbs_probe.py`, contract `STRUCTURAL.CBS`, 20/20, CBS-RESISTS) attacks the two scalar estimates the V6 list left — the structural CBS step (F4) and the band margin (F3) — with the same band machinery that made the Rayleigh step sharp in T122. The margin closes almost for free; the coupling does not, and the reason *why* is the content of the part.

The coupling anatomy, in closed form. The coarse block is the *other* half of the T122 construction: with $V = BP$ (P the piecewise-constant prolongation), V is an isometry too — $V^{\mathsf{T}}V = I$ and $V^{\mathsf{T}}W = 0$ exact — so the coarse, oscillation and coupling blocks are three compressions of *one* window Toeplitz form,

$$A_c = V^{\mathsf{T}}\mathrm{Toeplitz}_M(c)\,V, \qquad B_x = V^{\mathsf{T}}\mathrm{Toeplitz}_M(c)\,W, \qquad A_z = W^{\mathsf{T}}\mathrm{Toeplitz}_M(c)\,W$$

(identities to 3.4×10^{-15} on all 24 rows). Two new closed forms complete the T122 anatomy: the coupling is half a *first* difference of the lag sequence, $(B_x)_{jk} = \beta_{j-k} - (c_{K-2(j+k)} - c_{K-2(j+k)-2})/2$ with $\beta_m = (c_{|2m+1|} - c_{|2m-1|})/2$, verified to 1.8×10^{-15} (A_z was half a *second* difference), and in symbol form

$$b(\varphi) \;=\; -\tfrac{i}{2}\,\sin(\varphi/2)\,[\sigma(\varphi/2) - \sigma(\varphi/2+\pi)]$$

— the *aliasing difference* of the window symbol times $\sin(\varphi/2)$, verified to 5.7×10^{-16}: the coupling weight vanishes exactly where the coarse space lives. That explains κ_y quantitatively for the first time: Wy carries 99.64–100.00% of its mass at $d = |\theta - \pi| < \pi/2$ and only 8.9×10^{-6}–1.5×10^{-3} in the cell containing $\theta = 0$, while the worst-case y^* that attains γ^2 is a *middle*-band vector (81.22–100.00% of its mass exactly there) — the factor 2–109 between $\kappa_y = 0.0067$–0.4622 and $\gamma^2 = 0.7248$–0.9467 *is* that mismatch.

The certified κ bound: both routes fail for one reason in two guises. Route 1, band-split Cauchy–Schwarz (per frequency band separately, the strengthened-CBS mechanism of Axelsson–Vassilevski, including the exact two-grid factorisation that puts the $\sin(\varphi/2)$ zero into the certificate), is correct on 24/24 rows and non-vacuous on 0/24: it costs exactly one coarse condition number ($\mathrm{cond}(A_c) = 2.1 \times 10^4$–$3.1 \times 10^7$). Route 2 substitutes the certified minorant $T_M(\ell) \preceq T_M(\sigma^{(M)})$ into the Schur complement's minimum characterisation and delivers two new links:

$$(5\mathrm{R}^*)\quad y^{\mathsf{T}}A_z y \;\geq\; y^{\mathsf{T}}E_z y, \qquad (7\mathrm{R}^*)\quad y^{\mathsf{T}}S y \;\geq\; y^{\mathsf{T}}E_z y - (E_x y)^{\mathsf{T}}E_c^{-1}(E_x y),$$

with $E_c = V^{\mathsf{T}}T_M(\ell)V$, $E_x = V^{\mathsf{T}}T_M(\ell)W$, $E_z = W^{\mathsf{T}}T_M(\ell)W$. (5R*) holds with $\lambda_{\min}(E_z) = +0.303\ldots+5.017 > 0$ on *every* row of the ladder — uniform certified positivity of the oscillation block, at slack 1.0007–1.007, subsuming the whole T122 threshold sweep for free (since $\min(\ell, \mathrm{thr}) \leq \ell$ for every threshold) — while (7R*) is certified as soon as $E_c \succ 0$, which holds on 0/24 rows ($\lambda_{\min}(E_c) = -3.44\ldots-0.043 < 0$ everywhere). The asymmetry is the result of the part: $\lambda_{\min}(A_z) = 0.32$–$5.08$ is well conditioned, while $\lambda_{\min}(A_c) = 1.7 \times 10^{-5}$–$2.9 \times 10^{-4}$ sits three to five orders below the certified envelope margin (8.4×10^2–$4.0 \times 10^5 \times \lambda_{\min}(A_c)$, rising like $\alpha^{+4.202}$; against $\lambda_{\min}(A_z)$ the same margin is only 0.037–17). The near-null direction of the window form is *smooth*, so it lives in the coarse space, and its eigenvalue comes from a cancellation *inside* the form that no pointwise symbol minorant can see. The classical worst-case step $\kappa_y \leq \gamma^2$ (Yserentant) therefore *stays* in the chain — and V7 now states why it must.

The band margin closes; the certified balance; the ε_f lid. (F3) reduces by the same machinery: $y^\mathsf{T}Gy$ splits *exactly* over dyadic depth layers of the certified gap (closure 1.1×10^{-15}), and the two-factor bound [certified symbol depth] × [Parseval mass of Wy per band] overshoots by only 1.20–3.51 — the reduction to the band profile of Wy, the same object the coupling analysis needs, costs under 1%. The certified margin is positive on 5/24 rows (+0.0024–+0.3290 against the exact margin 0.070–0.560 $\sim \alpha^{-0.449}$); the honest lever limit: $\min \ell = -2187 \ldots -9.7$ at $M \le 2944$ — the deep dips of the frame's own resolution stay out of reach. The certified balance — the core number: the best fully certified supply is

$$S_6/\varepsilon_c \;\sim\; D^{-0.069\pm0.099}\,\alpha^{-0.544\pm0.113}$$

(fits; $\alpha = 0.98$–6.31, uniform in D and stable across the three resolutions) against the ceiling $S_4/\varepsilon_c \sim \alpha^{-0.044\pm0.076}$, so the gap is $\alpha^{+0.500}$ — and the residual split against S_6 has exactly *two* terms, closing to 2.8×10^{-17}: band profile $\alpha^{+0.000}$ (F3, now negligible) plus coupling $\alpha^{+0.499}$ (F4, all of it). The ε_f question then identifies the lid: $\varepsilon_c = \varepsilon_f + y^\mathsf{T}Sy$ is an identity ($\varepsilon_f/\varepsilon_c = 0.177$–$0.546 \sim \alpha^{+0.030}$, flat), and an additive chain that carries ε_f instead of discarding it would have ceiling exactly 1 — but ε_f needs its bound one level down, 4–8 recursion levels to an exact base case, and that base case is precisely (F4). The lid is not a modelling step; it is the *same object* as coarse-level positivity.

The theorem after T123 — four defects, restructured

Candidate (V7). The chain rewritten with the two-isometry completion: the Haynsworth identity; the exact CBS split $y^\mathsf{T}Sy = (1-\kappa_y)\,y^\mathsf{T}A_z y$ with $\kappa_y \le \gamma^2$ classical (Yserentant; Axelsson–Vassilevski) — the *only* worst-case step left in the chain, and V7 states why it stays: every certified route to κ_y needs coarse-level positivity from below; the two-grid identities $A_c = V^\mathsf{T}TV$, $B_x = V^\mathsf{T}TW$, $A_z = W^\mathsf{T}TW$ with $V = BP$, $W = BZ$ two orthogonal isometries ($V^\mathsf{T}W = 0$); the certified cell envelope; the uniform oscillation positivity (5R*); the certified band-margin bound; and the quoted demand fit.

The defect list — four items (T120 = T121 = V6 = V7 = 4), restructured:

(F1) *one sign of the corner increments* — unchanged, quoted from T120/T121; together with (F2) it is the *Harnack pair*, untouched by the band machinery;

(F2) *a uniform bound on the pairing ratio δ* — unchanged: 0.0126–0.1331 over 360 rows, any finite value suffices;

(F3) *one band-profile lemma* — the band margin reduces to the band profile of Wy at a cost of 0.7%, and (5R*) supplies the uniform certified positivity of the oscillation block for free; what remains is one analytic statement about how the mass of Wy distributes over frequency bands;

(F4) *coarse-level positivity — identified*, not estimated: every certified route to the coupling needs $\lambda_{\min}$ of the coarse form from below, that eigenvalue is a smooth-direction cancellation invisible to symbol minorants, and it is the same object as the ε_f lid that defines the chain's own ceiling.

Honest movement: the counter stays at four, but the terrain changed: the whole $\alpha^{+0.500}$ gap to the ceiling now sits in one named obstruction whose resolution cannot be a sharper two-level estimate. The next step is not sharper banding but the *recursion* — the level telescope $\varepsilon_\ell = \varepsilon_{\ell+1} + y_\ell^\mathsf{T} S_\ell y_\ell$ with certified per-level contributions.

Disposition after T124: the recursion was built, and the direction flipped. Written as a telescope, the rung contribution is a residual in the *inverse* norm — a maximum, not a minimum — so it is certifiable from *above*, where the certified envelope works: (F4) collapses from a quantitative estimate to the sign a coarse-to-fine induction already carries, the base case becomes pure semidefiniteness, and $\lambda_{\min}(A_c)$, $\mathrm{cond}(A_c)$ and γ^2 leave the chain entirely. The next subsection reports the measurements.

7.23 The direction flip: the telescope carries, and (F4) collapses to a sign (T124)

T124 (`multilevel_telescope_probe.py`, contract `MULTILEVEL.TELESCOPE`, 28/28, TELESCOPE-CARRIES) builds the recursion T123 demanded, and the exit turns out to be not the recursion's *length* but its *direction*: written as a telescope, the quantity each rung must supply is not a Rayleigh

quotient (a minimum, needing the form from below) but a residual in the inverse norm (a maximum, needing the form from above) — and from above, the certified envelope works, because Parseval is two-sided.

The telescope, exact and geometric. On the level chain $D_0 > D_1 > \cdots > D_L$ (halvings, α fixed) the ladder is *one* window form on nested spaces:

$$P^{\mathsf{T}} A^{(\ell+1)} P = A^{(\ell)}, \qquad P^{\mathsf{T}} b^{(\ell+1)} = b^{(\ell)}$$

exactly (2.4×10^{-14}; the pole-vector consistency is the identity $\sinh(a-d) + \sinh(a+d) = 2\cosh d \sinh a$) — the coarse block of the window form at $D_{\ell+1}$ *is* the window form at $D_\ell = 2D_{\ell+1}$, so the T122/T123 two-level system is literally one rung of the ladder. Every rung is the T118 saturation identity, $\varepsilon_\ell = \varepsilon_{\ell+1} + \delta_\ell$ with $\delta_\ell = y_\ell^{\mathsf{T}} S_\ell y_\ell$, and Galerkin orthogonality (Céa) makes it the energy of the level correction, $\delta_\ell = \|u_{\ell+1} - P u_\ell\|^2_{A^{(\ell+1)}}$ (verified to 3.7×10^{-8}) — nonnegative, hence the exact telescope $\sum_\ell \delta_\ell = \varepsilon_0 - \varepsilon_L$ (9.2×10^{-10}). The level distribution is geometric on average: the top rung carries 0.2722–0.8806 of ε_0, consecutive rungs fall by a median factor 0.316 (quartiles 0.229–0.473, against $2^{-\theta} = 0.289$ from the demand law), and at $L = 5$ the ceiling of the additive chain is 0.9911–0.9987 — against T123's two-level ceiling $\alpha^{-0.044}$. Rung law (a fit): $\delta \sim D^{+1.598}\alpha^{-5.900}$.

The certified rung contribution — the direction flip. The rung has two dual descriptions. Primal, $\delta_\ell = \min_{w \in V_\ell} \|u_{\ell+1} - w\|^2_A$: a *minimum*, every test vector an upper bound — the wrong direction for a supply. Dual: the level residual $r_\ell = t_{\ell+1} - A^{(\ell+1)} P u_\ell$ satisfies $V^{\mathsf{T}} r_\ell = 0$ (Galerkin), so $r_\ell = W \hat{s}_\ell$ lives purely in the oscillation space, and

$$\delta_\ell \;=\; \|r_\ell\|^2_{A^{-1}} \;=\; \hat{s}^{\mathsf{T}} S^{-1} \hat{s} \;=\; \max_v \frac{(\hat{s}^{\mathsf{T}} v)^2}{v^{\mathsf{T}} S v}$$

— a *maximum*: every test vector gives a lower bound, and the denominator wants S from *above*. That is where the certified machinery works: $S \preceq A_z$ needs only $A_c \succeq 0$ (Haynsworth monotonicity), $A_z \preceq U := Z^{\mathsf{T}} T_M(\mathrm{up}) Z$ is Parseval against the certified majorant up $\geq \sigma^{(M)}$ (the *mirror* of T123's $E_z = W^{\mathsf{T}} T_M(\ell) W$, on the block where it works), and Loewner inversion turns the upper bound on S into the lower bound on the rung,

$$\text{(8R)} \qquad \delta_\ell \;\geq\; \hat{s}^{\mathsf{T}} U^{-1} \hat{s},$$

with the optimal test vector $v^* = U^{-1}\hat{s}$ in closed form — no coarse inverse, no $\lambda_{\min}(A_c)$, no CBS constant. (8R) is valid on 400/400 rungs (80 chains × $L = 5$, 20 zones), recovering $\mathrm{cb}/\delta = 0.4739$–$0.9938$ at drift $\alpha^{-0.080 \pm 0.007}$ — seven times weaker than T123's $\alpha^{-0.544}$; of the loss, the coupling accounts for 0.5036–0.9967 and the envelope only 0.7895–0.9994. T123's obstruction remains and is *irrelevant*: $\lambda_{\min}(E_c) < 0$ on 399/400 rungs, and (8R) never forms E_c. The Levinson bordering route honestly *falls*: the innovation ledger $q = \sum_n \rho_n^2 / E_n$ is exact per level (9.6×10^{-12}; Levinson–Durbin–Delsarte–Genin), but refinement is not a bordering — the two filtrations are transversal (head/q_ℓ = 0.61–1.00, tail/δ_ℓ = 13–863) — so the rung must be read in the (8R) form.

Self-similarity, and why the recursion closes. The near-null direction of the window form stays *smooth* at every level (oscillation fraction 4.3×10^{-6}–1.2×10^{-1}) — T123's diagnosis, confirmed level by level — and it is self-similar along the ladder: prolonged, w_ℓ has Rayleigh quotient exactly $\lambda_{\min}(A^{(\ell)})$ at the finer level (an identity, from nesting), i.e. within a factor 1.30–10.11 of that level's own minimum, at subspace overlap $\|Q_K^{\mathsf{T}} P w_\ell\| = 0.944$–$1.000$. But (8R) asks only for the *sign* of A_c, and by the nesting identity that sign is the previous rung's own conclusion: positivity propagates fine → coarse for free (so the opposite direction would be circular — stated explicitly), and the non-circular induction runs coarse → fine, with base case $\varepsilon_L \geq 0$ — pure semidefiniteness of the smallest window form.

The balance; the honest negative. Telescope reading: $\sum_\ell \mathrm{cb}_\ell/\varepsilon_0 = 0.6404$–$0.9729$ against the ceiling 0.9911–0.9987 (fit $D^{-0.066}\alpha^{-0.084 \pm 0.033}$). Two-level comparison, like-for-like against T123's certified balance: $\mathrm{cb}/\varepsilon_c \sim D^{-0.090}\alpha^{-0.100}$ against T123's $D^{-0.069}\alpha^{-0.544}$ — the α exponent moves

by $+0.444$ of the $\alpha^{+0.500}$ gap. The honest negative, recorded as (F5): the *solution-free* version of the rung fails — the coupling term cancels the data high-pass rather than perturbing it ($|r_{\mathrm{corr}}| = 0.88$–$2.61$ of the explicit part), so any norm bound kills the certificate to exactly 0; what would be needed is a bound on a *direction*, not a size. (8R) therefore carries the coarse solution along — on the same bookkeeping standard as the whole chain: $\hat{s}$ is a function of the carried coarse solution exactly as y is in T122/T123's own certified supplies.

The theorem after T124 — (F4) collapses to a sign

Candidate (V8). The telescope chain: the nesting identity $P^{\mathsf{T}} A^{(\ell+1)} P = A^{(\ell)}$ (the ladder is one window form on nested spaces); the saturation telescope $\varepsilon_0 = \varepsilon_L + \sum_\ell \delta_\ell$ (exact); the dual residual form of the rung with the certified per-rung bound (8R) $\delta_\ell \geq \hat{s}^{\mathsf{T}} U^{-1} \hat{s}$, needing only $A_c \succeq 0$ (Haynsworth) and the Parseval majorant $U \succ 0$; and the coarse-to-fine induction whose hypothesis at rung ℓ is the conclusion of rung $\ell - 1$, with base case $\varepsilon_L \geq 0$.

The defect list — the counter reads differently now:

(F1) *one sign of the corner increments* — unchanged, quoted from T120/T121; with (F2) the *Harnack pair*, untouched by telescope and band machinery alike;

(F2) *a uniform bound on the pairing ratio δ* — unchanged: 0.0126–0.1331 over 360 rows, any finite value suffices;

(F3) *closed* — the T123 reduction to the band profile of Wy plus the uniform oscillation positivity (5R*); the telescope no longer needs it;

(F4) *collapsed* — no longer a quantitative estimate: (8R) needs only the sign of A_c, which the coarse-to-fine induction already carries; $\lambda_{\min}(A_c)$, $\mathrm{cond}(A_c)$ and γ^2 leave the chain entirely, and the base case is pure semidefiniteness;

(F5) *new, a bookkeeping convention, honestly stated* — the certified rung carries the coarse solution along (the solution-free version provably kills to zero: the coupling is a cancellation, not a perturbation); this is the same standard the whole chain already uses, but it is a convention, not a theorem, and it is listed.

Honest movement: the two-level worst case is gone — the last quantitative obstruction of the certification arc is replaced by a sign that the induction itself supplies, at the price of one named convention. What remains is the Harnack pair, one constant-not-rate loss per rung, and the assembly of the full chain end to end.

7.24 The grand assembly (T125): the chain composes

T125 (`grand_assembly_probe.py`, contract `GRAND.ASSEMBLY`, 34/34, ASSEMBLY-GREEN) is the series finale, and it does exactly one thing: it mounts every certificate of the arc on one ladder, end to end, and then states what the resulting object is. Five stages per zone — **[A]** the base-case Cholesky, **[B]** the telescope ascent with the (8R) rungs, **[C]** the ε lower bound on the target level, **[D]** the κ_{end}/parity chain, **[E]** the margin-free Albert handover — complete on 52 of 52 ladder zones, plus ten deep zones on stage [E] alone up to $n = 1331$ (h_{new} up to 1495).

The composing run. The construction that makes this an assembly rather than a list is ladder 1: 30 *consecutive* prime-power atoms $n = 2 \ldots 79$ on *one* common resolution $D = \min_k g_k/(2\nu) = 3.472446 \times 10^{-3}$ (with $\nu_{\mathrm{eff}} = 4.0$–$58.4 \geq 4$, so every zone of the run is admissible in the T105 sense). Because the resolution is common, the new window of zone k *is* the old window of zone $k+1$: $M_n(k) = M_o(k+1)$ on all 29 consecutive pairs as an *integer* identity, the incoming atom's triangle restricted to the old window is the exact zero matrix (bit-exact, $\max|\mathrm{entry}| = 0$), so $X = Q_{\mathrm{old}}$ with no truncation, and the leading $h_o \times h_o$ block of the next zone's balanced form equals that X to 2.9×10^{-14}. The output of every step *is* the input of the next. This is a constructive way *around* T114's [P2] seam rather than through it: 0 of 84 consecutive log-gap ratios are dyadic — per-zone frames have no common refinement — so the run chooses one frame for all of it, and pays for that with its length (30 zones, $n \leq 79$, because D is set by the smallest gap inside the run). Ladder 2 keeps the per-zone frames and reaches 22 further zones to $\alpha = 2.885$, with the inter-zone seam declared rather than closed.

The load-bearing finding. Reading the assembly stage by stage, the weakest stage of the *whole* mounting is [D] at WINDOW-CERT on every zone — but [D] is the second, independent route. The *load-bearing spine* [A][B][C][E] is at CHOLESKY-CERT on all 52 zones, its weakest stage being [A]. In other words: **the Harnack pair (F1)/(F2) no longer carries anything**. It survives only inside conclusion (iv) of the theorem below, which (i)–(iii) and (v) do not use.

The numbers, stage by stage. The four identities the assembly stands on hold on all 88 rungs of all 52 zones (nesting to 3.0×10^{-14} and 6.2×10^{-16}; the saturation/dual-residual/Galerkin trio to 4.7×10^{-7}; the exact piecewise-constant lag formula to 2.2×10^{-14}; the telescope $\sum_\ell \delta_\ell = \varepsilon_0 - \varepsilon_L$ to 3.3×10^{-7}). (8R) is certified on 88/88 rungs by its three completed Choleskys, at retention $\mathrm{cb}/\delta = 0.9071$–$0.9971$ — T124's band 0.4739–0.9938, reproduced on a *different* ladder (frame windows, not the T124 M-chain) — and the direction control holds: the primal minimum form returns values *above* δ on 88/88 rungs, i.e. it is an upper bound and can never be a supply. The certified ε lower bound lands on 52/52 zones, retaining 0.0013–0.9694 of the measured ε_0 but 0.9140–0.9929 of the L-level *ceiling* $1 - \varepsilon_L/\varepsilon_0 = 0.001295$–$0.993502$ that an L-level chain can supply at all, and it exceeds the *declared* floating-point bound $c_h u \,\mathrm{cond}(A_0)$ by a factor 3.2×10^1–8.7×10^{10}. The base case is an *equivalence*, not a bridge: $\operatorname{sign} \lambda_{\min}(Q^{(L)}) = \operatorname{sign} \varepsilon$ on 52/52 zones at $h = 416$–1384, with $\lambda_{\min}(Q^{(L)})$ sitting 4.4×10^5–5.6×10^{10} times its own floating-point floor. The margin-free handover certifies 62/62 steps ($n = 2 \ldots 1331$) with the exact $g_c \times g_c$ Schur complement at $\lambda_{\min} = 1.484 \times 10^{-3}$–$1.253 \times 10^{-1}$, a factor 3.0×10^{10}–4.6×10^{13} above its own Cholesky floor — while the norm-perturbation surrogate of the *same* Schur complement, the one that divides by $\lambda_{\min}(X)$ and *was* the T113 wall, is negative on 62/62 steps. In total: 430 completed Cholesky certificates and 440 verified identities.

Three gates corrected in the open. (a) The κ band: the unconditional pairing certificate (C1) dominates the measurement on 52/52 rows, but its worst case over the ladder is $C_1 = 0.20927$ — and that worst case is a *single* zone, the shallowest one ($n = 2$, $\alpha = 0.3507$, only $n_C = 12$ corner cells, too short for the pairing to average); drop that row and $C_1 \le 0.06721$ on the remaining 51, against T120's 0.04745 measured on deep zones only. (b) The head-room minimum is 31.6, at the thinnest zone ($n = 243$, $\alpha = 2.7492$, $h_0 = 679$), and it is thin because that zone's retention 0.6054 is itself pinned to its small L-level ceiling 0.629042 — not because the arithmetic is marginal; the factor stands over a *rigorous* bound, so any value above 1 certifies the sign. (c) The balance is fitted at *rung* level rather than on the collinear zone fit: $\mathrm{cb}/\delta \sim D^{-0.010}\alpha^{+0.001}$ (bands ± 0.004, ± 0.005, rms 0.012) — flat in *both* parameters, the strongest form of T124's quoted $\alpha^{-0.100}$ drift — and at zone level on the composing run, where D is common and only α moves, $\sum \mathrm{cb} / \sum \delta \sim \alpha^{-0.012}$: the certificate does not thin out along the run. The ratio reading $\mathrm{cb}/\varepsilon_c$ comes out at $D^{+0.345}\alpha^{+1.566}$ here and is explicitly *not* comparable with T124's $D^{-0.090}\alpha^{-0.100}$ (a different denominator: a 2–5-level frame chain against $M = 32$–2560 pairs) — reported, not claimed.

Five seams, named and measured. (S1) the inter-zone frame seam — *closed* on ladder 1 by the common-frame construction; (S2) the direction of the ε bound — structural and declared: the certified bound lands on the *coarse* level, so the target window must be the coarsest level of its chain and the base case the finest (a bound at the fine level would need certified *upper* bounds on the rungs, i.e. the CBS constant T123 proved resists); (S3) the matrix cap — cost, not a defect; (S4) the (F5) accounting standard — open, declared inside the theorem, with the solution-free variant measured to return exactly 0 of the rung ($|r_{\mathrm{corr}}| = 0.3888$–$14.8018$: a cancellation, not a perturbation); (S5) finiteness — open, *and it is the gap.*

Theorem V-final — the certified multilevel margin chain (T125)

Setting. Fix $\nu \ge 4$; let $n_i < \cdots < n_j$ be *consecutive* prime-power atoms with $u_k = \log n_k$ and log-gaps g_k, and put $D = \min_k g_k/(2\nu)$ (one common resolution), M_k the smallest even integer with $M_k D > u_k + D$, $\alpha_k = M_k D/2$, $A^{(k)}$ the reflection-odd window form on $h_k = M_k/2$ coordinates built from the exact Weil window lags, $b^{(k)}$ the odd pole vector, $\varepsilon_k = 1 - b^{\mathsf{T}} A^{-1} b$ and $Q^{(k)} = A^{(k)} - b^{(k)} b^{(k)\mathsf{T}}$; for each k let $D = D_0 > \cdots > D_{L_k}$ be halvings at fixed α_k.

Hypotheses. **(H-base)** $\varepsilon^{(k)}_{L_k} \ge 0$ at the *finest* level, equivalently $Q^{(k,L_k)} \succeq 0$ (Albert 1969) — pure

semidefiniteness, no rate, no margin. **(C-F5)** an accounting convention, *declared open*: the rung numerator $\hat{s}_\ell = Z^\mathsf{T} b^{(k,\ell+1)} - B_x^\mathsf{T} u^{(k,\ell)}$ is a function of the *carried* coarse solution, the standard T122/T123 already used; a solution-free rung is *not* claimed — it is measured to fail. **(H-F1)/(H-F2)** the Harnack pair, needed *only* for conclusion (iv).

Conclusions. (i) [IDENTITY] $P^\mathsf{T} A^{(k,\ell+1)} P = A^{(k,\ell)}$, $P^\mathsf{T} b^{(k,\ell+1)} = b^{(k,\ell)}$, hence $\varepsilon_0^{(k)} = \varepsilon_{L_k}^{(k)} + \sum_\ell \delta_\ell$ exactly. (ii) [CERTIFIED, given $A^{(k,\ell)} \succeq 0$] (8R) $\delta_\ell \geq \hat{s}_\ell^\mathsf{T} U_\ell^{-1} \hat{s}_\ell =: \mathrm{cb}_\ell > 0$; the rung is a *maximum*, so its denominator wants the form from above, and $v^* = U_\ell^{-1} \hat{s}_\ell$ is optimal in closed form — no A_c^{-1}, no $\lambda_{\min}(A_c)$, no CBS constant. (iii) [CERTIFIED, given (H-base) and (C-F5)] $\varepsilon_0^{(k)} \geq \sum_\ell \mathrm{cb}_\ell > 0$, i.e. $Q^{(k)} \succ 0$. (iv) [WINDOW-CERT, second and *independent* route] the κ_{end} chain, with $|R - 1| \leq 0.20927$, hence $R \leq 1.20927$ and $\kappa_{\mathrm{end}} \geq 0.45264$ on this run. (v) [CERTIFIED, margin-free] $X = Q_{\mathrm{old}}$ exactly, and $Q^{(k+1)} \succeq 0$ follows from Albert 1969 plus the Douglas range condition — nothing refers to the *size* of ε_k, only to its *sign*, which is (iii). (vi) [THE CHAIN] therefore (i)–(iii) and (v) compose: $Q^{(i)} \succeq 0 \Rightarrow \cdots \Rightarrow Q^{(j)} \succeq 0$, each step a completed Cholesky above its declared floating-point floor; (iv) is not used.

Attribution (25 lines). 10 IDENTITY, 9 CHOLESKY-CERT, 3 WINDOW-CERT (all three inside (iv)), 1 HYPOTHESIS ((C-F5)); every eigenvalue, condition number and retention is labelled MEASURED, every exponent law a FIT.

Verified instance. $D = 3.472446 \times 10^{-3}$, 30 consecutive zones $n = 2 \ldots 79$, $\alpha = 0.3507$–2.1494, $h_k = 101$–619, 88 telescope rungs, plus 22 non-composed reach zones to $\alpha = 2.885$ and 10 handover-only zones to $n = 1331$; all 430 Cholesky certificates completed.

What is not claimed. (a) No statement about all zones — the run is *finite* and its length is limited by $\min_k g_k = 0.027780$. (b) (H-base) is a Cholesky fact about *one* finite matrix, not a theorem. (c) (H-F1)/(H-F2) are window certificates, measured and bounded on the measured windows, with no uniform proof. (d) (C-F5) is a convention, not a solution-free bound. (e) No zeta function, no zero data, no explicit-formula positivity, *in either direction*: this is a statement about a given quadratic form at a given resolution.

Part IV
Honest negatives: the kill list and the series balance

This part is the collection point for the negative output, which the program treats as its more transferable product. Section 8 is the kill list of the first series: the readings and routes refuted by the program's own probes, each with the measurement that killed it. Section 9 is the bookkeeping of the certification arc — the reduction cascade, the certification degree, and the honest RH distance in six checkable statements. Later phases record their kills in place: the phase-2 breaks stay inside Section 10, and the frontier kills (reflection positivity on truncations, measured gap universality, the Sonin transfer, the UCP transport route) are collected in Section 12.6.

8 The kill list

A program that only reports what survived is not reporting. These are the readings and routes that were refuted *by the program's own probes*, each with the measurement that killed it. They are listed as a section rather than as footnotes because they are the more transferable output: each one tells a later attempt where not to go.

K1. The essential-singularity reading of the onset (T96, refuted T102). T96 read the counterfactual onset as super-polynomial, compatible with $\exp(-c/\delta')$, i.e. an essential singularity. T102 showed the onset is *anchored* at the crossing $\delta_c = 2w_k$ of two finite quantities ($R^2 = 0.968$), that exponential versus power behaviour is not separable at this resolution, and that the scatter of c (CV 103%) collapses to $c' = 1.52$ in the scale-free variable. The essential-singularity reading

is *compatible but not distinguished* — and it is no longer used.

K2. The C/g law as a causal law (T101, refuted T102). Triple negative: g_k is causally impossible (Q_{k-1} is blind to atom $k+1$), statistically dispensable (causal law $C \cdot u$: CV 23.3% versus 27.6%), and arithmetically only a ceiling ($C_k \le g_k \mu_k$ exactly; the 16-zone extrapolation violates its own ceiling from $k = 69$; checked on 18 120 prime-power atoms to $n = 200\,000$). *It was a proxy.*

K3. The naive margin route (T104). $M \ge m \cdot \mathrm{Id}$ holds in 0/16 zones; the coupling-mass to margin ratio is 10^3–10^6; $m \sim M^{-1.7}$ vanishes in the continuum (fit). The genuine form is nearly singular on the induction window and no margin rescues it. (The prolate wing basis recommended by T96/T103 was tested in both arms and also rejected.)

K4. The density chain (T106). It would need a hard gap $w_0 \ge 0.55$–2.44; measured 5.3×10^{-6}–1.1×10^{-3} — short by a factor 5×10^2 to 3.6×10^5. Dead for good.

K5. The Landau–Widom anchor (T106). The wrong classical anchor for this problem (rms 0.50–1.55). The right one is Grenander–Szegő on the Planck-coarsened symbol — averaging over the mode-spacing cell π/M — which hits 0.92–2.43 and is better on 67/72 pairs. This also explains why a negative symbol (dip $-21.3 \ldots -3.4$) still gives $Q \succeq 0$: the atoms oscillate exactly at the window's Planck scale.

K6. The accumulated invariant with self-propagation (T106). A no-go *with a measured mechanism.* The wing demand gives $\beta_{\mathrm{wing}} = 2.21$–$14.43 > 1$ on 16/16, but inherited demands give $\beta_{\mathrm{int}} \sim 10^{-2}$ on 15/15: an accumulated invariant with absolute anchoring is false. The reason is transport — in a larger window an old wing pair becomes an interior pair and draws 99.7–99.95% of its demand from the 64 softest modes (coupling 751–2653× stronger). No self-propagation either ($\beta_{\mathrm{on}} < 1$ on 15/15, loss 13–320×, $\theta_k = 0.9866$–0.9998). *The wing margin is an edge property of the window, not a property of the pair.* What survives is the window-wise family $\sigma_k(\delta_0) \ge 1.31(\mu_k/2)$, uniform in M with drift $\le 2.01\times$.

K7. The symbol route for the odd channel (T107). Structurally dead, not merely weak: the symbol lower bound for T_{odd} is certified but *negative* ($\lambda_{\mathrm{cert}} = -2.54 \ldots -0.81$) against a measured $\lambda_{\min}(T_{\mathrm{odd}}) = 4.7 \times 10^{-5} \ldots 6.4 \times 10^{-2}$ and a symbol infimum of $-22 \ldots -6$: five orders of magnitude and a sign change. The soft edge of T_{odd} is a pure finite-section effect that no symbol can see. Coarse certified chains (Cauchy–Schwarz against $\lambda_{\min}$) close 0/16 — and fail even with the *measured* $\lambda_{\min}$, so it is a category error, not a precision problem.

K8. Three certificate candidates for the avoidance norm (T108). Killed *quantitatively*, which is the useful form: the cancellation route ($\sum |k| = 401$ against $\sum k = 1$), the Bessel route (off by 10^7) and the pole Cauchy–Schwarz route (off by 10^6). ω_{cert} is blocked by the compression Schur distance alone (0.18–0.52). What replaced them is not a fourth bound of the same kind but a different object class: a boundary-decay statement about one explicit vector. T109 then killed the natural fourth candidate as well: **the Combes–Thomas resolvent bound at the boundary**, evaluated with the *exact* Green row so that no constant can rescue it, closes 1/16 — the mechanism at the edge is cancellation (44–4.6×10^4 on 15/16 zones), which no absolute-value bound survives. What works instead is the residual certificate that carries the cancellation (Section 7).

K9. The scalar step law for the margin (T110). Killed structurally, in three pieces: bordered Weyl closes 0/15 handovers; the Schur/Friedrichs scalar cap is vacuous on 15/15 (its scalar-only bound degrades back to Weyl); and the strictly scalar graded minorant ($n_{\mathrm{soft}} = \dim$) closes 0/15, with the strictly scalar *induction* compounding to a Schur factor of 2.7×10^{-36} by $k = 3$. The reason is a boundary layer: the new cells of a grown window attach at the edge where the pole source sits, so a one-number summary of the bordering is blind to the deciding direction. What survives is the graded ($n_{\mathrm{soft}} = 1$) Loewner minorant — one soft direction is exactly the amount of geometry the step needs.

K10. Two earlier structural kills, recorded here because they still bound the design space: the cross-block $\sqrt{\lambda_0}$ law is Douglas range inclusion and therefore circular (T98), and the cone-library route saturates at 5/24 coverage and is dead as a covering strategy (T72, promoted into v540 as a named limit). Likewise the seam route to the archimedean place (T20–T25) and the Kohnen-plus route to the central values (T44, promoted into v537 as a scope fence).

The finale re-counted this list against the whole certification arc and made it quantitative: 24 **refuted routes across** 18 **parts**, every one of them structural — an identity (the T123 ceiling, the T124 primal direction), an exact sign ($\lambda_{\min}(E_c) < 0$, the negative norm surrogate, the non-dyadic gap ratios), or a measurement that tuning cannot move (the vacuous band split, the empty energy route, the cancelling solution-free rung). The counting matters because *four* of the 24 died of the *same* cause — a bound that needs the coarse form from *below* — which is why the exit was a direction reversal and not a sharper estimate. T125 added one entry of its own: a certified ε bound at the *fine* level of a chain is the CBS wall again, which is why V-final mounts the target window as the *coarsest* level.

9 The series balance (T102–T125)

The finale's second half is bookkeeping about the series itself: what the hard core was at each stage, how much of the final chain is a certificate rather than a hypothesis, and how far that chain is from any infinite statement. All three are measurements against the same source, and they are reproduced here because they are the part a later reader can check without re-running anything.

9.1 The reduction cascade, seven stations

Each arrow is a strict reduction in dimension or in kind, and every one of them was paid for by a refutation in the list above:

1. **Matrix wall** (T104–T106) — a Loewner statement $M \succeq m\,\mathrm{Id}$ on $M/2$ dimensions, killed as a margin route and localised by the parity split to one statement (R) in the odd channel.
2. **Scalar** (T107–T108) — Sherman–Morrison/Woodbury reduce (R) exactly to one scalar ratio $r = \kappa/\varepsilon \le 1$, and ε itself becomes an identity (the square of the last Cholesky pivot).
3. **Ratio** (T108–T109) — the ratio splits into $\omega < 1$, certified unconditionally by a graded matrix cap, and one statement at an explicit vector.
4. **Boundary value** (T109) — what is left is literally one boundary value, carried by a residue certificate that *keeps* the cancellation instead of estimating it away.
5. **One inequality** (T110–T117) — the frame removes the ladder and ω walls, the margin wall turns out to measure the discretisation, the margin-free Albert step removes the division that was the wall; what remains is one lower bound on ε.
6. **Harnack pair + one sign** (T118–T124) — the ε bound becomes an identity plus a supply, and the supply needs either the corner route ((F1)+(F2)) or the telescope route ((8R), whose only requirement is the *sign* of A_c).
7. **One sign, one convention** (T125) — assembled: the spine [A][B][C][E] needs (H-base), a sign on *one* finite matrix, plus the (C-F5) accounting convention. **The Harnack pair survives as the second, independent route (iv) and is no longer a bottleneck of the spine** — this last station is what T125 added.

9.2 The certification degree

Every link of V-final counted once, classified by what actually establishes it:

Class	Links	%	Spine	%
IDENTITY	14	45.2	11	42.3
CHOLESKY-CERT	14	45.2	14	53.8
WINDOW-CERT	2	6.5	0	0.0
HYPOTHESIS	1	3.2	1	3.8
Total	31	100.0	26	100.0

90.3% of the whole chain is identity or completed Cholesky; on the load-bearing spine it is **96.2**%, with *zero* window certificates and exactly one hypothesis — the (C-F5) accounting convention. The three window certificates are confined to conclusion (iv).

9.3 The honest RH distance, in six checkable statements

(R1) No zeta input. The chain consumes exactly two arithmetic facts, both classical and both only for *admissibility* of the frame: $g_k \le \log 2$ (Bertrand–Chebyshev 1852) and $g_k \ge \log(1 + 1/n)$. No zeta function, no zero data, no zero-derived quantity — the probe's own AST firewall rejects the tokens, the imports and write-mode file access in its source.

(R2) RH is not used, in either direction. "RH ⇒ window Weil positivity" appears nowhere; (H-base) is a completed Cholesky of one explicit matrix of size $h = 416$–1384.

(R3) No infinite statement. V-final quantifies over a *finite* run of consecutive atoms whose length is bounded by $\min_k g_k = 0.027780$: here 30 zones, $n = 2 \dots 79$.

(R4) A measurement ladder is not all zones. 62 of 85 admissible handovers in the table, and 48 of 34 034 prime-power atoms up to 400 000 touched at all. Everything outside is untested, not implied.

(R5) The open items are uniformity, not size. What is missing for any infinite statement is a uniform-in-k version of stage [E]'s $g_c \times g_c$ Schur complement, a uniform lower bound on cb_ℓ, a proof rather than a measurement of (H-F1)/(H-F2) — classical addresses named (Widom 1974 / Fisher–Hartwig for the corner regularity of the inverse, Ostrowski 1937 / Varga 1962 for the sign) — and removal of the (C-F5) convention, which T124 measured to be a cancellation.

(R6) What it is. A machine-certified, finite, multilevel *lower* bound on the Galerkin prediction defect of a log-singular Toeplitz form with a prime-power comb, together with a margin-free bordering induction that transports its sign along a nested window chain. That is a statement in numerical analysis. It is not a statement about zeros.

The defect counter closes accordingly: (F1)/(F2) window-certified and now *outside* the spine, (F3) closed in T123, (F4) collapsed to a sign in T124, (F5) declared as (C-F5) — and one new entry, **(F6) uniformity**, the only thing between V-final and an infinite statement.

Part V
Toward uniformity: the second phase (T126–T199)

Section 10 is the longest section of the document and is deliberately chronological: the diary parts T126–T199 convert the one remaining inequality of the first series into theorem-shaped pieces, refute what breaks, and hand the exact cores to the verified layer as they stabilize (v542–v562 trace back to this arc). The subsection titles carry the findings, so the part can be skimmed by title alone. The consolidated end state — the closed-form theory — follows in Part VI.

10 Phase 2: toward uniformity (T126–T199)

The series closed at T125 with a mandate fulfilled and a distance named: what separates V-final from any infinite statement is *uniformity in the zone index*, not size. The first post-series part — T126 (`uniformity_seams_probe.py`, contract `UNIFORMITY.SEAMS`, 31/31, verdict SEAMS-CERTIFIED) — opens a second phase of the diary by attacking exactly that gap in its three components: the *segment seams* (consecutive runs carry different common resolutions, and the seam between them is one re-gridding), the *uniformity of the spine constants* (the (F6) entry of V-final, audited one constant at a time), and the *continuum argument* (finite-section positivity at every resolution ⇒ positivity of the continuum window form on a test class). All three move; what remains at the end is stated in Section 10.4 and is deliberately short.

10.1 The seam architecture: the partition question dissolves

The natural first question is arithmetic: can the prime-power axis be cut into segments such that each segment carries one common resolution $D_{\text{seg}} \propto$ min-gap and neighbouring segments have a resolution ratio the certified transport reaches? Since D_{seg} is set by the segment's minimum log-gap, this is a question about *minima of windows of the log-gap sequence* and nothing else — so it is decided *exactly*, by a dynamic programme over cut positions on the real gap table (9 700 zones, $n \le 10^5$; 33 DP runs in 2.71 s). The answer is a refutation with a replacement:

- **At T115's synthetic threshold $\rho^* = 1.83$ no partition exists.** The DP dies at zone $n = 127$, at the local gap pattern [0.0684, 0.0325, 0.0159, 0.0078, 0.0232, 0.0448] — a small gap directly behind a large one with no intermediate gap in the admissible window: the twin-after-a-large-gap mechanism, named exactly. The true requirement is $\boldsymbol{\rho_{\text{crit}} = 2.0258}$, identical on $n \le 10^4$ and $n \le 10^5$ — not an aggregate but essentially *one* quotient of neighbouring gaps, $g(125)/g(127) = 2.0238$, reproducing ρ_{crit} to 2.0×10^{-3}. A partition at ρ_{crit} is *constructed* (306 segments; the T112 frame lemmas hold on all 9 700 zone steps, $\min \nu_{\text{eff}} = 4.00$, zero violations).
- **A refinement-only partition exists at no ratio ≤ 64**, so the tight-cap partition forces *coarsening* seams — and coarsening fails for real: on real pairs the transported certificate fails at ratios 0.309 and 0.531 while the target floor itself stays positive on 9/9 (what fails is the certificate, not the positivity), and ladders of intermediate resolutions make it *worse* ($k = 1, 2, 3$ at $n = 47$: $-2.54 \times 10^{-2} \to -2.52 \times 10^{-2} \to -4.63 \times 10^{-2}$) — the obstruction is the *destination grid* on which every ladder must land; a finer subdivision cannot cure a bad target.
- **The decisive new measurement:** holding the worst-case ratio 2.0256 *fixed* and sweeping the zone, the transported certificate survives on only 12 of 22 zones. The seam dies at the *zone*, not at the ratio — so no ratio threshold can ever close the programme, and the lemma the seam architecture needs is not "a larger ρ" but a *zone-uniform* η/τ bound at ratio ≤ 2.026 (U5 below, sharpened).

What carries is the free-resolution route. Releasing the resolution from the per-segment cap and taking the running minimum $D_k = \min(\text{cap}_k, D_{k-1})$ makes monotonicity hold *by construction* — no DP, no gap theorem, not one coarsening seam anywhere. The largest single refinement drop over $n \le 10^5$ is 2.0256 (zone $n = 8191$ — the same binding quotient, now read in the good direction), and re-gridding happens at only **764** of 9 700 boundaries (7.88%): a drop needs a *record-small* gap, and every other boundary is a no-op at ratio 1. All 12 affordable actual seams of that route ($n = 3 \ldots 125$, $h \le 1222$, largest real ratio 1.943) are built and transported: 11 carry a positive certified floor in *one* re-gridding, the one at $n = 31$ (transported mass $\tau = 0.363$, unusually poor) closes through a three-stage ladder with composed floor $3.56 \times 10^{-2} > 0$; median retention 0.578, all 12 brackets valid. The seam device itself is sound throughout: the Haynsworth partial minimisation reproduces $\lambda_{\min}(S)$ to 2.8×10^{-14} on both grids and the transport bracket contains the fine value on 30/30 real pairs; and a *full* seam is certified end to end at the ratio the arithmetic demands — spine on grid A, one re-gridding (lower bracket end $8.671 \times 10^{-3} > 0$, retention 0.103), spine on

grid B — at $\rho = 2.251 \geq 2.0256$ on 3 of 4 attempts (the failures are the zone dependence above, printed and not hidden). The price of the route is honest and structural: the resolution overhead $D_{\text{free}}/\text{cap}$ has median 0.201, i.e. the *running minimum* of the gaps sets the window cost instead of the local gap — window size, not an assumption; certified seam by seam the route covers $n \leq 125$, an evidence limit rather than a gap in the argument.

10.2 The uniformity audit (U1–U6)

Every constant of the load-bearing spine is measured over zones *and* telescope depth (35 rungs, 14 zones, (8R) valid on 35/35 while audited; every fit jackknifed; the status rule declared before the numbers, and "measured-flat" is never upgraded to "uniform"):

Constant	**Finding**	**Status**
U1 envelope margin	forced by the construction's own doubling loop	self-certifying
U2 oversampling depth L/M	needs one α-uniform Bernstein/Markov bound	classical in form
U3 rung quality cb/δ	measured-flat, but the reason is *direction*, not spectrum: the worst pencil direction of (S, U) does not explain the flatness (alignment 0.866–0.987, median 0.975; $\mu_{\min}$ itself falls with the window)	genuinely new
U4 base-case head-room	a compute wall over the declared fp floor	no proof obligation
U5 Albert–Schur floor	$\lambda_{\min}(S)/\|A\|$ *falls* in the window direction — the one load-bearing gap; sharpened by the seam scan to a zone-uniform η/τ bound at ratio ≤ 2.026	genuinely new
U6 resolution monotonicity	the running minimum is monotone by construction	resolved

The audit's own finding is that (F6) is not one lemma but three — U2 (classical in form), U3 (a direction lemma on $\hat{s}$), U5 (a zone-uniform Schur floor) — of which exactly two are genuinely new mathematics.

10.3 The continuum step is a fractional Dirichlet energy

The chain one would write down first — density of piecewise-constant functions plus a continuity constant — is *refuted*: the archimedean kernel is Cauchy-type $\sim 1/(2|s|)$, not log-type, hence not locally integrable; its L^1 mass over the window and $\lambda_{\max}$ both diverge like $\log(1/D)$, so neither an L^2 nor a sup-norm continuity constant exists (measured). What makes the divergence harmless is an *identity*: with row sums w_r,

$$v^{\mathsf{T}} A v \;=\; \sum_{r} w_r v_r^2 \;+\; \sum_{r<s} (-A_{rs})\,(v_r - v_s)^2,$$

with 86.1% of the off-diagonal entries measurably negative and the row sums bounded — the window form *is* a fractional Dirichlet energy, and the divergent diagonal is the $\frac{1}{2}$-energy of a step function, a property of the test class rather than a defect of the kernel. The limit argument is therefore quantitative, and it is measured: on the dyadic tower (5 levels, $h = 63 \ldots 1008$, a $16\times$ spread in D) the Galerkin values converge at order $D^{1.798 \pm 0.057}$ for a C^∞ bump and $D^{1.802 \pm 0.065}$ for a Lipschitz hat — superlinear in both cases, exactly as the energy identity predicts — with every level strictly positive and Richardson limit $2.1256 \times 10^{-4} > 0$. What is classical here is the consistency estimate for a Cauchy-kernel Galerkin scheme (Calderón–Zygmund; Bramble–Hilbert 1970); what is measured is its rate on this kernel; the abstract density step is not needed and not used. The statement that would stand at the end is **positivity of the Weil window form on Lipschitz test functions supported in the reached window** ($\alpha = 1.5046$ on the tower, 1.6882 along the seam chain) — a finite-window statement whose bridge to RH is Weil's criterion (Weil 1952; Bombieri 2000), cited here and used in neither direction.

10.4 The full-proof map: sixteen points, two genuinely new

The definitive fail list after the three attacks has 16 points: 5 closed or classical, 9 conditional (constructed here, classical in form, or a machine limit), and exactly *two* genuinely new pieces of mathematics:

The two remaining inequalities (T126)

(U3) The direction lemma. Why the rung's data residual $\hat{s}$ sits in the good subspace of the pencil (S, U): the measured flatness of cb/δ is provably *not* the pencil's worst-case behaviour (alignment 0.975 while $\mu_{\min}$ falls), so a proof must control the *direction* of $\hat{s}$, not the spectrum.

(U5) The zone-uniform seam floor. A zone-uniform η/τ bound for the re-gridding transport at ratio ≤ 2.026 — the constant every seam consumes. Ladders provably cannot repair a bad destination grid, so the reach must be won per *zone*, not per ratio.

S4, the coarsening bracket — which this part opened and which would have been a third genuinely new statement — is *dissolved* by the free-resolution route rather than proved. Still open and unchanged: the (C-F5) accounting convention, and the all-α step — the one thing no computation can close (at the tight cap, 6 boundaries in 9 700 would need a double seam; for all n one needs a gap theorem, or the free-resolution relaxation, which is unconditional in the arithmetic). T127 (`two_inequalities_probe.py`, contract `TWO.INEQUALITIES`, 28/28, verdict BOTH-SHAPED) then took both inequalities head-on. The result — both come back proof-shaped, and *neither is the statement T126 named* — is the remainder of this section.

10.5 The separating variable is geometric, not arithmetic (T127)

The fixed-ratio sweep reproduces exactly: at the worst-case ratio 2.0256 the transported certificate survives on 12 of 22 zones, T126's list — and what dies is the *certificate*, not the positivity: the true target eigenvalue $\lambda_{\min}(S_f)$ is positive on all 22 zones. Of 29 candidate separating variables, exactly 5 separate the surviving from the dying zones perfectly (22/22; the rank statistic is a description of a 22-point sample and is never called a test), and all five measure the same *geometric* quantity: the right-hand protrusion $s_{l,r}$ — the two grids quantise the same window independently, their border blocks have equal width but sit offset by one rounding, and the transported mass τ_{dn} loses exactly what protrudes past the coarse block's inner edge (correlation +0.997 with τ_{dn}). Every arithmetic candidate fails (gap, α, atom weight, alias phase; best separation 0.727) — the separation is *not* the atom and *not* the gap. Deep arithmetic ($n \leq 300\,000$, 1 082 record seams): the fine border block has 4 cells everywhere, 97.60% of real seams have ratio ≤ 1.05 with median 1.00140 — real seams are *near-identity re-griddings* — and the 11 seams with ratio > 1.2 *all* have next-atom distance ≤ 2: Mersenne $(3, 7, 31, 127, 8191)$, Fermat-like $(4, 16, 256, 65536, 131071)$, one twin $(101, 103)$. Mihăilescu 2004 (Catalan) is cited as the classical address of that list and not used.

10.6 U5 as posed is refuted; the U5 band replaces it

As a zone-uniform certificate at ratio ≤ 2.026, U5 is *dead as posed*. The replacement is **U5-band**: all 22 zones survive for every ratio $\rho \leq 1.49531$, and at least one dies for every $\rho \geq 1.49766$ (bisected to 2.3×10^{-3}; binding zone $n = 19$) — a band that covers 99.26% of the route's real seams, the remainder being the 8 enumerated pairs above. The split of the form defect is an *identity* ($\eta = \eta_{\mathrm{cons}} + \eta_{\mathrm{proj}}$, exact), and η does *not* become small as $\rho \to 1$ (max $|\eta|/\lambda = 1.146$ at the *tightest* ratio): the band buys $\tau_{\mathrm{dn}} \approx 1$, not small η. The floor is measured at $c \geq 0.0090$ over 307 in-band transports with a positive single-step floor up to $n = 1331$, $h = 1496$. **The sharpest new finding:** the near-identity census (43 deep zones, all at ratio 1.00140) fails *once* — $n = 479$, $|\eta|/\lambda = 1.472$, within 0.14% of the identity — so the ratio band is only a *proxy* for the border geometry; the failure is repaired by a three-stage ladder (retention 0.5027), exactly as T126 repaired $n = 31$ — the fallback is part of the statement. And the risk is recognisable *before* any factorisation: the

indicator $\tau_{\mathrm{dn}}/\tau_{\mathrm{flat}}$ ranks the margin at Spearman +0.749, and the one failure is its argmax (τ_{flat} is a predictor, not a bound).

10.7 U3 defuses to a coarse floor: the harmonic-mean identity

Three identities restructure U3. The pole part of the rung data has the closed form $(Z^{\mathsf{T}}b)_j = -\frac{16}{\sqrt{2D}}\sinh^2(D/4)\cosh(\bar{x}^{\mathrm{c}}_j/2)$ (to 3.2×10^{-13}); the rung quality is a *harmonic mean*, $\mathrm{cb}/\delta = 1/\sum_i w_i/\mu_i$ (to 1.5×10^{-7}); and the weights split by Parseval (to 6.1×10^{-16}). The residual's mass on the good half of the pencil spectrum $\{\mu \geq \frac{1}{2}\}$ is $0.9665\ldots0.99998$ over 42 rungs (flat), so $\mathrm{cb}/\delta \geq 1/(2m + (1-m)/\mu_{\min})$ — a floor as *crude* as $\mu_{\min} \geq 1.62\times10^{-2}$ (one part in 62) already gives $\mathrm{cb}/\delta \geq \frac{1}{4}$, where T126 had to measure 0.192–0.307 sharply. The reading "$\hat{s}$ avoids the bad direction" is *refuted*: the flatness is generic — every smooth, flat, alternating or quasi-random direction has good-band mass above 0.50 — and exactly *one* direction type fails, a *boundary layer* at the window edge (concentration 3.1× there, against 0.03× for $\hat{s}$ and 0.001× for its closed-form pole part). The bad pencil directions are *not* edge-localised in the ℓ_2 sense (1.23×) — only the U-metric concentration separates.

10.8 The map V2: three inequalities and one enumeration

The map V2 has 16 points: 10 carried, 2 conditional-classical (one Bernstein/Markov bound, the declared fp floor), and the 2 genuinely new ones of T126 *reformulated*:

The remaining mathematics (T127, map V2)

(T) The border-retention bound. A lower bound for the transported mass τ_{dn} under the edge-geometry hypothesis — from predictor to bound, via the support of the S_f eigenvector; the ratio band $\rho \leq 1.50$ is only a proxy for it.

(E) The $|\eta|$ upper bound. Both parts of the form defect $\eta = \eta_{\mathrm{cons}} + \eta_{\mathrm{proj}}$, to be bounded *jointly* with (T): the two rank together at +0.749.

(M) The pencil-mass bound. The residual's good-band mass plus the crude floor $\mu_{\min} \geq 1.62\times10^{-2}$; the one direction type to exclude is named (a boundary layer at the window edge), and the closed-form pole part provably is none.

(L) The enumeration. The 8 out-of-band seams up to $n = 300\,000$ (Mersenne/Fermat pairs and one twin), a finite list; where a single step fails, a three-stage ladder composes — the fallback is part of the statement.

The craft judgement, labelled as one: the three remaining inequalities are of the *same craft* as the proven links of the chain — finite-dimensional inequalities between constructed matrices — and the residual risk is concentrated in the word *uniform*, which no amount of this probe's arithmetic can discharge. T128 (`teml_probe.py`, contract `TEML`, 27/27, verdict THREE-OF-FOUR) then worked that four-point list in order of cost, cheapest first. Three of the four points stand at their preregistered bars; the fourth — the protrusion concentration inside (T) — missed its own bar by 3.6%, systematically in the re-gridding ratio, which is exactly the kind of near-miss that distinguishes a bound from a fit. The remainder of this section is the four points in order.

10.9 (L) is derived and closed: the out-of-band table (T128)

The out-of-band class is no longer quoted but *derived*: applying the U5-band threshold $\rho > 1.49531$ (T127's number, quoted and never re-bisected) to the free-resolution record schedule yields *exactly eight* seams up to $n = 300\,000$, all at next-atom distance 1 and all with identical border geometry ($\mathrm{gc}_c = 2$ coarse border cells, $\mathrm{gc}_f = 4$ fine) — each is a pair $(2^k-1, 2^k)$ or $(2^k, 2^k+1)$ whose log-gap halves the previous record. Every seam carries a status and no residue:

seam n	ratio ρ	class	status	retention / required h_f
7	1.671	Mersenne	CERTIFIED-1STEP	0.179
16	1.943	Fermat-like	CERTIFIED-1STEP	0.156
31	1.910	Mersenne	CERTIFIED-LADDER (2 rungs)	0.091
127	2.024	Mersenne	BUDGET-OPEN	$h_f = 2\,476$
256	2.012	Fermat-like	BUDGET-OPEN	$h_f = 5\,694$
8191	2.026	Mersenne	BUDGET-OPEN	$h_f = 295\,253$
65536	2.001	Fermat-like	BUDGET-OPEN	$h_f = 2\,907\,297$
131071	2.000	Mersenne	BUDGET-OPEN	$h_f = 6\,177\,926$

The five open seams exceed the hard factorisation cap $h \le 1500$ by 1.7× to 4119×, and each is open for *one* reason only: a single Cholesky of size $h_f - \mathrm{gc}_f$ is missing — nothing conceptual. Each is characterised arithmetically and geometrically without any factorisation (ratio, both border cell counts, both window roundings, the flat transport factor), so the characterisation is available at any depth. "Closed list" means every item is enumerated and labelled, not that every item carries a Cholesky — that distinction is the point of the block. Mihăilescu 2004 (Catalan) remains the cited, unused address of the class; over a finite range a list is enumerated, not conjectured.

10.10 (T) becomes a bound, and its bar is missed honestly

The step from predictor to bound is an exact identity plus two explicit majorants. With y the transported S_f eigenvector, m_{prot} its mass on the protrusion, $m_{\mathrm{fill}} \ge 0$ the interior mass on the sliver the coarse block adds, and $V_{\mathrm{bord}} \ge 0$ the intra-coarse-cell variance of the transported function,

$$\tau_{\mathrm{dn}} \;=\; \|y\|^2 - m_{\mathrm{prot}} + m_{\mathrm{fill}} - V_{\mathrm{bord}}, \qquad \text{hence} \qquad \tau_{\mathrm{dn}} \;\ge\; 1 - m_{\mathrm{prot}} - V_{\mathrm{bord}},$$

verified to 9.0×10^{-13} against an independent full projection on all 126 transports. The bound is valid on 126/126 (slack = m_{fill}, median 0.000); with the two explicit majorants — the outer-cell majorant for m_{prot} and the discrete oscillation majorant $\rho(\lceil\rho\rceil+1)/2 \cdot \sum(\Delta y)^2$ for V_{bord} — it stays valid on 126/126 and positive on 125/126 (the exact form is positive on 126/126); total slack median 0.275, and the loose part is the oscillation majorant (median 7× overshoot), so what (T) still needs is a Bernstein/Markov-type statement about the discrete oscillation of one $\mathrm{gc}_f \times \mathrm{gc}_f$ eigenvector. **The honest miss:** the protrusion concentration $\kappa = m_{\mathrm{prot}}\,\mathrm{gc}_f/(t_l + t_r)$ — eigenvector mass on the protrusion against what a flat border vector would put there — measures 0.086–2.071 over the 120 transports with a nonzero protrusion, and *misses the preregistered bar* $\kappa \le 2$ *on* 1/120 *by* 3.6%. The excess is systematic in the ratio, not accidental: the group maxima rise monotonically, 1.889/1.965/2.071 at $\rho = 1.001/1.250/1.495$, with the argmax at the band edge. The bar is *not* moved — a bar moved after the numbers would make everything else here worthless; the shape the next preregistration should declare is $\kappa \le 2 + C(\rho - 1)$, the shape the U5 band already has. A side finding with a side: κ is 1.61–2.07 when the protrusion sits at the *inner* edge of the border block and 0.09–0.25 at the window edge — (T) is an *eigenvector* statement, not a rounding statement, contrary to what the +0.997 correlation invites one to think.

10.11 (E) and (T) jointly: the ℓ_2 route dies, the energy route carries

The ℓ_2 route to η_{proj} is structurally lossy where it matters: the norm bound overshoots the true defect by a median factor of 55, and since the available margin relative to λ_c is smaller still, it certifies 0 of 12 real seams. What carries is the *energy route*, an exact Cauchy–Schwarz chain in the Q-inner product: with Q_f certified PSD and $d = w_f - Pv$ the projection residual,

$$\sqrt{\lambda_f} \;\ge\; \sqrt{\lambda_c\,\tau^* - |\eta_{\mathrm{cons}}|} \;-\; \sqrt{e_d}, \qquad e_d = d^{\mathsf{T}} Q_f d,$$

with $e_d/\lambda_c = 0.041$–0.154 on the real seams. The joint chain certifies a positive floor on 9/9 **in-band seams** (floor ≥ 0.0850) and 11/12 overall. The one tear — $n = 31$, already on the (L)

list — is η-*limited*, not geometric: $|\eta_{\mathrm{cons}}|/\lambda_c = 0.392$ exceeds the whole geometric floor $\tau^* = 0.363$, so no sharper (T) would have helped; and shrinking the ratio is no alternative either, since the consistency defect does not vanish as $\rho \to 1$ (median 0.089 at $\rho = 1.0014$ over 43 zones). The tear is repaired by the declared two-rung ladder in which *every rung closes the chain by itself* — the fallback is part of the statement, not a patch.

10.12 (M) is earned: the exclusion is a proof, the floor is constructed

Three parts. *(a)* The boundary-layer exclusion for the pole part of $\hat{s}$ is now a *proof* from the closed form $(Z^{\mathsf{T}}b)_j = -\frac{16}{\sqrt{2D}}\sinh^2(D/4)\cosh(\bar{x}^{\mathrm{c}}_j/2)$: the adjacent-component ratio lies in $[e^{-D/2}, 1]$ and the outer-k-cell share is bounded by $kD(1+\cosh\alpha)/\sinh\alpha$ — pole edge mass $\Theta(D)$ against a layer's $\Theta(1)$, separation at least $84\times$ on all 32 (α, D) pairs, by elementary inequalities rather than a finite sample. *(b)* The crude floor is *earned*: $\mu_{\min}(S, A_z) = 1 - \gamma^2$ exactly (to 5.9×10^{-15}; the strengthened Cauchy–Schwarz constant of the two-grid splitting, Eijkhout–Vassilevski 1991), and $U \preceq (1+\varepsilon_{\mathrm{env}})A_z$ with $\varepsilon_{\mathrm{env}} \le 0.038$ Cholesky-certified, so the constructed floor is a true lower bound on all 31 rungs: **0.188** against the 1.62×10^{-2} the harmonic-mean argument demands — an 11.6× margin, where T127 could only measure. *(c)* The comb residual stays measured, but it is measurably *no* layer: half of its mass needs 12–39% of the half-window against one cell for a genuine boundary layer, and it vanishes with the cell width as $D^{+1.54\pm0.09}$ (a fit with a jackknife band).

10.13 The map V3: twenty-one points, five genuinely open

The map V3 (T128): what is left, by kind

21 points: **10 done** (T127's map still had the (M) floor, the (T) identity and the (L) list among its open items; all three are now closed), **3 certified on real objects** — valid on every seam, transport and rung the hard cap admits, wanting only the word "uniformly" — and **3 conditional-classical** (Bernstein/Markov for the border oscillation, Bramble–Hilbert for the consistency defect and the Q-invisible residual, and the continuum leg). **5 genuinely open**, and no two of the same kind:

M9 the κ bar — missed by 3.6%, systematically in ρ; next hypothesis $\kappa \le 2 + C(\rho - 1)$.
M17 the pencil mass of $\hat{s}$ on $\{\mu \ge \frac{1}{2}\}$ — one measured scalar per rung, no derivation.
M18 the comb residual is not a cell-scale layer — measured, not yet bounded.
M19 uniformity over *all* zones — the word "for all", which a finite probe structurally cannot supply.
M21 the RH address itself — Weil positivity as the address of the limit statement, cited and never used.

The honest distance, in the probe's own three sentences, quoted:

> *(1) After this block the finite-window chain has no step left that is merely plausible: every inequality it uses is either a completed certificate, an identity verified to machine precision, or an explicit inequality that has been evaluated — and found valid — on every seam, transport and rung the hard cap* $h \le 1500$ *admits, so the remaining mathematical content is three named classical estimates (Bernstein/Markov on one small Schur complement, Bramble–Hilbert twice) plus two measured scalars (the pencil mass, the comb's delocalisation). (2) The distance is nevertheless not small, and it is not small in a way no larger computation can shrink:* 5 *of the* 21 *map items are open because a finite list of objects cannot produce the word "for all zones", and one of them — the protrusion concentration* κ *— has just been measured outside its own preregistered bar by* 3.6 *per cent, systematically in the re-gridding ratio rather than accidentally, which is exactly the kind of near-miss that distinguishes a bound from a fit. (3) And even with all four TEML points closed, what would stand is a positivity statement about a finite window at the* α *actually reached: the continuum leg (M20) and the passage to Weil's criterion (M21) are untouched by anything in this probe, so the honest distance to RH is not "one uniformity lemma" but "one uniformity lemma, three classical estimates, a continuum limit, and a theorem nobody has".*

T129 (`kappa_deep_seams_probe.py`, contract `KAPPA.DEEP.SEAMS`, 28/28, verdict KAPPA-WILD) then did what the map demanded: it froze the constant of the κ law, tested the law on fresh transports, brought the two affordable deep seams under the cap, and bounded the comb. The verdict is the most productive break of the phase — *the fitted law falls, and the theorem underneath it stands.* The remainder of this section is that part.

10.14 The honest break: the κ law falls once, and the theorem underneath stands (T129)

The constant was frozen *before* the test: $C^* = 0.3506$, the least-squares slope of the T128 group maxima against $\rho - 1$ over the three calibration groups (jackknife SE 0.0955, intercept 1.8909, rms 0.0096) — a fit, labelled a fit, not rounded up, not padded, not refitted after the test; on the calibration set itself the law is violated 0/105, reported for completeness because it is *not* the test. On **331 new transports** at 14 ratios none of which is in the calibration set ($\rho = 1.05 \ldots 4.00$, $h_f = 29 \ldots 1450$, $n = 5 \ldots 1331$), the frozen law $\kappa \le 2 + C^*(\rho - 1)$ is violated **once** ($n = 7$ at $\rho = 3.00$, slack -0.0334), where the bare unmoved bar $\kappa \le 2$ would be violated 78 times; the sharper per-side law on κ_r alone is violated 1/293; and the T128 side asymmetry reproduces on entirely new data ($\kappa_r = 1.697$–2.913 against $\kappa_l = 0.054$–0.205). The bar is not moved — hence the verdict.

What makes the break productive is that the *structural ground* of κ is now solved, with identities rather than fits. Three pieces, in the order they stack. *(a)* The interval identity $\sum_i (1 - g_i) = t_l + t_r$ (verified to 7.1×10^{-13} on all 105 calibration transports) makes κ a *probability average of the border density*: the weights sum to 1 exactly, so the **flat** border vector sits at $\kappa = 1$ *exactly* — T128's bar 2 was *twice* the flat value, not the flat value. *(b)* The profile identity $p_{N-1} = 2 - p_0 + 2E$ (to 6.7×10^{-16}) makes a **linear** density profile with vanishing edge give exactly $\kappa = 2$: the bar 2 is the *linear-density limit*, and everything above it *is* the convexity of the border density ($E > 0$ on 219 of 436; Chebyshev's correlation inequality gives the sign). *(c)* The chain

$$\kappa \;\le\; 2 - p_0 + \tfrac{N-2}{N} \sum_i |\Delta p_i - \overline{\Delta p}| + (p_{\max} - p_{N-1})$$

is a *theorem with an explicit constant* — Hölder for the mean, telescoping for the profile identity, Abel summation for the majorant — and it holds on **all 436 transports** (calibration plus test; slack 0.0184–2.3126, median 0.3438). On 392 of 436 the density peaks at the inner end of the border block, the last correction vanishes, and the bound is the clean form $2 - p_0 + \frac{N-2}{N}\,\mathrm{CURV}$. The ρ-systematics T128 found is thereby *named*: $2E > p_0 \iff p_{N-1} > 2$ *exactly*, confirmed transport by transport (114 of 436 satisfy both, no exception) — a competition between the convexity term and the edge value, not a one-sided effect; the majorised curvature grows as $0.1445 + 0.1634(\rho - 1)$ (a fit) while p_0 shrinks with ρ, and both pull the same way. **The depth block:** 14 graded transports at $h_f = 1656 \ldots$ **17 669** — up to 12× the hard factorisation cap 1500, reached matrix-free through the two-scale assembly ($m/h = 0.064$–0.512), $\rho = 1.25$–2.50, $n = 79$–1019 — show **zero** violations of the frozen law, the curvature chain holds on 14/14 (slack 0.042–1.998), and κ does not drift upward with depth: whatever residual risk remains is a *ratio* effect, not a resolution effect, which is the one thing a finite computation can actually settle.

10.15 The deep seams carry — on the graded space, honestly downgraded

The machinery first, verified against the uniform reference at $n = 71$ ($h = 619$): the matrix-free two-scale assembly equals $J^{\mathsf{T}} Q J$ to 2.7×10^{-16} (and at $q = 1$ it equals the uniform form to 0.0, so the graded path *contains* the uniform path and no separate code is trusted); the compressed step identity $X = Q_{\text{old}}$ holds to 2.4×10^{-17}, so the step stays margin-free (Albert 1969); and the graded interval overlap reproduces the prolongation J itself to 6.4×10^{-14}. Then the calibration, printed *before* the seams: over 48 (zone, q) pairs at 12 zone/ratio settings the graded Schur floor is 1.534–5.181× *larger* than the uniform one (median 2.646) — Rayleigh–Ritz, the wrong direction for

a proof, with no exception — and on **4 of 48** pairs the graded floor is *positive where the uniform floor is not*: demonstrated **false positives** of the compression, a measured 8% rate at depths shallow enough that both sides can be computed. Both deep seams are then built and both fit: $n = 127$ and 256 ($h_f = 2476$, 5694) compressed to $m_f = 1256$, 1452 ($m/h = 0.507$, 0.255; $q_f = 2$, 4), every factorised block at or below the cap and the fine square matrix never formed; **both carry a complete graded seam certificate** (X positive definite by Cholesky, both Schur floors above the Wilkinson floor, transport bracket positive), retentions 0.3070/0.4165; as a κ test on *genuine record seams* ($\rho = 2.0238/2.0118$, outside the calibration range) they give 0 violations (chain slack $+0.079/+0.178$); and the floor survives 6 further coarsening rungs (0.166–0.883) — measured evidence against a one-merge artefact, which does *not* repair the Rayleigh–Ritz direction. The honest ledger, stated as the probe states it: $(L) = [\,3$ uniform-certified (T128), 2 GRADED-CERTIFIED here with the declared false-positive rate, 3 budget-open$\,]$ — emphatically *not* "5 certified, 3 open". The two deep seams moved from "no certificate at all" to "a complete certificate on a coarser Galerkin space, with the direction of the error known and its size calibrated" — strictly more than T128 had and strictly less than a uniform certificate.

10.16 The comb is bounded: the operator majorant works, the row bound is vacuous

Both candidate majorants for the comb's outer-k-cell share are majorants on all 40 zone × depth points (5 zones, depths $\times 1/2/4$, $h = 64$–946; the measured share itself is 1.96×10^{-3}–0.573, median 0.038). The unconditional Cauchy–Schwarz row bound is **vacuous**: below the bar 1 on 0 of 40, overshooting the truth by a median factor of 116 524 — reported as the failure it is, not as a footnote. The **S-procedure operator bound works**: below 1 on 40 of 40, running 0.170–0.774 and only 1.3–182× above the truth (median 7.8) — a statement about *every* coarse vector in the measured smoothness cone, not merely the one that happens to occur. Status CERTIFIED-CONDITIONAL: conditional on one measured cone parameter with a known law ($\theta_1 \sim D^{1.219}$, a fit with jackknife SE 0.088 — exactly the scaling of a fixed smooth profile sampled at cell width D). The honest limit: the bound *saturates* at $(kD_c)^{0.263}$ (a fit) instead of decaying like the pole part's closed-form $O(kD)$, because the cone still contains vectors far more edge-loaded than the coarse solve that occurs.

10.17 The map V4: two irreducibles, three new points, and the promotion assessment

The map V4 (T129): nine entries, and the two irreducibles named

3 reduced or bounded by this probe: **M9** the κ bar — *reduced*, not merely retested: κ is now an identity plus a theorem, and what remains open is a bound on the *curvature* of the border density on $\lceil t \rceil$ cells, a strictly smaller object than the original mass hypothesis; **L2** the two affordable deep seams — *graded*-certified with the direction of the error known and its 8% false-positive rate measured; **M18** the comb — *bounded* by the conditional operator majorant. **1 untouched**: **M17** the pencil mass (T128's measured 0.9665 and the constructed floor 0.188 stand). **2 irreducible here**: **M19** the word "for all" — the curvature chain is a theorem *per transport*; making it a theorem *for all* transports needs a zone-uniform curvature bound no finite computation can supply; **M21** the RH address itself — Weil 1952 / Bombieri 2000 / Connes 1999 cited and used for nothing. **3 new, created and named rather than buried**: **M22** the fitted constant C^* (it stays a fit until the curvature term is bounded in closed form — the same open question in different clothes); **M23** the graded-to-uniform conversion (needs the Céa/Strang defect $\|(I - \Pi)z^*\|^2$ of the fine partial minimiser; Q has an FFT matvec, so this is an *arithmetic* obstruction with a known route, not a conceptual one); **M24** the per-side law (κ averages a large right protrusion with a small left one, so T128's quantity is systematically milder than the mechanism it stands for).

Promotion readiness (T129): ready in principle, as a narrow module

The load-bearing content of this probe that would survive a `verification/vN_*` gate is exactly the part that is identity or theorem and needs no sampling: *(1)* the interval identity $\sum(1-g_i) = t_l + t_r$ and hence $\kappa_{\text{flat}} = 1$; *(2)* T128's four-term (T) identity; *(3)* the profile identity $p_{N-1} = 2 - p_0 + 2E$; *(4)* the Abel majorant $\kappa \le 2 - p_0 + \frac{N-2}{N}$ CURV; *(5)* the assembly identities (two-scale assembly $= J^{\mathsf{T}}QJ$, graded overlap reproduces J). Those five are checkable at fixed small sizes in seconds and are true per instance. **Not ready**: the constant C^* (a fit), the graded seam floors (a weaker Galerkin statement with a demonstrated false-positive rate), the comb-law exponent (a fit), and anything containing the word "uniform". A module built only on (1)–(5) would be load-bearing and small; a module that quoted C^* or a graded floor as a certificate would not be, and the ledger would be right to refuse it.

The honest distance, in the probe's own three sentences, quoted:

> *After K1 the retention bound (T) is no longer a hypothesis about eigenvector mass but a theorem reducing it to the curvature of the border density on $\lceil t \rceil$ cells, with $\kappa = 1$ the flat limit, $\kappa = 2$ the linear limit and an explicit Abel majorant for the rest. After K2 the (L) list has no item left that is open for a conceptual reason: two of the five deep seams are certified on a coarser space with the error direction known and its size calibrated, and the other three need one Cholesky each at a size that is a pure arithmetic fact. After K3 both parts of $\hat{s}$ — the pole part in closed form and the comb part by a certified majorant — are excluded from being boundary layers, so (M)'s one remaining direction type is gone; what is left is what was left before: the word "for all", and the distance from any finite window to the criterion of Weil, which nothing in this probe touches.*

T130 (`curvature_bridge_probe.py`, contract `CURVATURE.BRIDGE`, 30/30, verdict ONE-OF-TWO) then attacked the two named open pieces T129 left — the curvature bound (M22/M24) and the graded-to-uniform bridge (M23) — and exactly one of the two stands: *the bridge stands, the curvature bound does not.* The remainder of this section is that part.

10.18 The curvature term becomes a total variation — and the frozen shape band breaks (T130)

The structural gain of the curvature front is a new *exact* rewrite of what the term is:

$$\sum_j |\Delta p_j - \overline{\Delta p}| \;=\; \mathrm{TV}(P), \qquad P_k \;=\; p_k - (p_0 + k\,\overline{\Delta p}), \quad P_0 = P_{N-1} = 0,$$

the total variation of the density profile's own *chord deviation*, certified per transport by $\mathrm{TV}(P) \le (2n_{\text{run}} - 2) \cdot \mathrm{sag}$ with $\mathrm{sag} = \max_k |P_k|$ — equality when P has a single monotone hump, in which case the curvature term is *one* geometric number; on 83 of the battery's 765 transports that is literally the case. The border eigenvector never changes sign inside the border block on 765/765 transports (the T109 cancellation geometry lives outside the block), and the shape family is decided: the power profile $|w_i| \sim (i+1)^a$ wins on 635/635 transports against linear, exponential and reciprocal-pole candidates ($a = 0.703$–0.999, median 0.856 — T109's linear border profile, independently recovered). The Levinson coupling supplies the *structure*, not the bound: the T108 corner identity $u_0 = -\sqrt{2}\,\rho_{M-1}/E_{M-1}$ is independently reproduced (24 zones up to $M = 1236$, to 4.2×10^{-9}, exactly one negative pivot per zone), but the pole solve as a profile model is only a *proxy* (direction cosine 0.992–0.996, curvature ratio 1.5–5.3) — no majorant deployment. And the honest core: the shape band frozen on the calibration half ($a \in [0.734, 0.897]$, $S^* = 1.1926$) **breaks on 13 of 545 disjoint test transports** — the bar is not moved; with the per-transport measured exponent the same S^* holds on 520/545. **M22 is thereby reduced to a zone-uniform bound on *one* exponent — reduced, named, and not closed.**

10.19 The bridge stands as an identity: the matrix-form Céa/Strang defect

The graded-to-uniform bridge — the object M23 named — is implemented and it is an *identity*:

$$S_{\text{graded}} - S_{\text{uniform}} \;=\; R^{\mathsf{T}} X^{-1} R,$$

verified to 1.3×10^{-13} and evaluated entirely matrix-free (an FFT matvec for the fine form, a BPX-preconditioned CG, one graded Cholesky at or below the cap — the fine square matrix is never formed). The decisive distinction is scalar against *matrix* form: the scalar bound $\lambda_{\min}(S_{\text{graded}}) - \lambda_{\max}(G)$ is vacuous (it loses a factor 3–28), while the matrix form reproduces the *directly computed* uniform Schur floor on **84 compression pairs at 21 zones to** 2.5×10^{-11} **relative, without a single overshoot** — so GRADED-CERTIFIED becomes a one-sided statement about the *fine* space. T129's measured 8% false positives are thereby explained *completely*: 16/84 reproduce at the bracket level but **0/84 at the floor level** — they sat in the transport bracket (graded test vectors), an object the bridge makes unnecessary, not in the Schur floor. **Both deep seams carry**: $n = 127$ ($h_f = 2476$) $\to \lambda_f = 0.2255$ and $n = 256$ ($h_f = 5694$) $\to \lambda_f = 0.2614$, with positive bridged transport brackets, at 1.7× and 3.8× the factorisation cap, reached through FFT matvec and CG only. The status is honest: BRIDGED-CERTIFIED-CONDITIONAL — exactly *one* measured number per grid enters, a positive lower bound for $\lambda_{\min}(X_{\text{fine}})$, currently supplied by Lanczos (which bounds $\lambda_{\min}$ from *above*: a measurement, never a certificate); the Grenander–Szegő symbol route for that number is *dead* (the Weil lag symbol is indefinite at these windows, rigorous symbol floor $-76 \ldots -27$); and the robustness is benign: the bridge conclusion does not move when the assumed floor is shrunk by 100× or 10 000×.

10.20 The theorem battery, the map V5, and the promotion check list

The κ battery against the *theorem* chain rather than the fitted law: **zero violations of the exact chain on all 765 transports** of the new battery — the chain is proved per transport, so this is implementation verification, not a test. The fully explicit chain (the D1 candidate substituted for the curvature term) closes 635/635 of the shape transports — but under the bare bar 2 it closes *none*: measured $\kappa > 2$ on 130/765. Read honestly: the chain's own slack survives the 13 shape-band misses — that is robustness, not repair; the bound that would close M22 still does not exist.

The map V5 (T130): twelve items, one new gap, and the self-supply candidate

Certified: **I1** the interval identity $\sum(1 - g_i) = t_l + t_r$ (ω a probability measure, $\kappa_{\text{flat}} = 1$); **I2** κ as a weighted mean of the border density (Hölder); **I3** the profile identity with $2E > p_0 \iff p_{N-1} > 2$; **I4** the Abel chain; **I5 new: the TV rewrite** with the one-hump collapse and the $(2n_{\text{run}} - 2) \cdot \text{sag}$ bound. **Certified-conditional**: **I6** the Céa/Strang bridge and **I7** the two deep seams (one measured number per grid). **M22** reduced to a zone-uniform bound on one exponent; **M23** *conditionally closed* (the bracket-level false positives remain as a test-vector artefact; the bridge replaces the bracket rather than repairing it); **M25 new: a certified positive lower bound for** $\lambda_{\min}(X_{\text{fine}})$ — the only input I6 lacks, and it is small (the 10^4 robustness above); its elegant candidate is the *self-supply*: the chain's own telescope ε floor could deliver it, which is exactly what T131 tests. **M19** ("for all") and **M21** (the RH address) are untouched.

The promotion check list (T130): what a narrow vN module would carry

Exactly the certified block: *(1)–(5)* T129's five identity/theorem pieces (interval identity with $\kappa_{\text{flat}} = 1$; the mean representation with the Hölder step; the profile identity and its equivalence; the Abel chain; the TV rewrite with the sag bound), plus new from this probe *(6)* the defect identity $S_{\text{graded}} - S_{\text{uniform}} = R^{\mathsf{T}} X^{-1} R$ with its matrix bridge — pure Schur/Galerkin algebra, checkable on a random symmetric pair in three lines. **Not in**: the frozen shape band and S^* (a calibrated hypothesis, M22), the Lanczos inner floor (M25), and both deep-seam certificates (they inherit M25). The three remaining (L) seams are pure scale questions: only $n = 8191$ ($h_f = 295\,253$) admits a graded frame under the cap at all ($q = 256$, $m/h = 0.0042$ — far below the calibrated range), the other two at no merge factor $q \le 2048$.

T131 (`self_supply_probe.py`, contract `SELF.SUPPLY`, 25/25, verdict SUPPLY-PARTIAL) then tested exactly that candidate — the self-supply of $\lambda_{\min}(X_{\text{fine}})$ through the chain's own telescope ε floor, the hardening of the four M22 properties, the M17 composition, and a map V6 with the final promotion list — and the answer is: *the self-supply loop is built, one number short of closed.* The remainder of this section is that part.

10.21 The secular sandwich: the self-supply becomes a theorem (T131)

The $\varepsilon \to \lambda_{\min}$ chain is no longer a candidate but a *theorem*, derived from the secular equation of the rank-one downdate (Sherman–Morrison):

$$\frac{\varepsilon}{\|u\|^2 + \varepsilon/\mu_1} \;\le\; \lambda_{\min}(Q) \;\le\; \frac{\varepsilon}{\|u\|^2}, \qquad \varepsilon > 0 \;\wedge\; A \succ 0 \iff \lambda_{\min}(Q) > 0,$$

with $\mu_1 = \lambda_{\min}(A)$ — the upper half is convexity of the secular function, the lower half the Sherman–Morrison denominator, and the sign half is an *equivalence* (Albert 1969). On 10 dense windows ($h = 26$–434, $n = 3$–31): **zero violations**, predicted relative width $1 + \varepsilon/(\mu_1\|u\|^2) = 1.2663$–$1.3894$, and the true value sits at 1.06–1.31× the lower bound (median 1.22) — the bound is not merely valid, it is *sharp*. The ε side is supplied *without* a fine factorisation: the same Céa/Strang defect that carried the bridge carries ε (a CG-tightened certified lower bound), valid on 10/10 and retaining 1.0000 of the true ε. Two side findings, both named: *(a)* the **direction obstruction** — the mirror rung lacks a Loewner *minorant*, $T(\ell)$ is indefinite on 4/4 windows ($\min \ell < 0$ wherever the lag symbol dips: the same indefiniteness that killed the Szegő route), so the telescope runs coarse-from-fine only; *(b)* a **correction to T130**: the Lanczos Ritz value overestimates $\lambda_{\min}(X)$ by a factor of up to **7.9** — it is an upper bound by construction (Rayleigh–Ritz), so T130's conditional hung on a number that is systematically *too large*; a chain floor below the Lanczos value is not a loss but the difference between an estimate and a bound. The bridge test: on **84 bridge pairs the chain floor replaces the Lanczos floor with zero brackets lost** (bar 0, declared), retaining 0.766–1.059 (median 0.875) against the genuine certified Cholesky floor; **both deep seams are chain-positive** ($n = 127$: $\varepsilon \ge 2.63 \times 10^{-7} \to$ floor 1.08×10^{-6}, $\lambda_f = +0.2255$; $n = 256$: $3.83 \times 10^{-8} \to 2.52 \times 10^{-7}$, $\lambda_f = +0.2614$); and the residual input is robust — μ_1 may be underestimated by **nine decades** before the floor dies, because μ_1 enters only as a correction. **M25 is thereby reduced-to-pole-free**: what was "certify $\lambda_{\min}$ of the near-singular inner block at $h = 5694$" is now "certify positivity of the *pole-free* odd Toeplitz–Hankel section, with any floor within nine decades of the measured value".

10.22 The Perron sign theorem, the honest one-hump break, and the M17 vacuity (T131)

On 575 transports, one of the four M22 properties becomes a theorem and one honestly breaks. *(i)* **Sign constancy is a theorem** — not via Metzler (the border block has all off-diagonals nonpositive on only 11/575, so that route fails as a hypothesis) but via **Perron–Frobenius on** S^{-1}, which is entrywise positive on 575/575 transports (Perron root $= 1/\lambda_{\min}(S)$); the eigenvector is strictly sign-constant *and* $\lambda_{\min}$ is simple — exactly what T130's 765/765 could only record. *(ii)* The **one-hump property breaks at depth**, revising T130: on the deeper sweep P has more than two monotone runs on 530/575 transports, so the curv $= 2 \cdot$ sag identity is not generally available; what carries instead is the certified $(2n_{\text{run}} - 2) \cdot$ sag majorant, valid on 575/575 with no hump hypothesis at all, plus the run-count chain $n_{\text{run}} \le s_2 + 2$. *(iii)* The exponent band stays a *measurement* ($a = 0.727$–0.999 over 492 transports, 55/492 outside the old calibration band; the drifts are fits, not bounds). *(iv)* S^* stays a measurement — and the deeper sweep *raises* it: the frozen 1.1926 is exceeded 16 times, and the constant this sweep would need is **1.8472**. M22 thereby collapses onto *one* object — the border profile exponent — but by a different route than expected: (i) is proved, (ii) is dropped rather than reduced. And M17 is **assembled — and vacuous, with the loss attributed**: the mass bound is mounted from its two parts (the pole part in closed form

to 1.8×10^{-13}, the harmonic-mean identity to 2.0×10^{-11}), majorises the true bad mass on 27/27 rungs — and reaches the bar on 0/27. The dominant loss is the **U-metric mismatch** 2.18 (the assembly measures in the U-metric but bounds the comb in the Euclidean one); the named fix is M18 run in the whitened basis — not more measurement. Status: ASSEMBLED-VACUOUS.

The map V6 (T131): thirteen items, and the nine-item promotion list

Theorem/identity (3): the $\varepsilon \to \lambda_{\min}$ sandwich (M25a), the Perron sign theorem (M22a), the Céa/Strang bridge identity (M23). **Certified(-conditional)** (2): ε at a deep grid without a fine factorisation (M25b), the chain floors on the 84 bridge pairs and both deep seams (M25c). **Measurement/hypothesis/fit** (4): $\lambda_{\min}(A) > 0$ for the pole-free section (M25d — the one number the whole loop now rests on), the exponent a (M22c), the constant S^* (M22d), the quantifier (M19). **Dead/broken** (3): the mirror rung (M25e), the one hump (M22b), the RH address (M21 — cited, never used). The **promotion list grows from six to nine items**, each a per-instance certificate, none zone-uniform — new here: *(7)* the $\varepsilon \to \lambda_{\min}$ sandwich, *(8)* sign constancy from $S^{-1} > 0$ (Perron–Frobenius), *(9)* the run-count chain $n_{\text{run}} \le s_2 + 2$. The shortest honest remaining list has four points: *(1)* $\lambda_{\min}(A) > 0$ pole-free (nine decades of slack), *(2)* a zone-uniform bound on the exponent and the run count, *(3)* $S^{-1} > 0$ structurally, *(4)* the quantifier.

10.23 The reverse flow: the toolkit pays the theory side back (T132/T133)

For the first time the direction of the program inverts. Two *reverse-flow* probes carry the certification toolkit built for the prime front back to the theory side of the compiler, and both land.

T132 (`bd_seam_probe.py`, contract `BD.SEAM`, 21/21, verdict SPECTRUM-ONLY) reads the seam Dirichlet-to-Neumann operator through the Beurling–Deny triad (jump kernel, killing measure, diffusion part) and uses it as an *operator* discriminator where the spectrum alone is blind. The Beurling–Deny identity is exact to 2.4×10^{-16} at every N; the triad reconstructs the operator, not just its form (1.4×10^{-14} at $d = 256$); the diffusion part vanishes identically (a finite atomic state space carries no strongly local part, and the continuum DtN is the pure-jump $\frac{1}{2}$-Laplacian, Caffarelli–Silvestre 2007, fitted decay exponent 1.980); and the killing measure *is* the μ_4 mark-sum curvature profile pointwise (1.3×10^{-14}) — the marks enter the operator exclusively through the killing term. The Markov remainder is not small but *exactly zero*, with a closed jump-weight form $J_{rs} = 1/(d \sin^2(\pi(r-s)/d))$ for $r - s$ odd and exactly 0 for $r - s$ even, which is what explains T126's 86.1% count fraction (the even lags are the zeros, not defects). The discriminator then separates: the model DtN and the μ_4-graded free contraction share their spectrum to 7.5×10^{-13} but are *not* the same operator, and the gap sits in the killing measure at 0.1746 at every N (spread 1.9×10^{-14} relative) while the jump gap shrinks monotonically ($7.7 \times 10^{-3} \to 3.5 \times 10^{-4}$) — so the difference is not a truncation artefact. The mark-preserving lattice symmetries (the clock rotation and the sheet reflection, generating D_4) leave the triad exactly invariant (1.6×10^{-14}), while a random orthogonal or a generic μ_4-equivariant conjugation destroys it: the triad is canonical relative to the *marked boundary coordinate*, not to the marks alone — which is a coupling statement, `QGEO.MARKS.01` ↔ `QGEO.KERNEL.01`. **Ceiling, stated:** this is the *model* DtN, not the raw RP seam, so `QGEO.KERNEL.01` is *not* closed.

T133 (`cert_floor_probe.py`, contract `CERT.FLOOR`, 23/23, verdict MIXED, 840 control validations on the certificate machinery itself) turns the toolkit on the load-bearing suite's own PSD rows and produces one finding that changed a module in this same round. The 10×10 Hankel matrix v379 builds is, *as a matrix of doubles*, certifiably *not* positive semidefinite — a certified negative direction $x^{\mathsf{T}} \widehat{M} x = -7.75 \times 10^{-18}$ at 60 digits — and the module's own tolerance ($\lambda_{\min} > -10^{-10}$) passes it. The reason is priced, not guessed: cond $\approx 1.8 \times 10^{18}$ against a certifiability ceiling of 1.0×10^{13} at $n = 10$, and the entry bar 7.6×10^{-12} from evaluating a transcendental kernel in binary64 dominates every floor the floating-point route could produce; the blind band is exhibited explicitly (a matrix with a certified negative direction passes the test). The *mathematical* matrix is fine — at 40 digits it carries a certified floor 2.7×10^{-25}, while rounding its entries to doubles moves it by 3.5×10^{-17}, eight orders more — and the exact repair needs no precision at all: the same object is

a *positive-mixture Gram* $M = V^{\mathsf{T}}\mathrm{diag}(1/1600)V$ with $V_{ki} = e^{-\varepsilon_k \tau_i}$, so $x^{\mathsf{T}}Mx = \sum_k w_k (v_k \cdot x)^2$ is a sum of squares and $M \succeq 0$ holds in exact arithmetic with no bar. The probe's recommendation table asks for method and wording, explicitly *not* a marker move (v171's Vandermonde Gram is exactly certified already; v527/v519 certify at the 40 digits they already run). **That repair is executed in this same change:** v379 now asserts the exact mixture identity (1.1×10^{-15}, at the rounding level) plus a sum-of-squares certificate, the eigenvalue check is retained and relabelled a diagnostic *measurement*, and the ledger row `SEAM.S3.RP.01` is reworded accordingly — marker [**E**] unchanged.
The narrow identity module of this phase is promoted: v542, Section 4.8.

10.24 The pole-free floor: existence closes, and every cheap route fails by sign (T134)

T134 (`pole_free_floor_probe.py`, contract `POLE.FREE.FLOOR`, 21/21, verdict PARTIAL; sub-verdicts FLOOR-CERTIFIED-PER-WINDOW / CLASS-IDENTIFIED / MASS-VACUOUS-METRIC-FIXED) attacks the three items of T131's rest list, and nothing else. The first answer closes the existence question: the floor is *never* an existence problem. Every Cholesky pivot of the *pole-free* form is positive on 79/79 windows — T119's famous negative pivot belonged to the form *with* the pole — and the per-window Cholesky certificate carries R4 on 79/79 and R2b on 68/68. But the route table is unanimous: every cheap lower-bound route fails by *sign*, not size, so the nine decades of slack T131 measured are worthless to them — they die at a sign that no margin can heal.

Cheap route	**Certifies**	**Failure mode**
Dirichlet + Gershgorin	0/68	sign
global Gershgorin	0/68	sign
Jacobi dominance	0/68	sign
Ostrowski (1937)	0/68	sign
path comparison	infeasible	congestion up to 4.7×10^5

The anatomy names the culprit. The good half $\mathrm{diag}(w) + L_N$ of the T126 fractional Dirichlet split is positive definite on 68/68 windows, with $\lambda_{\min} = 0.13$–3.06 — that is 8–85,000× *above* the target; the *entire* loss is the positive-off-diagonal Laplacian L_P at the long lags: $\lambda_{\max}(L_P)/\mu_1$ up to 4.7×10^5 at effective rank $k_{\mathrm{eff}} = 24$–397, so L_P is neither small nor low-rank, and T131's rank-one trick does not generalise. R5 is measured rather than hoped: the compression *and* the Schur floor both sit *above* $\lambda_{\min}$ on 56/56 pairs — no coarse recursion can ever supply a lower bound here — and the certified floor degrades like $D^{2.26}$ (a fit, labelled a fit). The second answer gives the inverse-positivity its *class*: the naive split stays definite on only 659/900 blocks, but the **lumped Stieltjes comparison matrix** $S_B = S + L_\Delta$ (the positive off-diagonals lumped onto the diagonal) is Stieltjes *and* positive definite by construction, with Jacobi spectral radius $\rho = 0.709$–$0.990 < 1$ on 900/900 blocks — so S_B^{-1} is a convergent nonnegative Neumann series, a discrete Green function: the M-matrix ground of the sign constancy that T131 could only record via Perron. The naive killing reading is refuted (row sums negative on 900/900), the Neumann margin is a valid majorant on 558/558 (a-priori coverage 26.9%), entrywise positivity of S^{-1} is now *certified* a posteriori on 900/900 (residual $\le 2.6 \times 10^{-11}$ against smallest entries $\sim 10^{-2}$), and the weakening "reflection positivity on the floor eigenspace only" is *refuted* (535/900): the full statement is needed, the weaker one is false. The third answer honestly corrects T131's diagnosis: the 2.18 U-metric mismatch was *not* the dominant loss — in the one honest norm the *comb dominates the pole* by 4.7–81×, invisible to T131 because its comparison mixed two metrics. With the exact normaliser the whitened mass bound gets a *finite* number for the first time: 2.81–5.04 (sharp 2.52–4.26) against the bar 0.50 — a factor 5.6–10, no longer orders of magnitude — and $\sigma_b = 0.77$–1.00 exhausts the budget alone, so what is missing is a *localisation* statement about the bad pencil subspace, not more measurement.

After T134: fifteen promotion items, five remaining points

The promotion list grows from nine to **fifteen** per-instance items (new: the per-window Cholesky certificate of the pole-free form, the lumped Stieltjes construction, the $\rho(\text{Jacobi}) < 1$ series, the a-posteriori $S^{-1} > 0$ certification, the row-sum refutation of the killing reading, the whitened mass identity). The remaining list has **five** points: *(1)* D-uniformity of the floor via the M-matrix question (under attack in T136), *(2)* a zone-uniform bound on the exponent and the run count, *(3)* $\rho(K)$ a priori rather than measured, *(4)* the M17 localisation, *(5)* the quantifier.

10.25 The reverse flow, continued: a bounded seam state where the Weil window has none (T135)

T135 (`comb_compressibility_probe.py`, contract `COMB.COMPRESS`, 13/13, verdict BOUNDED-STATE (MATCHED)) is the third reverse-flow trace: does the T116 boundary-state machinery, which *failed* at the Weil window because the off-band mass sits in the prime comb, *succeed* on the seam DtN of T132/v210, whose comb is an arithmetic progression? The port is verbatim — the T116 Riccati recursion runs unchanged on Λ_Σ (bordered pole identity 1.5×10^{-16}, the banded m-march reproducing the dense state to 8.2×10^{-16}), with the pole candidate the rank-8 Calderón polarisation of v113 on the 16 residue classes, documented as a *model* (v210's DtN carries no rank-one defect of its own). The census is not a kill: 100.00% of the off-band mass sits on the mod-4 stripes (the complement is machine zero), and the stripe count grows exactly like $\lfloor W/4 \rfloor = \Theta(W)$ (fitted exponent 1.0000 ± 0.0000) where the true prime comb grows like $\pi(W)$ (0.726 ± 0.018). The core number: $\mathbf{m_{cert} = 12}$ — the recursion error is 3.3×10^{-16} of the margin exactly ($h = 64$–1500) and 2.1×10^{-17} certified on the deep march ($h = 1000$–$100{,}000$, *falling* in h) — against T116's 1.3×10^{2}–1.1×10^{6} *above* the margin at the Weil window; growing the state $m = 4 \to 48$ divides the error by 4.7×10^{8}, where T116 saw no decay at all. The match is honestly labelled: 12 lies in the pre-declared look-elsewhere set $\{8, 12, 16, 24, 32\}$ (the search ladder contains five non-member values), so the verdict is MATCHED, not "derived" — and the value is bar-fragile, with only the band $8 \le m_{\text{cert}} \le 16$ defensible. The controls name the driver: an artificial mod-3 seam *also* closes ($m = 8$), so the causal driver is **weight summability**, not the comb period as such; and the *true* prime comb *fails* under identical machinery (no h-uniform floor, error/margin 0.16–0.27 and rising, 12× more state buying only 1.69×) — non-vacuity established: the same recursion separates the two combs. **Sober, three times over:** `QEC.SEAM.01` is *not* advanced (the driver is not the period), m_{cert} is partly circular (the 16 channels are set by hand), and this is the model DtN, not the raw reflection-positive seam. What is new and non-circular: the finite-dimensionality of the seam is visible in a *recursion* for the first time, not only in a spectrum, with the Weil window as the proved contrast.

10.26 The M-matrix pair: one item closes, and the bookkeeping is exact (T136)

T136 (`m_matrix_pair_probe.py`, contract `M.MATRIX.PAIR`, 30/30, verdict ONE-CARRIES) attacks the M-matrix question T134 named, and one of the three goals carries cleanly. The classical row-sum criterion — the thing one would normally invoke to get $\rho(\text{Jacobi}) < 1$ — is *vacuous* at the border block (the row sums are negative on 900/900, the same sign failure that killed T134's cheap routes). It is not needed: for the lumped Stieltjes comparison, **Varga's regular splitting turns convergence into an identity**, $\rho(J) = \tau/(1+\tau)$ with $\tau = \rho(A_B^{-1} N_0)$, and the Collatz–Wielandt bound at the M-matrix anchor vector reproduces the measured $\rho = 0.709$–0.990 with a gap shortfall of only 1.00–1.03, *flat* in D and in the zone on 900/900 blocks (a priori $\rho \le 0.7160$–0.9903 against the declared bar 10). Two solves replace a spectrum, and rest item *(3)* — "$\rho(K)$ a priori rather than measured" — is **closed**.

The other goal resists, but its anatomy is now exact. The pair is the right tool and runs the right way; the anchor half is certifiable and loses at most 1.51. What kills it is the margin: $1 - \rho(E)$ is only 2.5×10^{-6}–2.5×10^{-2} wide while *every* cheap upper bound on $\rho(E)$ sits at 1.82–3.11. The shifted ladder is empty *by monotonicity of M-matrix inverses*, not by measurement. And the

three-way bookkeeping splits the degradation exactly:

$$\lambda_{\min}(A) \;\sim\; D^{2.27} \;=\; \underbrace{D^{-0.56}}_{\text{anchor}} \times \underbrace{D^{2.72}}_{\text{margin gap}} \times \underbrace{D^{0.12}}_{\text{slack}} \qquad \text{(exact split; the exponents are fits).}$$

The anchor *improves* with depth; the slack is flat; the entire loss is the gap. Finally M17 closes *negatively*: the bad pencil subspace is **delocalised** (participation ratio 0.54–0.98, 91% of the cells carrying 90% of the mass, flatness 0.98 on every rung), neither a border layer nor the comb stripes, with a dimension tracking the *atom count* rather than the grid ($n_{\text{bad}}/n_{\text{at}} = 1.64$ median). Hence $\sigma_b(k) \geq 0.646$ on the whole ladder and no choice of cut brings M18 under its 0.50 bar (best 2.76–7.00): what M18 needs is a localisation *statement*, and there is none. The identity block of this part is promoted: v543, Section 4.9.

10.27 The long lags: the support is arithmetic, and the absolute value is the culprit (T137)

T137 (`long_lag_probe.py`, contract `LONG.LAG`, 22/22, verdict BOTH-RESIST) names the perturbation, and the name is arithmetic. The archimedean half of the kernel is nondecreasing outside its near field and its odd section carries *no* positive off-diagonal (0 of 40 windows), so the whole bad half L_Δ is **comb-generated**, supported on the anti-diagonal stripes $r + s = M - 1 - \text{round}(u_j/D)$ addressed by the prime-power atoms of the window (1.00–1.00 of the positive entries within one grid cell of an atom, 4–192 stripes on 3–99 atoms, 1.33–1.97 stripes per atom), with the certifiable amplitude law $\Delta \leq c_{\text{comb}}(\text{anti})$ — while the single-atom form $\mu_j/2$ is only *typical* (worst ratio 2.268). Each stripe is a perfect matching, so L_Δ is explicit.

Half the question then closes, and it closes as a *certificate of death*. Gershgorin bounds, row-sum norms and Collatz–Wielandt quotients at positive weights are all upper bounds for $\rho(|E|)$, so a single *lower* bound decides them all: the Collatz–Wielandt lower end gives $\rho(|E|) \geq 1.3218$–2.2963, above 1 on 35/35 windows. **No member of the absolute-value envelope family can ever produce a floor** — not "has not yet". The second family (edge/Gram bounds built on the *second differences* of the Green function) is alive but does not beat the margin: 1.1530–2.3502 against a true ρ of 0.9752–1.0000 and a target of 0.998879–0.999999, clearing it on 0 of 35 windows (overshoot 24–7.1×10^5 gaps). The coarsening ladder says why it fails structurally — the bound drops below 1 only when the entire Gram is a single block, so *the inter-stripe coupling is the object* — and the measured decay of those second differences is algebraic ($\text{dist}^{-0.63} \pm 0.02$, a fit) where a Demko–Moss–Smith argument would need a geometric one, A_B being dense so that theorem does not apply at all.

Rest item *(2)* is delivered without being closed. The Neumann series of the regular splitting gives $(S_B^{-1})_{rs} \geq \max_{k \leq 6}(J^k)_{rs}/d_s$ *with no error term* — only a discarded nonnegative tail — the partial sum recovering 0.011–0.406 of the true minimum entry, and the anchor-weighted ceiling overshooting the maximum by only 2.49–12.24: the Green function is two-sidedly bracketed by path quantities. But the certificate for $S^{-1} > 0$ still stops at 563/900 blocks (716/900 with the sign-aware k-exact split, 66/900 from the structural bracket alone) because $\rho(|F|) \geq 1$ on 77 blocks — the same wall. **The diagnosis is one sentence:** the absolute value destroys the cancellation that the seam lives on, and what is missing on both sides is a *sign-preserving* bound. The structure block of this part is promoted: v544, Section 4.10.

After T137: thirty promotion items, and a rest list of three

The promotion list grows from fifteen to **thirty** per-instance items (7 new in T136, 8 in T137); thirteen of them are now load-bearing as v543 and v544. Of T134's five remaining points, *(3)* $\rho(K)$ a priori is **closed** (Varga plus Collatz–Wielandt) and *(4)* the M17 localisation is **closed negatively** (the bad subspace is delocalised, so no cut works); the Green lower bound asked for under *(1)* is delivered two-sidedly. What is left is *(1)* D-uniformity of the floor — now sharpened to a single named requirement, a *sign-preserving* bound on $\rho(E)$, every absolute-value route being certifiably dead — together with *(2)* the zone-uniform exponent and *(5)* the quantifier.

10.28 The sign law is interval geometry, and the m-paired certificate removes the arithmetic wall (T138)

T138 (`sign_compensation_probe.py`, contract `SIGN.COMPENSATION`, 26/26, verdict PAIR-EXACT) keeps the cancellation instead of majorising it, and finds the mechanism. The sign of an inter-stripe coupling is decided by the **interval geometry** of its two edges — nested pairs positive on 0.76–0.94 of the area, crossing pairs on 0.59–0.63, disjoint pairs on only 0.06–0.36 (i.e. mostly *negative*) — and *not* by the lag distance (the alternation reading is false, flip rate near 0.5). The couplings cancel globally to $|\Sigma|/\Sigma|\cdot| = 0.29$–$0.70$: that one number is the reason for every absolute-value overshoot of T134–T137. The honest negatives stand beside it: Karlin total positivity *fails* (only 0.54–0.65 of the TP_2 minors nonnegative), and the Green function is rank-one dominated but *not* semiseparable — the Gantmacher–Krein one-pair form stays a candidate, not a theorem.

The best *certified* signed bound (band-exact by completed Cholesky, tail by block row sums) reaches 0.9758–1.1020 against a target of 0.998556–0.999979 — clearing 1 of 16 windows, overshoot 0.02–565 gaps against T137's 27–8×10^5: three orders of magnitude better, and still no floor. The clamp is measured, not conjectured: the band part alone crosses the target at bandwidth $b = 2$–8 while the certified tail there is still 0.46–0.70 — the two terms are never small together; the far tail is *net negative* (truncation *raises* ρ on 59/142 rungs, so the tail is no small remainder — it carries compensation); the Leibniz pairing buys nothing; and the exact two-stripe blocks are thin ($\le 0.6\%$ gain, three-body terms up to 2.5×): **the compensation is a many-body effect**.

The gain of the part is the **m-paired Neumann certificate**. From the identity $(I-F)^{-1} = (\sum_{j<m} F^j)(I-F^m)^{-1}$ only the *outer* factor is majorised, so the condition is $\rho(|F^m|) < 1$ — genuinely weaker than $\rho(|F|) < 1$, because the signs inside the m-fold product are kept ($m = 1$ is verbatim T137's certificate). It removes the arithmetic wall on *all* 77 dead blocks and certifies 52 of them; the pool rises from 563 to 875/900 (mechanism: $\max|F^m|/|F|^m$ falls to median 0.295). And M17 gets its characterisation: whitened, $\mathrm{St} = I - W_S$ with $W_S \succeq 0$, so $\lambda(\mathrm{St}) \le 1$ structurally, and the *direct* Kantorovich harmonic-mean route needs no bad subspace anywhere, relaxing the required margin from 0.5 to 0.8284 (854/900) — HARMONIC-PARTIAL. The honest conclusion: the signed route does *not* inherit the absolute-value wall — the m-ladder is a real structural gain — but the margin question returns one level down, as an upper bound on $\rho(W_S)$.

10.29 The decay lemma is refuted at its hypothesis, and the sign law is derived (T139)

T139 (`green_decay_probe.py`, contract `GREEN.DECAY`, 30/30, verdict DENSE-RESISTS, bars declared in the docstring before any number) asks whether the classical dense-matrix literature supplies the decay lemma T138 named as the one missing input — and the answer is an *arithmetic* refusal. The lemma fails **at the hypothesis**: the certified kernel envelope of the lumped pair decays only like $d^{-0.15}$–$d^{-0.04}$ where Jaffard 1990 needs $p > 1$ — the archimedean lags sit *on* the borderline ($\sim 1/d$) and the prime-power atoms lay $O(1)$ spikes across the whole window, so no entrywise envelope can see the thinness of the comb. The constant chain is built constructively rather than asserted: Demko–Moss–Smith is *verified* where it applies (75/75 banded rungs — the failure on the dense object is a statement about the object), the operator gate reads 0.97–1.83 (1/75 below 1), the algebra gate 3.4–1.6×10^3 (0/75); and the exponent is *not* inherited (kernel $d^{-1.5\ldots-1.1}$ against inverse $d^{-0.52\ldots-0.37}$). The positive replacement object is an identity, not a fit: the exact double telescoping $b_e^{\mathsf{T}} G b_{e'} = \sum_{\mathrm{box}} H$ (4×10^{-15}), with the mixed second-difference kernel H negative on 0.71–0.87 of the off-diagonal and positive on 100% of the diagonal — **T138's entire sign law follows from one kernel plus box geometry** (disjoint pairs have boxes missing the diagonal), derived instead of measured; the Gantmacher–Krein one-pair form is quantitatively dead.

The never-truncating layer series is then killed *from below*. It delivers 3.388–6.171 against a target of 0.999182–0.999972 (0/15, overshoot 681–46900×), and triple direction pedantry pays: no layer may be dropped (zero trace forces $\lambda_{\max} > 0$, and a negative *mean* is not a Loewner sign), Gram $\preceq \mathrm{Band}_b$ holds *never* (85/85), and by Weyl every layer series is $\ge \max_b \mathrm{Rayleigh}(\mathrm{Band}_b) =$

1.0303–1.1116, above the target on 15/15 windows — **no decay lemma of any strength can rescue the family**, and the overshoot *is* the effective layer count, growing with the window. The clean-up localises the rest: on a rebuilt pool of 620 border blocks the m-paired ladder out to $m = 48$ converges on all 620 and certifies **619/620** — the one open block (need 2.15) has its tightness at index distance 15, far-carried 1.000: the single item for which an H-decay statement would be the right tool. One level down, $\rho(W_S) = 0.667$–0.939 inherits the structure verbatim; the three-body cumulant is negative on 0.93–1.00 of 4200 stripe triples — certified *benign* (it pushes upper bounds down). The three-sentence verdict, quotable: the lemma does not exist classically for this object, for an arithmetic reason; the right object is now identified exactly (the sign law is derived), but every absolute-value envelope built from it is certified dead; **the measured core has shrunk from "a decay law is missing" to "one signed inequality at stripe distance $b \leq 16$ is missing"**.

After T139: forty-seven promotion items, and a rest list led by one signed inequality

The promotion list grows from thirty to **forty-seven** per-instance items (7 new in T138, 10 in T139). The sign-preserving bound T137 asked for is now *located* rather than merely named: everything that fails, fails at stripe distance $b \leq 16$ — the far layers are certifiably harmless (the tail is net negative, the three-body remainder certified benign, the border pool at 619/620) — so *(R1)* is **one signed small-bandwidth inequality** that keeps the cancellation among the first few stripe layers (a signed block-Rayleigh or interlacing step, not a norm). Behind it: *(R2)* a replacement for the Weyl step (any $O(nb)$ triangle-inequality route is dead by construction), *(R3)* an optional certified decay statement for H (which by construction cannot close R1 — every entrywise envelope is an absolute-value route), the same margin question one level down ($\rho(W_S)$), and, unchanged, the zone-uniform exponent and the quantifier.

10.30 The signed inequality acquires an exact finite core, and the D-dependence is located (T140)

T140 (`signed_band_probe.py`, contract `SIGNED.BAND`, 31/31, verdict FINITE-CORE, bars declared in the docstring before any number) attacks exactly the one signed inequality at stripe distance $b \leq 16$ that T139 left — and nothing else. The central move lifts the H-telescope from the *entries* of the Gram to its *form*: Gram $= CHC^{\mathsf{T}}$ exactly (3×10^{-15}), with $C = \mathrm{diag}(\sqrt{\Delta})M$ and M the edge-interval incidence matrix, so $x^{\mathsf{T}}\mathrm{Gram}\, x = u^{\mathsf{T}}Hu$ with $u = C^{\mathsf{T}}x$ the occupation function of the edge system — Abel summation over the box structure, with no absolute value and no factorisation. Three exact consequences follow. First, $\mathrm{rank}(\mathrm{Gram}) \leq h - 1$ (measured 44–202 against 128–1499 edges, a 2–10× reduction): the whole spectral question lives on the index grid. Second, **the exact finite-core reduction** $\rho(W) = \lambda_{\max}(K^{1/2}HK^{1/2})$ (6×10^{-14} on every window), with $K = M^{\mathsf{T}}\Delta M$ the *covering kernel* in closed geometric form $K_{rs} = W([r \wedge s,\, r \vee s])$ — entrywise nonnegative, monotone by inclusion, Green-like but *not* one-pair (the off-tridiagonal defect of K^{-1} is 0.08–0.43). The signed inequality is thereby a statement about two matrices of size $h - 1$ per prime-power zone, reached with no truncation, no envelope and no series anywhere. Third, the energy reordering $H = \mathrm{diag}(s) + L_N$, exact to 5×10^{-16}: T139's sign law read as a mass term plus a long-range *Dirichlet form* with positive weights on 0.71–0.87 of the off-diagonal. The direct certified chain (every Loewner step a completed Cholesky) then locates the failure exactly: it **tears at step (i)** — dropping the 13–29% *positive* part of H's off-diagonal alone costs the whole margin (6.4–$239 \times \rho(W)$), because that positive part is precisely the compensation the sign law lives on; the discrete Hardy step adds only 6.4–9.0× on top, with the first-moment input $\max_k Q_k = 18.5$–339; below the target on 0/16 windows for all four routes.

The checkerboard split solves R2. With stripe groups of length b the band condition forces adjacent groups, the adjacent pairs two-colour into two families of disjoint supports, and $\mathrm{Band}_b = D + A_{\mathrm{even}} + A_{\mathrm{odd}}$ holds *entrywise exactly* — so **three D-independent Weyl steps replace the $O(nb)$ ones** whose count T139 identified as the overshoot itself (value 0.76–$1.79 \times \rho(W)$). And the zone question is answered by measurement: the stripe block norms are *zone-uniform* (group

diagonal $\sim D^{0.13\pm0.01}$, cross blocks $\sim D^{0.02\pm0.01}$, fits) while $\lambda_{\max}(K) \sim D^{-2.99}$, $\lambda_{\max}(H) \sim D^{+0.33}$ and $\max_k Q_k \sim D^{-2.20}$ against a flat $\rho(W) = 0.9962$–0.9999 — **the entire D-dependence of the core sits in the geometry, not in the stripe blocks**, so only a *joint* bound on the pair can be D-uniform. Floquet is not applicable (the stripes sit at atom positions; group spread 0.15–0.69, quasi-periodic); Bendixson certifies 1.07–$1.08 \times \rho(W)$ without any Gram (the spectrum of W is real), below the target on 0/16. Two hard negatives close the additive readings for good: the band *from below* exceeds the target on 16/16 windows (Rayleigh 0.3894–1.1116 on 229 (window, b) pairs — the mechanism is *rank inflation*: truncation inflates the rank by 1.2–10.1×, cutting exactly the directions in which the boxes compensate), and the new index-grid band family — the only one in which a truncation could have been legitimate — fails the Loewner test too: the discarded far part is certified Loewner-*positive* ($\lambda_{\max} \geq 1.14 \times 10^{-1}$) on 256/256 pairs. The clean-up inherits and localises: of 420 rebuilt border blocks the m-paired ladder to $m = 24$ certifies 402; the 18 open ones carry need 1.01–10.99 with argmax at index distance 11–15 and *far mass fraction* 0.635–0.912 — genuinely far-carried, so a decay statement is the right tool *there*, unlike R1; and the finite core is inherited one level down ($\rho(W_S) = \lambda_{\max}(K_S^{1/2} H_S K_S^{1/2})$ to 4×10^{-15}; $\rho(W_S) = 0.541$–0.925).

After T140: the rest list is led by one zone-uniform discrete Hardy inequality

The map V12 has 18 rows and the promotion stock grows from forty-seven to **sixty-four** per-instance items (17 new). The rest list is the shortest of the series: *(R1′)* a **joint** bound on $\lambda_{\max}(K^{1/2} H K^{1/2})$ — not factor-wise (the D-exponents -2.99 and $+0.33$ cancel against each other), not after dropping the positive part of H (costs 6.4× alone), not after truncating H in either grid (both far parts are Loewner-positive); *(R1″)* the ingredient, *named and located*: a **zone-uniform discrete Hardy inequality** comparing the long-range Dirichlet form of $N = -\text{offdiag}(H)$ against the form K^+, with the first-moment weight $\max_k Q_k = 18.5$–339 as its natural input; *(R3′)* a first-moment bound on $(-H)_+$ rather than a decay exponent — only that enters the certified chain; *(R4)* the 18 far-carried border blocks; *(R5)* nothing for the layer / envelope / Jaffard / one-pair families — certified dead, not to be retried.

10.31 The Hardy ingredient resists, and the resistance has an address (T141)

T141 (`discrete_hardy_probe.py`, contract `DISCRETE.HARDY`, 22/22, 51.2 s on 60 windows with $h = 50 \ldots 1173$, zones $n = 11 \ldots 529$ and $D = 2.31 \times 10^{-3} \ldots 2.60 \times 10^{-2}$, verdict HARDY-RESISTS, bars declared in the docstring before any number) attacked exactly the ingredient T140 named — the zone-uniform discrete Hardy inequality for the pair (Dirichlet form of N, form K^+) — and nothing else. It did not close, and the way it fails is the content of the part.

Four exact identities put the question in classical shape. The reduction of T140 becomes a **Rayleigh quotient against the pseudoinverse**, $\rho(W) = u^\mathsf{T} H u / u^\mathsf{T} K^+ u$ at $u = K^{1/2} v$; the Dirichlet part is a **signed crossing kernel**, $L_N = D^\mathsf{T} B D$ with $B_{kl} = \sum_{r \leq k \wedge l,\, k \vee l < s} N_{rs}$ — the same closed covering formula as K, one grid further in; T140's Cauchy–Schwarz weight is *identified*, $Q = B_+ \mathbf{1}$, so that step is the Gershgorin diagonal step on B_+ and its price is attributable; and $DKD^\mathsf{T} = L_\Delta$ on the interior nodes, whose diagonal is the **endpoint edge mass** — a *local* quantity, not $\lambda_{\max}(K)$. Together these give the two-weight Muckenhoupt quantity in closed form, $A_{M0} = \max_k Q_k (\Delta \mathbf{1})_{k+1}$, computed by two independent routes. The identity block of T140 and T141 is now load-bearing as v545, Section 4.11.

The separation is the finding. The certified Muckenhoupt constant is *not* zone-uniform: it grows like $D^{-0.366\pm0.036}$ (a fit) against a preregistered bar of 0.25. But the object it is supposed to bound — the exactly computed Dirichlet share of the finite core — is **strictly uniform on the same surface**, $D^{-0.229\pm0.007}$. The growth therefore does not sit in the inequality being false; it sits in the *diagonal profile* of the certificate, i.e. in the weight one puts on the endpoint mass. That is a repairable defect in a way a growing target would not be.

Both bound shapes are typed, and both types are negative. The **additive** shape is dead *as a shape*, not by a weak constant: its own exact Weyl floor — the best any mass-plus-Dirichlet splitting can ever reach — already sits at 1.694–3.855× the target, so no sharpening of the Hardy step can rescue it. The **joint** shape $\Lambda(H, Y) \cdot \Omega(Y)$ survives the eigenvalue side and fails at the

normalisation alone, $\Omega = 20.7$–2724: Gantmacher–Krein returns as the obstruction one grid down, and the whole question is now a question about the profile Y.

After T141: R1 collapses to R1b, a single conductance profile

The rest list contracts from "a joint bound" to **R1b: exhibit a closed conductance profile Y with $Y \preceq K^+$ in the Loewner order and $\Omega(Y) \approx 1$.** Both requirements are per-instance checkable and both are already met separately — Y positive definite with the geometric profile, and $\Omega \approx 1$ for profiles that lose the Loewner step. The classical addresses are named, not invoked as authority: Muckenhoupt 1972 and Bradley 1978 for the two-weight criterion, Opic–Kufner 1990 for the profile calculus, Miclo's spectral-gap weights for the conductance reading. *(R3′)* is now shaped as well: what the Hardy route consumes is a **first moment** of $(-H)_+$ and not a decay exponent — a strictly weaker thing to have to prove — and keeping the signs in the crossing kernel is worth a factor 31.95–15561.41 against the 6.4–239× the T140 Loewner drop costs. Border pool: of 256 blocks the m-paired ladder to $m = 24$ certifies 245; the 11 open ones are far-carried (far mass fraction 0.835–0.902 at index distance 13–15), unchanged in kind from T140 — a rebuilt pool, so the anatomy transfers and the tally does not.

Verdict integrity. The negative half of this part rests on a comparison across zones, so the surface matters: the finite-core reduction of T140 is what makes an 11.21× span in D affordable at all, and on a narrower surface the separation $D^{-0.366}$ against $D^{-0.229}$ would have been within its own error bars. The verdict HARDY-RESISTS is recorded with that span attached. Thirteen statements of T140/T141 were mature enough to promote (v545); the sandbox stock stands at seventy-five items.

10.32 The optimal weight is found exactly, and the comparison path closes (T142)

T142 (`conductance_profile_probe.py`, contract `CONDUCTANCE.PROFILE`, 24/24, 155.4 s on 26 windows with $h = 210 \dots 878$, zones $n = 71 \dots 317$ and $D = 2.90 \times 10^{-3} \dots 1.08 \times 10^{-2}$, verdict PROFILE-RESISTS, bars declared in the docstring before any number) does not guess a fourth profile. It *constructs* the optimum, from the classical address (Miclo 1999; Maz'ya's capacity criteria; Muckenhoupt 1972 and Opic–Kufner 1990 as addresses, not as authority) — and the construction settles the search half of R1b outright. With $J = DKD^{\mathsf{T}} = L_\Delta$ the endpoint Laplacian, $x = K^{-1}\mathbf{1}$ the equilibrium charge and $\mathrm{cap} = \mathbf{1}^{\mathsf{T}}K^{-1}\mathbf{1}$ the capacity,

$$K^{-1} \;=\; D^{\mathsf{T}}J^{-1}D \;+\; \frac{xx^{\mathsf{T}}}{\mathrm{cap}}\,,$$

the capacity decomposition of the optimal Hardy weight — verified as an identity to 5.5×10^{-12} on 26/26 windows, never asserted. Its Dirichlet half is an *orthogonal projection* under the congruence ($X = J^{-1/2}DK^{1/2}$ has $XX^{\mathsf{T}} = I$), so $\Omega(D^{\mathsf{T}}J^{-1}D) = 1$ **exactly** (certified 1.0000001, the excess being the declared Cholesky floor) — precisely the object T141 was missing, in closed form, against the guessed profiles' $\Omega = 20.7$–2724. The weight was never a free choice: it is the Green function of the endpoint Laplacian plus the equilibrium rank one, and the exact capacity Rayleigh form follows with no inequality taken anywhere, $\rho(W) = \sup_u u^{\mathsf{T}}Hu/[(Du)^{\mathsf{T}}J^{-1}(Du) + (x^{\mathsf{T}}u)^2/\mathrm{cap}]$.

The failure is a constant, not a power. The variational optimum over the strict conductance class (a heuristic search whose every reported number is re-certified by Cholesky; 11 windows) reaches $\Lambda \cdot \Omega = 1.3160$–$1.4235 = 1.32$–$1.42 \times \rho(W)$ with a fitted D-exponent of -0.029 ± 0.014 — inside the uniformity bar, so the class fails by a *bounded factor at every depth*, a different (and harder) situation than T141's $D^{-1.87}$ blow-up. The stable shape is found: $c_k \sim 1/J_{kk}$, the reciprocal endpoint mass (log-log correlation 0.988–0.995), and the optimum wants essentially *no* mass — a pure Dirichlet form. The certified chain of the best closed member reads $\Lambda \cdot \Omega = 2.2671$–$2.4536$ against a target of 0.999983–0.999999 (0/26, median shortfall 2.3376, flat in D at $D^{0.006 \pm 0.004}$); the control run with T141's guessed profile on the same machinery gives 74.84–320.03, so the capacity construction beats guessing by 31.9–137×. The single-direction class barrier is saturated ($\beta = 1.000000$, the argmax being the top mode itself) and is recorded as a dead route with its reason.

The rank ladder closes the comparison path. On the anchored ladder — the diagonal minorant plus the r dominant residual modes, a certified Loewner minorant at every rank with $\Omega \le 1$ throughout — the chain stalls at 1.408 ($r = 1$) … 1.391 ($r = 128$) and first crosses the hurdle 1 only at $r^*/m = 0.995$–0.998: the tautology $Y = K^{-1}$ itself (end-to-end anchor: at $r = m - 1$ the chain collapses to $\rho(W)$ to 1.7×10^{-11} relative). The structural consequence deserves its own sentence: since $\rho(W) = 1 - \Theta(D^3)$ and any comparison bound is $\ge \rho$ with equality only at $Y \propto K^{-1}$, a class bound would have to reproduce the optimal weight to relative accuracy $O(D^3)$ — **no comparison argument can deliver D-uniformity**. The next move is forced, not chosen: the *sharp* route, estimating the exact capacity Rayleigh form directly, with no Loewner step taken at all.

After T142: the comparison branch is closed, and the rest list is led by the sharp route

(R1c) **the sharp capacity–Rayleigh route**, which *replaces* the comparison rather than continuing it: estimate $\rho(W) = \sup_u u^\mathsf{T} H u / [(Du)^\mathsf{T} J^{-1} (Du) + (x^\mathsf{T} u)^2/\mathrm{cap}]$ directly (Miclo 1999, Maz'ya capacity — a Hardy inequality for the Green form of the endpoint Laplacian, no profile choice left). What sends the next probe there is measured here: the best closed minorant is short by 2.27–2.45, the whole conductance class by 1.32–1.42, and no truncation below $r = m/2$ ever helps. *(R1d)* a multi-direction dual certificate for the class (LP/SDP over the conductance cone), without which every negative statement about the class stays *measured*. *(R3′)* the corrected R3′: T141's line was backwards — only 0.03% of the first moment of $(-H)_+$ sits *inside* index distance 8 (support radius 204–873), so R3′ survives as a *long-range* inequality (the Loewner step $L_{N_-} \preceq T_{Q^-}$ is certified with the full weight on every window) whose constant $\max_k Q_k^- = 3.20 \times 10^2$–$3.46 \times 10^3 \sim D^{-1.73}$ is not D-uniform. *(R4)* border pool rebuilt: 72 blocks, 68 certified, 4 open and far-carried — the required far shrink factor, 0.656–1.052, is the one number a decay statement has to beat. Map V14; nine statements ripe (stock 75 → 84), 0 promoted here. The verdict, stated precisely: a resistance of the comparison *method* at a known optimum, not a failure to identify the weight. T143 (`sharp_capacity_probe.py`) is running at exactly R1c.

10.33 The sharp route carries: D-uniformity is reduced to one named inequality (T143)

T143 (`sharp_capacity_probe.py`, contract `SHARP.CAPACITY`, 24/24, 18.5 s on 26 windows with $h = 142 \dots 1173$ (core $m = 141 \dots 1172$), zones $n = 31 \dots 529$ and $D = 2.54 \times 10^{-3} \dots 1.28 \times 10^{-2}$, verdict SHARP-CARRIES, bars declared in the docstring before any number) estimates the exact form instead of comparing it — exactly R1c, with no Loewner step taken anywhere. The T140/T142 identities assemble into one quotient, verified as an *identity* on all 26 windows (numerator to 1×10^{-11}, denominator to 5×10^{-12}, eigenvalue reproduction 3×10^{-6} relative under the explicitly declared 1/gap conditioning factor):

$$1 - \rho(W) \;=\; \inf_{v} \frac{d^\mathsf{T}(J^{-1} - B)\, d \;+\; (x^\mathsf{T} v)^2/\mathrm{cap} \;-\; \sum_k s_k v_k^2}{d^\mathsf{T} J^{-1} d \;+\; (x^\mathsf{T} v)^2/\mathrm{cap}}\,, \qquad d = Dv\,.$$

The ingredient bookkeeping at the exact minimiser is structurally new: the minimiser is *orthogonal* to the equilibrium charge (to 10^{-5} — the capacity rank one carries *nothing*); the mass share is *negative* (-0.0698 to -0.0029 — the site masses *help* the gap); and the gap itself is the cancellation between the Green share (exactly 1) and the crossing share (1.0029–1.0698). The naive split $\rho \le \rho_{\mathrm{mass}} + \rho_{\mathrm{long}}$ is certified dead at 6.19–66.63, i.e. 6–67 × ρ itself: any capacity criterion must be applied to the *gap* form, where its factor of four costs the coefficient and not the scale.

The core number: Maz'ya's criterion lands inside its window. Brought to the gap form by a congruence (the capacities in closed form, $\mathrm{cap}_E(A) = \mathbf{1}^\mathsf{T}[(E^{-1})_{AA}]^{-1}\mathbf{1}$, verified against the constrained minimisation itself), Maz'ya's ratio gives $\Phi_{\sup}\lambda = 0.5438$–$0.6457$ on 26/26 windows — inside the window $[1/4, 1]$ the criterion forces for a Dirichlet form — although this form is *not* one (72–84% positive off-diagonals). The lower bound $\lambda \ge 1/(4\Phi_{\sup})$ captures 0.3872–0.4597 of the true gap, and the route's **loss factor is zone-uniform**: $D^{-0.048 \pm 0.010}$ against the preregistered 0.25 bar (the gap's own scatter, $D^{0.000 \pm 0.079}$, belongs to the arithmetic and not to the criterion); as a coefficient of $D^{2.829}$ the bound reads $c = 4.95$–67.70 against the envelope constant 12.04. The supremum is not an oracle: the closed families (Green diagonal, transported potentials, index

intervals) recover 0.9606–1.0000 of the best value found, a full add/remove local search improves them by at most 1.0410×, and in the *node* coordinates the near-optimal sets are **intervals** of window lags, dominating the extremal level sets by a factor 8.3–129.5 — the one-dimensional structure Muckenhoupt's 1972 closed form needs. The honest half: Miclo's constructive capacitary *chain* loses a factor 46–2561 ($\kappa = 19.9$–1144.9, drifting like $D^{-2.13}$), because the top dyadic level sets are small sets of $O(1)$ capacity against an eigenvalue of 10^{-5} — the criterion's *conclusion* holds while its classical *proof mechanism* does not, and that is precisely where the remaining work sits.

The rest, typed. R1d is negative with a reason: the LP dual over the conductance cone (39 Λ and 35 Ω directions, a hand-verified dual point on 4/4 windows) certifies a class floor of $\rho(W)$ itself to a factor 1.000000 — saturated at the trivial floor, exactly as T142's single-direction barrier was; a finite direction sample of Ω cannot bind the cone, so R1d needs the full SDP constraint $K^{1/2}YK^{1/2} \preceq I$ (recorded so the sampling version is never retried — and after the core number above, no longer on the critical path). R4: 36 border blocks rebuilt, 32 certified by the ladder, 4 open; on the 6 largest blocks *all* 255–16383 subsets were enumerated, so $\Phi_{\sup}$ is *exact* there — $\Phi_{\sup}\lambda = 0.6087$–$0.9016$, again inside the window, the only non-family-restricted supremum of the probe — and the capacity argmax set is *entirely* the far tail (1.000 of its indices at block index ≥ 8), the exact object a decay statement has to act on, with T142's far shrink 0.656–1.052 still the number to beat.

After T143: what is missing is a proof, not a number — one named inequality

(S1) **the one named inequality**: $\mathrm{cap}_E(A) \geq |A|\,\lambda_0/c_0$ for *all* sets A with an absolute c_0 — equivalently $\Phi_{\sup}\lambda \geq 1/c_0$. Measured here at 0.5438–0.6457 on 26 windows and *certified by enumeration* on the small border blocks; for a Dirichlet form this is Maz'ya's theorem with $c_0 = 4$, and what is needed is the criterion for a *non-Markovian* form ($\sim 80\%$ positive off-diagonals). *(S2)* **the Muckenhoupt route to S1**: in the node coordinates the near-optimal sets are intervals, so a comparison of the gap form to a one-dimensional path form *for the purpose of the capacity only* reduces the sup over all sets to a closed two-weight sup (Muckenhoupt 1972) — and T142's obstruction does *not* apply, because the capacity only has to be right to a bounded factor, not to $O(D^3)$. *(S3)* a closed formula for $\Phi_{\sup}$ or a closed lower bound on $\mathrm{cap}_E(A)$ (the families already recover 0.9606–1.0000: bookkeeping, not a search). *(R1d′)* the class dual as an SDP, no longer on the critical path. *(R4)* the 4 open border blocks, their capacity argmax entirely far. Map V15; ten statements ripe (stock 84 → 94), 0 promoted here. The honest state: D-uniformity is reduced to a capacity verification, and the verification is a Muckenhoupt-type statement about *intervals*, not another profile. T144 (`capacity_inequality_probe.py`) is running at exactly S1/S2.

10.34 The interval route carries: the chain has exactly one unproven input (T144)

T144 (`capacity_inequality_probe.py`, contract `CAPACITY.INEQUALITY`, 31/31, 91.9 s on 34 windows with core $m = 142 \ldots 1393$, verdict INTERVAL-CARRIES, bars declared in the docstring before any number) attacks exactly the first three items of T143's rest list — S1, the interval route S2, and the closed capacity bound S3. The probe's single comparison licence is the *Jacobi whitening* — a denominator comparison, which T142's obstruction does not touch — certified at $\kappa_{\mathrm{up}} = 1.1195$–$1.3162$ per window (Gershgorin would have allowed ≤ 1.983); it puts the criterion in coordinates where the measure is the counting measure. Two different interval-reduction statements then separate, and only one survives. The *pointwise hull comparison is false*: $\sup_A \Phi(A)/\Phi(I(A))$ reaches 425 and drifts like $D^{-1.137}$, with T143's scattered level sets as the offenders — so S2 cannot be proved by transporting each set to its convex hull; it must be a *testing condition*. The *best-interval comparison holds*: $c_{\mathrm{glob}} = \Phi_{\sup}/\Phi_{\mathrm{int}} = 1.0000$–$1.6876$, zone-uniform at $D^{-0.068\pm0.024}$ against the preregistered 0.25 bar. The exhaustion behind these numbers is honest: 136 blocks of size $g = 14$ with *all* 16383 subsets each, 2.24 million sets in total; and capacity *monotonicity* holds on all of them (measured), although the form is not Dirichlet.

The two-weight sup, exact rather than sampled. The interval class is exhausted *exactly* on every window — 11 390 676 intervals via a Cholesky nestedness, $\mathrm{cap}_E([a,b))$ a prefix sum of squared

triangular-solve entries, verified to 10^{-11} against an independent solve. The core number of the part: $B_{\mathrm{res}}\hat{\lambda} = 0.6694$–$0.7813$ inside the Maz'ya window $[1/4, 1]$ on the whole surface, fitted $D^{0.013\pm0.005}$ — zone-uniform and coordinate-robust (T143 measured 0.5438–0.6457 in the unwhitened coordinates). The free half passes its falsification test, $\Phi_{\mathrm{sup}}\hat{\lambda} \le 1$ on all 2.24 million sets, as the theorem requires. The chain then closes modulo one constant,

$$\lambda \;\ge\; \frac{1}{c_0\,\kappa_{\mathrm{up}}\,c_{\mathrm{glob}}\,B_{\mathrm{res}}}\,,$$

certified per window at 0.1002–0.2653 of the exact gap — with the remaining D-dependence sitting in the Jacobi *transfer* ($D^{-0.178}$), not in the capacity calculus. Dead with a reason: the naive Muckenhoupt *transcription* through trial-function energies misses the interval supremum by 89–6995 ($D^{-1.885}$ — the equilibrium potential spreads over the whole window, and no local trial function sees it); closed rankings bound the value but do not point at the argmax. And the honest counter-side, computed exactly: $\Phi_{\mathrm{int}}\hat{\lambda} = 0.0928$–$0.3578$ drifts like $D^{0.654\pm0.016}$ — the intervals saturate S1 only from *above*, no interval-only argument will produce the constant; what carries is the closed resistance sup above them, a two-weight statement about the Green function and not about a family of sets.

The rest, typed. (i) The Markov perturbation route is *certified dead*: $\lambda_{\min}(E-P) = -0.340\ldots-0.274 < 0$ on every window, and the largest certified t with $E - tP \succ 0$ shrinks like $t^* \sim D^{3.96}$; only 3–12% of the off-diagonal *entries* are positive, but they carry 36.7–64.5% of the *mass* — the positive couplings are load-bearing, so S1 must be proved without the Markov hypothesis. (ii) S3 returns *stronger than asked*: the Cauchy–Schwarz floor $\mathrm{cap}_E(A) \ge |A|^2/(\mathbf{1}^{\mathsf{T}} R_{AA}\mathbf{1})$ makes Φ_{sup} a *maximum density subgraph* value for the Green kernel $R = E^{-1}$ (Charikar 2000; Goldberg 1984), so the sup over *all* 2^m sets is bounded with a cited absolute constant: $\Psi_{\mathrm{all}}\hat{\lambda} = 0.7399$–$0.8515$ at $D^{0.015\pm0.005}$ — the flattest number of the probe, with no family restriction, no interval reduction and no Markov hypothesis anywhere; and the argmax density set lives on *intervals* too, an independent confirmation of the reduction, with B_{res} only a factor 1.078–1.129 below. (iii) R4: 22 border blocks (a new count), 19 certified, 3 open; on 5 blocks *all* subsets were enumerated, so $c_{\mathrm{glob}} = 1.0000$ *exactly* there.

After T144: one unproven input — the absolute Maz'ya constant

(S1′) **the leading object, the sharpest available shape of S1**: it suffices to prove the capacitary strong-type inequality for forms whose Green function satisfies a *mean-density bound* ($\Psi_{\mathrm{all}}\hat{\lambda} \le 0.8515$ measured here) — a hypothesis of *Muckenhoupt* type rather than of Markov type. *(S1)* unchanged in statement, reduced in scope: the family restriction is gone (the density bound covers all 2^m sets), the Markov hypothesis is unrestorable (the positive couplings are load-bearing), and the coordinates are fixed at a certified $O(1)$ cost. *(S2′)* the best-interval comparison $c_{\mathrm{glob}} \le 1.6876$ is measured, not proved; a proof would be a testing condition, and the exhaustive R4 blocks show it is not vacuous. *(R4)* the 3 open border blocks. *(R5)* the whitening sandwich $\lambda/\hat{\lambda} = 1.213$–$2.175$, the last $O(1)$ factor of the chain, removable by a two-sided estimate of κ. Map V16; nine statements ripe (stock 93), 0 promoted here. The honest state, in one sentence: the *only* unproven input of the certified chain is the absolute constant c_0. T145 (`mazya_proof_probe.py`) is running at exactly S1′.

10.35 The proof attempt: one step missing, and it has a name (T145)

T145 (`mazya_proof_probe.py`, contract `MAZYA.PROOF`, 33/33, 8.0 s on 64 windows $m = 26\ldots1173$, verdict ONE-STEP-MISSING, the verdict rule hard-wired and checked *before* the verdict is printed) attacks exactly S1′: it does not look for a better chain, it takes the classical *proof* of the capacitary strong-type inequality apart, transcribes it onto the non-Markovian form line by line, and reads off what survives. The step list, declared before it is measured:

Step	Content	Status here	Factor
M1	dyadic level decomposition of the minimiser	theorem, verified	1.649–2.670 (≤ 3)
M2	$\lvert A\rvert \leq \Phi_{\sup}\,\mathrm{cap}(A) \leq \Psi\,\mathrm{cap}(A)$ (Green floor)	theorem (Cauchy–Schwarz)	$\times 1$
M3	$\mathrm{cap}(A_{k+1}) \leq 4^{-k}E(f_k)$, truncation admissible	theorem, verified	$\times 4$, slack ≤ 1.036
M4	$\sum_k E(f_k) \leq E(f)$ — *the* Markov line	**void** on this form	measured

M1–M3 are theorems for *any* positive definite form (verified here in the direction they are used, on 168 levels of 24 windows for M3). M4 is the only step that uses the Markov property, and it uses it twice — the sign of the conductances and the sign-coherence of the truncation increments. On this form it is *void*, not merely unproved: on all 64 windows 3.2–14.9% of the off-diagonal entries of E are positive and carry 18.3–58.7% of the off-diagonal mass, the conductance part of the minimiser's energy is itself *negative* on 61 of 64 windows ($-0.380 \ldots 0.593$ of $E(\psi)$) — and the same code on the Stieltjes surrogate $E - P$ reproduces the classical step, so the death belongs to the form and not to the measurement. T143's Miclo chain loss of 46–2561 is the same death, now localised in *one line* of the proof instead of in a construction.

But M4 splits, and only one half is Markovian. By the pair (Beurling–Deny) decomposition the step is the sum of a *conductance* half, which needs a nonnegative weight and fails above, and a *mass* half, which rests on the pointwise theorem $\sum_k f_k^2 \leq \psi^2$ for the dyadic truncations (measured ratio 0.2639–0.2995 ≤ 1) — and the mass half *dominates* (0.407–1.380 of $E(\psi)$), so the full step $\sigma_{\mathrm{tot}} = \sum_k E(f_k)/E(\psi) = 0.2145$–$0.4425$ is below 1 on every window although its Markov half is dead. That one exact number per window is all the classical proof ever needed. Two routes then make the constant *explicit*: the **energy route** (the classical proof with M4 replaced by its exact value at the minimiser) gives $c_0 = 12\,\sigma_{\mathrm{tot}} = 2.5734$–$5.3099$; the **sign-free Green route** (entrywise domination $x^{\mathsf{T}}Rx \leq \lvert x\rvert^{\mathsf{T}}\lvert R\rvert\lvert x\rvert$ for the first use of Markov, nested-set domination of a nonnegative matrix for the second — no sign hypothesis anywhere) gives $c_0 = G_{\mathrm{dy}} = 2.1559$–$6.3937$, the geometric level constant of the minimiser's own layer cake. The better route:

$$c_0 \;=\; 2.2480\text{–}4.2265, \qquad \text{fit } D^{0.028\pm0.017} \text{ (zone-uniform)},$$

against Maz'ya's classical Dirichlet value 4 and the minimum 1.067–1.352 the measured density sup allows. And S1′ stands as a *certified window inequality* on all 64 windows: $\mathrm{cert\_lam\_max}(R) \leq c_0\,\Psi$, with the certified side independent of whether the computed vector is the minimiser. The price of dropping the sign, as a number: the density hypothesis must be stated for $\lvert R\rvert$ rather than R^+, a factor 1.12–1.41 — and the direction error this fences ($x^{\mathsf{T}}Rx \leq \lvert x\rvert^{\mathsf{T}}R^+\lvert x\rvert$ is *false*) carries its own built-in witness, $R = \left[\begin{smallmatrix} 1 & -1/2 \\ -1/2 & 1 \end{smallmatrix}\right]$ at $x = (1,-1)$, where $3 > 2$.

The no-go, and why the hypothesis list is tight. The "direct spectral argument" is not an independent bypass: $\Psi(A)$ is a Rayleigh quotient of R at $\mathbf{1}_A$, so $\Psi_{\mathrm{all}}\hat{\lambda} \leq 1$ is a theorem with *no content* for S1′ — the load-bearing direction is the lower one, a statement about the minimiser's *profile* (participation ratio 0.3026–0.4588, $K = 35$ levels, delocalised and sign-structured). And the explicit form $R = aa^{\mathsf{T}} + \varepsilon I$ with $a_i = i^{-1/2}$ — positive definite, entrywise *nonnegative*, with the *proved* density ceiling $4 + \varepsilon$ (prefixes by rearrangement) — has $\lambda_{\max} \sim \log m$: $\Psi_{\mathrm{all}}\hat{\lambda}$ decays $0.7023 \to 0.4882$ along $m = 64 \to 1500$ while G_{dy} grows $7.6 \to 36.8$ (flat 1.69–4.16 on the Markovian control, 2.25–4.23 on the TFPT windows). So positivity, a nonnegative Green function and a bounded density sup do *not* imply S1′ with any absolute constant: a hypothesis on the minimiser's *level profile* is necessary, and the step list is not merely sufficient but tight.

The verdict discipline, as its own statement. The verdict rule is checked before the verdict is printed: PROOF-SHAPED requires every step factor to be a theorem or a certified bound, the replacement of M4 is a *measured* constant at the minimiser, no a-priori bound for it is available here — so exactly one step of the transcribed proof is open and the verdict is ONE-STEP-MISSING. Boundedness (max 4.2265 ≤ 16, the preregistered ceiling) and flatness ($D^{0.028\pm0.017}$ against the 0.25 bar) of that constant are *evidence* for the missing lemma and are reported as such, never as the lemma. The rest, typed: S2′ is *retired as a requirement* (the chain never mentions an interval, so c_{glob} is not an input; the finite test family brackets the all-sets sup with $c_{\mathrm{test}} = 1.0782$–$1.1327$

against Charikar's cited 2); R5 is certified on *both* sides per window but the sandwich width $\kappa_{\mathrm{up}}/\kappa_{\mathrm{lo}} = 2.668$–$28.532$ drifts like $D^{0.543}$ — downgraded, not retired, the used side $\kappa_{\mathrm{up}} = 1.1491$–$1.4230$ staying flat ($D^{0.065}$); R4: all 24 border blocks of this pool close directly and inherit exactly the one missing input. Map V17, 11 new statements (stock 104). The identity spine of the whole cycle T142–T145 — the capacity decomposition, the exact capacity–Rayleigh identity, the Cholesky nestedness, the whitening certificate and the M4 split with the certified chain — is load-bearing as v546 [verification/v546_capacity_chain_identities.py] (Section 4.12).

After T145: one named lemma — L1, the level lemma

(L1) **the only mathematical item left in the S1′ branch**: an a-priori bound, uniform in D, for *one* constant of the minimiser's own dyadic layer cake — either $\sigma_{\mathrm{tot}} = \sum_k E(f_k)/E(\psi) \le C$ on the energy route (measured 0.2145–0.4425) or $G_{\mathrm{dy}} \le C$ on the sign-free route (measured 2.1559–6.3937). Either one alone closes S1′ with an explicit c_0; the no-go proves some such bound is *necessary*; and both are delocalisation statements about the minimiser (participation ratio 0.303–0.459). *(L2)* the visible route: the T143 profile (orthogonal to the charge, masses help) plus a perturbation estimate, with the level count $K = 35$ and the sign share 0.348–0.672 as the profile's address. *(R4)* bookkeeping only. Retired with reasons: S2′ (the chain has no interval step), the Markov perturbation search (T144 certified it dead, and the proof needs Markov in exactly one line with two replacements), and the reading of $\Psi_{\mathrm{all}}\hat{\lambda} \le 1$ as evidence (a content-free theorem). The honest state, in one sentence: T144's unproven input was an absolute constant with no address — after T145 it is a bounded, flat, exactly computable quantity attached to a named object with a necessity proof behind it, which is a strictly better place to stand and still not a theorem. T146 (`level_lemma_probe.py`) is running at exactly L1.

10.36 The level lemma stands (T146)

T146 (`level_lemma_probe.py`, contract `LEVEL.LEMMA`, 30/30, 7.6 s on 64 windows $m = 26 \ldots 1173$, verdict LEVEL-CARRIES, the verdict rule hard-wired on 9 preregistered conditions: a constant *measured at the minimiser* counts as *not* a priori, and the rule cannot be argued out of that) attacks exactly L1 — and closes it on the measurement surface as a chain of theorems and certified window inequalities, with no step reading the minimiser.

The profile defeat is the point. Four hand-written closed profiles — the endpoint sine mode, the linear ramp, the frame's own pole vector, the equilibrium potential of the lumped partner — lose against the true minimiser by *five to nine orders of magnitude* in the Rayleigh quotient: T143's cancellation anatomy seen from the other side, since every guessed profile sees $O(1)$ while the gap is $O(D^3)$. What survives is the *Green column* itself — one solve of the closed endpoint-Laplacian geometry, a legal test vector (Rayleigh–Ritz) with $\lambda_{\mathrm{up}}/\hat{\lambda} = 1.0497$–$1.3764$, tens of percent and not orders. The transfer step one would write next is instrumented and then *discarded*: Davis–Kahan is informative on 0 of 64 windows (bound 1.28–87.4), because the bottom of the whitened spectrum is a near-degenerate *block* ($\lambda_2/\hat{\lambda} = 1.2542$–$3.4471$ — the separation sits in the denominator); and even where informative it would not help, since transferring a level-set functional needs an anti-concentration hypothesis, a second unproved input. L1 bypasses the transfer entirely because $\psi = \lambda R\psi$ is an *identity*. The step classification, declared before it is run and then earned: S1 a certified window object, S2 a theorem (Rayleigh–Ritz), S3 a theorem (the counting lemma, form-independent), S4 a theorem (the resolvent identity) plus certified column norms with the solve residual paid for — and S5, the transfer, *not needed*.

Delocalisation, read off the form. The lever of the whole part is one display:

$$\|\psi\|_\infty \;\le\; \lambda_{\mathrm{up}} \max_j \|Re_j\| \,,$$

verified on every window within a factor 1.055–1.426 of the truth — the $\Theta(D^3)$ smallness of the gap, everywhere else in this line the obstacle, is here the *tool*: the minimiser is λ times the Green function applied to itself, so it cannot concentrate where the Green columns do not. Made quantitative: the trivial ceiling $\max_j \|Re_j\| \le 1/\lambda_{\min}$ would give only $\Gamma = \sqrt{m}\,\lambda_{\mathrm{up}} \max_j \|Re_j\| \le \sqrt{m} = 5.1$–$34.2$ (useless); measured, $\Gamma = 1.8356$–2.2797 — the column norms sit a factor 2.7–16.0 *below* the trivial

ceiling, and that factor *is* the delocalisation, a functional of the form and never of the minimiser. The localisation contradiction (restricted Gershgorin over all $\binom{m}{l}$ sets, $\ell^* = 2$–5) stays a theorem with no quantitative content (leak tolerance $\leq 2.38 \times 10^{-3}$ — it forbids only exact localisation), and Perron–Frobenius applies only to the lumped partner (sign purity 1.0000), not to E (0.348–0.672 positive entries): delocalisation does not come from positivity on this surface.

The composite constant, and the two structural findings. The a-priori level constant

$$c_0^{\mathrm{ap}} \;=\; 2b^2\,\Gamma\,\min(1,\Gamma_1) + \varepsilon \;=\; 3.9042\text{–}4.8488 \qquad (b = 1.031,\ 64/64 \text{ windows})$$

dominates T145's *measured* 2.156–6.394, and the chain $\lambda \geq 1/(\kappa_{\mathrm{up}}\, c_0^{\mathrm{ap}}\, \Psi_{\mathrm{abs}})$ holds with loss factor 0.0422–0.1586 of the exact gap — every factor a functional of the form alone. Finding (a): *the cake base is free.* The chain consumes only the lower domination $|\psi| \leq \psi_t$, so Maz'ya's classical dyadic 8 was never a price of the theorem, only of the convention: $2b^2 \to 2$ as $b \to 1$, a gain of 3.76 at zero cost, because the union identity makes the level constant an $O(m)$ closed form and the levels are never materialised. Finding (b): the spectrum bottom is a *block*, which kills every perturbation route and costs the identity route nothing — the resolvent identity holds for every eigenvector at the bottom. Remarkably, c_0^{ap} lands at the size of Maz'ya's classical Dirichlet value 4. The stress tests discriminate in both directions: on T145's no-go form Γ grows $3.676 \to 13.800$ ($m^{0.420}$, *unbounded* — the preregistered criterion) while the theorem half survives there (slack 1.087–1.242), and on the Dirichlet path-Laplacian control Γ stays flat (1.7183–1.7315); κ_{up} counts as a certified building block with its qualification stated (per-window completed Cholesky, maximum 1.4230; the D-flatness is a fit, so what is certified is a finite list — the same status the level lemma has, and the chain is homogeneous). The border pool of this part: 24 blocks, the direct Schur route closes 24/24 and the a-priori chain is positive on all 24 ($c_0^{\mathrm{ap}} = 2.855$–$3.176$); T145's 3 open blocks are not reproducible from a rebuilt pool — the anatomy transfers, the count does not. The four a-priori pieces — the union identity, the counting lemma at a general base, the resolvent delocalisation bound and the composite level lemma with its no-go discriminator — are load-bearing as v547 [`verification/v547_level_lemma_identities.py`] (Section 4.13).

After T146: the level lemma stands — one remainder, all-D

(G1) **the one genuine remainder**: D-uniformity for *all* D rather than on a finite surface — Γ is a certified per-window number (fit $D^{-0.016\pm0.005}$, a fit and never load-bearing), and turning the finite list into a theorem needs an asymptotic bound on $\max_j \|Re_j\|\,\lambda_{\min}$, i.e. the delocalisation of the *Green columns* themselves. *(G2)* σ_{tot} on the energy route stays measured — *not* a hole (either constant alone closes S1′, and the Green route is the certified one), recorded so nobody mistakes the asymmetry for a gap. *(G3)* the *actual* 3 open R4 border blocks from the T134 assembly, rather than a proxy pool — bookkeeping. The honest state, in one sentence: T145's missing lemma is now a theorem chain on the measurement surface with an explicit a-priori constant of classical size, and what separates it from a statement for all D is exactly one asymptotic delocalisation bound. T147 (`green_asymptotic_probe.py`) is running at exactly that asymptotics.

10.37 The identity and the named mechanism (T147)

T147 (`green_asymptotic_probe.py`, contract `GREEN.ASYMPTOTIC`, 39/39, 27.2 s on 85 windows $m = 26 \ldots 1491$ spanning a factor 15 in D, verdict ASYMPTOTIC-SHAPED, the verdict rule wired constructively: "proved" cannot be returned in any branch, because a finite surface proves no statement for all D) attacks exactly the one remainder of T146 — and the last open object becomes an *exact identity*,

$$\Gamma \;=\; \sqrt{Q^\star}\cdot S_w\,, \qquad S_w = \lambda_{\mathrm{up}}\|R\|_F\,, \qquad Q^\star = m\,\max_j (R^2)_{jj}/\operatorname{tr}(R^2)\,,$$

verified to 2.3×10^{-16} on all 85 windows and on both stress forms: the D-uniformity of the level constant is *equivalent* to the boundedness of two named scalars and to nothing else — S_w purely spectral (the effective bottom multiplicity $n_{\mathrm{eff}} = \sum_k (\lambda_{\mathrm{up}}/\lambda_k)^2 = 1.251$–$2.831$, certified $S_w \leq 4.6438$

at a certified cut of a preregistered ladder, the count confirmed by a completed $\mathrm{LDL}^{\mathsf{T}}$ inertia and no eigenvector read), and $Q^{\star}$ purely geometric and scale-free (one for a perfectly flat Green diagonal, m for a localised column; certified ≤ 2.8634 against the localised extreme $m = 1491$).

The classical route dies first, by computation. Demko–Moss–Smith 1984 gives the decay rate $q = 0.994815$–0.999999 on this surface, so the envelope $q^{m/2} = 0.9346$–0.9995 sits above the 0.05 bar on *every* window — the culprit is the condition number $1/\Theta(D^3)$, not the form (E itself decays off-diagonally with Jaffard exponent 0.06–0.82) — and the Green columns are *delocalised* (half-mass radius 0.048–0.105 of the window width), so no decay statement can be the mechanism: the delocalisation itself *is* the bound. The norm anatomy locates it — 0.9864–0.9996 of $\|Re_j\|^2$ at the largest column comes from the bottom block alone ($|B| = 3$–6, certified by inertia) — and the equidistribution constant $Q_B = m \max_j(\Pi_B)_{jj}$, whose exact floor is $|B|$, measures $Q_B/|B| = 1.375$–1.839: the naive alternative $Q_B \sim m$ is wrong by a factor 5–258, and T146's unexplained factor 2.7–16.0 is *identified* as $\sqrt{m/Q_B}$ up to $\lambda_{\mathrm{up}}/\lambda_{\mathrm{lo}}$ — growing like $\sqrt{m}$, exactly the rate that makes Γ bounded. The honest negative is reported with it: the crude free-tail split (drop the mid-spectrum with $\|\Pi^{\perp}e_j\| \leq 1$) is valid but its tail piece grows like $m^{0.538\pm0.020}$ — on record, *not* used.

The mechanism, named and made quantitative. The form is a *Toeplitz-minus-Hankel section* ($A_{rs} = c_{|r-s|} - c_{M-1-r-s}$), whose bottom eigenvectors are the Fourier modes at the symbol's near-zero by Szegő/Widom theory (Kac–Murdock–Szegő 1953; Widom 1958; Basor–Ehrhardt 2009) — and a Fourier mode is *exactly* equidistributed. The sharp prediction $Q_B \leq 2|B|(1 + o(1))$ *holds* (1.375–$1.839 \times |B|$), and the chain down to it is pure arithmetic: $Q_B \leq \sum_{k\in B} m\|\psi_k\|_\infty^2 \leq 2\sum_B \|a_k\|_1^2$, with sparsity ≤ 8.773 per bottom mode and 0.999+ of each mode's energy in the lowest $4|B|$ Fourier modes. The surface is stratified in four D-strata with their own certified maxima; six trend exponents are all flat against the preregistered 0.25 bar and all labelled fits, never load-bearing. Discarded with a reason: the congruence structure of the zones does not carry (scatter 0.465 — the equidistribution is a property of the frame, not of the zone arithmetic).

Clean-up, in both directions. σ_{tot} on the energy route does *not* fall, but now has an address: truncation is a nonlinear operation and leaves the invariant subspace (block leakage up to 0.9627 against the 2.7×10^{-7} that boundedness through the split would need) — not a hole, the Green route is the certified one. **The 3 open R4 border blocks close**: rebuilt in this probe's coordinates, 8/8 border windows carry a valid positive lower bound through the T146 chain plus the new spectral bound ($c_0^{\mathrm{ap}} = 2.9135$–$3.0383$) — the same mechanism, no border-specific argument; what stays open there is the same all-D step, not a border gap. The mandatory stress separates the forms: on T145's no-go form both constants *diverge* as required ($Q_B = 13.49 \to 190.10$ over $m = 64 \to 1500$) and hit the closed-form prediction $Q_B = m/H_m$ to 1.0000, while on the Dirichlet path-Laplacian control — whose bottom modes are half-sines, exactly the Fourier modes the mechanism predicts — both stay flat ($Q_B/|B| = 1.4142$–1.4376 against the closed sine value 2). A bound that stayed finite on the no-go would be too strong and therefore wrong.

After T147: an exact identity, two certified factors — one lifting statement

(H1) **the one lifting statement**: a Szegő-type theorem for the *diagonally reweighted* Toeplitz-minus-Hankel section — that the bottom block of $\Lambda^{-1/2}A\Lambda^{-1/2}$ is Fourier-sparse with an m-independent ℓ^1 norm. That single statement lifts the whole chain from the stratified surface to all D (address: Widom 1958; Basor–Ehrhardt 2009; the measured obstacle is that the mechanism's hypothesis is translation covariance, which the Jacobi whitening breaks — Toeplitz defect 0.21–0.54). *(H2)* an a-priori bound for the spectral half S_w, i.e. for the eigenvalue counting function of the form near zero: the factorisation pays $S_w \geq 1$ whatever $Q^{\star}$ does, so a bottom block whose width grows with D would break the bound even at perfect equidistribution — a separate question from the shape of the eigenvectors. *(H3)* the σ_{tot} classification — either an energy-side analogue of the split or the explicit statement that the Green route is the only certified one. The honest state, in one sentence: what stands after T147 is a theorem-shaped task with two addresses — the all-D claim remains an extrapolation over 85 stratified windows and a factor 15 in D until the two statements are proved. T148 (`szego_bottom_probe.py`) is running at exactly the Szegő statement.

10.38 The lifting statement dissected (T148)

T148 (`szego_bottom_probe.py`, contract `SZEGO.BOTTOM`, 31/31, 13.0 s on 48 windows $m = 96 \dots 1077$ in four D-strata, stress ladders to $m = 1500$, verdict ONE-INPUT-MISSING, the verdict rule wired constructively: "proved" cannot be returned in any branch) attacks exactly T147's two rests — and closes one of them, at the price of an honest negative, while the other resists with its hypothesis reduced to a single named scalar.

The step classification, every sub-statement typed. The passage from the pure Toeplitz-minus-Hankel section to the reweighted one $E = \Lambda^{-1/2} A \Lambda^{-1/2}$ runs in seven steps: *(1)* the order-2 model ladder $4k^2/\pi^2 \le \lambda_k/\hat{\lambda} \le k^2$ is a THEOREM (Kac–Murdock–Szegő 1953), verified exactly with $C \le \pi^2/4$; *(2)* the Fourier sparsity of the *pure* section's bottom mode is a THEOREM (Widom 1958; Basor–Ehrhardt 2009), measured flat on the model ladder ($\nu = 4.236$–4.257); *(3)* the ℓ^1 *ceiling* — the lever — is CERTIFIED: from the identity $\mu_k^p a_k = \langle s_k, L_0^p \psi \rangle$ the coefficient bound $|a_k| \le \min(1, \|s_k\|_\infty \|L_0^p \psi\|_1 / \mu_k^p)$ and, with $\mu_k \ge 4k^2/(m+1)^2$, the *m-free* closed form $\|a\|_1 \le 2\sqrt{\nu} + 1$, $\nu = (m+1)^{3/2} \|L_0 \psi\|_1 / (2\sqrt{2})$ — every power of m cancelled, ν alone carries the m-dependence (largest excess -2.6×10^{-6}, i.e. slack); *(4)* the chain $Q^\star \le 2\langle \|a\|_1^2 \rangle_w \le 2\langle \mathrm{ceil}^2 \rangle_w$ is CERTIFIED on 48/48; *(5)* the commutator/Davis–Kahan route is DEAD BY COMPUTATION (perturbation norm from *below*, the correct direction for a vacuity claim: factor 5.4×10^2–5.3×10^5, vacuous by 2.7–5.7 orders — the reweighting is not a perturbation); *(6)* the live route is weighted Szegő via the *Liouville transform* — $E\psi = \lambda\psi \Leftrightarrow A\varphi = \lambda \Lambda \varphi$ with $\varphi = \Lambda^{-1/2}\psi$, an exact change of variables — $m\|\psi\|_\infty^2 \le 2\kappa_\Lambda \, \mathrm{ceil}(\hat{\varphi})^2$ with $\kappa_\Lambda = \max \Lambda / \min \Lambda = 1.733$–$3.933$ certified (a ratio of two diagonal entries of the form, a priori), buying a factor 50.6 in the smoothness functional of the Green-weighted modes; *(7)* ν bounded by a functional of the form: THE OPEN INPUT. *The hypothesis isolated*: a controlled weight-class experiment on a model section with a genuine order-2 zero shows both bounded-variation weight classes flat (ν trends $x^{0.000 \pm 0.001}$ and $x^{-0.007 \pm 0.002}$) while the rough class (TV $\sim m$) grows as $x^{1.994 \pm 0.002}$ — at *identical* $\kappa_\Lambda = 4$: it is the roughness, not the conditioning, and $\mathbf{TV}(\log \mathbf{\Lambda})$ is the named scalar. On the real surface the a-priori $Q^\star$ ceiling measures 81.4–469.0 and grows $m^{0.615 \pm 0.016}$; ν_L would need $\lesssim m/(8\kappa_\Lambda) \approx 34$ at the largest window, measured 282 (trend $x^{0.166 \pm 0.025}$); TV$(\log \Lambda) = 3.63$–11.93, trend $x^{0.444 \pm 0.006}$, *not* flat — the real whitening is rough in the experiment's sense.

S_w closes — and the symbol falls. A layer cake over LDL$^\mathsf{T}$ inertia counts at a preregistered threshold ladder (no sorted eigenvalue list; Sylvester 1852) certifies $\mathbf{S_w \le 1.3905}$–$\mathbf{1.9587}$ against the true 1.2820–1.6825, trend $x^{0.007 \pm 0.007}$ *flat*, certified per D-stratum (4×12); $n_{\mathrm{eff}} = S_w^2 = 1.644$–$2.831$. The honest negative, asserted rather than hidden: the Toeplitz symbol of the arithmetic lag sequence has $\mathbf{f(0) < 0}$ **on 48/48 windows** ($-87.3 \dots -22.5$, curvature 1.9×10^5–9.7×10^7) — the positive definiteness of the section is a *finite-section effect* of the minus-Hankel part and the Stieltjes lumping, not a symbol property; the KMS order-2 hypothesis is not available at the symbol level, and the bottom scaling survives only as a *measurement* ($\alpha = 1.643$–1.989 inside the preregistered band $[1.5, 2.5]$). The mandatory stress separates exactly where it must: the no-go violates precisely the smoothness hypothesis ($\nu = 48.7$–4.28×10^3, divergent; its spectral half is innocent, $n_{\mathrm{eff}} = 1.0000$), and the Dirichlet control is exact ($\nu \to \pi$, $m\|\psi\|_\infty^2 \to 2$).

The whole chain, end to end, with the big number. The chain $\lambda \ge 1/(\kappa_{\mathrm{up}} c_0^{\mathrm{ap}} \Psi)$ with $\Gamma = \sqrt{Q^\star} S_w$ (T147's identity, re-verified in this file's code path to 4.4×10^{-16}) is a certified *lower* bound on all 48 windows and delivers **8.36**–**15.86**% **of the true gap** (median 11.74%, no window losing more than a factor 12); the same chain with the a-priori-shaped factors substituted is still a lower bound everywhere at a factor 10.5 (fraction 0.72–1.56%) — the price of a-priority is a number, and the loss sits entirely in the looseness of the ℓ^1 ceiling, not in the architecture. The uniformity balance over all ten factors: six CERTIFIED, two MEASURED (ν_L, TV), two FITS — and the fits enter no inequality; the load-bearing distinction is now certified-per-window versus certified-*and*-*m*-free, and exactly one factor sits on the wrong side. σ_{tot}: RETIRED AS A ROUTE — the Green route is the only certified one, no compiler statement waits on it, and later passes should not re-open it as a gate. The identity/certificate core of T147+T148 — the factorisation identity, the Q_B floor, the ℓ^1 ceiling, the layer-cake S_w certificate and the Liouville transform with its weight-class discriminator

— is load-bearing as v548 [`verification/v548_green_szego_identities.py`] (Section 4.14).

After T148: S_w closes on the surface — one scalar input remains

(I1) **the load-bearing rest**: a bound $\nu_L \le F(\text{form})$ with F m-free for the bottom block of the *reweighted* section — equivalently bounded TV($\log\Lambda$), or any weaker smoothness of the multiplier that still controls $\|L_0\hat{\varphi}\|_1$; the target is $\nu_L \lesssim 34$ at the largest window, measured 282. That single scalar statement is the whole rest of the lifting statement Z1. *(I2)* cosmetic: a theorem behind the order-2 bottom ladder directly from the minus-Hankel structure (Basor–Ehrhardt 2009 is the address) — the S_w certificate does not need it (it is an inertia count), but without it the m-freeness of the KMS constant rests on a measurement. The honest state, in one sentence: what stands after T148 is a certified end-to-end chain that delivers 8.4–15.9% of the true gap on a stratified surface of 48 windows, with exactly one scalar — ν_L, equivalently the roughness of the whitening weight — between the finite list and a statement for all D. T149 (`weight_smoothing_probe.py`) is running at exactly that scalar: whether a smoothed whitening $\tilde{\Lambda}$ with a certified sandwich removes the TV roughness.

10.39 The smoothing freedom (T149)

T149 (`weight_smoothing_probe.py`, contract `WEIGHT.SMOOTHING`, 30/30, 61.4 s on 44 prime-power windows $m = 277 \dots 1393$ in four D-strata, 9 preregistered gauges, verdict PARTIAL-SMOOTHING, the verdict rule wired constructively: "proved" cannot be returned in any branch) attacks exactly T148's one open input, through the observation that the whitening diagonal is a *free gauge*: for any positive diagonal $\tilde{\Lambda}$ the chain runs unchanged — an identity plus one Rayleigh step, licensed and verified before use — so every gauge gives a valid lower bound and the maximum over the family is valid too.

The three numbers, under the preregistered winner rule (8 admissible of 9 candidates; the winner is `const`, the geometric mean of the Jacobi diagonal): the sandwich is CERTIFIED at $\sigma \le 5.5789$ (trend $x^{0.154\pm0.022}$, bar 8.0 held); **TV($\log\tilde{\Lambda}$) is exactly 0.0000** — T148's blocking number, 11.93 growing as $x^{0.444}$, is not reduced but *eliminated*; and $\nu_{\tilde{L}} = 222.70$ ($x^{0.170\pm0.047}$) meets the target $m/(8\kappa)$ only on **21/44** windows ($\rho = 1.347$ at $m = 277$ falling to 0.485 at $m = 1393$) — PARTIAL: the small-m end is missed. The price of the gauge: $\tilde{\kappa}_{\text{up}} \le 2.3146$ against Jacobi's 1.2647 (a factor 1.83, paid once and linearly, certified). *The real finding is the two-regime decomposition, and it refutes the hypothesis*: the flutter-smoothing gauges (flutter removed, macro profile kept) are *cheap* (sandwich 1.129–1.572, TV → 1.27–1.73) yet move $\nu_{\tilde{L}}$ by at most **0.9**%; only the macro-removing gauges improve $\nu_{\tilde{L}}$ at all (factor 1.27) — the roughness the smoothness functional responds to was never in the multiplier: TV($\log\Lambda$) *was the mislocated hypothesis*, withdrawn and relocated — the roughness sits in the *form* itself.

The lifted chain. All 178 assembled chains are valid lower bounds; $\Gamma = \sqrt{Q^\star} S_w$ holds gauge-invariantly at 4.4×10^{-16} over 89 assemblies. Certified, the family-maximum chain delivers **9.52–18.45% of the true gap** (median 12.68%; T148's 8.36–15.86% reproduced on a longer surface), a priori 0.800–1.566% (an improvement of up to 1.284×); on the certified side the Jacobi gauge still wins (44/44 — its sharper $\tilde{\kappa}_{\text{up}}$ outweighs the smoother $Q^\star$ ceiling: the gauge freedom is a tool for the a-priori side, not a free win). The $Q^\star$ ceiling falls 697.9 → 149.1 with its exponent falling 0.551 → 0.330 — bounded yes, *flat no*, and the driver is named: $\nu_{\tilde{L}} = 265.2$ ($x^{0.772\pm0.048}$) on the Green-weighted modes *above* the bottom block.

Stress, and the corrected question. The mandatory no-go's own multiplier is already smooth (best sandwich 1.2629 over the whole family), and its ν diverges as $x^{1.420\pm0.004}$ under *every* gauge — the break stays visible in the form, smoothing rescues nothing and cannot manufacture equidistribution (passed, preregistered). The Dirichlet control has a constant diagonal, all 9 candidates are the identity on it (5.9×10^{-14}), $\nu \to \pi$ exactly, $m\|\psi\|_\infty^2 \to 2$ — and the control *corrects the question*: classically $\nu_k = \pi k^2$ *exactly*, so the right form is the ladder $\nu_{\mathbf{k}} \le \mathbf{Ck^2}$ **with** m**-free** C (measured in the `const` gauge: $C = 18.66$–44.61, $x^{0.272\pm0.021}$, attained at $k = 1$). The TV anatomy of the Jacobi diagonal locates the cheapness: the macro part is 0.78–1.52 (8.5–14.2% of the total), the flutter part 6.42–13.02 — many *small* steps (amplitude 0.064–0.182, *flat* at $x^{-0.007\pm0.028}$, largest single log-step ≤ 0.193) spread over the whole window, with the frame edges not the source (enrichment 0.75–1.20):

TV grows through the step count while the sandwich pays only the amplitude. The order-2 bottom ladder from the minus-Hankel structure stays NAMED OPEN, cosmetic ($C_{\text{bot}} = 5.6$–14.1; Basor–Ehrhardt 2009 does not cover a sign-changing symbol; the chain does not wait on it — S_w is certified by an inertia count). Seven promotion candidates are named and stay unpromoted.

After T149: TV eliminated, the missing input relocated — a ladder remains

(J1) **the load-bearing rest**: an m-free constant C in the ladder $\nu_k \le Ck^2$ for the modes of the smoothed section (measured $C = 18.66$–44.61, $x^{0.272\pm0.021}$) — with the constant gauge this is *exactly* a question about the deep modes of the pure Toeplitz-minus-Hankel section, no multiplier, no whitening, no arithmetic weight; it makes the $Q^\star$ ceiling m-free and the whole measurement-surface chain a priori. *(J2)* **the second lever, which would close (J1) outright**: an a-priori bound on the flutter *amplitude* of $\log \operatorname{diag}(A_B)$ — measured 0.064–0.182 and *flat* ($x^{-0.007\pm0.028}$); it would make $\kappa_{\tilde\Lambda} = 1$ and $\tilde\kappa_{\text{up}} \le 2.3146$ a priori rather than certified per window. *(J3)* cosmetic: the order-2 ladder theorem from the minus-Hankel structure (unchanged from T148). The honest state, in one sentence: after T149 the smoothing freedom has retired TV($\log\Lambda$) as a hypothesis — eliminated exactly, at a certified price — and the one input between the finite list and a statement for all D is now the smoothness of the deep modes of the pure section, in ladder form. T150 (`mode_ladder_probe.py`) is running at exactly that ladder, through the flat flutter amplitude.

10.40 The mode ladder and the parity mechanism (T150)

T150 (`mode_ladder_probe.py`, contract `MODE.LADDER`, 36/36, 48.0 s on 72 prime-power windows $m = 50 \ldots 1393$, verdict ONE-TERM-MISSING, the verdict rule wired constructively: "proved" cannot be returned in any branch) attacks the ladder $\nu_k \le Ck^2$ through the flat flutter amplitude — T149's rest (1) via rest (2) — and comes back with the mechanism's *name*.

The parity compression. The reflection-odd form that every probe since T106 works with is *exactly* the compression of the full symmetric Toeplitz section onto its antisymmetric parity sector:

$$U_-^{\mathsf T}\, T_M(c)\, U_- \;=\; T - H\,, \qquad U_+^{\mathsf T}\, T_M(c)\, U_+ \;=\; T + H\,, \qquad U_+^{\mathsf T}\, T_M(c)\, U_- \;\le\; 4.0 \times 10^{-16}$$

also on real windows (Basor–Ehrhardt 2009: the parity sectors are the natural objects of the Toeplitz+Hankel algebra) — and the inertia *localises*: **the entire negative inertia of the sign-changing symbol sits in the even sector** (odd 0, even 1 on 72/72, LDL$^{\mathsf T}$ certified). T148's honest negative $f(0) < 0$ is thereby *explained rather than worked around*: it is a statement about the *other* parity sector. The matched parity sines $t_k(r) = \frac{2}{\sqrt{N}} \sin(2\pi k(r{+}1)/N)$, $N = 2m{+}1$ (exact eigenpairs of the parity Laplacian, corner 3 forced by the antisymmetry; control model exact, $\nu_k = \pi k^2$ to 0.0002) give a third valid ceiling, improving the ladder constant by 1.000–1.130 over the Dirichlet+Neumann minimum of T148/T149.

The zone diagonal, and who carries the positivity. The whitening diagonal is an *explicit zone function*, $\Lambda_r = c_0 - c_{M-1-2r} + \sum_{s\neq r} \max(c_{|r-s|} - c_{M-1-r-s}, 0)$ (residual 0.0 on 72/72), and the lag vector splits exactly as $c = c^{\text{arch}} + c^{\text{atom}}$ with $c^{\text{atom}} \le 0$. The flutter amplitude becomes a certified *form* functional, no fit anywhere: $\text{amp}_{\text{loc}} = 0.0606$–$0.1993$ at trend $x^{-0.165\pm0.015}$ (T149's fit-based 0.064–0.182 reproduced fit-free), with the certified chain $\text{amp}_{\text{loc}} \le \text{half} \cdot d_1^{\log} \le \text{half} \cdot d_1^{\lambda} / \min\Lambda$ (1.345–3.954) and the closed atom budget $\|c^{\text{atom}}\|_1 \le 4B\sqrt{N}$. The honest negative beside it: the archimedean diagonal is *not* the macro profile — the level shift $\|\log(\Lambda/\Lambda^{\text{arch}})\|_\infty = 0.734$–$2.647$ grows as $x^{0.244}$; the flutter is small and a priori, the *level* is not. And the perturbation step is precisely dead, for two independent reasons: the multiplicative (Loewner) step transfers scale-free with $\exp(\text{amp}) \le 14.11$ — the gauge step *closes* — while the additive step (Weyl 1912 / Bauer–Fike 1960 / Davis–Kahan 1970) compares $\|A^{\text{atom}}\| = 4.3$–$12.3\%$ of the section against a $\Theta(D^3)$ bottom and misses by 2.4–5.8 *orders of magnitude*; moreover the archimedean-only section is itself *indefinite* (2–7 negative eigenvalues, LDL$^{\mathsf T}$ certified) while the full section has none — **the atoms are co-responsible for the positivity**, not a perturbation of it.

The one gate, with its number. The chain delivers 0.0680–0.1845 of the true gap certified (the best window reproduces T149's 0.1845 exactly), a priori 8.16×10^{-3}–1.74×10^{-2}; the ladder-shaped $Q^\star$ ceiling is valid by monotonicity (38.76–261.3 against the certified minimum 36.98–144.6,

price $\leq 2.555\times$ for the reduction to *one* constant). But $C = 14.24$–43.39 $(= 13.81\pi)$ with trend $x^{0.258\pm0.009}$: the gate "ladder constant non-growing" falls — the only one of all. The maximum sits at $k = 1$ (67/72), the growth is carried by the deeper modes, and the gap $\pi \to 13.81\pi$ is exactly the *arithmetic leakage* of the bottom mode (ℓ^1 mass 1.41–1.79, $m\|\psi\|_\infty^2 = 2.45$–$3.41$ against the model's 2). The stress is now localised: because the amplitude and the ladder are separate statements, the T145 no-go can be asked *which* it violates — the answer is the ladder alone (its multiplier is smooth, TV ≤ 0.2334; its ladder constant grows $x^{1.840\pm0.130}$), while the Dirichlet and parity controls reproduce $C = \pi$ to 0.0004/0.0002. The identity/certificate core of T149+T150 — the gauge freedom with the const gauge and its certified sandwich, the two-regime discriminator, the explicit zone diagonal with the closed atom budget, the parity compression with the inertia localisation, and the parity-sine calibration with the per-window ladder certificate — is load-bearing as v549 [`verification/v549_gauge_parity_identities.py`] (Section 4.15).

After T150: the mechanism is named — one inequality remains

(K1) **the load-bearing rest**: $\nu_k \leq Ck^2$ with an m-free C for the *odd parity sector* of a sign-changing-symbol Toeplitz section — *the* one inequality; measured $C \leq 43.391 = 13.81\pi$ certified per stratum, trend $x^{0.258\pm0.009}$ against the flatness bar 0.25. *(K2)* a closed lower bound on $\min \Lambda^{\text{arch}}$ from the smooth kernel — it would make the flutter-amplitude ceiling closed rather than per-window certified. *(K3)* the extension of Basor–Ehrhardt 2009 to a symbol with $f(0) < 0$, or an explicit local model at the symbol minimum inside the odd sector. The honest state, in one sentence: after T150 the mechanism has a name — parity — T148's honest negative is explained (the negative inertia lives in the even sector), the flutter amplitude and the zone diagonal are certified form objects with the atoms co-responsible for positivity, and the one input between the finite list and a statement for all D is a single named inequality about the low modes of one explicitly described finite matrix. T151 (`odd_ladder_probe.py`) is running at exactly that odd-sector ladder.

10.41 The odd ladder and the Sobolev reroute (T151)

T151 (`odd_ladder_probe.py`, contract `ODD.LADDER`, 27/27, 43.4 s on 72 prime-power windows $m = 96 \ldots 1491$, verdict ODD-CARRIES, the verdict rule wired constructively: "proved" cannot be returned in any branch) attacks the one remaining inequality — and closes it by *rerouting off the ladder*.

The grid step-over, and the matrix-versus-symbol lesson. The positivity of the odd sector is a *grid* fact: the parity sines live on the shifted grid $\theta_k = 2\pi k/N$ with $\mu_k^P = 2 - 2\cos\theta_k$ exactly, and the symbol's negative window (width $\theta_c = 7.06 \times 10^{-4}$–$1.32 \times 10^{-2}$) lies *below* the first odd grid point θ_1 on every window ($\theta_c/\theta_1 = 0.328$–$0.407$, trend $x^{-0.057}$): the grid steps over it on 72/72 — stepped over rather than cancelled. But the pointwise symbol *language* fails at exactly this resolution: $f''(0) = 3.8 \times 10^5$–6.8×10^8, f moves by a relative 0.47–0.85 across *one* mode spacing, the section form averages the symbol against a Fejér kernel of exactly that width instead of sampling it (mode-by-mode deviation 1.39–1.83 at $k = 1$), and the one-parameter symbol margin is vacuous ($\rho \geq 1$) on 72/72. **T150's rest item 3 — extending Basor–Ehrhardt 2009 to $f(0) < 0$ — is thereby answered in the negative**: the local model at the symbol minimum exists, but as a *matrix* statement, not a symbol one — $\lambda_k(A) \leq S\mu_k^P$ for $k \leq 8$ certified by completed LDL^T counts (Sylvester 1852) with $S = 1.1019$–2.3870: the odd effective minimum sits at θ_1, order 2, $O(1)$ prefactor, $f(0) < 0$ *absorbed* by the Rayleigh floor $L_P \geq \mu_1^P I$. The leakage T150 left as a bare number is anatomised: a *low-k rotation* of the bottom mode away from t_1 ($\sin\angle(\psi, t_1) = 0.8156$–$0.8943$, band shares 0.355 at $k = 1$, 0.687 at $k = 2$–4, 0.005 beyond $k = 64$; sideband peak at $l^* = 2$) — not an arithmetic sideband.

The Sobolev step, and the linear bound. The multiplicative mode comparison T150 asked for *is* the discrete Sobolev step: an odd-sector vector vanishes at the virtual node, so $\|\psi\|_\infty^2 \leq 2\|\psi\|_2\|\nabla\psi\|_2$; the certified pencil floor $\kappa = 0.2273$–0.4502 (*one* completed Cholesky per window, floor carried,

trend $x^{0.047\pm0.009}$) bounds the gradient energy by λ_k/κ, and

$$b_k \;:=\; m\|\psi_k\|_\infty^2 \;\le\; 2m\sqrt{\frac{K_{\mathrm{bot}}\,\mu_k^P}{\kappa}} \;\le\; C_S\,k$$

with $C_S = 11.5137$–19.5731 (median 13.82, trend $x^{0.020\pm0.007}$): **linear in k and non-growing** where T150's quadratic ladder grew ($C = 13.81\pi$ at $x^{0.258}$) — nothing additive anywhere, so the $\Theta(D^3)$ bottom that killed Weyl / Bauer–Fike / Davis–Kahan never enters, and the $\sqrt{m}$ that kills every rotation route (the ν conversion costs $4\sqrt{m}$ and overshoots π by 4.2×10^3–1.0×10^6) is never paid; at the bottom the gain over the ν route is a factor 7.5–34.9. The layer cake buys only 1.00–2.37 and does not change the trend. The honest negative beside it: the *global* pencil ratio grows $x^{2.695}$ (useless — L_P saturates at 4 while the section's top does not); everything is carried by the *bottom* ratio $R = K_{\mathrm{bot}}/\kappa = 3.3634$–$9.7108$ at $x^{0.037\pm0.015}$ — the *one* remaining fit in the chain.

The large numbers, and the localised stress. T150's rest item 2 is closed *and attained*: $\min_r \Lambda_r^{\mathrm{arch}} = c_0^{\mathrm{arch}} - c_{M-1}^{\mathrm{arch}} = 2.5928$–$4.8981$ at relative deviation 0.0 (two per-window-checked shape properties). $Q_{\mathrm{sob}}^\star = 12.39$–$23.94$ ($x^{0.029\pm0.008}$) beats the ℓ^1 ceiling 43.66–158 by 2.69–10.13; end to end moves from 7.88×10^{-3}–1.74×10^{-2} to 2.01×10^{-2}–3.52×10^{-2} of the true gap (gain 1.64–3.18×), and the dominant loss is now $\Psi = 4.7\times10^3$–6.1×10^6 — never before the bottleneck. The stress localises exactly where it must: the T145 no-go breaks *in the pencil and only there* (R grows $x^{1.986}$ to 3.0×10^7; its localised bottom mode has parity Dirichlet energy $\sim 1/\log m$ instead of μ_1^P), while the controls are exact (parity pencil $(1,1)$ to 3.73×10^{-12}, $C/\pi - 1 \le 2.5\times10^{-4}$; the model Sobolev price is *exactly* π — the honest, m-free cost of leaving the ℓ^2 world), and the Dirichlet-versus-parity pencil is 1.985–1.999: one rank-one corner mismatch, bounded. Verdict ODD-CARRIES: all six preregistered gates met, every layer typed — *theorem* (parity compression, Weyl minimax, Sylvester, exact μ^P), *certified* (κ, K_{bot}), *elementary-verified* (the Sobolev step) — and the one fit named. The identity/certificate core of T151 — the grid step-over with the certified symbol vacuity, the certified bottom ladder, the Sobolev step with the π-price calibration, the closed-and-attained archimedean minimum, and the linear per-mode bound with the no-go discriminator — is load-bearing as `v550` [`verification/v550_odd_sector_identities.py`] (Section 4.16).

After T151: the bound is linear — one scalar remains

(L1) **the load-bearing rest**: the m-freeness of the bottom pencil ratio $R = K_{\mathrm{bot}}/\kappa$ — *one* scalar, no eigenvector, the only non-certified link left in the chain; measured 3.3634–9.7108, trend $x^{0.037\pm0.015}$, and the T145 no-go breaks exactly there ($x^{1.986}$). *(L2)* a closed lower bound on $\kappa = \lambda_{\min}(A, L_P)$ from the lag vector — it would make the Sobolev route closed rather than per-window certified; the symbol route to it is *vacuous* and the reason is now known, so this needs the arithmetic of the atoms, not an asymptotic. *(L3)* Ψ, the new dominant end-to-end loss now that $Q^\star$ has been improved. The honest state, in one sentence: after T151 the one inequality closes by rerouting off the ladder — the odd grid steps over the symbol's negative window, the bottom spectrum is certified against the parity Laplacian, the Sobolev step makes the per-mode bound linear with a non-growing constant — and the whole distance between the finite list and a statement for all D is the m-freeness of a single printed scalar. T152 (`pencil_ratio_probe.py`) is running at exactly that scalar.

10.42 The pencil ratio and the cancellation (T152)

T152 (`pencil_ratio_probe.py`, contract `PENCIL.RATIO`, 37/37, 33.2 s on 60 prime-power windows $m = 149\ldots1445$, verdict ONE-TERM-MISSING, the verdict rule wired constructively: "proved" cannot be returned in any branch) attacks the one scalar T151 left as a fit — the m-freeness of the bottom pencil ratio $R = K_{\mathrm{bot}}/\kappa$ — and returns the sharpest structural fact of the phase.

Both hoped-for gifts refuted: positivity is a cancellation. A^{arch} is *not* positive in the odd sector on any of the 60 windows ($\lambda_{\min} = -2.8132$–-1.8396, certified negative by a completed Cholesky of $A^{\mathrm{arch}} - tI$ with the floor carried): the odd sector removes the constant mode but not the negative archimedean mass near the origin. In pencil normalisation the failure is catastrophic, not marginal — $\kappa_{\mathrm{ar}} = \nu_{\min}(A^{\mathrm{arch}}, L_P) = -5.28\times10^5$–$-4.00\times10^3 = O(-m^2)$, because the lowest parity sine has

L_P-energy $\mu_1^P = O(m^{-2})$ while it sees an $O(1)$-negative form. And the atom part is negative too ($\nu_{\min}(A^{\text{atom}}, L_P) = -1408$–$-25.6$ on 60/60): *neither* summand of the exact split is positive, and the sum is — **the positivity of the odd section is a cancellation between the archimedean kernel and the prime-power atoms, not a property of either half** (exhibited exactly at t_1: an archimedean part -5.26×10^5–-3.98×10^3 plus an atom part 8.19×10^3–2.88×10^6 add to a near-$O(1)$ quotient — the atoms even *overshoot* the archimedean hole). The additive Weyl route misses κ by a measured 1.5×10^4–1.8×10^6 and the closed ℓ^1 budget route by a further 2.3×10^5–9.9×10^7 — the $O(m^2)$ conversion $1/\mu_1^P$ kills both. But the archimedean diagonal is negative on only an *m-independent* number of parity modes ($k \le 4$–7 of up to 1445), and above them its pencil quotient is ≥ 0.989 on 60/60: **the Schur two-block criterion (Schur 1917 — an equivalence, with both Cholesky floors subtracted) with a fixed 16-mode low block certifies $\kappa \ge t = 0.2250$–0.2500 on every window**, at $m^{0.006 \pm 0.036}$ with identical quartile medians — the floor is no longer a fit. It consumes *one* unproven inequality — $B_{HH} \succeq tI$ on the bulk block — and entrywise routes are provably insufficient for it: Gershgorin returns -8.99×10^5–-1.3×10^3 on the full whitened parity block and still -7189–-58 after cutting at the last arch-negative mode, the median row coupling being 3.9–19.6 against a diagonal of 0.36–0.89. What is needed is a *block* statement, not a bound on individual prime-power sums.

The ratio, certified on both ends. The archimedean part does not carry the *ceiling* either: $K_{\text{bot}}^{\text{arch}} = 24.6$–$1326$ at $m^{1.519}$, one to two orders of magnitude above the true $K_{\text{bot}} = 1.102$–2.118 and growing where the true one is flat — the atoms do not perturb an archimedean alignment, they *create* it. The new number: $R \le K_{\text{bot}}/t = 4.408$–$8.471$ **at $m^{0.099 \pm 0.167}$, certified on both ends** (inertia ladder above, Schur floor below), with a quartile walk of 1.269 over a 4.54× m-range. Two structural m-free routes to the ceiling lose four to six orders of magnitude: Courant–Fischer on the parity basis (the atom overshoot is $O(m^2)$), and the two-sided bulk block — whose KMS ratio step *is* a theorem ($\sin a / \sin b \le a/b$, hence $\mu_{k+16}^P / \mu_k^P \le 289$, verified 285.95–$288.97 \le 289$ on the table) but whose ingredient $T_{HH} = 26.8$–3116 grows $m^{2.044}$: the section's low *eigenvalues* track the model's while its low *eigenvectors* do not. The no-go reconstruction breaks exactly at the unclosed step (the floor survives; ceiling and R grow $m^{1.994}$) — the gap is real, not a tool loss; the control (L_P against itself) is $(1, 1, 1)$ to 3.5×10^{-11}.

The Ψ map, and the honest dual reading. $\Psi \le 8.70 \times 10^3$–$6.24 \times 10^5$ *is* $1/\lambda_{\min}(E)$ to within 1.12–1.19 — the spectral bottom of the whitened section and essentially nothing else; it grows $m^{2.18}$ because $\mu_1^P = O(m^{-2})$, not because κ degrades. The reserve: 89–92% **of** $\text{trace}(E^{-1})$ **sits on the eight lowest modes — exactly those the certified ladder already controls**; rebuilding the subset supremum into a ladder sum is worth 3.46–5.29× end to end (it would lift T151's 2.01×10^{-2}–3.52×10^{-2} into the 8×10^{-2}–1.4×10^{-1} range) — now the dominant loss, outranking any further sharpening of R. The verdict is kept honest by printing both readings: on the band estimator alone gate B would fail and the verdict would read RATIO-RESISTS; the quartile-median reading (15 windows per stratum, high variance of the band fit) is the declared one, and both are in the output. Also on the record: T151's R exponent 0.037 was fitted against the zone variable (not monotone in m) — the same quantity refit in m is 0.099, so part of that flatness was the fit variable rather than the object. Ten map entries are named; *none is promoted* — the strongest candidates consume the unproven block positivity $B_{HH} \succeq tI$ and stay pending with exactly that reason.

After T152: the floor is structural — one term and one rebuild remain

(M1) **the block inequality**: $B_{HH} \succeq tI$ on the bulk block — the single unproven input the Schur certificate consumes; entrywise/Gershgorin bounds provably cannot deliver it (-8.99×10^5–-58 even after the cut), so it must be a block statement. *(M2)* **the load-bearing rest**: an m-free ceiling on K_{bot} — every structural route loses four to six orders of magnitude because the low eigenvectors are not the model's, and the T145 no-go breaks exactly there ($m^{1.994}$). *(M3)* the Ψ rebuild, subset supremum → ladder sum — worth 3.46–5.29× end to end, now the dominant loss and ahead of any further work on R. Only after these: frames beyond A, zones beyond prime powers. The honest state, in one sentence: after T152 the positivity of the odd sector is known to be a cancellation between geometry and arithmetic rather than a property of either half, the pencil floor is certified by a fixed-size Schur block instead of a

fit, the ratio is certified on both ends with one m-free ceiling missing — and the largest remaining loss sits on modes the certified ladder already controls. T153 (`psi_ladder_probe.py`) is running at exactly that rebuild and that block inequality.

10.43 The Ψ pinning and the two remaining terms (T153)

T153 (`psi_ladder_probe.py`, contract `PSI.LADDER`, 33/33, 7.6 s on 18 prime-power windows $m = 96 \dots 1430$, block anatomy on 11 and the ceiling on 12 of them, verdict REBUILD-RESISTS, the gates *evaluated* rather than chosen) carries out the rebuild T152 mapped — and refutes it by exactly its own factor.

The pinning: the reserve was never there. The rebuild itself is real and comes in three rigorous forms, each an upper bound on the same Ψ: the mode-resolved ladder sum (cost 1.195–1.761× against the T149 route), the arithmetic-free operator form $(\mathrm{const}/t)\,\Psi_P$ against the pure Kac–Murdock–Szegő model (cost 3.73–9.70×), and the theorem-grade collapse $\Psi \le \mathrm{const}/(t\,\mu_1^P)$ (cost 3.73–9.70×). But the decisive measurement is a *lower* bound: for the subset that maximises the first mode, every term of $\sum_k \lambda_k^{-1}\langle v_k, \mathbf{1}_S\rangle^2$ is nonnegative, so $a_1/\lambda_1 \le \Psi \le 1/\lambda_1$ with $a_1 = 0.7695$–0.8306 — exactly the $8/\pi^2 = 0.8106$ of the first parity sine. **Ψ is pinned to within** $1/a_1 = 1.204$–1.300**, so T152's mapped** 3.46–5.29× **reserve never existed**; the head/tail replacement T152 proposed lies 2.31–6.12× *below* the hard lower bound and is not an upper bound at all — the maximising subset is aligned with v_1 itself, so the head *is* the value of Ψ, not a redundancy. The bookkeeping is exact: the rebuild retires the level constant $c_0^{\mathrm{ap}} = 4.157$–$4.870$ and pays the collapse cost 3.73–9.70; on this surface the two cancel, the net end-to-end gain is 0.50–1.11× (median 0.62), and the certified fraction moves from 2.01×10^{-2}–3.52×10^{-2} to 1.01×10^{-2}–3.92×10^{-2} — not the targeted 8×10^{-2} band. The collapse cost is named exactly: $\lambda_1(A)/(t\,\mu_1^P) = 3.25$–$8.40$ — the pencil minimum is not attained at the bottom mode, so a floor uniform over all modes is lossy for mode 1 alone. What the rebuild does buy is *uniformity*: the collapse form contains nothing beyond an explicit arithmetic sum, the one open block inequality t, and the exact μ_1^P — it retires Charikar's licence and every per-window diagonalisation from the Ψ step. **Ψ is closed as a term.** The balance is clean: t is flat ($x^{-0.004}$, certified 0.2400–0.2500 on all 18 windows) and the gain is flat ($x^{-0.039}$).

The sign reversal, the four dead candidates, and the live route. $B_{HH} \succeq tI$ is the original pencil inequality restricted to $\mathrm{span}\{t_k : k > 16\}$ — an identity, not an analogy: the Schur route relocates the question onto a subspace, it does not ease it. The find of the zone reverses a T152 sign: **on the bulk parity block the archimedean part is positive** ($\lambda_{\min}(B_{HH}^{\mathrm{arch}}) = 1.0362$–$1.4143 > t = 0.25$), after an m-free peeling of at most 8 modes on every window — the archimedean negativity lives entirely in the lowest parity modes, and on the bulk it is the *atoms* that are negative (-484.6–-14.95): the obstruction has moved from a negative smooth kernel to an arithmetic remainder. Four block candidates die: entrywise Gershgorin (-1929–-21.35 on the bulk), block Gershgorin (Feingold–Varga 1962) on a dyadic partition (-333.4–-7.68; the coupling is long-range), a mode-index Toeplitz-plus-Hankel fit (residual 0.70–0.95 of the Frobenius mass — no second-level Szegő/Widom argument exists), and a recursive Schur step, which certifies the same $t = 0.25$ at levels 0/1/2 — *scale-invariant*, so recursion alone can never terminate. The one route that produces positivity at all is a relative (Kato-type) bound in the archimedean metric: $1+\lambda_{\min}(C) = 8.3\times10^{-4}$–$2.9\times10^{-2}$, certified positive on every window where all three additive candidates return a negative number — but it shrinks as $x^{-1.778}$: positivity per window, not a uniform floor. The whole of t sits in how far inside -1 the arch-whitened atom operator lies, and that distance is of the same order as μ_1^P itself.

The inverse-iteration find. No family fixed in advance carries the K_{bot} ceiling: the fixed-size band replacement grows exactly like the bulk ceiling it replaces (T_{band} at $x^{2.017}$ against T_{HH} at $x^{2.019}$), the parity sines and the Hann taper grow at $x^{2.47}$ and $x^{2.46}$, and the Fejér family degenerates (a sine is an eigenvector of every convolution). The atom overshoot is located exactly: the Toeplitz half of the atom response of the first parity sine is -1.99 (the atoms *help*) while the Hankel reflection half is $+9.45$ — a boundary-reflection effect, not a bulk-atom effect. But **two**

inverse-iteration steps $(A^{-1}L_P)^2 t_k$ **give a flat** $K^F = 1.432$–2.369 $(x^{0.116})$ against the certified true $K_{\text{bot}} = 1.102$–1.896, on a certificate of fixed size 8: the ceiling *is* carried by a fixed-size family — just not by one written down in advance. Nine map entries are named (v580–v588); *nothing is promoted* — the probe is exploration only, and the T149–T152 candidates stay pending.

After T153: Ψ is closed — two terms remain, and they are one object

(N1) **the block inequality** $B_{HH} \succeq tI$ on the bulk parity block — after T153 its sharp form is *one* number: the distance of the arch-whitened atom operator inside -1 (8.3×10^{-4}–2.9×10^{-2}, shrinking as $x^{-1.778}$), to be bounded against the m-free archimedean reserve 1.04–1.41 the bulk block now carries. *(N2)* **the Green/alignment estimate**: a structural description of $(A^{-1}L_P)^2 t_k$ for $k \le 8$ — it closes the ceiling (the flat $K^F = 1.43$–2.37 is already certified *by* that family) *and* recovers the collapse factor 3.25–8.40 that the uniform floor throws away: **both remaining terms lose to the same missing object** — where the bottom eigenvector of the section sits. The Ψ rebuild is off the list: the theorem-grade collapse needs no certificate beyond t. The honest state, in one sentence: after T153 the hoped-for reserve is known to have been a mismap — Ψ is pinned to within 1.204–1.300 by the first parity sine and closed as a term — the archimedean part is positive on the bulk block after an m-free peeling of at most 8 modes, and the two remaining terms are one and the same missing object, a Green estimate for the bottom of the section. T154 (`green_align_probe.py`) is running at exactly that estimate.

10.44 The ceiling closes at fixed size, and the obstruction becomes geometry (T154)

T154 (`green_align_probe.py`, contract `GREEN.ALIGN`, 29/29, 5.0 s on 12 prime-power windows $h = 50 \ldots 1077$, the Kato anatomy on 10 of them, verdict ALIGN-RESISTS, the gates *evaluated* rather than chosen) attacks the Green/alignment estimate both T153 terms lose to — and the ceiling half closes, exactly.

The ceiling, closed with a direction lesson. The part opens with a correction to its own commission, stated explicitly because it is easy to get backwards: the ceiling needs *no residual argument at all.* Ritz values are *upper* bounds for the eigenvalues of the *same index* (Courant–Fischer 1920 / Cauchy 1829 interlacing), so a Ritz quotient is a ceiling by a theorem; Temple 1928 / Kato 1949 is a *floor* device and is needed only there. The certificate: $\operatorname{span}\{t_1..t_8\} + A^{-1}L_P \operatorname{span}\{t_1..t_8\}$ — sixteen columns, *fixed* size, the whole certificate 8 completed Choleskys of matrices $\le 8 \times 8$ — carries $K^F = 1.1019$–1.9964, flat $(x^{0.094 \pm 0.058})$, and **agrees with the inertia-certified** K_{bot} **to a relative** 5.17×10^{-7} **on every window**: the sixteen-column certificate does not merely bound the true bottom ladder, it *attains* it, and the eight $\mathrm{LDL}^{\mathsf{T}}$ counts of size m per window are retired from the ceiling step [`verification/v551_ritz_ceiling_certificate.py`]. The parity sines alone overshoot as $K^F = 11.2$–2.1×10^4 at $x^{2.47}$ (T152's $O(m^2)$ reproduced), and the geometry says why the Green step repairs them: the true bottom eight of A lie inside the twice-iterated subspace to $\le 0.33°$, the Green columns have only 1.8–4.9% of their mass outside the sine block, and their pencil Rayleigh quotients sit at 0.287–0.681, just above the certified floor $t = 0.25$ — precisely what an inverse-iteration step should do (Kaniel 1966 / Paige 1971, quoted as the explanation, never used as a bound).

The floor half, refuted at fixed size and recovered at full size. The Temple/Kato residual argument — with the separation $\beta = t\,\mu_9^P$ that the containment of the parity sines makes free, $\mu_9^P/\mu_1^P \to 81$ exact — fails by $\|R\|^2/(\beta\lambda_1) = 6.3 \times 10^3$–$5.6 \times 10^9$, growing as $x^{4.50}$: any sine-containing subspace has residual $O(\|A\|) = O(1)$ against a target $\lambda_1 = O(\mu_1^P) = O(m^{-2})$, for *every* fixed containment depth (the preregistered scan over $k_c = 8 \ldots 48$ closes 1 of 12 windows, and that one needs $k_c/m = 0.64$ — not fixed size in effect; the scalar Temple form is vacuous on 12/12). **The obstruction is named**: seven of the eight bottom directions of A agree with the bottom of L_P to 0.15–1.35° (median), *one* sits at 82.93–89.79° — and that single misalignment *is* the collapse price. One completed Cholesky of $A - \gamma I$ per window recovers it *in full*, 4.408–7.985 — certified per window, size m, not m-free, labelled as such. The missing fixed-size ingredient reduces to *one arithmetic-free number*: the L_P floor on the complement of the eight bottom Ritz directions — a

question about the tridiagonal L_P and one eight-dimensional subspace, with no prime-power input anywhere — measured flat at 5.93–$8.25\,\mu_1^P$, and feeding the measured value into the fixed-size Temple step returns 4.408–7.617 of the price 4.408–7.986: worth 91–100%, conditionally.

The Kato anatomy, and the double refutation. The loss of the live R1 route is *not* the floor: $\lambda_{\min}(B_{HH}) = 0.266$–$0.415 > t = 0.25$, yet the Kato product delivers only a factor 1.97–121 less ($x^{1.635}$), because the two minima are attained in different directions, 24.9–89.8° apart — the minimiser that ruins the relative bound is a perfectly harmless direction for the block inequality itself. Its mass sits 96.5–100% on the eight modes *directly above* the peel cut (only 0.9–11.9% at the boundary), so T153's Hankel-reflection culprit is refuted *twice*: at the minimiser both halves hurt (Toeplitz $-100.3\ldots-0.248$, Hankel $-72.69\ldots-0.90$), and removing the reflection destroys positive definiteness altogether ($\lambda_1 = -7.49\ldots-2.64$) — the reflection half is load-bearing, not a nuisance term. Boundary peeling buys 1.08–1.38×, the norm route is dead by two to three orders, and the only repair that moves the number needs a peel of 0.40–0.69 of the window — m-proportional, for a scale-free reason: the minimiser always sits on the lowest eight modes of whatever block is left.

Two end-to-end numbers, never conflated. m-free in shape: *unchanged* at 1.01×10^{-2}–3.92×10^{-2} — the ceiling closure buys uniformity and cost (eight size-m factorisations retired), not size, because T153 already used the certified K_{bot} and the certificate attains exactly that number. Per window certified, with the collapse price recovered by one Cholesky: $\mathbf{4.45\times10^{-2}}$–$\mathbf{3.13\times10^{-1}}$ — inside the target band 3×10^{-2}–3×10^{-1}. The honest reading: the *size* target is met and the *uniformity* target is not, and those are different targets. The obligatory stress separates cleanly: on the T145 no-go reconstruction the floor survives ($t = 0.05$) while K_{bot} explodes at $x^{1.99}$ and the certificate explodes *with* it ($x^{2.29}$, overshooting the true constant by 1.12–2.19) — so the exact attainment is a property of the *real* section, not of the instrument — and the collapse price separates the families by orders of magnitude (1.8×10^3–2.1×10^5 against 4.4–8.0); the parity control is exact to 3.5×10^{-6} and the Courant–Fischer direction is violated on 0/26 positive-definite cases.

After T154: the ceiling is closed — two uniformity floors remain

(O1) **R1′, the m-free block floor**: an m-free lower bound for $\lambda_{\min}(B_{HH})$ on the bulk parity block, now *direction-aware* — every relative route is dead with a number (the Kato number $1+\lambda_{\min}(C) = 3.3\times10^{-3}$–$2.3\times10^{-1}$ shrinks as $x^{-1.69}$ against a requirement 0.19–0.30, and the loss is the misalignment, not the floor), and the anatomy says where to look: 96.5–100% of the offending direction sits on the eight modes immediately above the cut, wherever the cut is put. *(O2)* **R2′, the m-free bottom-mode floor**: an m-free lower bound for $\min_{v\perp\text{bottom}} v^{\mathsf T}L_P v/v^{\mathsf T}v$ — arithmetic-free, only the tridiagonal L_P and one eight-dimensional subspace, measured flat at 5.93–$8.25\,\mu_1^P$ and worth 91–100% of the collapse price at fixed size. Both are now *uniformity* terms: the numbers are certified per window, and what is missing is the m-free argument. The verdict rule was applied honestly: ONE-TERM-MISSING was deliberately *not* taken — the two terms are of the same type but not the same statement (the pencil minimum demonstrably does not sit at the bottom mode, which is exactly what the collapse price measures), and merging them would be a silent upgrade. The honest state, in one sentence: after T154 the ceiling of the bottom ladder is closed at fixed size and closed exactly, the collapse price is a named geometric object recovered per window by one Cholesky, and what stands between the per-window band 4.45×10^{-2}–3.13×10^{-1} and an m-free statement is the uniformity of two floors. T155 (`bottom_floor_probe.py`) is running at exactly those two floors.

10.45 The complement floor becomes exact at fixed size, and the block floor excludes every symbol argument (T155)

T155 (`bottom_floor_probe.py`, contract `BOTTOM.FLOOR`, 27/27, 9.0 s on 16 prime-power windows $h = 142\ldots1293$, the block surface on 14 of them, verdict FLOORS-RESIST, the gates evaluated from the numbers) attacks the two uniformity floors T154 left — and reduces the first to its smallest form, exactly.

The exact 12×12 reduction. The single m-sized object left in the bottom-mode chain was the complement floor $L_c = \min_{v\perp Q_8} v^{\mathsf T}L_P v/v^{\mathsf T}v$. The new certificate is one identity plus one Rayleigh

bound, a theorem for every m and every subspace W with orthonormal basis Q_W:

$$\min_{v\perp W,\ \|v\|=1} v^{\mathsf{T}} L_P\, v \;\geq\; \mu_{K+1}^{P} - \lambda_{\max}(M^{1/2}\,(I - GG^{\mathsf{T}})\,M^{1/2}), \qquad M = \operatorname{diag}(\mu_{K+1}^{P} - \mu_k^{P})_{k\leq K},$$

with $G = T_K\, Q_W$ and T_K the first K exact parity sines — the direction is checked at every window (the Cholesky-certified $\lambda_{\max}$ is *subtracted* from a lower bound, and the certificate never exceeds the exact value). It loses nothing: the ratio certificate/exact is **0.999999783–0.999999958** on 16 of 16 windows, and $K = 12$ suffices everywhere (preregistered ladder $8 \ldots 64$) — a 12×12 eigenvalue problem in the exact Kac–Murdock–Szegő numbers plus one 12×8 overlap matrix replaces the m-sized eigenproblem T154 had to solve. Both closed-form controls are hit: $W = \operatorname{span}\{t_1..t_8\}$ returns μ_9^P to 1.05×10^{-12}, and $W = \operatorname{span}\{t_2..t_9\}$ — the configuration in which the certificate is *worthless* — returns μ_1^P to 8.89×10^{-11}: the instrument reports its own worthlessness rather than something flattering. (This certificate is now load-bearing: it is promoted, together with T157's three instruments, as v552, Section 4.18 — [verification/v552_angle_instruments.py].)

The defect is localised, and the 83–90° *direction is demystified.* If Q_8 covered the eight lowest L_P modes, the floor would be $\mu_9^P = 80.74\text{–}81.00\,\mu_1^P$ (m-explicit, KMS); measured it is 5.82–7.86, a factor 10.31–13.92 lost — and the loss is *not* spread over the eight modes: the largest uncovered fraction $1 - \|\mathrm{row}_k(G)\|^2$ sits at $k = 1$, the bottom mode itself, at **0.539–0.611** (flat, $x^{-0.063\pm0.016}$). The reason is visible in the same object: T154's misaligned direction (82.08–89.70° on this surface) carries only 2.8×10^{-5}–1.9×10^{-2} of its mass on modes 1–8, has L_P Rayleigh quotient $81.2\text{–}103.4\,\mu_1^P$ and peaks at mode 9–10 — the certificate spends one of its eight directions on modes 9–12 and pays for it at mode 1, which is exactly why it needs $K = 12$ and not $K = 8$. The payoff: feeding the fixed-size floor into the unchanged Temple step recovers 3.250–7.226 of the collapse price 3.250–8.471 — **78.8–100.0**% *at fixed size* (in full on 12/16 windows, at least ninety percent on 14/16; the two remaining windows are residual-limited, not floor-limited), with 32 Green columns, an 8×8 Ritz block, one 12×12 certificate and a Temple bisection — nothing in the chain has a size depending on m, where T154 needed one Cholesky of size m. Two repairs are refuted with their mechanism: the pencil-ceiling chain — a theorem in every step (projector-split identity, Davis–Kahan 1970 transfer $s = 5.2 \times 10^{-4}$–1.0×10^{-2}) — dies on its one new input $\kappa = \lambda_{\max}(B) = 3.1 \times 10^3$–$2.7 \times 10^6$ growing as $x^{2.82}$ (the two-sided pencil is maximally ill-conditioned exactly at the bottom the floor is about; peeling does not cure it), and widening the floor subspace to $r = 12/16/24$ raises the certified floor $\times 1.19$–3.71 while the residual rises $\times 12.4$–8514 — the Temple step returns a floor on 0 of 32 widened configurations: floor and residual are bought from the same account, and at $r = 8$ the account is balanced.

The block floor, and the impossibility of a symbol argument. $\lambda_{\min}(B_{HH}) = 0.2430\text{–}0.4249$ is certified positive on every window (the inequality holds against $t = 0.25$ on 13/14; on the rest the Schur ladder backs off to a smaller certified t — reported, not smoothed). First, a correction *of target*: T154's anatomy (the minimiser sits 95.3–99.6% on the eight modes above the cut, confirmed here) belongs to the minimiser of the arch-whitened operator C — the minimiser of B_{HH} *itself* puts only 1.5×10^{-5}–0.69 of its mass there: a decomposition aimed at the C-minimiser is aimed away from $\lambda_{\min}(B_{HH})$. The direction-aware 2×2 decomposition dies on the coupling (both diagonal blocks healthy at 0.3518–0.5245 and 0.2431–0.4273, but $\|B_{wd}\| = 5.81\text{–}389.41$ against the allowed 0.292–0.456 — too large by 16.8–908, growing $x^{1.94}$: the atoms couple the mode index long-range); the local atom norm on the eight-mode window exceeds the local arch reserve by 1.50–5.12 on every window; and the deeper cut does not terminate (the deep block's own floor is 2.1×10^{-3}–$6.0 \times 10^{-2} < t$, with a Kato factor of the same order as at the original cut — moving the cut up by 8 modes moves the problem up by 8 modes). The one positive result is a mechanism: **the arch reserve *is* the symbol infimum** — $\min_{\theta\geq\theta_{k_b+1}} f^{\mathrm{arch}}(\theta)/(4\sin^2(\theta/2))$ equals $\lambda_{\min}(B_{HH}^{\mathrm{arch}})$ to 1.000022–1.000521 on 14/14 windows (band 1.0004–1.4049, a factor 4.0–5.6 above the target) — a theorem *candidate* (Szegő 1915 / Widom 1958 / KMS 1953): proving it means proving an inequality about one function of one variable. And the same instrument delivers the strongest negative statement of the series: **the full symbol infimum over the same bulk range is** $-714.2 \ldots -7.6$**, negative on every window** (the atom symbol alone reaches -1148.5), while the section floor is

positive — any Szegő/Widom-type argument returns the symbol infimum and therefore a negative number here, so *no symbol argument can ever produce R1′*: whatever carries it must be a genuinely finite-section mechanism (the Fejér averaging of the atom spikes against the section), and the diagonal already shows the section is doing the work ($\min_k (B_{HH})_{kk} = 0.3840$–$0.7444$, a factor 1.33–1.89 above the true minimum).

The honest miss, and the stress. The end-to-end fraction reaches 3.28×10^{-2}–2.83×10^{-1} with the label *certified at fixed size* (T154 left 1.01×10^{-2}–3.92×10^{-2} m-free in shape / 4.45×10^{-2}–3.13×10^{-1} certified per window with a size-m Cholesky). The declared bar was 4×10^{-2} and the bottom of the band is 3.28×10^{-2} — **missed by a factor** 1.22, reported rather than repaired: the pairing of two band minima from different and wider surfaces is deliberately pessimistic, and the like-for-like number — fixed size against size m, same window, same Temple step — is the share 78.8–100%, which is what this probe moved. Fixed size is *not* m-free: the certificate does not grow, but its value is measured on 16 windows, not bounded by an argument. Balance: 5 theorem, 3 certified-fixed-size, 2 certified-per-window at size m, 1 measured, 1 theorem candidate — T154's balance had zero fixed-size entries below the ceiling row. The obligatory stress breaks on two axes: on the T145 no-go family the ceiling explodes at $x^{2.010 \pm 0.003}$ (flat on the real family), and the new instrument separates too — the mode-1 uncovered fraction reads 0.999–1.000 (real: 0.539–0.611), the misalignment is exactly 90.00°, and the Temple recovery is 0/8 against 16/16 real. The parity sines are exact L_P eigenvectors to 4.85×10^{-14}, and the Dirichlet negative control hits its own rank-one prediction $2/\sqrt{N}$ to 4.1×10^{-7}.

After T155: both floors resist — two smaller open terms remain

(O1) **R2″, the m-free overlap defect**: an m-free *upper* bound for $\lambda_{\max}(M^{1/2}(I - GG^{\mathsf{T}})M^{1/2})$ at $K = 12$ — equivalently, a bound on how much of t_1 the eight bottom Ritz directions of A may miss (measured 0.539–0.611, flat $x^{-0.063 \pm 0.016}$). The term has shrunk to *one* 12×8 matrix: everything else in the bottom-mode chain is a theorem or a fixed-size certificate, and the one m-free-shaped route this probe could construct — the two-sided pencil — is refuted by a factor 10^{-3} for a structural reason. *(O2)* **R1″, the m-free atom part on the bulk**: excluded are relative routes (T154), direction-aware two-block splits (coupling too large by 16.8–908), local atom norms (short by 1.50–5.12), deeper cuts (scale-free), and *every symbol argument* (the full symbol infimum is negative while the section floor is positive); the one surviving shape is a finite-section Fejér cancellation, and the arch half is a theorem candidate (it *is* its symbol infimum to 5×10^{-4}). The verdict rule was evaluated from the numbers: the R2′ gate reads 10^{-3} against a required 0.5 and the R1′ gate reads -5.5 against $t = 0.25$, hence FLOORS-RESIST; merging the terms would be false — R2′ is a question about L_P and a subspace with the arithmetic compressed into one 12×8 matrix, R1′ a question about prime-power sums against a finite section, now provably outside the reach of the method that settles the first. The honest state, in one sentence: after T155 the complement floor — the one m-sized object of the bottom-mode chain — is reproduced exactly by a 12×12 certificate and the collapse price is recovered 78.8–100% at fixed size, while the block floor has acquired an obstruction that closes off a whole family of methods; this probe bought size and structure, not uniformity. T156 (`twelve_by_eight_probe.py`) is running at exactly those two objects.

10.46 Both terms become single scalars: the t_1 loss is one angle, the block positivity is alignment (T156)

T156 (`twelve_by_eight_probe.py`, contract `TWELVE.EIGHT`, 37/37, 6.1 s on 16 prime-power windows $h = 142 \ldots 1293$, the block surface on 12 windows $h = 50 \ldots 1077$, verdict TERMS-RESIST, the gates evaluated from the numbers: TERMS-CLOSE needs zero measured rungs, ONE-TERM-MISSING exactly one — there are three) attacks the two rest objects T155 left, and reduces both to single scalars.

The exact $F(P, r)$ reduction. The subspace S_2 contains t_1 *and* $y_1 := A^{-1} L_P\, t_1$, and the bottom Ritz direction of A mixes them — that mixing is where t_1 is lost. On $\mathrm{span}\{t_1, y_1\}$ the mixing closes *exactly*, because $L_P t_1 = \mu_1^P t_1$ turns the coupling into an identity with no A in it:

$$t_1^{\mathsf{T}} A\, y_1 \;=\; t_1^{\mathsf{T}} L_P\, t_1 \;=\; \mu_1^P,$$

so the t_1-loss of the two-dimensional Ritz pair is a *closed* function $F(P, r)$ of exactly two dimensionless scalars, both Rayleigh quantities of A at the single vector t_1: the Kantorovich product $P = (t_1^\mathsf{T} A t_1)(t_1^\mathsf{T} A^{-1} t_1) \geq 1$ and the inverse moment ratio $r = \|A^{-1} t_1\|^2 / (t_1^\mathsf{T} A^{-1} t_1)^2 \geq 1$ (the reduction agrees with the four-scalar form to 2.2×10^{-16} and with the true 2×2 Ritz pair to 4.4×10^{-16} — an identity, not a fit). Measured: $P = 5.6 \times 10^2$–1.0×10^6 (enormous — A and L_P differ by orders of magnitude at t_1, so F is essentially $F(\infty, r)$) and $r = 2.7158$–3.4089, flat ($x^{-0.047 \pm 0.020}$). The box route over the admissible (P, r) region is *vacuous*, despite a new m-free bound: the exact spectral-defect identity yields $P - 1 \geq L^2 s^2 (r - 1)$ (with $L = \lambda_1(A)/\mu_1^P$ and $s = \mu_1^P \, t_1^\mathsf{T} A^{-1} t_1$; it holds on 16/16 windows with slack 2917–4.87×10^6) and cuts the region — but $\sup F$ over what remains is 1.0000 where the chain needs at most 0.632, attained at $P \to 1$ with $r - 1$ of the same order: a genuine configuration the pencil does not exclude. The best route is a two-line theorem,

$$r \;\leq\; \frac{1}{Ls} \;\leq\; \frac{1}{p_1}, \qquad Ls = \sum_j p_j \, \frac{\lambda_1}{\lambda_j} \in [p_1, 1],$$

and its ceiling $1/(Ls) = 2.930$–3.967 is *tight* to a factor 1.03–1.16: **R2″ is one m-free lower bound on the overlap** $p_1 = \cos^2 \angle(t_1, e_1(A)) =$ **0.2010–0.3282** (55.05–63.36°, flat, $x^{0.054 \pm 0.029}$). The loss at the cruder $1/p_1$ ceiling is 0.6718–0.7990 against the model's 0.6318–0.7067 against the truth d_1(8 Ritz directions) = 0.5392–0.6316 — the chain survives the cruder of the two ceilings. The model *dominates* the truth on 16/16 windows (ratio 0.833–0.917) — measured, no monotonicity theorem, and the stress below is exactly about that. The interlacing route is refuted in both of its forms: at $K = 12 > \dim W = 8$ the matrix $I - GG^\mathsf{T}$ carries the eigenvalue 1 *exactly* (a rank hole — the $\sigma_{\min}$ form returns μ_1^P and nothing more, identically, for every m), and at $K = 8$ it is not vacuous but empty: $\sigma_{\min}(T_8 Q_W) = 1.7 \times 10^{-2}$–$1.4 \times 10^{-1}$ yields a floor of 1.023–2.579 μ_1^P against the 5.78–7.83 the *same* overlap matrix gives through the T155 certificate. T155's anatomy is reproduced (object ratio 0.9533–0.9654, $d_1 = 0.5392$–0.6316, the defect direction peaking on modes 9–10).

The J2a correction — an honest negative. The one-variable statement T155 expected is *false*: the infimum of $f^{\mathrm{arch}}(\theta)/(4 \sin^2(\theta/2))$ over the bulk range is 0.8226–1.3973 ($x^{0.171 \pm 0.009}$), **below** 1 **on** 3 **of** 12 **windows** — T155's quoted 1.000022–1.000521 was the upper surface of the arch-reserve-equals-symbol-infimum identity (reproduced here to 3.6×10^{-5}–1.8×10^{-3}), not the floor of a ≥ 1 statement. What survives, and what the chain actually needs, is the *weaker* one-variable statement $\inf \geq t = 0.25$: a factor 3.29–5.59 above target, non-decreasing in h, grid-stable to 1.4×10^{-5} under fourfold refinement, with the minimum at $\theta/\pi = 0.990$–1.000 — the alternating lag sum, which is where a proof would have to work. Status: a certified-per-window grid fact about a function of one variable, not a theorem.

The mechanism of the block floor: alignment, not mass. The lag decomposition $v^\mathsf{T} A v = \sum_l c_l \, a_l(v) - \sum_k c_{M-1-k} \, b_k(v)$ is exact to 6.9×10^{-15} on every window (Fejér 1915 — the weights below are the section's true weights, not a model). The Fejér damping is real but worth *one order of magnitude*: on the minimiser the realised atom form is 3.2×10^{-2}–1.9×10^{-1} of the naive ℓ^1 budget ($x^{-0.501 \pm 0.024}$), a factor 5–31, where the split needs 3–1981 *and growing*; and the additive split is *divergent*: $\|B_{HH}^{\mathrm{atom}}\| = 2.2$–$2673.4$ grows like $h^{2.31}$ ($x^{2.314 \pm 0.082}$) and exceeds the arch reserve by 2.7–1.98×10^3, closing 0 of 12 — no bound of the shape $\lambda_{\min}(B_{HH}^{\mathrm{arch}}) - \|B_{HH}^{\mathrm{atom}}\|$ can close R1″ for *any* damping of the atom mass, because what is being subtracted is not bounded at all. The mechanism, identified: **on the block minimiser an arch part of** 1.4685–91.7130 **and an atom part of** $-91.2694 \ldots - 1.1151$ **cancel to** $\lambda_{\min}(B_{HH}) = 0.2661$–$0.4436$**; the minimiser sees only** 6.16×10^{-3}–5.03×10^{-1} **of the atom operator (**$x^{-1.044 \pm 0.109}$**), at** 52.7–90.0° **from the atom-extremal vector — the positivity is an *alignment* fact, not a size fact**, and diagonal dominance fails as well ($\min_k (B_{HH})_{kk} = 0.3997$–$0.8186$; the off-diagonal atom coupling eats 38–63% of the diagonal).

The balance, the stress, and the unchanged end to end. The end-to-end fraction is 3.28×10^{-2}–2.83×10^{-1}, *unchanged*: J1 and J2 are statements about what is missing, and neither adds a certified inequality to the chain. The recovery, paired like for like ($K = 12$ fixed by contract, a

wider surface, the pairing declared before the number): 49.5–100.0%, median 99.8%, at or above 75% on 12/16 windows. The balance: 9 theorem rungs (3 new: the $F(P, r)$ reduction, the chain $r \le 1/(Ls) \le 1/p_1$, the lag decomposition), 6 certified, **3 measured** — the zero-measured target is *not* met, two of the three are the open terms themselves, and the third (the model domination) is a new debt created by the reduction, counted rather than hidden. The obligatory stress breaks on *three* axes, two of them new and both J1's own: the ceiling explodes at $x^{2.047\pm0.009}$ (T154 confirmed); p_1 — the one scalar J1 reduces to — *collapses* to 7.4×10^{-15}–6.6×10^{-10} on the no-go family ($x^{-4.818\pm0.080}$) against 0.2010–0.3282 real, so the one number J1 asks for is exactly the number that separates the families; and the r-ceiling tightness separates too (12.5–24.6 against 1.03–1.16). The floor survives ($t = 0.05$), so a floors-only probe would have seen nothing. **And the stress kills J1's one measured step**: the domination of the eight-dimensional defect by the 2×2 model holds on 16/16 real windows and *fails* on 8/8 no-go sizes (model 0.977–0.982 against truth 0.9990–0.9996, a narrow undershoot of 1.7×10^{-2}–2.2×10^{-2} — which is the point): it is not a theorem and cannot become one by more measuring on the real family. Controls exact: parity 4.8×10^{-14}; μ_9^P to 2.3×10^{-13}; μ_1^P to 2.0×10^{-11} in the *worthless* configuration $W = \mathrm{span}\{t_2..t_{13}\}$ (the certificate reports its own worthlessness); the Dirichlet negative control hits its own $2/\sqrt{N}$ prediction to 7.4×10^{-5}.

After T156: both terms resist — two angles remain

(O1) **R2″, one m-free lower bound on p_1**: the squared overlap of the bottom parity sine with the bottom eigenvector of the section, measured 0.2010–0.3282 (55.1–63.4°), flat. Everything above it is now a theorem: the 2×2 reduction to $F(P, r)$ is an identity, $r \le 1/(Ls) \le 1/p_1$ is two lines, and F is explicit — a bound $p_1 \ge p^*$ would give the m-free loss $F(P, 1/p^*)$ directly (0.672–0.799 at the measured p_1). Separate second debt: the step from the two-dimensional model to the eight Ritz directions, which holds on 16/16 real windows and fails on 8/8 no-go sizes — a good separation instrument, not a theorem. *(O2)* **R1″, one m-free alignment statement**: a lower bound for the arch Rayleigh quotient *on the subspace where the atom form is large*, equivalently an m-free angle between the two extremal vectors — *not* a mass bound of any kind (refuted with numbers: symbol arguments, additive splits, ℓ^1 budgets, diagonal dominance); the arch half is ready for it (a one-variable grid infimum 0.8226–1.3973, non-decreasing, factor 3.29–5.59 over t). The verdict was evaluated from the numbers: 2 open terms and 3 measured rungs, hence TERMS-RESIST. T155's 12×12 certificate is strengthened as an *instrument* (exactness controls to 2.0×10^{-11}, self-reported worthlessness, the no-go breaking on three axes) and weakened as a *uniformity claim* (its value is governed by the single scalar p_1, and the load-bearing inequality of the reduction step fails on the no-go family) — the honest recommendation: promote it as a fixed-size certified instrument with its per-window numbers, and promote nothing about uniformity. The honest state, in one sentence: after T156 both remaining terms are single scalars — one angle each — whose m-freeness is not a matter of measuring more windows but of proving one inequality each. T157 (`angle_floor_probe.py`) is running at exactly those two angles.

10.47 Neither angle falls, both change shape: the p_1 floor becomes a theorem times one number, the arch half is certified (T157)

T157 (`angle_floor_probe.py`, contract `ANGLE.FLOOR`, 32/32, 3.9 s on 16 prime-power windows $h = 142 \ldots 1293$, the alignment surface on 12 windows $h = 50 \ldots 1077$, verdict ANGLES-RESIST, the gates evaluated from the numbers: p_1 m-free FALSE, p_1 non-trivial TRUE, arch half certified TRUE, alignment m-free FALSE — terms = 2) attacks the two angles T156 left, and neither falls — but both change shape. The instrument cores below are promoted as [`verification/v552_angle_instruments.py`].

The best p_1 route is the resolvent, and it splits into a theorem times one number. Write $\gamma = Te_1(A)$ for the parity coordinates of the bottom eigenvector and split the modes at $k_b = 16$. The tail is a *theorem*, one line from the certified pencil floor $A \succeq tL_P$ and the certified ladder ceiling $\lambda_1 \le S\mu_1^P$ alone:

$$\|\gamma_H\|^2 \;\le\; \frac{\lambda_1}{t\,\mu_{17}^P} \;\le\; \frac{S/t}{\rho_{17}} \;=\; 0.0165\text{–}0.0293, \qquad \rho_{17} = \frac{\mu_{17}^P}{\mu_1^P},$$

so $e_1(A)$ lives 97.1–98.4% inside the first sixteen parity sines, *m-freely in shape* — T146's measured "98 per cent in the sine block" now has a one-line proof, and the lesson is the ceiling: substituting the pencil ceiling κ (3.1×10^3–2.7×10^6, growing $x^{2.821\pm0.038}$) for the flat ladder $S = 1.1676$–2.1178 ($x^{0.010\pm0.092}$) would make the bound vacuous (43.5–38632). What remains measured is *one* number: $\hat{g}_1^2 = 0.2010$–0.3282 (flat, $x^{0.054\pm0.029}$), the first coordinate of the bottom vector of a 16×16 Schur null problem (solved to a relative residual 8.4×10^{-15}), and the resulting floor $p_1 \geq \hat{g}_1^2(1 - \text{tail}) = 0.1968$–$0.3228$ is 97.9% of the measured p_1. Three other routes are refuted with numbers: the classical Rayleigh angle bound $p_1 \geq (\lambda_2 - \text{RQ})/(\lambda_2 - \lambda_1)$ is *empty already in its measured form* (16/16 negative — $\text{RQ}(t_1)$ exceeds λ_2 by 818–519458, and the bottom is nearly degenerate, $\lambda_2/\lambda_1 = 1.73$–$5.87$: the shape fails, no certification can rescue it); its block version answers the T156 question — the chain *does* admit the block, and the block mass is spectacular, $p_{\text{blk}}(2) = 0.9369$–$0.9993$ (t_1 lives almost entirely in the bottom *pair*) — but the block Rayleigh bound stays empty at every J of the ladder; and the angle-free Cauchy–Schwarz route $Ls \geq L/\hat{a}$ is a theorem that loses exactly the Kantorovich product $P = 5.6 \times 10^2$–1.0×10^6, beating the p_1 floor on 0 of 16 windows.

The structural gem: the whole first term is one diagonal entry. There is an identity nobody had used: $s = \mu_1^P \, t_1^{\mathsf{T}} A^{-1} t_1 = (B^{-1})_{11}$, hence $1/s$ is carried by the $(1,1)$ entry of the *same* fixed-size 16×16 Schur complement $S_L = B_{LL} - B_{LH} B_{HH}^{-1} B_{HL}$ that the T152..T157 chain already forms for its floor certificate — and $1/s = 2.3359$–6.2049 is *flat* ($x^{-0.088\pm0.008}$), an $O(1)$ number, against $\hat{a} = 2.8 \times 10^3$–$2.4 \times 10^6$ for the same entry *without* the Schur subtraction. R2″ is thereby an m-free *upper* bound on one diagonal entry of a fixed-size matrix instead of a lower bound on an angle between two m-sized vectors. The obvious certificate fails for a named reason: the inversion-free Cauchy–Schwarz ceiling on $b^{\mathsf{T}} B_{HH}^{-1} b$ is off by 5.4×10^2–5.2×10^5 — the cancellation in that one bilinear form is nearly complete, and Cauchy–Schwarz cannot see it.

The arch half is now certified, and the mechanism is localised. The adaptive Lipschitz ceiling is *executed*, not asserted: interval bisection with the local Lipschitz constant (the global θ_{lo} constant would need up to 7.8×10^6 uniform grid points, $x^{3.365\pm0.147}$) certifies $\inf f^{\text{arch}}/(4\sin^2(\theta/2)) \geq t$ on 12/12 windows — certified floor 0.2500–0.5353 in 255–3139 symbol evaluations, cost growing only like $h^{0.850\pm0.097}$: the arch half carries a certificate for the first time (T156's 1.4×10^{-5} grid stability was evidence; this is a certificate). And the extremals are located exactly: the *atom* extremal sits at $\theta/\pi = 0.0158$–0.3762, the *arch* extremal at 0.9901–0.9995 — opposite ends of the band (separation 0.61–0.98), with the bulk minimiser going with the atom. The strategy turn this forces: the arch Rayleigh quotient is *smallest* at $\theta = \pi$ and *grows* as θ falls, so a proof of R1″ must use the *θ-growth* of $f^{\text{arch}}/(4\sin^2(\theta/2))$, not its infimum at π — the one-variable infimum is the wrong number to feed it. The shape $\lambda_{\min}(B_{HH}) \geq \inf_{\text{arch}} \cos^2\varphi - \|\text{atom}\|$ is refuted as a shape (on the minimiser the arch form is 1.47–91.71, nowhere near its infimum); what replaces it is the domination form $B_{HH}^{\text{arch}} - tI \succeq (-B_{HH}^{\text{atom}})_+$, which holds per window with quotient 1.0003–1.0907 — but the margin at the largest window is 7.3×10^{-4} with a shrinking trend ($x^{-0.018\pm0.015}$): a certificate, not a uniform statement.

The balance, the stress, and the price of the substitution. Substituting the resolvent bound into the first row of G costs a factor 1.00–1.62 (direction-clean and conservative: the substituted complement floor 4.37–$6.20\,\mu_1^P$ never exceeds the measured 5.84–7.88). The balance: 9 theorem rungs, 2 certified-uniform (the ladder S; the executed arch ceiling), 2 certified-per-window (the pencil floor t; the R1″ domination), **3 measured** ($\hat{g}_1^2$, the eleven other rows of G, the T156 model domination) — the target of one is missed, but all three measured steps are now *fixed-size in their statement*, which is the change T157 bought. The obligatory stress breaks on *five* axes: p_1 collapses ($x^{-4.818\pm0.080}$, T156 reproduced); the ladder S explodes ($x^{2.289\pm0.079}$) against flat on the real family; the confinement dies ($(S/t)/\rho_{17} = 7.9$–1.6×10^3, $x^{2.253\pm0.120}$ — the no-go bottom eigenvector is *not* confined, and the m-free bound correctly reports vacuum); the resolvent floor is *identically zero* there (the route reports its own vacuum); and $(S_L)_{11}$ explodes ($x^{2.268\pm0.041}$) against flat. The floor survives ($t = 0.05$), so a floors-only probe would have seen nothing. Controls: parity 4.8×10^{-14}; μ_9^P to 2.3×10^{-13} and μ_1^P to 2.0×10^{-11} in the worthless configuration; $s = (B^{-1})_{11}$ to 1.9×10^{-13}

with $s = 1$ exactly on $A = L_P$; the Dirichlet negative control hits its own $2/\sqrt{N}$ prediction to 7.4×10^{-5}.

After T157: both angles resist — three named rests remain

(O1) **an m-free upper bound on one number**: $(S_L)_{11} = 1/s = 2.3359$–$6.2049$, the $(1,1)$ entry of the 16×16 Schur complement of B — equivalently an m-free lower bound on $\hat{g}_1^2 = 0.2010$–0.3282. Cauchy–Schwarz on $b^{\mathsf{T}} B_{HH}^{-1} b$ is refuted (off by 5.4×10^2–5.2×10^5), so what is needed is a bound that *sees* the cancellation in that one bilinear form. *(O2)* **an m-free R1″ domination with a margin that does not shrink**: the certified quotient is 1.0003–1.0907 with trend $x^{-0.018 \pm 0.015}$ and a margin of 7.3×10^{-4} at $h = 970$; the pointer is the mechanism finding — use the θ-growth of $f^{\mathrm{arch}}/(4\sin^2(\theta/2))$ as θ falls, not its infimum at π. *(O3)* the T156 debt T157 did not touch: the two-dimensional model dominating the eight-dimensional defect — measured on the real family, refuted on the no-go family, and the only remaining measured step whose *statement* is not fixed size. What stands a priori on the measurement surface, precisely: the KMS spectrum and the exactness of the parity sines, the ratio floors ρ_k, the identity $F(P,r)$ with $r \le 1/(Ls)$, the identity $1/s = (S_L)_{11}$, the sine-block confinement, the classical inequalities the chain is assembled from, and the arch one-variable infimum with an *executed* Lipschitz certificate per window — and nothing else: neither p_1 nor the alignment margin is available before the window is built. The four instrument candidates of T155/T157 (the 12×12 complement certificate, the sine-block confinement, the Schur-entry identity, the adaptive Lipschitz ceiling) are promoted as v552 — per-instance instruments, no angle closed, both open terms typed open. T158 (`schur_entry_probe.py`) is running at exactly the cancellation-seeing bound.

10.48 The cancellation-seeing bound exists, and it is a theorem: the entry as a Dirichlet maximum, the growth pointer refuted (T158)

T158 (`schur_entry_probe.py`, contract `SCHUR.ENTRY`, 36/36, 8.2 s on 28 prime-power windows $h = 254 \ldots 1393$, the band surface on 21 windows $h = 254 \ldots 1031$, verdict ENTRY-RESISTS, the gates evaluated from the numbers: entry ceiling m-free FALSE — the cancellation depth grows $x^{2.135 \pm 0.021}$ where m-free needs non-growing; entry ceiling sharp-and-uniform TRUE — tightness ≤ 1.2738, flat; domination margin non-shrinking FALSE — trend $x^{-1.649 \pm 0.643}$ — terms = 2) attacks exactly the cancellation-seeing bound T157 demanded, and it exists — as a theorem.

The entry as a Dirichlet maximum, and the ladder of positive terms. The object that makes trial spaces legitimate in the right direction is the dual (Thomson) form of the entry (Maz'ya 1985; Miclo 1999 — s is a *conductance* of mode 1 into the bulk, and every admissible flow is a floor):

$$s \;=\; (B^{-1})_{11} \;=\; \max_{x} \left(2x_1 - x^{\mathsf{T}} B x\right),$$

so *every* trial vector is a lower bound on s and hence an upper bound on $1/s$ — the entry R2″ must ceiling. This is exactly the variational structure Cauchy–Schwarz lacks: CS evaluates $b^{\mathsf{T}} B_{HH}^{-1} b = \max_z (2b^{\mathsf{T}} z - z^{\mathsf{T}} B_{HH} z)$ at the *single* direction $z \parallel b$, and it missed by 3.13×10^3–5.18×10^5 (reproduced with its direction intact on 28/28: what fails is sharpness, not sign — the cancellation is spread over the low modes, and one direction cannot see it). The Cholesky ladder $g_K = \sum_{j \le K} y_j^2$ with $y = L_K^{-1} e_1$ (the nested-Schur identity) writes s as a sum of *strictly positive* terms with monotone partial sums on 28/28 windows, and $1/g_1 = \hat{a}$ is exactly T157's route: the ladder starts where T157 ended. At $K = 16$ — the same low block the T152..T157 chain already forms — the certified ceiling is $1/g_{16} = 2.9670$–7.9664 against the true $1/s = 2.3359$–6.3868, *tight to* 1.1323–1.2738, ceiling trend $x^{0.059 \pm 0.017}$ (at $K = 8$ already inside a factor 1.767). Directions checked before use: (M1) $1/s \le (S_L)_{11}$ with slack 8095–1.013×10^5 on 28/28 (Schur 1917 / Haynsworth 1968 — T157's entry is a ceiling on $1/s$, an inequality and not an identity), and no trial ever overshot s.

The entry is two-dimensional. The Green-column trial space is not a bound at all — it is the identity: $\mathrm{span}\{t_1, A^{-1} L_P t_1\}$ attains s exactly ($g/s = 1$ to 10^{-9} on 28/28), for a theorem reason — $L_P t_1 = \mu_1^P t_1$ (KMS 1953), so $A^{-1} L_P t_1$ *is* the Dirichlet maximiser. This settles that the entry is a *two-dimensional* quantity the moment one Green column is granted; that column costs an m-sized solve, and the fixed-size sine truncation pays exactly the factor 1.1323–1.2738 for not needing it.

The object the chain actually wants: $L\,g_{16} = 0.2143$–0.2724 against the true $Ls = 0.2521$–0.3413, and with the certified pencil floor t in place of L, $t\,g_{16} = 0.0314$–0.0826 — the Dirichlet ladder lands in the band of T157's *measured* $p_1 = 0.2010$–0.3282, without an angle.

The growth pointer, refuted — an honest negative. The growth T157 pointed at is real and sub-quadratic: $f^{\mathrm{arch}}/(4\sin^2(\theta/2)) \sim \theta^{-q}$ with $q = 1.259$–1.438 ($q < 2$ says f^{arch} itself decays like $\theta^{0.56\text{–}0.74}$ — a partial cancellation of numerator and denominator, not a clean pole), and the binding vector of the domination pencil sits exactly where T157 said: 0.6818–0.9988 of its mass in the *lowest* dyadic band (centroid $\theta/\pi = 0.0191$–0.0845 — the localisation half of the pointer is confirmed). What is refuted is that the growth can pay for it: the atom negative mass grows like $\theta^{-1.546\ldots-1.744}$, *faster* than the certified arch floor's $\theta^{-1.203\ldots-1.458}$ — the low-θ limit belongs to the atom, and the growth is a liability, not a resource. In the lowest band the arch floor covers only 0.4161–0.4507 of the atom mass, band-local domination *fails in every band* on 21/21 windows, and the off-band coupling (Frobenius 1.612–27.97, the conservative direction) exceeds the available margin by 660–7.71×10^5: what carries the inequality is precisely the structure a dyadic argument throws away. T157's adaptive Lipschitz ceiling, re-run band by band, certifies 7 of 116 (band, window) pairs — exactly the large-θ bands where the growth is not the resource. The margin is 3.57×10^{-5}–2.86×10^{-3} with trend $x^{-1.649\pm0.643}$, shrinking: R1″ stays a per-window certificate, and the verdict gate reads it numerically.

The debt relabelled, the balance, the stress. The containment $\mathrm{span}\{t_1, A^{-1}L_P t_1\} \subset S_{16}$ is exact (5.84×10^{-16} on every window) — nothing in the T156 debt is about the spaces — but it cannot be cashed: Ritz defects are not monotone in the trial space (smallest principal angle to the eight bottom Ritz directions 65.04–73.04°). With the Dirichlet floor in place of the measured angle, $F(P, 1/(L\,g_{16})) = 0.7276$–$0.7857$ dominates the true defect $d_1 = 0.5392$–0.6241 with margin 1.1969–1.3568 on 28/28 ($x^{0.052\pm0.007}$) — both sides now fixed-size certified numbers, so the debt moves MEASURED → CERT-UNIF (not a theorem, and the probe does not pretend otherwise). End to end, priced like for like: the substitution is direction-clean and conservative (the substituted complement floor 4.3904–$6.5357\,\mu_1^P$ never exceeds the measured 5.7763–8.6051), the Temple recovery goes from 0.4951–1.0 to 0.2872–1.0 of $\lambda_1(A)$ — a cost of 1.000–1.724, above the 4×10^{-2} bar. The balance: **6 theorem** / 3 certified-uniform / 4 certified-per-window / **1 measured** (T157 stood at 5/2/3/3): p_1 is *retired* — the ladder replaces the angle outright — the model domination is relabelled, and the one remaining measured row ($\lambda_1(A)$ itself, needed only by the sharp L variant) vanishes entirely if $t\,g_{16}$ is used: the measured count falls from three to one. The obligatory stress breaks on *five* axes: p_1 collapses ($x^{-4.818\pm0.080}$), the true entry $1/s$ explodes to 1185–2.547×10^5, the fixed-size ceiling $1/g_{16}$ explodes *with* it ($x^{2.267\pm0.042}$ — the ceiling tracks the collapse instead of hiding it; the tightness stays at 1.001, so what breaks is the size of the thing bounded, not the sharpness), the floor $L\,g_{16}$ collapses ($x^{-0.275\pm0.047}$), and the tail bound reaches 1.56×10^3 — against flat on the real family. Controls: parity 4.8×10^{-14}; μ_1^P to 2.0×10^{-11} in the worthless configuration; new — for $A = L_P$ the ladder returns $s = 1$ to 1.9×10^{-13} *and* every partial sum $g_K = 1$ to 4.0×10^{-15}, exactly constant in K; the Dirichlet negative control hits its own $2/\sqrt{N}$ prediction to 7.4×10^{-5}.

After T158: the entry ceiling is a theorem plus one Cholesky — two rests remain

(O1) **an m-free upper bound on one quadratic form**: $x^{\mathsf{T}} B_{LL} x = 2.9670$–$7.9664$ for a nearly universal fixed sixteen-vector (pairwise alignment 0.8599–0.9976 across the windows, normalised to $x_1 = 1$) — nothing else is needed for R2″. The obstruction, named: the archimedean half carries $-9.323 \times 10^5 \ldots - 2.394 \times 10^4$ and the atom half $+2.395 \times 10^4 \ldots + 9.323 \times 10^5$, both of one sign across the surface, so the bound must be relatively accurate to 3.5×10^{-6}–2.3×10^{-4} — a requirement growing as $x^{2.135\pm0.021} \approx h^2$, which is the $1/\mu_1^P = N^2/(4\pi^2)$ normalisation of the pencil block: the explicit formula's own cancellation, localised in sixteen coordinates, not an artefact of the trial vector. *(O2)* **an m-free lower bound on $\lambda_{\min}(B_{HH})$ that is not band-diagonal**: band-diagonal minorants are refuted (the band-local inequality is false in every band), and pricing the off-band coupling by its norm is refuted too (660–7.7×10^5 times the available margin); what is left is the *sign structure* of the off-band arch entries. *(O3, optional)* a certified floor on $\lambda_1(A)/\mu_1^P$ better than t (the gap between $L\,g_{16}$ and $t\,g_{16}$ is a factor 3.25–8.47 — the only measured row left). What stands a priori on the measurement surface,

precisely: for every m and every K, $1/s \le 1/g_K$ with g_K a sum of K strictly positive terms read off one Cholesky of the leading $K \times K$ block of B — the inequality, its direction, its monotonicity in K and the exactness of the two-dimensional span are a priori; the *size* of the terms is not. The five T158 candidates (the dual form plus the positive ladder; the two-dimensionality; the $1/g_{16}$ sharpness; the negative result on the growth pointer; the relabelling) were bundled with T159, and the theorem cores are now ledger-carried as v553. T159 (`exact_form_probe.py`) executed the cancellation algebraically — next subsection.

10.49 The cancellation executed algebraically: seven identities, the gauge identity, and the form is the weight (T159)

T159 (`exact_form_probe.py`, contract `EXACT.FORM`, 41/41, 2.9 s on 23 of 24 prime-power zones $h = 142 \dots 1293$ (70–24574 atoms; the sign surface on 18 windows $h = 142 \dots 963$), verdict FORM-RESISTS, the gates evaluated from the numbers: R2″ m-free FALSE — every pricing of the transformed sum grows, best rung $h^{+3.604}$, every closed vector grows, frozen $h^{+3.510}$; R1″ m-free FALSE — the sign structure sits entirely on the archimedean side — terms = 2) executes T158's rest O1 symbolically. The algebra is delivered; the bound is not: *the cancellation is localised, and it is the weight.*

The symbolic sixteen-form: seven machine-checked identities. All seven hold to 10^{-12} against the absolute scale $\sum_d |c_d\, w_d|$ — the honest bar, because relative to the cancelled total the same identities hold only to 1.0×10^{-12}–1.2×10^{-8}: the cancellation eats 4.0–8.1 decimal digits in double precision, and that measured loss *is* the quantitative reason no m-free bound can be read off numerically. (P1) $x^{\mathsf T} B_{LL} x = y^{\mathsf T} A y$ with $y = \sum_k (x_k/\sqrt{\mu_k^P})\, t_k$ — the pencil is gone (2.8×10^{-17}–1.4×10^{-15}). (P2) the *exact lag sum* $x^{\mathsf T} B_{LL} x = \sum_d c_d\, w_d$ with $w_d = T_d - H_d$ (T the autocorrelation, H the convolution of y): one explicit arithmetic sum with arithmetic (c) and geometry (w) separated. (P3) w_d in *closed form*: a 256-term Dirichlet-kernel expression (Dirichlet 1829, product-to-sum) reproduces the assembled weights to 1.42×10^{-15}–2.85×10^{-15} of $\sup|w|$ at every lag of every window — w_d is a fixed-size trigonometric function of (d, h), no m-dependence beyond $N = 2h + 1$. (P4) the *gauge identity*, the content of the module:

$$\sum_d w_d \;=\; 0 \qquad \text{exactly,}$$

because $A_{rs} = c_{|r-s|} - c_{M-1-r-s}$ annihilates constant lag vectors identically (0.0–9.4×10^{-14} of $\sup|w|$): *the form is blind to the lag mass of c and sees only its variation* — and the lag mass is exactly where the archimedean and atom halves are h^2-sized and cancel. (P5) the two closed scalars $w_0 = \sum_k x_k^2/\mu_k^P$ (the $N^2/4\pi^2$ factor itself, 2.655×10^5 at $h = 1119$, relative error $\le 4 \times 10^{-16}$) and $2w_0 - w_1 = \|x\|^2$ exactly — because L_P is the odd Toeplitz $(2, -1, 0, \dots)$ and the t_k are its exact eigenvectors (KMS 1953): the huge and the $O(1)$ scale sit in the *same two weights.* (P6) the p-fold Abel identity exact at every rung $p = 1..5$ (2.6×10^{-20}–3.3×10^{-16}). (P7) the Toeplitz-minus-Hankel signature $A_{r,s} - A_{r+1,s+1} = f(r+s)$ (1.1×10^{-16}–1.5×10^{-16}) — the premise of P4. Reproduction: $x^{\mathsf T} B_{LL} x = 1.7648$–$4.9008$ flat ($h^{-0.130}$; same object as T158's 2.967–7.966 on a different surface), halves $-9.605 \times 10^5 \dots -9.828 \times 10^3$ against $+9.831 \times 10^3 \dots +9.605 \times 10^5$, cancellation depth 1.86×10^{-6}–3.56×10^{-4} falling $h^{-2.234}$; the sixteenth Cholesky rung stays sharp, $(1/g_{16})/(1/s) = 1.0694$–$1.2509$.

m-free not reached: four routes closed with exponents. The core negative first: **the h^2 does not telescope away** — |arch| and |atom| both grow $h^{+2.104}$, i.e. like $w_0 = 1/\mu_1^P$ itself. The h^2 is the common *weight* of both halves, not an archimedean artefact algebra could clear, and it cancels only in the sum — the gauge identity says why: each half alone carries the lag mass against an h^2-sized weight. The four routes: (R1) **T158's fixed sixteen-vector, refuted** — frozen at one window the form grows $h^{+3.510}$ (3.025–6.997×10^5), with the reason closed: $\mathrm{cond}(B_{LL}) = 1.223 \times 10^5$–$1.386 \times 10^8 \sim h^{+2.887}$, so a direction error ε costs $\varepsilon^2 \lambda_{\max} \sim \varepsilon^2 h^2$; a closed direction would have to be accurate to $\kappa^{-1/2} = 8.5 \times 10^{-5}$–$2.9 \times 10^{-3} \sim h^{-3/2}$, while the true optimiser drifts by $1 - \cos = 2.52 \times 10^{-2}$–$1.47 \times 10^{-1}$ between neighbouring zones — a refutation, not a difficulty.

(R2) one sine refuted: $B_{11} = 1/g_1 = 1.9 \times 10^4$–$2.1 \times 10^7 \sim h^{+3.066}$, the price of collapsing sixteen rungs to one is 4224–1.3×10^7 — the sixteen-dimensional minimum is irreducible, the cancellation lies *across* the sixteen. (R3) the preregistered ansatz family (4 forms × 10 exponents, no fitting): the best member grows $h^{+3.001}$ — no closed power direction is flat. (R4) $\ell^1 \times$ sup pricing of the Abel sum at every rung refuted: best rung $p = 1$ with bound 1.54×10^7–$3.95 \times 10^{10} \sim h^{+3.604}$, missing the target 1.765–4.901 by 5.63×10^6–2.21×10^{10}, higher rungs worse ($p = 5$: $h^{+7.718}$) — the transformation removes the lag mass (ℓ^1 masses fall) but the surviving weight tails grow exactly as fast ($\sup|W^1| = 3.45 \times 10^5$–$2.00 \times 10^8 \sim h^{+2.906}$, and $W_1^1 = -w_0$ identically, so $\sup|W^1| \geq w_0 \sim h^2$ is unavoidable: the transformation *moves* the cancellation from the lag mass into the weight tail, it does not remove it). Two m-free-shaped new pieces survive: $\|\Delta c^{\mathrm{arch}}\|_1 = 3.6487$–$5.3788$ **flat** ($h^{+0.165}$) — the T115 kernel has uniformly bounded variation ≤ 6.00 in the zone — and the atom budget is T150-Chebyshev-correct, $\|\Delta c^{\mathrm{atom}}\|_1 = 37.0$–$191.7 \sim h^{+0.727}$ inside $\sum \mu_j \leq 4B\sqrt{X}$ at the *atom cutoff* $X = e^{2\alpha}$ (a pedantic correction: not the window size $N = 2h+1$ — a different integer), occupancy 0.8767–0.9602.

The sign law: the archimedean block is a Z-matrix, exactly. (S1) holds raw: B_{HH}^{arch} **is a symmetric Z-matrix** — 100% of the off-band archimedean entries nonpositive, without similarity, on 18/18 windows; (S2) the checkerboard sign pattern is refuted (0.4890–0.4936 mass fraction = a coin flip). Collatz–Wielandt bites on the archimedean half: the *closed sign-based floor* $\min_k(B_{HH}^{\mathrm{arch}}\mathbf{1})_k = 0.8194$–$1.1647$, flat ($h^{+0.176}$), comfortably above the target $t = 0.25$ — row sums of a Z-matrix, an m-free-shaped object (Gantmacher–Krein 1950; the Perron root of the off-band part sits a factor 2.2740–2.7262 below its row-sum norm: the signs pay). And it is not enough: the atom block obeys *no* sign law (0.3067–0.4891 nonpositive = noise), its off-band mass exceeds the archimedean floor by 32.44–4555 growing $h^{+2.330}$, and on the full block the M-matrix criterion is vacuous: best power test vector $\min_k(B_{HH}v)_k/v_k = -9.324\ldots - 0.6086$ against the exact $\lambda_{\min}(B_{HH}) = 0.0742$–$0.1974$. The verdict without hedging: the archimedean sign structure is real, exact and usable, and it does not suffice, because the object it would have to dominate has no sign structure.

Assembly, stress, and the numerical horizon. End to end the chain still certifies $\lambda_1(A) \geq t\,\mu_1^P$ per window with $t = 0.0705$–0.1583, recovery 0.0686–0.2164 of the exact $\lambda_1(A)$ (a deeper zone surface than T158's 0.287–1.0 — a surface difference, not a regression; and the decisive reading: M1/M2 did *not* move it — the new theorems are structural, not quantitative). The T145 no-go breaks 5/5 (no Z-law, signature defect 7.79×10^{-2}, M-matrix criterion vacuous despite positive definiteness, the localisation referee m/H_m diverges $h^{+0.821}$, diagonal constancy violated); three directions re-verified before use (Thomson maximum never overshot, Schur complement $\geq 1/s$, Ritz ceiling); Dirichlet and parity controls at 8×10^{-15} with the $2/\sqrt{N}$ bound *saturated* to 0.999708–0.999995. The balance moves 6/3/4/1 → **13 theorem** / 6 certified-uniform / 4 certified-per-window / **1 measured**: seven new theorems P1–P7 and three new certified-uniform rows (U1 the Z-law, U2 the archimedean floor ≥ 0.81, U3 the bounded variation ≤ 6), zero new per-window rows, zero new measured rows. And a **numerical horizon, declared as its own measurement**: one window dropped because $\mathrm{cond}(B_{LL}) > 10^{12}$, the first at $h = 1292$ — beyond it the sixteen-block is singular in double precision and the per-window optimiser does not numerically exist. That is the same obstruction the cancellation depth reports, seen from the other side. The theorem cores of T158 + T159 are ledger-carried as `v553` (`[verification/v553_exact_form_identities.py]`): the dual form with the positive ladder, the two-dimensional span, the gauge identity with the signature, the closed scalars with the lag sum, and the Z-law with the Collatz–Wielandt floor — per instance, with the pairing bound and m-freeness explicitly typed open.

After T159: one fully explicit pairing inequality remains

(O1) R2″ is now **one fully explicit inequality**: the form equals $\sum_{d\geq 1}(\Delta c)_d\, W_d^1$ with $\|\Delta c^{\mathrm{arch}}\|_1 \leq 6$ (U3, m-free-shaped), $\|\Delta c^{\mathrm{atom}}\|_1 = O(h^{0.73})$ inside the Chebyshev budget, and $\sup|W^1| = 53.88$–$504.72 \times w_0$. What is missing is a bound on the *one pairing* that uses the *correlation* between Δc and W^1 instead of their sizes: $\ell^1 \times$ sup is off by $h^{3.7}$, and the cancellation to be recovered is exactly h^2. *(O2)* R1″ needs a

third device for the atom off-band block that is neither a norm (refuted in T158 by 660–7.7×10^5) nor a sign (refuted here, purity 0.31–0.49). *(O3)* two cheap proofs would upgrade rows: U1 from the Hankel half of the closed T115 kernel, U3 as a total-variation bound on the same integral. Permanently off the list: the fixed sixteen-vector, the single sine, power ansätze, Abel-ℓ^1 at every rung, the checkerboard, the full-block M-matrix. T160 (`pairing_probe.py`, contract `PAIRING`) executed exactly that attack — next subsection.

10.50 The pairing attacked through correlation: the sampling identity — the last term is a prime exponential sum (T160)

T160 (`pairing_probe.py`, contract `PAIRING`, 46/46, 2.8 s on 19 of 20 prime-power zones $h = 199 \dots 1256$ (187–19593 atoms per window; the R1″ sub-surface on 8 windows $h = 199 \dots 534$; one window dropped at the declared T159 horizon $\mathrm{cond}(B_{LL}) > 10^{12}$, first at $h = 1292$ — not fitted through), verdict PAIRING-RESISTS, the gates evaluated from the numbers: arch half inside the bar TRUE (slack 1.95×10^{-8} of the $O(1)$ total); arch half non-growing at a fixed witness FALSE (0 of 120 fixed pairs — but floor-limited, see below); atom half inside the bar FALSE (best device 4.78×10^6); third R1″ device positive FALSE (floor -7.93) — terms = 2) attacks the one pairing $\sum_{d\geq 1}(\Delta c)_d\, W^1_d$ through its correlation structure. Two terms remain open — the same *count* as T159, and that is the honest headline — but both are now *located*, and one of them is no longer an algebra problem.

No oscillation: the sign-block count is closed, and the block device is refuted by its own numbers. W^1 has $J = 3$ sign blocks on every window, flat ($h^{+0.000}$), with a *closed* bound $J \leq 4(2\cdot 16)+2 = 130$ from the trigonometric zero count (w_d is a piecewise trigonometric polynomial of degree ≤ 32; Féjer 1915): W^1 is one smooth *three-lobe profile* with quasi-period 0.332–0.333 of the window, not an oscillation. The prefix ratio $\sup|\Psi|/\sup|W^1| = 83.5$–$569.8$ grows as $h^{+1.022}$ — the quantitative reason iterated Abel loses: each further summation by parts trades a factor h of tail size for a factor h of difference smallness, neutral in the exponent and strictly worse in the constant. Consequently the Leibniz block device the contract asked for is refuted *quantitatively*: block-Hölder overshoots as $h^{+4.115}$ and block-Abel as $h^{+4.131}$ — 5.2–16.5 times *worse* than the already-refuted $\ell^1 \times \sup$ pricing at $h^{+3.595}$, because with three blocks the block masses are of prefix size. The head peel is refuted too: no proper split clears the bar on any window ($K^* = M$ on 19/19), and the best improper one ($K/M = 0.97$) still overshoots the true pairing $Q = 1.7527$–6.0895 by 60.3–4.80×10^3, growing as $h^{+2.428}$ — the h^2 cancellation itself, essentially undiminished.

The moment laws, and the second numerical horizon. What carries on the smooth half are three *closed sign-definite moment laws*, new theorems, machine-checked: $m_0 = \sum_d w_d = 0$ (the T159 gauge identity), $m_1 = \sum_d d\, w_d = -[S_0^2 + 2\sum_j P_j^2] \leq 0$ (the $\max(r,s)$ identity: minus a sum of squares; 7.1×10^{-16}), and $m_2 = \sum_d d^2 w_d = -[2S_1 - (M-1)S_0]^2 \leq 0$ (a *perfect square*; 3.0×10^{-16}), with closure of m_p in the moments of y for every even p (verified at $p = 2, 4$): the pairing cannot see the lag mass, and it sees a linear or quadratic trend only with a sign known in advance. Against a *fixed* polynomial witness (fraction 0.2 of the lags, degree 48, the same pair on every window — a device, not a fit) the smooth archimedean half of the pairing is *reproduced* to 0–2.44×10^{-15} relative, worst slack 1.95×10^{-8} of the $O(1)$ total on all 19 windows. And a **second numerical horizon is declared** rather than hidden: the measured residual is 4.12–428 times the double-precision floor $\varepsilon \sup|c^{\mathrm{arch}}|\, \|w\|_1$, so the surface reports the arithmetic floor of the measurement and *not* the witness error — the arch half is honestly reported as *neither established nor refuted* on any double-precision surface; only a symbolic Chebyshev bound on the T115 kernel could decide it (rest item R-A below). Atom sparsity is refuted in one number: the support is *not* sparse — it fills 0.3226–0.6377 of the lag range, because the prime-power gaps $\log(1 + 1/n)$ fall below the grid spacing D as soon as $n > 1/D$; even the support-localised ℓ^1 (exact position *and* exact local value of every atom, absolute values taken) misses by 4.78×10^6–2.64×10^9 growing as $h^{+3.505}$, while the required relative precision on the atom half is the cancellation depth itself, $|\mathrm{total}|/|\mathrm{atom}| = 2.19 \times 10^{-6}$–$1.12 \times 10^{-4} \sim h^{-2}$.

The core of T160: the sampling identity. The atom half of the pairing is a *weighted sampling* of the closed weight vector at the prime-power positions — a theorem, machine-checked to 1.0×10^{-16}–

2.9×10^{-15} of the absolute scale on every window:

$$\sum_d c_d^{\mathrm{atom}}\, w_d \;=\; -\frac{1}{2} \sum_{n \le X} \frac{2\Lambda(n)}{\sqrt{n}}\, \widehat{w}(\log n) \quad \text{(plus one reflected term below the grid)},$$

with $\widehat{w}$ the linear interpolant of the closed weight vector — and by T159's Dirichlet-kernel form, $\widehat{w}(u)$ is a *fixed* trigonometric polynomial with frequencies $\pi(k \pm l)/\alpha + O(1/h)$, $k, l \le 16$. So **the atom half of R2″ is, identically, a finite combination of sums $\sum_{n \le X} \Lambda(n)\, n^{-1/2} \cos(t \log n)$ at thirty-two explicit frequencies** t — needed to relative depth 2.19×10^{-6}–$1.12 \times 10^{-4} = h^{-2}$, and measured to cancel only to 0.0000–0.3659 of the trivial mass bound. The consequence, stated with the fence in place: *the h^2 cancellation is the intrinsic arithmetic hardness of the problem*, not an artefact of the T115 assembly — the geometric half is now evaluated down to the arithmetic floor, so nothing geometric is left to trade against the prime sum. $\sum_n \Lambda(n)\, n^{-1/2-it}$ is the *shape* of a partial sum of a Dirichlet series on the line $\mathrm{Re}\, s = \frac{1}{2}$; that is reported as a *diagnosis* of where the h^2 lives, never as an input: the sums are assembled from the finite von-Mangoldt table and nothing else, no zero of any L-function is read, generated or approximated, no equivalence is claimed, and Weil 1952 / Bombieri 2000 remains an address.

The third R1″ device: $\rho > 1$, flat — and its inequality form is refuted. On the sub-surface (8 windows, $\lambda_{\min}(B_{HH}) = 0.0748$–$0.2023$; the Z-matrix and its Collatz–Wielandt floor 0.8811–1.0988 reproduced exactly): the rank/support route is refuted — the effective negative rank of the atom block is 0.455–0.634 of the block dimension, so no fixed-size corner can contain it; the corner device is refuted — it prices the coupling by $\|M_{12}\|_2 = 1.554$–71.447 and is T158's refuted norm route in disguise. The surviving fact: $\rho = \min_{0 \ne v \in V_-} (v^{\mathsf{T}} B_{HH}^{\mathrm{arch}} v)/(-v^{\mathsf{T}} B_{HH}^{\mathrm{atom}} v) =$ **1.0036–1.0140** > 1 on *every* window, flat ($h^{-0.011}$) — the archimedean block dominates the atom block on every destructive direction, an *angle* statement no norm can see, and the first R1″ fact that is neither a norm nor a sign nor a rank (MEASURED: a generalised eigenvalue of two size-h blocks). Its inequality form $B_{HH} \succeq B_{HH}^{\mathrm{arch}} - (1/\rho)\, P_- B_{HH}^{\mathrm{arch}} P_-$ is a theorem given ρ, with no arithmetic on the right — and as a device it is refuted: floor $-7.9261 \ldots -1.0919 < 0$ on 8/8, because the margin $(1 - 1/\rho)\lambda_{\min} = 3.76 \times 10^{-3}$–$1.24 \times 10^{-2}$ loses to the archimedean cross-coupling 2.808–18.15 between V_- and its complement by a factor 10^4. Any future device must use ρ *together with* the geometry of V_- inside B_{HH}^{arch} — pricing that geometry by a norm is exactly what fails.

Two cheap theorems, the reassembly, the stress. **U3 is upgraded cert-unif → theorem**: c^{arch} is nonpositive and monotone increasing on $d \ge 1$ (both hypotheses machine-checked; fraction 1.0000 of 25458 pairs), so the total variation equals the total rise and $\|\Delta c^{\mathrm{arch}}\|_1 \le |c_0^{\mathrm{arch}}| + 2 \sup_{d \ge 1} |c_d^{\mathrm{arch}}| < 6$ — measured 3.9101–5.4153 against the closed bound 3.9105–5.4153: *four digits*, the constant closed and window-independent. U1 is *partially* upgraded, and the honest statement is the interesting one: $K_{kl}(0) = 0$ and $\sum_d K_{kl}(d) = 0$ exactly, so $R_{kl}(0) = R_{kl}(1) = 0$ and the Abel step has no boundary term — the Z-law of B_{HH}^{arch} is *equivalent* to one *prime-free* trigonometric inequality $\sum_{d \ge 2} (\Delta c^{\mathrm{arch}})_d\, R_{kl}(d) \le 0$ (the weight nonnegative by U3); it holds on every sampled pair, but the pointwise strengthening $R_{kl}(d) \le 0$ is refuted (it holds at only 0.4848 of the lags): the monotone weight is essential, and the statement is *not* yet a theorem and not reported as one. End to end, the number is the verdict: the composite chain (arch half by the fixed witness, atom half by the support-localised ℓ^1) recovers 3.79×10^{-10}–2.10×10^{-7} of the exact per-window value — *if the atom half were evaluated as exactly as the arch half now is, the same chain would recover* 1.0000: the whole deficit of R2″ is carried by one term, of size 4.77×10^6–2.64×10^9. T159's certified per-window band 0.287–1.0 is unchanged — T160 moves the obstruction, it does not sharpen the number. The identities roll up worst-case at 2.95×10^{-12}; the T145 no-go breaks on **6/6** axes including the two new ones (A5 the gauge/moment structure — the no-go form does not annihilate constants, so the Abel step has no analogue on it; A6 the complementarity — ρ is undefined on it); Dirichlet and parity eigenpair controls at 8.2×10^{-15} with the $2/\sqrt{N}$ bound saturated. The balance moves 13/6/4/1 → **14 theorem** (6 new) / 4 certified-uniform / 3 certified-per-window / 3 measured, with 8 refuted device families; the candidates T160-P1…P7 (the moment laws; the sampling identity; the block count with its refutation; the U3 upgrade; the U1 partial upgrade; ρ;

the second horizon) are all PENDING — no promotion here, and v553 is untouched.

After T160: two classical rests, one arithmetic object, and the circularity question

(R-A) one classical estimate: a symbolic Chebyshev bound for $s \mapsto A(s, D)$ on $[0.2\cdot 2\alpha,\ 2\alpha]$ (the Bernstein ellipse of the T115 integral, whose only singularity is the $1/s$ head at the origin) — it decides the *arch* half of R2″ m-freely, which no double-precision surface can (T160-P7). *(R-B)* one trigonometric inequality, prime-free: $\sum_{d\geq 2}(\Delta c^{\mathrm{arch}})_d\, R_{kl}(d) \leq 0$ — it turns the Z-law (T159-U1) and with it the Collatz–Wielandt floor into a theorem. *(R-C)* the arithmetic term, and it is not an exercise: a bound on $\sum_{n\leq X}\Lambda(n)\, n^{-1/2}\cos(t\log n)$ at the thirty-two explicit frequencies $t = \pi(k \pm l)/\alpha$, to relative depth h^{-2} — by the sampling identity (T160-P2) this *is* the atom half of R2″, and nothing geometric is left to trade against it. *(R-D)* a fifth device for R1″: ρ together with the geometry of V_-, without pricing that geometry by a norm. *(R-E)* optional: closed odd-p moment functionals. T161 (`classical_closure_probe.py`, contract `CLASSICAL.CLOSURE`) executed exactly that programme — the two classical rests R-A/R-B plus the *circularity triage* for R-C — next subsection.

10.51 The circularity triage: the hardness sits in the split, not in the primes (T161)

T161 (`classical_closure_probe.py`, contract `CLASSICAL.CLOSURE`, 35/35, 1.2 s on 19 log-spaced prime-power zones of 75 admissible, $h = 50 \ldots 1445$ — a 28× lever arm — with $\alpha = 2.3979$–6.4938 and atom cut-off $X = e^{2\alpha} = n_{\mathrm{zone}}^2$ exactly $= 121$–4.37×10^5; $\psi(x)/x \leq 1.038830$ *verified* at every jump point up to $n = 400000$, the unconditional input the triage prices; verdict CLOSURE-RESISTS, triage verdict NEEDS-BEYOND-RH-STRENGTH) attacks R-A and R-B head-on and prices R-C against the classical ladder.

R-A: the analytic half closes m-free, and the head needs no peeling. Two theorems anchor it: the T115 arch kernel is *exactly* $A(s, D) = D\,\widehat{A}(s, D)$ with $\widehat{A}$ the triangle smear of $g(w) = e^{-w/2}/(1-e^{-2w})$ (4.2×10^{-16}–2.0×10^{-13} against the assembly on 19/19), and the analyticity width is set by the $1/s$ head alone — the poles of g sit exactly at $w = ik\pi$, and on $[0.4\alpha, 2\alpha]$ the first complex pole lands outside the Bernstein ellipse by margin 1.29–3.29 on every window. The Bernstein parameter is therefore *closed and scale-free*, $\rho^* = (3+\sqrt{5})/2 = 2.618034$ (it depends only on the interval ratio 5; on the surface $\rho(D) = 2.5593$–2.6160), with the ellipse constant $M = 3.06$–5.93 uniform in h and the measured coefficients obeying the envelope at 0.055–$0.074 \leq 1$ — tight to 1.1 orders. The new structure T161 contributes: the smeared pole integrates in *closed form*, so for every lag $d \geq 1$

$$c_d^{\mathrm{arch}} \;=\; \Psi_d + D\,\widehat{G}(dD, D), \qquad \Psi_d \;=\; -\tfrac{1}{2}\big[(d{+}1)\log(d{+}1) - 2d\log d + (d{-}1)\log(d{-}1)\big],$$

with Ψ closed, D-free and m-free (the $\log D$ cancels identically; agreement 1.7×10^{-16}–4.8×10^{-16} at *every* lag of every window) and $\widehat{G}$ the smear of the regularised $g - 1/(2w)$ — no head peel, no fitted cut; on the full interval $[0, 2\alpha]$ the closed rate $\rho_{\mathrm{reg}} = 2.05$–$3.38$ is *not* scale-free ($\to 1 + \sqrt{\pi/\alpha}$). The "fixed degree" hope is refuted with a number: both routes give a degree *schedule* $K(h) = O(\log h)$ ($K = 12$–19 peeled, 9–25 head-free; $dK/d\log h \approx 2.0$ and 4.5), because $\|w\|_1$ grows as $h^{+2.85}$ (fit) against $D \sim h^{-1}$. The residual of R-A, named exactly: the *log-moment* $\sum_d \Psi_d\, w_d = 2.15$–$3.14$ of the arch half $= -\frac{1}{2}\sum_d (d\log d)(\Delta^2 w)_d$ — prime-free, classical (a Clausen/Lerch-type object), and *not* in the polynomial moment ladder.

R-B: refuted in all four readings — and the fraction bound survives. The contract's inequality $\sum_{d\geq 2}(\Delta c^{\mathrm{arch}})_d\, R_{kl}(d) \leq 0$ is tested in all four of its readings before any proof is attempted, and all four fail: per pair directly on 3168 of 4320 pairs (worst +0.92 of $\|\Delta c^{\mathrm{arch}}\|_1$), per pair in the Abel-tail reading on 1229, the *aggregate* by *sign* — the arch half is negative on every window while its off-diagonal block is positive, exactly the harmful direction for the needed upper bound — and the Abel route's sufficient condition on 3801 of 4320 (worst +763). The reason is structural: R_{kl} is a difference of Dirichlet kernels and changes sign in d, and the coefficients $a_k a_l$ change sign too — a one-sign statement was never the right object. The by-products are positive: $(\Delta c^{\mathrm{arch}})_d \geq 4.5 \times 10^{-8} > 0$ and decreasing, with a total-positivity reason from the Ψ split (Ψ is minus the triangle smear of the completely monotone $1/w$). What survives is weaker and usable:

|off-diagonal|/|Q^{arch}| = 0.1035–0.1713, flat in h (exponent -0.116, fit) over a factor 24 — a certified *fraction* bound with the constant $\frac{1}{4}$, costing the chain a factor $1+\frac{1}{4}$ in the constant and nothing in the structure; its m-free version is a 16×16 Gram positivity statement (Fejér 1915, Schur 1917), entirely prime-free.

The triage — and the answer to the circularity question. A new theorem first: the thirty-two frequencies satisfy $t\,(2\alpha)=2\pi j$ *exactly* for $j=1\dots32$ — they are precisely the *Fourier harmonics of the log-window* $[0,2\alpha]$, so the atom half of the pairing is the vector of the first 32 Fourier coefficients of the measure $\sum_n(2\Lambda(n)/\sqrt{n})\,\delta_{\log n}$. The measured 0.00–0.37 cancellation of T160 is then *identified*: at the harmonics $\theta=t\log X=2\pi j$ collapses the partial-summation main term to the closed, parameter-free prediction $(\sqrt{X}-1)/(\frac{1}{4}+t^2)$, which matches $S(t)$ to 0.014–0.232 of the trivial mass, improving as $X^{-0.39}$ (fit) — the $1/(\frac{1}{4}+t^2)$ is the Mellin factor of $x^{-1/2}$, *not* an arithmetic saving; the genuinely arithmetic part is the residual against $d\psi(x)-dx$. The δ arithmetic, compactly (the demand is $h^{-2}=X^{-\delta}$ with $\delta=\log h/\alpha$ exactly, since $\log X=2\alpha$):

strength	δ **it buys**
trivial / Chebyshev 1852	0
zero-free region (de la Vallée Poussin)	$c/\sqrt{\log X}\to0$ (no power saving)
van der Corput 1921 / Vinogradov 1937	inapplicable ($t=0.5$–42 is $O(1)$; $j\le32$ turns)
harmonic-frequency geometry ($\sin\theta=0$)	0.29–0.92 (measured suppression 7.0–19.8)
RH strength	$\frac{1}{2}-o(1)$
needed (arch/atom split)	**1.1482–1.8809**
needed (arch+smooth / fluctuation re-split)	**0.9839–1.3767**

The sharpest form of the same fact: the required *absolute* precision is 0.012–0.31 of the *last term* of the sum — below the boundary term $|\psi(X)-X|\,X^{-1/2}$ that every partial-summation bound carries, so no strengthening of the $\psi(x)-x$ input (zero-free region, RH, or beyond) lowers the floor of the method to it. And the re-split proves the demand is *split-dependent*: subtracting the smooth prime term $-\int e^{u/2}\widehat{w}(u)\,du$ cancels the arch half to 0.50–0.76 (the explicit-formula pairing, confirmed), and the demand moves from $\delta=1.148$–1.881 to $\delta_{\text{eff}}=0.984$–$1.377$ — it *moves* with the split, but stays above $\frac{1}{2}$ on 18/18.

The honest reading, stated prominently and soberly: the chain is not circular. It does not secretly need RH — it needs *more* than RH-strength input would supply, and the demand is not even reachable by the partial-summation framework at any input strength. The measured cancellation at the thirty-two frequencies is fully the prime-number-theorem main term, with no arithmetic surplus; the frequency geometry is genuinely special (worth $\delta=0.29$–0.92, already inside the measured numbers) and does not close the gap. So the h^2 is located in the *split*, not in the primes: T161 refutes two splits with numbers, and finding a third — one with $\delta_{\text{eff}}<\frac{1}{2}$ — is the whole remaining R2″ task. The controls are re-verified rather than inherited (Dirichlet kernel identity 1.3×10^{-15} including the degenerate branch, KMS eigenpairs 7.1×10^{-15}, Chebyshev reconstruction 6.1×10^{-15}, odd_toeplitz(c^L) $=L_P$ exact); the balance moves by +6 theorem / +6 certified-uniform / +1 certified-per-window / +6 measured / +3 refuted to **20**/10/4/9/3. The closed cores of T160+T161 — the sampling identity, the moment laws with the TV theorem, the harmonic-frequency theorem with the PNT prediction, the head split with ρ^*, and the fraction bound with the sign refutation as its wired negative control — are load-bearing as `v554` [`verification/v554_sampling_harmonics_identities.py`] (Section 4.20); the δ triage itself is a comparison of strengths, a documentation item, and is promoted nowhere.

After T161: one moment, one matrix inequality, one split, one device

(R-A′) one closed prime-free moment: the log-moment $\sum_d\Psi_d\,w_d=-\frac{1}{2}\sum_d(d\log d)(\Delta^2w)_d$, 2.15–3.14 of the arch half and outside the polynomial moment ladder (Clausen/Lerch address) — everything else in R-A is done, and a *fixed* witness degree is provably not available, only the schedule $K(h)=O(\log h)$. *(R-B′)* one matrix inequality, prime-free: the m-free version of the fraction bound |off-diagonal| $\le\frac{1}{4}|Q^{\text{arch}}|$ — a positivity/norm statement for the 16×16 Gram form $(\Delta c^{\text{arch}},R^1)_{kl}$ (Fejér 1915, Schur 1917), certified

on every window today. *(R-C′)* one *split*: not an exponential-sum bound — the demand sits below the boundary term of every partial summation — but a split of the pairing with $\delta_{\text{eff}} < \frac{1}{2}$; the arch/atom split gives 1.148–1.881 and the arch+smooth/fluctuation split 0.984–1.377, both above $\frac{1}{2}$, and finding a third is the whole remaining R2″ problem. *(R-D)* a fifth device for R1″: $\rho = 1.0036$–$1.0140 > 1$, flat, stays the surviving structure fact. T162 (`third_split_probe.py`, contract `THIRD.SPLIT`) executed exactly that search — next subsection.

10.52 The third split: the Mellin ladder saturates, and the Fejér split goes sub-RH at a price (T162)

T162 (`third_split_probe.py`, contract `THIRD.SPLIT`, 30/30, 5.2 s on the T161 surface rebuilt *independently* from the declared recipe: 19 log-spaced prime-power zones of 75 admissible, $h = 50 \ldots 1445$, of which 18 windows sit inside the declared condition horizon $\text{cond}(B_{LL}) \le 10^{12}$, $h = 50 \ldots 1218$; the pairing identity re-checked to 9.98×10^{-18}–3.01×10^{-14} of the large scale and the gauge identity to 2.5×10^{-16}; $\psi(x)/x \le 1.038830$ verified at every jump point up to $n = 400000$, i.e. $\kappa = 0.038821$ — the *one* unconditional arithmetic input every number below is priced against; verdict DELTA-REDUCED) searches for the third split, and the search returns both the split and its limit.

The currency, declared before any candidate. Two numbers are kept strictly apart from here on, because T161 used only the first: $\delta_{\text{val}} = \log(|R|_{\text{meas}}/|Q|)/\log X$ is the measured residual *value* — a floor — and $\delta_{\text{bnd}} = \log(E_{\text{triv}}\, C_w/|Q|)/\log X$ is what a *proof* must beat, with E_{triv} the Chebyshev-strength (unconditional) size of the arithmetic factor. RH strength is $\frac{1}{2}$ in both currencies, as a comparison of strengths and never as a claim; always $\delta_{\text{bnd}} \ge \delta_{\text{val}}$. The taxonomy, in both currencies (smooth part closed on every row):

split	δ_{val} **(floor)**	δ_{bnd} **(demand)**
C0 arch / atom	1.1482–1.8809	— (no factorisation)
C1 arch+smooth / fluctuation	0.9839–1.3767	— (no cell-level input)
C2 = C1 + Mellin ladder at K^*	0.6247–0.9307	— (same)
C5 = C2 + one Abel step	0.6247–0.9307	1.3965–1.7867
C3 spectral (ground pair, ≤ 16 modes)	—	0.4602–1.3145 (price $\sim h^{+2.93}$)
C4 Fejér taper (`fejer_2`)	—	**0.1332–0.4169** (price $\sim h^{+2.86}$)
frontier, price ≤ 2		1.319–1.650
frontier, price $\le 10^2$		0.412–1.126
frontier, price $\le 10^4$		−0.079–0.681

The Mellin ladder is the third genuine gain, and it is an asymptotic series. Past the PNT main term the smooth part of $d\psi$ is the derivative of the *elementary* closed function $-\frac{1}{2}\log(1 - x^{-2})$, so the k-th rung pays $+\sum_d w_d\, C_d(-(2k + \frac{1}{2}))$ in closed cell moments (Mellin 1896) — no zero data anywhere. The residual is minimised at $K^* = 2$ on all 18 windows (relative drop 0.9015–0.9256 — a genuine factor), and past K^* it *rises* by 15.5–25.1 up to $K = 20$: the predicted log-divergence of the formal sum $\sum_k x^{-2k-1} = 1/(x(x^2 - 1))$, not a numerical artefact. What it buys: $\delta_{\text{val}} = 0.6247$–$0.9307$ against C1's 0.9839–1.3767, a forced (not fitted) gain of 0.359–0.446 — and it *cannot* be iterated to the missing $X^{0.12}$–$X^{0.43}$, because the series diverges. C0 and C1 reproduce T161's 1.148–1.881 and 0.984–1.377 to 0.01 at both ends on the independently rebuilt surface.

One Abel step, and R-C′ stops being an arithmetic problem. Level 0 admits no unconditional input at all (no cell-level Chebyshev bound — T161's granularity finding restated); one Abel step, licensed by the gauge identity, gives $|V_d| \le 2\kappa\sqrt{X}$ by two partial summations, and the ladder identity $\sum_d v_d w_d = \sum_d V_d^{(k)} (\Delta^k w)_d$ is machine-checked at every level $k = 1 \ldots 4$ (3.9×10^{-15}–3.1×10^{-14}, boundary free). The optimal level is *exactly one* on 18/18, for a closed reason: each further step multiplies the envelope by $\sim 2/D$ and divides $\|\Delta^k w\|_1$ by $\sim \omega_{16}$, and the product is $32\pi/\alpha = 16.7$–$41.9 > 1$ — partial summation is a single step, and it has been taken. At that level

the demand factorises *exactly* as

$$\delta_{\mathrm{bnd}} \;=\; \tfrac{1}{2} + \frac{\log\!\big(2\kappa\,\|\Delta w\|_1/|Q|\big)}{\log X},$$

so every arithmetic input sits in the single constant κ and the remaining question is the closed, *prime-free* ratio $\|\Delta w\|_1/|Q| = 2.25\times10^{3}$–$1.15\times10^{6}$, growing as $h^{+1.769}$ (fit); an RH-strength input misses the corresponding test statistic by 4.8–8.2 orders of magnitude.

The Fejér split, and the price of going below one half. The Thomson dual makes every trial vector x with $x_1 = 1$, $Q(x) > 0$ admissible at the price $P(x) = Q(x)/Q(x^*) \geq 1$ in the $1/s$ ceiling — and the objective must be priced, or it is gamed: an *unpriced* search does find $\delta_{\mathrm{bnd}} < 0$ here, by making $Q(x)$ huge and the bound vacuous. Priced (778 trial vectors per window, the fast path re-assembled against a literal double loop at 0.0), the result is an exchange rate: a sub-$\frac{1}{2}$ demand *exists* on every window, at minimum price $P_{\min} = 7.2\times10^{1}$–$8.2\times10^{4}$ growing as $h^{+2.0}$ (fit), and every unit of δ costs 1.17–3.31 *decades* of price. The structured families beat the sweep: the spectral truncation C3 reaches 0.4602–1.3145 (below $\frac{1}{2}$ on 5/18), and the **Fejér taper C4** — x_k weighted by $1 - k/n_b$ (Fejér 1915), which by self-adjointness of the smoothing *is* the contract's "pairing against Fejér-averaged Λ-mass" — reaches $\delta_{\mathrm{bnd}} = \mathbf{0.1332}$–$\mathbf{0.4169}$, *below* $\frac{1}{2}$ *on all* 18 *windows*, lowering the demand by 1.15–1.51 in δ against the untapered optimiser. The catch, stated without hedging: its price grows as $h^{+2.86}$ (fit), so the sub-RH demand is bought with an asymptotically diverging loss in the $1/s$ bound, and a per-window certificate at growing price is not a uniform statement — R2″ is *relocated* into the price, not closed.

R-A′ closes, R-B′ is refuted. The log-moment $L = \sum_d \Psi_d w_d$ — the one non-polynomial moment of the arch half — agrees on three independent routes: the direct sum; the double-Abel form *with* its Wronskian boundary term, $L = -\frac{1}{2}\sum_d (d\log d)(\Delta^2 w)_d - \frac{1}{2} M \log M\, w_{M-1}$ exactly (2.1×10^{-15}–4.2×10^{-14}; the boundary term is 3.2×10^{-6}–3.5×10^{-2} of L and not optional — T161's identity is completed here, not quoted); and the Lerch/Frullani integral

$$L \;=\; -\tfrac{1}{2}\int_0^\infty \frac{(1-e^{-y})^2}{y^2}\,G(y)\,dy, \qquad G(y) = \sum_{d\geq 1} w_d\, e^{-(d-1)y} \text{ closed},$$

agreeing with the direct sum to 0.0–6.0×10^{-16} — machine precision — and converging at $y = 0$ for exactly one reason, the gauge identity; the $d = 1$ term peels in closed form via $\int_0^\infty ((1-e^{-y})/y)^2 dy = 2\log 2$, and the same constant reproduces $\Psi_1 = -\log 2$ to 0.0, an independent confirmation of the whole derivation (Frullani 1828 / Lerch 1887 / Clausen 1832). R-A′ moves from open to certified-per-window; it is m-indexed, and no m-freeness is claimed. R-B′ is *refuted*, and the refutation is a result: the 16×16 Gram form *is* the arch half ($a^{\mathsf{T}} G a = Q^{\mathrm{arch}}$ to 0.0–1.9×10^{-15}, a theorem) and it is *indefinite on every window*, $\lambda_{\min}/|\lambda_{\max}| = -13.34\ldots-1.59$, zero of 18 windows semidefinite in either sign; the mechanism is measured — the diagonal is strictly positive but small, the unweighted Gershgorin ratio is 7.97–580.76, so Gershgorin 1931 does not apply and no semidefiniteness certificate exists. What survives is the a-weighted quarter bar: off-diagonal mass against the arch half = 0.1435–0.2902, flat (exponent −0.102, fit), holding on 14 of 18 windows and exceeded by 16.1% on four — reported honestly as 0.29 and not as $\frac{1}{4}$: a bound on a *contribution*, not a spectral property.

The assembly, the stress, and the anti-fitting refusal. The full chain $Q = (Q^{\mathrm{arch}} + Q_K^{\mathrm{smooth}}) + R$ closes end to end at 6.1×10^{-17}–1.8×10^{-15} of the large scale $\max(|\mathrm{arch}|, |\mathrm{atom}|)$ — one unbroken chain of identities followed by exactly one inequality, which holds with 2.0–3.2 orders of slack; the gap $\delta_{\mathrm{bnd}} - \delta_{\mathrm{val}} = 0.77$–$0.86$ *is* the price of factorising, and it is bigger than the 0.12–0.43 separating the residual value from RH strength: the factorisation, not the arithmetic, now dominates the loss. The must-break controls break — uniform surrogate prime powers (same mass multiset) drive the total to 5.2×10^{2}–4.6×10^{7} of $|Q|$, and violating the gauge breaks the Abel identity by *precisely* $c_0 \sum_d w_d$ (reproduced to 1.9×10^{-14}, a positive control on the mechanism) — while a control that did *not* break is declared as information: 4× under-resolution leaves the total $O(1)$ (0.41–1.83), so

the cancellation does not rest on resolving individual prime powers. And the anti-fitting refusal, the most uncomfortable number of the probe, reported in full: the exponent $\beta = \frac{1}{2}$ in the smooth prime term is *forced* by the $n^{-1/2}$ weighting; the empirical minimiser over $[0.2, 0.8]$ is $\beta^* = 0.600$–$0.800 \neq \frac{1}{2}$, where the residual would look up to a hundred times better — and it is *refused*: the drift is a property of the residual being large (50–76% of the object), not evidence about the exponent. The verdict, in the two currencies: the δ trajectory $1.88 \to 1.38 \to 0.93$ is real and monotone and does *not* reach $\frac{1}{2}$ (the best provable demand after the one Abel step is $X^{0.90}$–$X^{1.29}$ beyond RH strength), and the exhaustion *saturates rather than converges* — three independent ladders all turn around at their first or second step. The balance of T162: 10 theorem / 2 certified-uniform / 3 certified-per-window / 9 measured, zero fit rows (the only fitted numbers are h-exponents, labelled in place); the candidates T162-P1…P6 (the closed log-moment; the Abel form with its boundary term; the quarter bar; the Gram indefiniteness — a negative result that closes a route; the Abel-level theorem; the exchange rate) are all PENDING — no promotion here, and v554 is untouched.

After T162: one front, one substitute, one device

(R-C″) the Pareto front, now prime-free: find an admissible trial vector x with $\delta_{\mathrm{bnd}}(x) < \frac{1}{2}$ at a price $Q(x)/Q(x^*)$ the T159/T160 chain can pay — such x exist at price $\geq 7.2 \times 10^1$ growing as $h^{+2.0}$, and both ways out are closed questions: fix the affordable price from the chain and read the frontier, or enlarge the search space beyond the 16 low modes, where the total variation may drop at constant price. *(R-B″)* a substitute for the refuted Gram positivity: the chain wanted a *sign* and has a contribution bound (the a-weighted quarter bar, certified-uniform via flatness). *(R-D)* a fifth device for R1″: $\rho = 1.0036$–$1.0140 > 1$, flat, untouched by T162. T163 (`pareto_front_probe.py`, contract `PARETO.FRONT`) executed exactly that survey — next subsection.

10.53 The Pareto front: the front resists as a theorem, and the h^2 is the parity spectral gap (T163)

T163 (`pareto_front_probe.py`, contract `PARETO.FRONT`, 32/32, 1.9 s on a surface rebuilt with declared caps: 28 log-spaced prime-power zones of 70 admissible, $h = 142 \ldots 1445$ — a 10× lever arm, with $H_{\min} = 128$ declared as twice the largest mode block $K = 64$ — of which 27 windows sit inside the condition horizon $\mathrm{cond}(B_{LL}) \leq 10^{12}$; the closed weight vector re-derived as a *correlation form*, $w_0 = A_0$, $w_d = 2A_d - H_{M-1-d}$ with A the autocorrelation and H the self-convolution of $v = T_K^{\mathsf{T}} a$, to 1.6–2.0×10^{-15} — $O(h^2)$ and block-size-free, which is what makes the sweeps affordable; pairing identity to 5.9×10^{-17}–9.5×10^{-14} of the large scale, cancellation headroom ≥ 10 decades over double precision, declared; $\psi(x)/x \leq 1.038830$ verified at every jump point up to $n = 400000$, $\kappa = 0.038821$; verdict FRONT-RESISTS) fixes the affordable price *from the chain* and reads the front — and the answer arrives as a theorem rather than as a search result. The theorem-shaped cores of T162 and T163 are promoted as v555 [`verification/v555_pareto_tv_identities.py`] (Section 4.21).

The price axis is chain-derived, not invented. $Q(x^*) = 1/g_{16}$ certifies that the T160 optimiser *is* the top rung of the T158 Cholesky ladder (2.3×10^{-13}–2.1×10^{-8}, below the conditioning bar the solve carries); the rung prices $P_{\mathrm{rung}}(K) = g_{16}/g_K \geq 1$ are prices the chain has *already paid*, since $1/g_K$ is an upper bound on $1/s$ it already accepts; and the affordable price is preregistered as $P_{\mathrm{aff}} = P_{\mathrm{rung}}(1) = g_{16} B_{11} = 3.93 \times 10^3$–$1.01 \times 10^7$, growing as $h^{+3.049}$ (fit) — the T157 route-(0) certificate $1/s \leq B_{11}$, the loosest rung the chain has ever used. The Fejér knob $t_k(\sigma) = \max(0, 1 - k/\sigma)$ (Fejér 1915; 50 geometric grid points on $[1, 4096]$ plus ∞) has *both endpoints in the chain*: $\sigma = 1$ is exactly the $K = 1$ rung (0–3.9×10^{-15}) and $\sigma = \infty$ the untapered optimiser, i.e. the $K = 16$ rung. Price and demand are *measured* monotone in opposite directions along the knob at all 27×50 grid points (every point admissible), so the knob curve *is* the Pareto front: there is no interior optimum to find, and the only question is where the chain can afford to sit. Read at the declared prices, the flat caps 1.25/2.00/10.00 give $\delta_{\mathrm{bnd}} = 1.310$–$1.756$ / 1.263–1.686 / 1.102–1.375, and the flat rung tiers the same — 0/27 **windows below the yardstick $\frac{1}{2}$ at any flat price**; only the full P_{aff} reaches $\delta_{\mathrm{bnd}} = -0.004$–$0.019$ on 27/27. The crossing price is $P_{\mathrm{cross}} = 8.05 \times 10^2$–$1.01 \times 10^5 \sim h^{+1.906}$ (fit), and $P_{\mathrm{cross}}/P_{\mathrm{aff}} = 0.0066$–$0.2047$: the crossing sits 5–152× *strictly inside* the certificate the chain

already accepts, with the margin *widening* in h — the one genuinely new positive of the part, stated without inflation, because a per-window bound at a price growing like h^2 is not a uniform statement.

The price anatomy is an identity, not a fit. Substituting $P(x) = g_{16}|Q(x)|$ into the definition of the demand gives, for every admissible trial vector and with no asymptotics,

$$\delta_{\mathrm{bnd}}(x) \;=\; \tfrac{1}{2} + \frac{\log(2\kappa\, g_{16}\,\mathrm{TV}(x)/P(x))}{\log X}, \qquad \delta_{\mathrm{bnd}}(x) < \tfrac{1}{2} \iff P(x) > 2\kappa\, g_{16}\,\mathrm{TV}(x),$$

machine-checked at all 27×50 grid points (0.0–7.0×10^{-14}): **price and demand are two coordinates on one object**, the trial vector enters only through the single ratio $\mathrm{TV}(x)/|Q(x)|$, and the "exchange rate" T162 measured is a change of variables, not a second lever. The crossing exponent *decomposes additively* — TV at the crossing grows as $h^{+2.013}$, g_{16} as $h^{-0.061}$, sum $+1.952$ against the directly fitted $+1.906$ — so the h^2 sits in the total variation and nowhere else. The taper's structural gain in TV is real and *bounded*: a factor 2.75–4.64, h-flat, worth at most 0.276 of δ and decaying like $1/\log X$; where the price moves, $\tau_{\mathrm{TV}} = -0.0191$ (median) says every remaining unit of δ is paid one-for-one out of the price. And the second half of the anatomy identifies the object: the cancellation quality $\mathrm{CANC}(x^*) = |Q|/\max(|Q^{\mathrm{arch}}|, |Q^{\mathrm{atom}}|) = 2.2 \times 10^{-6}$–$3.6 \times 10^{-4}$ ($h^{-2.039}$) jumps to 0.449–0.934 at $\sigma = 2$ — *the taper destroys the arch/atom cancellation*, and the price is the cancellation depth read backwards.

The search space, and then the theorem that makes the sweeps unnecessary. Enlarging the block, $K = 16 \to 32 \to 64$, lowers TV only by 0.868–0.958 at a price *dividend* (0.81–0.94) and makes the demand *worse* (up to $+0.0135$); the TV exponent is unmoved ($+1.832/+1.820/+1.810$); the best closed candidate — the Fejér-weighted 64-mode sum — gives a TV factor 0.843–0.931 and no crossing; and the pooled flat-price best over everything built is $\delta_{\mathrm{bnd}} = 1.331$–$1.788$. Then the sweeps can be put aside, because the question they were probing has an exact answer:

$$\begin{array}{llllllll} \textbf{(T1)}\ w_0(x) & = & \|a(x)\|^2 & \textbf{(T2)}\ \mathrm{TV}(x) & \geq & |w_0(x)| & \textbf{(T3)}\ x_1 & = \\ 1 & \Rightarrow & \|a\|^2 & \geq & 1/\mu_1^P & \Longrightarrow & \mathrm{TV}(x) & \geq \quad \dfrac{1}{4\sin^2(\pi/N)} \quad \sim \quad h^2 \end{array}$$

for *every* admissible trial vector in the parity sector — (T1) because the parity rows are orthonormal (0–5×10^{-16}), (T2) by telescoping with $w_M = 0$, (T3) because the Thomson normalisation is not free. Verified on all 1674 trial vectors built anywhere in the probe (50 taper widths $\times$ 4 weightings $\times$ 3 block sizes), with slack $\mu_1^P\mathrm{TV} = 5.00$–$23.20$ — the floor is tight to a factor of order ten — and the floor itself 2.06×10^3–$1.77 \times 10^5 \sim h^{+1.997}$ (fit; exactly h^2 in closed form since $N = 2h + 1$). **(T4)** Combined with the crossing criterion: every x with $\delta_{\mathrm{bnd}} < \frac{1}{2}$ must pay $P > 2\kappa g_{16}/\mu_1^P = 5.46 \times 10^1$–$5.87 \times 10^3 \sim h^{+1.937}$, already 5.5–590× above the largest declared flat cap — *a flat-price sub-$\frac{1}{2}$ demand is provably impossible on this surface*, for every admissible trial vector and not merely for the families swept. R-C‴ ("bounded total variation at bounded price") is thereby **closed negatively**, by an inequality.

Never in the primes — the sober reading. The relocation T162 suspected is exact, and its h^2 is now identified: the $h^{2.86}$ of T162's Fejér price, the $h^{1.91}$ of the crossing price and T161's granularity are *all* the reciprocal smallest parity eigenvalue — the spectral gap of the parity Laplacian (Kac–Murdock–Szegő 1953) meeting the entry normalisation $x_1 = 1$. Sub-$\frac{1}{2}$ demand at flat price is impossible in the parity sector as long as $1/s$ must stay $O(1)$, which is the whole point of the T152–T160 chain; the statement involves no prime, no taper and no truncation — and the mode sweep confirms it from the other side, since $16 \to 64$ modes buys a price dividend, a worse demand, and leaves the TV exponent at $h^{+1.81}$, a bounded factor above the theoretical floor.

R-B″, R-D, the stress and the anti-fitting control. The chain never needed the sign: in the upper direction $1/s \leq Q(x)$ is Legendre duality plus an *evaluated* positive number, with headroom $\geq 10^{14.9}$ over the rounding floor on every window; T162's indefiniteness is reproduced ($a^{\mathsf{T}}Ga = Q^{\mathrm{arch}}$ to 9.3×10^{-13}, $\lambda_{\min}/|\lambda_{\max}| = -14.6\ldots-2.2$); what the sign would have bought is *uniformity* — the certified cancellation quality at the operating point drifts ($h^{-0.172}$) — so R-B″ reduces to R-B‴:

make the positivity margin h-uniform (T162's a-weighted quarter bar is the right shape of object). R-D is a type mismatch: the taper acts on the trial vector while the block floor $1/g_K$ is a property of the *operator*; smoothing the operator instead (a Fejér 3-tap average of the lag coefficients) degrades the floor by 2.3×10^3–$2.7 \times 10^5 \sim h^{+2.036}$ — the same h^2 again, for the same reason — and bounds a different number: a measured negative, not a device. The must-break controls break: the uniform surrogate drives the total to 7.7×10^2–4.7×10^7 of $|Q|$, the gauge violation to 3.4×10^{-2} of $\|w\|_1$, and the T145 resolution no-go breaks *monotonically* over $\nu = 4 \to 2 \to 1 \to 0.5 \to 0.25$ with damage $1.0 \to 9.1 \to 25.4 \to 129 \to 434$ (maxima; the $\nu = 4$ column an exact null control) — monotone in the collision rate, which is what makes it the T145 mechanism rather than a generic loss of accuracy. And the anti-fitting control is enforced rather than promised: a single globally fixed knob setting $\sigma = 3$, calibrated on the smallest window alone, delivers $\delta_{\text{bnd}} = 0.176$–$0.380$ out of sample on all 26 remaining windows at 0.14–0.23 of P_{aff} — no per-window tuning anywhere. The balance of T163: 11 theorem / 1 certified-uniform / 4 certified-per-window / 10 measured, zero fit rows (the only fitted numbers are h-exponents, labelled in place).

After T163: one successor with two prime-free arms

(R-C‴) **closed, negatively, on this construction**: bounded total variation at bounded price is impossible in the parity sector — $\mathrm{TV}(x) \geq 1/\mu_1^P \sim h^2$ for every x with $x_1 = 1$, an inequality over the whole admissible set. *(R-E)* the successor the theorem points at, both arms prime-free: *arm A* — show that the downstream T159/T160 chain tolerates a *growing* $1/s$ ceiling (a question about the chain, not about the pairing); *arm B* — represent the entry functional in a sector whose lowest eigenvalue does *not* vanish like h^{-2}. *(R-B‴)* make the positivity margin h-uniform (the certified CANC at the operating point drifts, exponent -0.172; the a-weighted quarter bar is the right object). *(R-D)* a fifth device for R1″: $\rho = 1.0036$–$1.0140 > 1$, flat, untouched by the Fejér perspective for the type reason above. T164 (`sector_change_probe.py`, contract `SECTOR.CHANGE`) decided both arms of exactly that successor — next subsection.

10.54 The sector change: the $O(1)$ gate discharges to a quantifier, and sectors are a gauge (T164)

T164 (`sector_change_probe.py`, contract `SECTOR.CHANGE`, 28/28, 31.7 s, the T163 surface rebuilt and re-verified: 28 log-spaced prime-power zones of 70 admissible, $h = 142 \ldots 1445$ inside the declared caps, 27 windows inside the condition horizon $\mathrm{cond}(B_{LL}) \leq 10^{12}$; pairing identity to 2.2×10^{-16}–1.0×10^{-13} of the large scale, T163's TV floor reproduced at the untapered optimiser ($\mu_1^P \mathrm{TV} = 13.77$–$23.20$); $|\psi(x) - x| \leq \kappa x$ with $\kappa = 0.038821$ verified at every jump point in $[100, 4 \times 10^5]$; verdict TOLERANCE-CARRIES (arm A) / SECTOR-RESISTS (arm B)) decides both arms of the successor R-E, and the two answers reshape the map rather than merely extending it.

Arm A — the tolerance analysis, and the power is exactly one. The chain does not use $1/s$; it uses a ceiling U with $1/s \leq U$, and T164 instruments every downstream station with the direction of its arrow. ST1, the direction check: $s \geq g_{16} \geq g_1$ window by window, i.e. $1/s \leq 1/g_{16} \leq 1/g_1 = B_{11}$ — the T158 ladder really is a chain of *upper* bounds on the entry ceiling and the T157 route-(0) rung really is the loosest; $P_K = \hat{a}\, s \geq 1$ is the Kantorovich admissibility the T156 kernel needs. ST3, the heart: at $U = 1/g_{16}$,

$$\frac{d \log(1-F)}{d \log U} = -1.0005 \ldots - 1.0000, \qquad \frac{d \log r}{d \log U} = +1.0000 \text{ exactly},$$

so inflating the ceiling by λ shrinks the T156 loss margin by $\lambda^{-1.00}$: **the kernel spends the ceiling at power exactly one**, and a ceiling growing like h^{ε} costs a margin decaying like $h^{-\varepsilon}$, algebraically, for every $\varepsilon > 0$ (measured for the power, theorem for the monotonicity). The end-to-end ε-scan confirms it — the exponent of the declared proxy $T_{\text{marg}} = (1-F)\, L$ falls by exactly one unit per unit of ε ($+0.391$ at $\varepsilon = 0$ down to -1.609 at $\varepsilon = 2$) — and the declared one-sided rule returns $\varepsilon^* = 0.50$, which the probe *disowns* at once: the $\varepsilon = 0$ baseline already carries a tailwind of $h^{+0.391}$, bought entirely from the measured drift of $L = \lambda_1(A)/\mu_1^P$ ($h^{+0.196}$) on this surface, and ε^* is

nothing but that tailwind divided by power one. No structural slack; the $O(1)$ requirement was real.

The quantifier collapse — the key result of arm A. But who asked for a growing ceiling? T163's price grows because it was buying a *sub-yardstick demand*; the ceiling the chain actually consumes is a different object, logically independent of it, and the ladder certifies it flat: feeding the *single constant* $U_{\mathrm{ref}} = 4.9008 = \max_h 1/g_{16}$ into the downstream map — no h-dependence anywhere in the substitution — gives $T_{\mathrm{marg}} = 7.1 \times 10^{-2}$–$4.0 \times 10^{-1}$ with exponent $h^{+0.391}$, non-decreasing against the one-sided bar and positive on all 9 sub-surface windows, out of sample. The ceiling itself is $1/g_{16} = 1.7527$–5.3286 over the full 27-window surface, $h^{+0.061}$ over a 9× lever arm (split halves $h^{-0.010}$ / $h^{+0.103}$, spread 3.040), and every ladder g_K is strictly increasing in K on every window (Schur 1917 nested complements), so $1/g_{16}$ is the tightest rung and the certificate is monotone as well as flat.

> **The $O(1)$ gate is not open as a quantity** — it is discharged window by window by a Cholesky identity. What a proof still needs is exactly one thing: $\sup_m 1/g_{16}(m) \;<\; \infty$ — the m-freedom of a certified flat list.

A certified flat *list* is not a certified flat *function*: nothing here upgrades the per-window certificate to a uniform one, and the split-half test is a consistency check, not a proof of the sup. But the residue now has the same grammatical form as every other open uniformity item in the chain — a per-window certificate plus a quantifier over windows — and it is no longer an existence question about trial vectors, which is what R2″ was before T163.

Arm B — the sector change is negative by a theorem, and the theorem organises all three candidates. A sector change, in the only sense the chain admits, replaces the entry spectrum μ by another positive spectrum μ' and leaves the lag Gram alone. The structural fact, stated as the theorem it is: $x_1 = 1$ fixes $a_1 = 1/\sqrt{\mu_1}$, i.e. only the *scale* of a; Q and TV are both homogeneous of degree two in a; hence $\mathrm{TV}(a)/Q(a)$ depends only on the direction of a and **is the same number in every sector — the entry normalisation is a gauge** (machine-checked to 0.0–8.8×10^{-10} over 5670 sector-by-trial-vector combinations — 35 sector spectra × 6 taper widths × 27 windows — against the declared cancellation horizon 2.6×10^{-8}, the residual sitting 29× below it). The crossing criterion $Q/\mathrm{TV} > 2\kappa = 0.0776$ does not contain the normalisation and cannot be moved by any sector change. Per candidate: (i) the *full space* (dropping the parity projection) is settled in closed form and is strictly worse — $\mu_0 = 0$ exactly (the constant vector), so the T3 step has no finite floor at all, and even excluding it the lowest full-space eigenvalue never exceeds μ_1^P; (ii) a *shifted or weighted sector* does flatten the floor — $\mathrm{TV} \geq 1/(\mu_1^P + \kappa_s)$ is $h^{+0.000}$ at $\kappa_s = 4$, so T163's obstruction really is absent there — but the certified value scales by $\theta = \mu_1^P/(\mu_1^P + \kappa_s)$ and recovering a bound on the original $1/s$ costs the transfer factor $1/\theta \sim h^{+1.997}$, with

$$\text{floor} \times \text{transfer} \;=\; \frac{1}{\mu_1^P} \;\sim\; h^{+1.997} \quad \text{identically}$$

at every shift on the declared grid (the two exponents sum to +1.997 to 10^{-9}) — an exact product identity: **the h^2 is conserved, not removed — it lives in the ratio, never in the floor**; (iii) the *Mellin/frequency side* becomes, through the invariance, a well-posed normalisation-free problem, treated next.

The unconstrained ascent, and the TV floor holds where T163 never searched. Summation by parts (Abel 1826, $w_M = 0$) gives the identity $Q = \sum_e (\Delta w)_e\, C_e$ with C the partial sums of the lag coefficients (checked to 4.7×10^{-13}–4.8×10^{-10} inside its declared horizon), hence $Q/\mathrm{TV} \leq \|C\|_\infty = 30.27$–$930.60 \sim h^{+1.185}$ — the ceiling on the gauge-invariant demand ratio is a window-length quantity that *grows*, so nothing on the frequency side forbids the crossing, and the natural coordinate is the alignment efficiency $\eta := (Q/\mathrm{TV})/\|C\|_\infty$. A declared first-order ascent on $Q(v)/\mathrm{TV}(v)$ over the full window (all 142–1320 coordinates, 17 restarts × 250 steps, no truncation, no normalisation) reaches $Q/\mathrm{TV} = 2.38$–74.57 against the bar $2\kappa = 0.0776$ — 31–960× **over, on** 9 **of** 9 **windows** ($\delta_{\mathrm{bnd}} = -0.124 \ldots -0.090$, $\eta = 7.9$–11.1%, cancellation headroom 3.8×10^{15}) — and

then the price is put back: normalising the ascent output to $x_1 = 1$ gives TV $\mu_1^P = 8.30$–$11.72 \geq 1$ — **T163's total-variation floor holds on the unconstrained optimiser too**, a family T163 never searched and the strongest available confirmation that (T1)–(T3) is an inequality rather than a property of the swept families — at a price $P_{\mathrm{pr}} = 1.39 \times 10^4$–$4.41 \times 10^7 \sim h^{+3.299}$, above the T163 crossing floor on every window and 3.5–7.6× above P_{aff}: worse than T163's own crossing price $h^{+1.91}$. Dropping the normalisation moved the difficulty from one coordinate to the other without reducing it; the binding axis was never the demand and never the sector — it is the *alignment* between Δw and the partial sums C, and that quantity is itself gauge-invariant.

R-B‴ narrowed, R-D settled in type, the stress. R-B‴ turns out to be a *naming* problem — two different quantities were being called the same name: the T162 a-weighted quarter-bar object $|\sum_{k\neq l} a_k a_l G^{\mathrm{arch}}_{kl}|/|Q^{\mathrm{arch}}| = 0.1001$–$0.1623$ carries the $h^{-0.172}$ (T163's exponent reproduced to 0.0004) and sits under the 0.25 bar on all 27 windows with 35% of margin, while the raw arch/atom cancellation is a different number ($h^{-2.04}$). The declared two-variable regression puts -0.046 of the quarter-bar drift on $\log h$ against -0.477 on $\log \alpha$ — 10× more on the *zone scale* — and on this recipe α and h are coupled by construction (the window comes from a prime gap), so a regression on h alone misattributes an α-effect to h: the -0.172 is *not* established as an h-gate, and separating it needs a surface on which α and h vary independently — a narrowing, not a closure. R-D is untouched in value ($\rho = 1.0036$–$1.0140 > 1$ flat, T161, quoted) and settled in *type*: under the shift the three kernel inputs move in exactly compensating directions — $s \to s/\theta$, $\hat{a} \to \theta\hat{a}$, $L \to \theta L$ — so P_K, the r-ceiling and $F(P_K, r)$ are unchanged to 3.5×10^{-16} on 54 window-by-shift combinations: every object the chain consumes downstream is a ratio of equally-homogeneous forms, hence *no fifth device can come from a sector change*. The must-break controls break: the uniform surrogate drives the total to 1.5×10^3–4.9×10^7 of $|Q|$, the T145 no-go breaks monotonically over $\nu = 4 \to 2 \to 1 \to 0.5 \to 0.25$ with damage $1.0 \to 9.1 \to 25.4 \to 129 \to 434$ (maxima; exact null control, damage ≥ 26.1 at the coarsest setting on every window), the gauge violation costs 3.4×10^{-2} of $\|w\|_1$; the classical controls are exact (Dirichlet kernel $\leq 2.0 \times 10^{-14}$ on 64 random triples, parity Laplacian exact, orthonormality 3.6×10^{-15}), and the correlation form is verified on *random* v (3.8×10^{-16}) — the ascent measures the same object the chain does. The balance of T164: 11 theorem / 1 certified-uniform / 4 certified-per-window / 6 measured, zero load-bearing fit rows (the only fitted numbers are h-exponents and the one two-variable regression, labelled in place); candidates T164-Q1…Q9 all PENDING, no promotion here (bundled with T165 into v556, Section 4.22).

After T164: one quantifier, one alignment question

(R-E-A) **reduced to a quantifier**: the chain tolerates no growing ceiling (power one), but the ceiling it consumes is $1/g_{16}$, certified per window by a Cholesky identity and measured flat — what remains is $\sup_m 1/g_{16}(m) < \infty$, the m-freedom of a certified flat list. *(R-E-B)* **closed, negatively, by a theorem**: the entry normalisation is a gauge, the demand ratio and the whole T156 kernel are sector-invariant, the full space is worse in closed form, and every floor-flattening shift pays the identical exponent back as a transfer factor. *(R-F)* the genuinely new successor, gauge-invariant and prime-free: is there a v with $Q(v)/\mathrm{TV}(v) > 2\kappa$ *and* $Q(v)$ comparable to $1/s$ — equivalently, can the alignment efficiency η stay above $2\kappa/\|C\|_\infty$ at bounded price? *(R-B‴)* narrowed: the drift belongs to the quarter-bar object and mostly to the zone scale; a surface with independent α and h is needed. *(R-D)* untouched in value, settled in type: no fifth device from a sector change. T165 (`alignment_eta_probe.py`, contract `ALIGNMENT.ETA`) closed exactly that alignment question by certificate — next subsection.

10.55 The alignment question: the successor closes by certificate, and one quantifier remains (T165)

T165 (`alignment_eta_probe.py`, contract `ALIGNMENT.ETA`, 30/30, 15.1 s, the T163/T164 surface rebuilt and re-verified with the window ladder up to $h = 1445$, 27 windows inside the condition horizon; the dashboard $R = Q/\mathrm{TV}$, $\eta = R/\|C\|_\infty$, P_{pr}, $\mathrm{TV}/(t_1 v)^2$ and the quarter statistic all checked gauge-invariant under $v \to tv$; $\kappa = 0.038821$ verified at every jump point; verdict ETA-RESISTS in the strong form) answers R-F — and the answer is a structure statement, not a failed

search. R-F asked for a trial vector with alignment efficiency η above $2\kappa/\|C\|_\infty$ at bounded price; no such vector exists on anything measured, and the reason is an *identity* plus a *theorem*.

The one equation the part is about. Write the four gauge-invariant factors side by side; dividing the price by the demand and the floor gives, with no inequality anywhere,

$$P_{\mathrm{pr}}(v) \;=\; g_{16}\cdot R(v)\cdot \frac{\mathrm{TV}/(t_1 v)^2}{\mu_1^P}, \qquad R \;=\; \frac{Q}{\mathrm{TV}} \quad \text{(machine-checked to } 3.3\times 10^{-16}\text{)}.$$

Four factors, each a named object of the chain: g_{16} is the R-E-A quantifier, R is the demand, $\mathrm{TV}/(t_1 v)^2 \geq 1$ is the T163 theorem, and $1/\mu_1^P = 1/(4\sin^2(\pi/N))$ is the Kac–Murdock–Szegő 1953 spectral scale — exactly the h^2 the whole chain has been paying since T152. Demand and price are *one* equation, so the two clauses of R-F cannot be chosen independently; with T163's floor, every crossing vector pays $P_{\mathrm{pr}} \geq 2\kappa g_{16}/\mu_1^P$ — the old h^2. The anatomy of the free optimum, measured: it is a *low-harmonic* object (93.7–96.8% of its energy in the first 32 parity modes against an equidistribution baseline of 2.4–22.5%, lift 4.2–39.7), essentially orthogonal to the PNT main term (cosine $-0.070\ldots-0.031$), with heavy edges on prime-power-occupied cells (lift 1.36–2.12); the alignment is a *cancelling* sum over the whole window (participation 8–18%, mass centroid 0.72–0.92).

The exact price decomposition. The identity turns the price exponent into a sum of four exponents with no remainder, and the decomposition closes to 0.0:

$$h^{+3.261} \;=\; \underbrace{+1.997}_{\text{KMS } h^2 \text{ (since T152)}} \;+\; \underbrace{+1.279}_{\text{demand overshoot}} \;+\; \underbrace{+0.093}_{\text{floor}} \;-\; \underbrace{0.108}_{g_{16}},$$

i.e. 61% of the exponent is the same h^2 the series has met since T152 (reproduced to 0.003 of the exact -2), and the rest is the ascent overpaying for demand the chain never asked for; the bar-tight counterfactual price stays $h^{+1.982}$. The η(Cap) profile is two-sided and the ceiling is a theorem (identity + floor): at the preregistered tolerance Cap = 10 the certified ceiling is 2.95×10^{-7}–4.69×10^{-4} against the requirement $2\kappa/\|C\|_\infty = 1.16\times 10^{-4}$–$2.56\times 10^{-3}$ — **short by** 5.5–392× **on every window,** 0/27 **capped-ladder points reach the bar**.

The equivalence — R-F was never independent. Dividing T163's exchange law by the identity above, the T163 normalisation drops out completely: $P_{\mathrm{pr}} > 2\kappa g_{16}(\mathrm{TV}/(t_1 v)^2)/\mu_1^P \iff R > 2\kappa$ — the demand half of R2″ and the gauge-invariant crossing condition are *the same predicate* (105/105 probe vectors). R-F is therefore strictly *stronger* than the R2″ demand (it is demand *and* affordability), and the two clauses collide by the identity plus the floor alone: **R-F is empty, and its m-uniform form *is* R-E-A**. T2 localises the quantifier further: the m-free ingredients of the chain (the head split, the atom budget $4(1+\kappa)\sqrt{X}$, the moment laws) bound B_{11} *sharply* (factor 21–26, flat $h^{+0.039}$) — but it is the wrong object: $1/g_1$ grows $h^{+3.110}$ while $1/g_{16}$ is flat $h^{+0.061}$, and the exponents close exactly ($+3.110 - 3.049 = +0.061$); the entire certification lives in the 15 ladder increments (half the gain by rung 2–5). The residue is still one quantifier, but smaller: *bound the cascade gain g_{16}/g_1 from below by $h^{3-\varepsilon}$*, one scalar per window — and provably not reachable from absolute-value budgets, because it is a *cancellation*.

The decoupled surface — the quarter-bar confound settled, and one constant honestly retired. The frame-A recipe couples α and h by construction; T165 decouples them with ν as a knob (at fixed zone it multiplies h against α, with the gap and the atom set fixed): 25 admissible windows over 6 zones, $\nu \in \{4,5,6,8,11,16\}$, collinearity of $\log\alpha$ against $\log h$ falling $0.87 \to 0.68$. Finding: **the quarter-bar drift is zone depth, not window length (factor** 19) — the same answer as T2.2 from independent statistics, settling the R-B‴ confound T164 named. Two honest costs, reported openly: $\sup 1/g_{16}$ rises to 5.7327 off-recipe — exceeding T164's $U_{\mathrm{ref}} = 4.90$ by 17%, so that constant was frame-A-specific and is *retired* as a surface constant (an errand: replace it wherever a chain station quotes it) — and the h-coefficient of $1/g_{16}$ is no longer negligible at free ν. End-to-end on the union of both surfaces (52 windows, $h = 142$–1320): every crossing vector has $P_{\mathrm{pr}} \geq 54.6$–$5865 > \Theta_{\mathrm{TOL}} = 10$, and in closed form for every $h > 84.9$. Stress: the T145 coarsening staircase is monotone (1.0/2.8/12.1/27.5, up to 102×); the uniform surrogate breaks by

8.6×10^3–6.9×10^4; the gauge perturbation breaks; Dirichlet/KMS controls exact to 10^{-15}. The balance of T165: 11 theorem / 1 certified-uniform / 4 certified-per-window / 6 measured — the theorems carry more than the measurements; six new candidates, all PENDING. The theorem-shaped cores of T164+T165 — the gauge theorem, the h^2 conservation law, the power-one spend, the P_{pr} identity with the predicate equivalence, and the Schur-cascade direction with its exact exponent closure — are promoted as v556 [verification/v556_gauge_ppr_identities.py] (Section 4.22).

After T165: exactly one genuine open object

(The quantifier) $\inf_m g_{16}(m) > 0$ — equivalently a lower bound on the sixteen-step Schur-cascade gain, $g_{16}/g_1 \geq c\,h^{3-\varepsilon}$ uniform in m: a quantifier over a certified flat list, one scalar per window, a *cancellation* and hence provably beyond absolute-value budgets. Everything else that looked open at the end of T164 has changed type rather than moved: *(R-F)* closed negatively by the P_{pr} identity plus the T163 floor (the η route is not a device, and its uniform version is the same quantifier); *(R-B‴)* settled as a zone-depth effect on a genuinely decoupled surface, at the cost of retiring the out-of-sample constant $U_{\mathrm{ref}} = 4.90 \to 5.7327$. No missing term, no missing device, no sector. T166 (`schur_cascade_probe.py`, contract `SCHUR.CASCADE`) dissected exactly that cascade lower bound — next subsection.

10.56 The cascade dissected: the gain is a Gram-determinant cancellation, and one inequality in three dresses remains (T166)

T166 (`schur_cascade_probe.py`, contract `SCHUR.CASCADE`, 30/30, 2.1 s, the two surfaces rebuilt as a *union*: 27 frame-A windows ($h = 142$–1320, the T163–T165 recipe and density) plus 36 decoupled-ν windows (6 zones $\times$ $\nu \in \{4, 5, 6, 8, 11, 16\}$, $h = 96$–1371) = 63 windows with a 14$\times$ lever arm on which zone depth and resolution are *independent* knobs ($\log h$ against $\log \nu$ correlated at only 0.33 — an exponent read on the union is a statement about m, not about the recipe); T165's three numbers reproduced on frame A ($1/g_1 = B_{11} \sim h^{+3.110}$, $1/g_{16} = 1.7527$–$5.3286 \sim h^{+0.061}$, gain $\sim h^{+3.049}$, exponents closing exactly), the union reading $h^{+2.759}/h^{+0.193}/h^{+2.566}$; $\kappa = 0.038821$ verified at every jump point up to $n = 4 \times 10^5$; verdict CASCADE-RESISTS) attacks the one open object of T165 head-on. The answer is a complete anatomy, an honest exponent gap, and a sharper statement of what is missing than the line has ever had.

Four identities before any number. Written as the Cholesky ladder $g_K = \sum_{j \leq K} y_j^2$ the increments look like fifteen unrelated positive numbers; four elementary identities — all theorems, re-verified on all 63 windows and five rungs each — turn the ladder into one readable object: *(I1)* the prefix property ($(L_K^{-1} e_1)_j$ does not depend on K, so *one* Cholesky yields all sixteen rungs; 1.2×10^{-15}); *(I2)* the minor form $g_K = \det Q_K^{(1)} / \det Q_K$ (Schur 1917) — the increments are closed objects in the block entries; *(I3)* the regression form,

$$\frac{g_K}{g_1} \;=\; \frac{1}{1 - R_K^2}, \qquad R_K^2 \;=\; \frac{b^{\mathsf{T}} B_{HH}^{-1} b}{B_{11}} \;\in\; [0, 1) \qquad \text{(checked to } 2.1 \times 10^{-9} \text{ on } 63/63\text{)},$$

so the open object *is* a near-collinearity statement: $g_{16}/g_1 \geq c\,h^{3-\varepsilon}$ says mode 1 is explained by modes $2 \ldots 16$ up to a residual fraction $1 - R_{16}^2 \leq h^{-3+\varepsilon}$ — a far more specific request than "a cancellation"; and *(I4)* the diagonal invariance,

$$\frac{g_K}{g_1} \;=\; B_{11}\,(Q_K^{-1})_{11} \;\text{ is invariant under }\; B \;\to\; DBD \;\text{ for every positive diagonal } D,$$

so the gain does *not* see the KMS $1/\sqrt{\mu_k^P}$ normalisation at all: the h^3 is a property of the arithmetic Gram block $G = TAT^{\mathsf{T}}$ alone (Kac–Murdock–Szegő 1953) — which kills a whole family of "it is an artefact of the normalisation" objections before it is raised. *The rung profile is extreme:* rung 2 alone carries a median 59.26% of the whole gain (range 0.14–0.85), rungs 2–5 a median 71.79%, $K_{\mathrm{half}} \in \{2..5\}$ on 63/63 windows, and the chain's flat object first appears at rung $K = 6$ — the h^3 is never spread over the block. Rung 2 in closed form: $y_2^2/y_1^2 = r_{12}^2/(1 - r_{12}^2)$ with $1 - r_{12}^2 = 1.3 \times 10^{-7}$–$4.5 \times 10^{-4} \sim h^{-2.92}$ on frame A ($h^{-2.25}$ on the union) — *one* rung multiplies g by 2207–7.8×10^6, and the price of certifying it is a number: an absolute-value budget would have

to resolve B_{11} against B_{12}^2/B_{22} to that relative accuracy, where the m-free budgets are sharp only to a flat factor 21–26. The Krylov reading is exact: the T158 bridge reproduces the entry s over $\mathrm{span}\{t_1, A^{-1}L_P t_1\}$ to 5.5×10^{-10} (a theorem — the maximiser lies in the span), $g_{16} \le s$ on 63/63, $g_{16}/s = 0.69$–0.95 (median 0.88), and the bottom eigenvector of A is confined to 0.9999–1.0000 in the sixteen low parity sines (T146/T157 reproduced): the cascade is the mode-truncated Ritz approximation of the Green column.

The cancellation finding — a third mechanism. The block was set up to distinguish two stories: an arch-against-atom cancellation in the lag sums (the T159/T160 mechanism — then no closed object sees it) or a mode-against-mode effect in the archimedean kernel alone (then the mechanism is explicit and m-free). **Neither holds**: no entry of the 2×2 bottom block cancels (G_{11}, G_{12}, G_{22} each keep 60–99% of their larger half), and neither half alone is collinear (the archimedean block has $1 - r_{12}^2 = 0.98$ — no degeneracy whatsoever — the atom block 0.04–0.50) — yet the *sum* is degenerate to seven digits. The polarisation split (a theorem, checked to 6.7×10^{-14}) locates it:

$$\det(G_{\mathrm{ar}} + G_{\mathrm{at}}) \;=\; \det G_{\mathrm{ar}} + \det G_{\mathrm{at}} + \Pi, \qquad \Pi \;=\; G_{11}^{\mathrm{ar}} G_{22}^{\mathrm{at}} + G_{11}^{\mathrm{at}} G_{22}^{\mathrm{ar}} - 2 G_{12}^{\mathrm{ar}} G_{12}^{\mathrm{at}},$$

with pieces of size 2.6–630 cancelling to a full determinant of 1.0×10^{-3}–1.7×10^{-2} — a ratio of 2.6×10^{-6}–5.1×10^{-4} decaying as $h^{-1.45}$. **The cancellation lives in the 2×2 Gram determinant**: the same arch-against-atom mechanism as T159/T160, one level up, in a *quadratic* functional of the lag sums rather than in any lag sum. And it is pinned to the arithmetic twice over: the L_P no-go control breaks *completely* (for $A = L_P$ the normalised block is the identity to 3.6×10^{-15} and the cascade gain is exactly 1.0000000000 — no cascade at all), and the anti-fitting scramble (uniform random atom positions at the *same* total mass) destroys the near-degeneracy by a factor 4569 — the mechanism hangs on *where* the prime powers sit, not on the atom budget, the kernel or the discretisation.

The best lower bound, and the one inequality left. Every route is an instance of one theorem ($1/g_K = \mu_1^{P\,-1} \min\{u^{\mathsf{T}} G u : u_1 = 1\}$ — a trial vector bounds g_K from *below*, the only useful direction). The two-rung route with a closed β has its admissible window stated as a number — relative accuracy 3.57×10^{-4}–2.13×10^{-2} on β^* — and all three declared closed candidates miss it (by 12.4–2540; 0/63 windows), the best (from the atom block) certifying gain $h^{+1.319}$ against the $h^{+2.566}$ the exact ladder delivers. Combining the minor form with the diagonal invariance gives the sharpest restatement the line has ever had:

$$\frac{1}{g_K} \;=\; \frac{\det G_{[1..K]}}{\mu_1^P \det G_{[2..K]}}, \qquad \mu_1^P = 4\sin^2(\pi/N) \quad \text{(checked to } 3.9 \times 10^{-9} \text{ on 252 evaluations)},$$

first flat at $K = 6$ on the union — so exactly *one* uniform inequality is missing: $\det G_{[1..6]} \le U \cdot \mu_1^P \det G_{[2..6]}$. The errands close: U_{ref} is redone off-recipe over the union with a declared safety factor — $\sup 1/g_{16} = 6.4781$, attained at a *deep* zone at moderate ν (the union supremum is driven by zone depth, not resolution), $U_{\mathrm{ref}}(\mathrm{union}) = 1.15 \times 6.4781 = 7.45$, superseding T164's 4.90 and T165's 5.7327 (CERT-WINDOW, never CERT-UNIF); the multi-covariate fit separates the knobs (gain $\sim h^{+2.92}\,\nu^{-1.53}$); and the full chain $g_1 \le$ best-of-routes $\le g_{16} \le s$ is verified on 63/63. A $K = 64$ test shows the confinement surrogate's shortfall is *truncation* (the same v_0 projected on 64 modes improves $h^{+1.963} \to h^{+2.574}$) — good news about the mechanism, not a certificate at either K.

Two honest contract corrections. (K1) Form F2 (gain $\ge c\,h^{3-\varepsilon}$) is strictly *weaker* than form F1 ($\inf_m g_{16} > 0$) on this surface, because B_{11} grows $h^{+3.110}$ and not h^3 — F1 would need $\varepsilon \le -0.110$, so honouring F2 is not the same as closing the quantifier; every verdict distinguishes the two. *(K2)* The confinement route was a *sub-surface artefact*: $h^{+2.714}$ on frame A restricted to $h \le 780$ (an earlier pass's eigen-horizon — it would have returned ONE-TERM-MISSING) against $h^{+1.963}$ on the full union, exposed by raising the eigen-horizon to the cap instead of leaving it as a to-do; the $+0.751$ of optimism exceeds the whole ε budget — a concrete reason to distrust any exponent quoted on a sub-surface. The verdict criterion is an *exponent gap* so it cannot be argued about: CASCADE-CARRIES needs a closed route reaching the measured B_{11} exponent $+3.110$; ONE-TERM-MISSING needs the gap to be a flat factor (below 0.05 in the exponent); measured: best closed

route $h^{+1.319}$, target $h^{+3.110}$, **gap** $h^{+1.791}$ — **an exponent, not a constant: the one missing inequality *is* the cancellation**, the same intrinsic hardness as the h^2 cancellation of T159/T160. The balance of T166: +5 theorem / + 3 certified-per-window / + 4 measured, and deliberately *no* new certified-uniform row — the honest statement: nothing here became uniform in m; nine candidates P166.1…P166.9, all PENDING (no promotion; to be bundled with T167).

After T166: one inequality, three dresses

(The inequality) an m-free upper bound on the Gram-minor ratio $\det G_{[1..K]}/(\mu_1^P \det G_{[2..K]})$ at the first flat K — equivalently a closed β at $K=2$ to relative accuracy 3.6×10^{-4}, equivalently a closed near-null vector of A in the low modes: the *same* inequality in three dresses, and it is the whole quantifier — everything else in the reduction is now theorem. *(Bookkeeping)* every statement carrying U_{ref} names its surface (7.45 on the union, off-recipe, CERT-WINDOW). Off the list: "a better trial space" (the contract object *is* the optimum over the sixteen modes), "which rungs matter" (rungs 2–5, measured, rung 2 closed), "arch-vs-atom or mode-vs-mode" (arch against atom, in the Gram determinant). T167 (`null_vector_probe.py`, contract `NULL.VECTOR`) attacked exactly that most constructive dress — next subsection.

10.57 The closed vector is exact at $K=2$: three dresses are one inequality (T167)

T167 (`null_vector_probe.py`, contract `NULL.VECTOR`, 39/39, 1.4 s, the 63-window union of T166 rebuilt bit for bit (27 frame-A windows $h=142$–1320 plus 36 decoupled-ν windows, $h=96$–1371, lever arm 14×); T163–T166 reproduced as fits, never as inputs (frame A $B_{11}\sim h^{+3.110}$, gain $\sim h^{+3.049}$; union $h^{+2.759}/h^{+0.193}/h^{+2.566}$, closure 4.4×10^{-16}); $\kappa=0.038821$ verified at every jump point up to $n=4\times10^5$; verdict VECTOR-RESISTS) attacks the most constructive of the three dresses — the closed fast-null-vector — and returns the strongest reduction of the series.

The pivot identity, and the exact vector. Twenty-two candidate families are *preregistered* before any number is seen (the Neumann/Jacobi and Kato/Rayleigh–Schrödinger perturbation routes, the Gram route, the moment route, and the KMS/arch/exact controls), and all of them are evaluated through one instrument:

$$u=x_K^*+\delta,\quad \delta_1=0 \implies u^{\mathsf T}Q_K u \;=\; \frac{1}{g_K}+\delta^{\mathsf T}Q_K\delta \qquad \text{(theorem; 10206 evaluations, } 3.2\times10^{-9}\text{)},$$

because $Q_K x_K^* = e_1/g_K$ and $\delta_1=0$ kills the cross term — so the relative excess $\rho = g_K\,\delta^{\mathsf T}Q_K\delta \ge 0$ of every candidate is an *exact number*, never an estimate, and by the trial theorem ($u^{\mathsf T}Q_K u \ge 1/g_K$ for every $u_1=1$; 252 random trials, Schur 1917) every candidate is a certified *lower* bound. The central positive result:

$$u=\Big(1,\,-\tfrac{Q_{21}}{Q_{22}}\Big)\ \text{attains}\ \frac{1}{g_2}=Q_{11}(1-r_{12}^2),\qquad B_{11}\,g_2=\frac{1}{1-r_{12}^2}\qquad \text{(exact on 63/63; } 8.7\times10^{-10}\text{)},$$

with $r_{12}^2=Q_{12}^2/(Q_{11}Q_{22})$: **at $K=2$ the fast-null-vector needs no approximation at all — the vector is free**, $\rho=0$ as a theorem, and the certified gain exponent is $h^{+2.921}$ on frame A — that is $h^{3-0.079}$, already inside the preregistered ε budget — and $h^{+2.246}$ on the union, with rung 2 carrying 96% of the exponent: dress (c) collapses onto dress (b).

Why perturbation theory dies — and it is not what one would guess. The Jacobi map genuinely diverges (spectral radius 1.0–12.7 at $K=2/6/16$), so the Neumann route fails honestly; but the Kato reduced-resolvent map has radius only 0.067 — **it converges, fast, to the wrong object**: the Rayleigh–Schrödinger series builds the *eigenvector* attached to level 1 (Kato 1949), and the constrained minimum $1/g_K$ is not an eigenvector problem. The diagnostic that proves it: the exact bottom eigenvector of Q_6 is itself a useless trial ($\rho=6.04$), because its overlap with mode 1 is only $|v_1|=0.083$ and the trial theorem divides by v_1^2; the underlying structure is the relative spectral gap 5.4×10^{-7} — the RS denominators are meaningless, the fast degeneracy makes the level assignment meaningless. This closes off the perturbation route in *every* order. The moment route reaches $h^{+1.320}$, reproducing T166's $h^{+1.319}$ to three digits (flagged, not interpreted).

The thresholds, and the monotonicity against the hypothesis. Two thresholds are separated and both envelopes verified on 189 random perturbations each: the vector accuracy $\mathrm{eps}_{\mathrm{vec}} = \sqrt{\rho/(g_K\lambda_{\max})}/\|x^*\|$ and the entry accuracy $\mathrm{eps}_{\mathrm{ent}} = \rho\,(1/g_K)/S_K$ — its *square* (the square law is exactly why the vector dress looked cheaper, and at $K = 2$ genuinely is). **The threshold sharpens monotonically in K — against the working hypothesis it is mildest at $K = 2$** (by 1.90 in the median and $h^{+0.353}$ in trend: $\mathrm{eps}_{\mathrm{ent}} \sim h^{-2.246}$ at $K = 2$ against $h^{-2.600}$ at $K = 6$), the *only* K at which the closed vector is exact — the line should be worked at $K = 2$, and the first flat $K = 6$ is the worst place to stand: there no closed family hits (best separation $h^{+1.138}$, best ratio already 264× — the achieved and required accuracy curves *diverge*). The anchor: $\mathrm{eps}_{\mathrm{ent}}(2) \to (1 - r_{12}^2)/4$ — T166's independently derived 3.57×10^{-4} is literally the quarter of $1 - r_{12}^2$, same exponent.

End-to-end and stress. The best closed candidate certifies gain 2207.3–7.84×10^6 ($h^{+2.921}$ on frame A against the preregistered $\varepsilon_{\mathrm{carry}} = 0.50$ bar); gram2's ρ at $K = 16$ stays in $[0.17, 6.16]$ and $(1 + \rho)$ carries $h^{+0.320}$ — **the deficit is the higher rungs, not the vector**. All four must-breaks break, and the scramble breaks *harder* than T166 saw: 8/8 **windows lose positivity of the 2×2 diagonal itself** ($Q_{11} = -1.35 \times 10^5 \ldots -1.02 \times 10^4$) — at $K = 2$ the collapse under scrambled prime-power positions is a change of *type*, not a factor: g_2 does not even exist there. Headroom 5.7×10^8–2.0×10^{12}. The anti-fitting control (power-fitted coefficients — the most generous cheat) reaches only $h^{-0.823}$, *worse* than the honest candidate. The off-recipe sweep (1.6× denser, same cap) reads $1 - r_{12}^2 \sim h^{-2.740}$ against $h^{-2.921}$ (+0.181): the exponent is a property of the arithmetic Gram block, not of the zone recipe.

The unification — the key result. The entry threshold is an identity at every K of the ladder, to machine precision:

$$\mathrm{eps}_{\mathrm{ent}}(K) \;=\; \rho_{\mathrm{carry}}\,\frac{1/g_K}{S_K}, \qquad S_K \;=\; |x_K^*|^{\mathsf{T}}\,|Q_K|\,|x_K^*| \quad \text{(machine precision at every } K),$$

and $(1/g_K)/S_K$ — the certified value against the absolute-value mass — *is* the one object of all three dresses: T166's determinant ratio against its polarisation pieces (dress a), the $1 - r_{12}^2$ of dress (b) up to the closed factor $1 + 3r_{12}^2 \to 4$ (T166's independently derived quarter is this factor), and the entry threshold of dress (c). **There is one inequality, not three.** The verdict is VECTOR-RESISTS, with ONE-TERM-MISSING deliberately *not* rendered: exactly one term is missing and it has a number, but that term *is* dress (b) itself — rendering it would misread a reduction as progress on the inequality. The balance of T167: 6 theorem / 0 certified-uniform / 2 certified-per-window / 7 measured — certified-uniform zero *by construction*: T167 supplies a map, not a step towards the bound. The theorem-shaped cores of T166 and T167 — the cascade identities, the diagonal invariance, the exact $K = 2$ identity, the pivot identity with the unification, and the trial theorem with both must-breaks — are promoted as v557 [verification/v557_cascade_vector_identities.py] (Section 4.23).

After T167: one scalar

(R1, the only item) an m-free upper bound on the single scalar $1 - r_{12}^2 = 1 - Q_{12}^2/(Q_{11}Q_{22}) \le C\,h^{-3+\varepsilon}$ — three closed lag sums (the T159 apparatus, no zeros, no L-evaluation, finite Λ-sums only); required relative accuracy on r_{12}^2: 1.25×10^{-5} at the median union window, 3.19×10^{-8} at the worst; the measured exponent $h^{+2.921}$ ($\varepsilon = 0.079$) sits comfortably inside $\varepsilon_{\mathrm{carry}} = 0.50$. Nothing else is missing: the vector is exact and the gain is the inverse of this scalar. *(Closed off permanently)* perturbation theory of every order around the KMS bottom mode (the Kato series converges to the wrong object); position-blind m-free surrogates (the archimedean-only minimiser certifies $h^{-0.017}$, i.e. nothing; the type-change scramble). *(Redirected)* work at $K = 2$, not at the first flat $K = 6$ (milder by 1.90 and $h^{+0.353}$). T168 (`lagrange_minors_probe.py`, contract `LAGRANGE.MINORS`) attacked exactly that scalar through the Lagrange identity — the determinant as a sum of squares — next subsection.

10.58 The Lagrange identity: the sum of squares is real, and the hardness is self-similar (T168)

T168 (`lagrange_minors_probe.py`, contract `LAGRANGE.MINORS`, 39/39, 2.3 s, the same 63-window union as T163–T167, recipe untouched, $h = 96$–1371; T167's scalar reproduced bit-compatibly

before it is attacked ($1 - r_{12}^2 = 1.28 \times 10^{-7}$–$4.53 \times 10^{-4}$, $h^{-2.921}$ on frame A); $\kappa = 0.038821$ verified at every jump point up to $n = 4 \times 10^5$; verdict MINORS-RESIST) attacks R1 through the Lagrange identity — the determinant as a sum of squares — and returns a fully mapped relocation.

The anatomy is exact. $Q = X^{\mathsf{T}} A X$ with $X = [u, v]$ the two lowest parity modes in the A-metric (3.9×10^{-15}); the mode vectors are closed trigonometry, m-free (1.0×10^{-14}), and — the first anatomy fact — **Euclidean-orthogonal**: the Euclidean r_{12} is 0 to 6.2×10^{-17}, so the near-parallelism $r_{12}^2 = 1 - O(h^{-3})$ is created *entirely* by the arithmetic metric A and not at all by the vectors. The Wronskian telescopes exactly,

$$W_r - W_{r-1} = (\mu_1 - \mu_2)\, u_r v_r \quad (1.7 \times 10^{-13};\ W_{h-1} = 0 \text{ to } 2.7 \times 10^{-18}),$$

so the near-diagonal minors are one closed one-parameter family — a Dirichlet partial sum; the minor sup is closed and m-free ($\sim N/\pi^2$, attained to 0.770); and Cauchy–Binet is verified as algebra (1.1×10^{-15} on random indefinite A): the second compound of A is the exact kernel of the Lagrange form.

The finding. **On every one of the** 63 **windows the** $h \times h$ **arithmetic kernel** A **is positive definite** ($\lambda_{\min} = 5.0 \times 10^{-6}$–$8.5 \times 10^{-4}$, 8–12 orders above the solver floor), so the second compound is PSD and the Lagrange identity is a genuine sum of squares with strictly positive weights — $\rho_{\mathrm{LAG}} = S_+ / \det Q = 1.000$ exactly, no sign cancellation. Hard fence, stated with the finding: this is a statement about one finite matrix per window, certified on *this* surface ($h \le 1371$); the Weil criterion is not tested, not assumed and not reverse-inferred.

The key measurement — it splits by basis. In the closed basis the minors are *maximal*:

$$\tfrac{1}{2} \sum_{r,s} M_{rs}^2 = \|u\|^2 \|v\|^2 - \langle u, v \rangle^2 = \frac{1}{\mu_1 \mu_2} \quad (\text{exact, } 6.9 \times 10^{-16};\ \sim h^4),$$

— the Wronskians carry zero arithmetic and the *largest* norm they could have: all smallness sits in the PSD kernel evaluated at one closed vector. In the eigenbasis the sum is *thin*: 90% of the weighted sum of squares from 4–101 pairs ($\le 8 \times 10^{-4}$ of the h^2 eigen-pairs), the top pair always ($\lambda_{\min}, \lambda_{\max}$) carrying 6.8–41.7% alone, and the pairs are *raw* ($\kappa_w = 0.30$–0.94; a typical minor cancels only ∼100:1) — no internal cancellation, the thinness is the story.

The exponent ledger closes. $1 - r_{12}^2 = \nu_1 \nu_2 / (\hat{a}_{11} \hat{a}_{22})$ (1.0×10^{-9}, Poincaré separation) with $\nu_1 \sim h^{+0.81}$, $\hat{a}_{11} \sim h^{+0.76}$, $\hat{a}_{22} \sim h^{+0.91}$ and $\nu_2 \sim h^{-1.38}$: **every factor but one at the right power**. The best m-free-shaped bound $B_1 = \lambda_1 \lambda_2 / (\hat{a}_{11} \hat{a}_{22})$ reads $h^{-0.548}$ on the union / $h^{-0.880}$ on frame A against the required h^{-3} — short by $h^{2.45}/h^{2.04}$; the literal count-times-sup budget costs $h^{+2.049}$, twelve orders above target, closed so it is never tried again. The open factor is identified exactly: ν_2, with minimiser $t^* = Q_{12}/Q_{11}$ (factor 1.0000), needed to relative accuracy 6.05×10^{-3} median / 2.82×10^{-4} worst, sharpening $h^{-1.146}$ — T167 measured the same requirement from the other end, and **by T168-TH7 this factor *is* the target rewritten**: the Lagrange route does not lower the accuracy demand, it relocates it (which is why ONE-TERM-MISSING is deliberately not rendered — it would misread a reduction as progress on the inequality). No closed constant candidate works ($1/\sqrt{2}$ misses by up to 0.194 relative; t^* drifts $h^{+0.076}$), no single-mode trial (overshoot 285–1.5×10^6), the ground state is only 0.982-confined (δ does not decay), and the Turán band is neither sign-definite nor telescoping.

Chain and stress. $1/g_2 = Q_{11}(1 - r_{12}^2)$ verified to 1.2×10^{-8} on 63/63; certified end-to-end gain $\ge$ 1.00–2.13 (weak, valid, direction-verified). **The scramble breaks harder than T167:** 8/8 **windows lose** $A \succ 0$ ***and*** **the Gram property** (n_{neg} up to 73) — no window survives, and on those windows the Lagrange identity is no longer a sum of squares at all; the L_P control has $Q_{12} = 0$ and $1 - r_{12}^2 = 1$ exactly, the sum collapsing onto its diagonal pair; the KMS/Dirichlet/correlation controls are exact. **The conclusion: the hardness is self-similar under exact reformulation** — determinant → PSD compound at a closed vector → near-null Rayleigh value → one ratio of two closed lag sums, same accuracy price — and the seven no-gos say that no position-blind budget, no m-free trial vector, no h-independent constant and no count-times-sup budget can pay it. The balance of T168: 8 theorem / 0 certified-uniform / 3 certified-per-window / 11 measured / 7 no-go.

After T168: one ratio

(The open factor) ν_2, equivalently the single ratio $t^* = Q_{12}/Q_{11}$ of two closed lag sums, to relative accuracy 6.05×10^{-3} median / 2.82×10^{-4} worst ($h^{-1.146}$) — by T168-TH7 the target itself in a new notation. *(Closed off)* the count-times-sup budget ($h^{+2.049}$), h-independent constants (t^* drifts $h^{+0.076}$), single-mode trials, the ground-state route (δ does not decay), the Turán band. T169 (`tstar_ratio_probe.py`, contract `TSTAR.RATIO`) attacked exactly that ratio as a preregistered candidate table — next subsection.

10.59 The candidate table closes the loop: the self-similarity is proved (T169)

T169 (`tstar_ratio_probe.py`, contract `TSTAR.RATIO`, 41/41, 3.0 s, the same 63-window union, recipe untouched; T168 reproduced bit-compatibly ($t^* = 0.5700$–$0.7898 \sim h^{+0.076}$); the whole 2×2 block verified as a lag sum (4.7×10^{-16}) with an exact arch/atom split (5.6×10^{-16}); verdict RATIO-RESISTS) attacks T168's one open factor as a preregistered candidate table — and the table speaks with two voices.

The negative half. Every genuinely closed candidate fails by $O(1)$: the archimedean ratio at 0.84 relative median, the lag heads at ∼1, the Toeplitz half at 1.0; the best closed candidate is T168's constant $1/\sqrt{2}$ at 0.15/0.24 (hitting 1/63 windows, with the wrong sign of trend). The structural reason: the block is **atom-dominated** ($|\text{atom/arch}| = 3.5$–43 on the diagonal, 14–461 off it) and $\hat{a}_{11}^{\text{arch}} < 0$ **on** 63/63 — the archimedean half is not even PSD, so the budget interval is vacuous *by sign*; the atom shares are correlated (0.994) but not proportional ($s_{12}/s_{11} = 4.1$–10.7), and $|s_{12} - s_{11}|$ grows $h^{+0.84}$ against a threshold sharpening $h^{-1.146}$ — a divergence of $h^{+1.99}$.

The key result — the loop closes as an identity. The only candidate that meets both thresholds, $K7 = \sqrt{\hat{a}_{22}/\hat{a}_{11}}$ (2.49×10^{-5} median / 2.27×10^{-4} worst, on 63/63 windows individually, right trend $h^{-1.379}$ against the target $h^{-1.380}$), does so by a new theorem (T169-TH7):

$$R\Big(\sqrt{\tfrac{\hat{a}_{22}}{\hat{a}_{11}}}\Big) \quad = \quad \frac{2\sqrt{\hat{a}_{11}\hat{a}_{22}}\,\det \hat{A}}{(\hat{a}_{11}+\hat{a}_{22})\big(\sqrt{\hat{a}_{11}\hat{a}_{22}}+\hat{a}_{12}\big)} \qquad (1.1 \;\times\; 10^{-9}),$$

— it meets the threshold precisely because it puts $\det \hat{A}$ back into the numerator. **The t^*-language is provably exactly as hard as T167's determinant — no cheaper, no dearer: T167's scalar, T168's factor and T169's candidate are the same object, and the self-similarity is now proved, not observed.** In its usable form: $\nu_2 \le 2 \det \hat{A}/(\hat{a}_{11} + \hat{a}_{22})$, tight to a factor 2.000.

The real gains. **R3 is booked, the first certified-uniform row in weeks**: $\nu_1 \le \max(\hat{a}_{11}, \hat{a}_{22}) + |\hat{a}_{12}|$ (Gershgorin, unconditional, uniform in h, closed in the three entries), holding on 63/63 and losing only 1.10–1.19 at the same exponent — the certified-uniform column moves off zero. R2 is refuted and honestly downgraded from cheap to open: the absolute-value budget gives $\hat{a}_{11} \ge -1.7 \times 10^2 \ldots -13$ — it cannot even certify positivity; a lower bound at $h^{+0.76}$ needs the sign structure of the atom lag sum, i.e. R1's difficulty again. R4: Cholesky on 63/63 — certified-per-window, permanently; three uniform routes refuted (the Gershgorin margin is negative on 63/63; the Z-/M-matrix route cannot start, A has positive off-diagonals on 63/63; the perturbation split is off by 6.4×10^5). **The Weil fence, prominent: uniform-in-h positivity of A_h is RH-equivalent-shaped and is never routed, never claimed, never reverse-inferred — instead the chain is rebuilt R4-free (T169-TH9): it does not need A-positivity.** And by T169-TH3 the $1/\sqrt{\mu}$ factors cancel exactly — $\hat{a}_{ij} = t_i^{\mathsf{T}} A t_j$ are single-sine lag sums against the closed T159 Dirichlet weights, so the cond $\sim 10^8$ ladder leaves the file.

The R4-free chain, end to end.

$$1 - r_{12}^2 \quad \le \quad \frac{\big[\max(\hat{a}_{11}, \hat{a}_{22}) + |\hat{a}_{12}|\big]\, R(\hat{t})}{\hat{a}_{11}\hat{a}_{22}} \qquad \text{(valid on 63/63; loss 1.10–1.19 with } \hat{t} = K7),$$

following the true scalar exponent for exponent: **frame-A trend $h^{-2.948}$, i.e. $h^{-3+\varepsilon}$ with $\varepsilon = 0.052$ inside the 0.5 carry window** (union $h^{-2.262}$ against the true $h^{-2.246}$); with the best

closed $\hat{t} = 1/\sqrt{2}$ it flattens to $h^{-0.990}$. Stress: the L_P control is the sharpest line of the file ($\hat{a}_{12} = 4 \times 10^{-17}$, $t^* = 0$ exactly as T168 predicts, overshoot exactly $4\cos^2(\pi/N)$ — the t^*-recipe is a near-degeneracy artefact: its tightness *measures* $\det \hat{A}$, it does not bound it); scramble 8/8 (K7 accuracy $2.5 \times 10^{-5} \to 4.6$); flat-weight null 8/8 (both halves of the arithmetic input are necessary); the parity-pair control is only 6.7× — honestly reported as partial (the near-degeneracy is a low-spectrum property, not specific to (t_1, t_2)). The balance of T169: 10 **theorem** / 2 **certified-uniform** / 1 **certified-per-window** / 4 **no-go**.

After T169: one bilinear von Mangoldt sum

(R1, in standard shape for the first time) an m-free upper bound on $\det \hat{A}$ at $h^{-3+\varepsilon}$, now a **bilinear von Mangoldt sum against closed Dirichlet weights** (equivalently the diagonal ratio $\hat{a}_{22}/\hat{a}_{11}$ to 1.2×10^{-2} median / 5.6×10^{-4} worst). *(Housekeeping)* R2 = R1 (it needs the same sign structure); R3 closed (off the list); R4 not needed (the chain is R4-free). The strategic point: T161's beyond-RH triage applied to *linear* sums — for *bilinear* forms the large sieve gives unconditional square-root cancellation, so for the first time the one open object stands where an unconditional classical tool is in principle competent. T170 (`bilinear_sieve_probe.py`, contract `BILINEAR.SIEVE`) ran the unconditional sieve toolbox (Montgomery–Vaughan, the Vaughan decomposition) on the bilinear form — next subsection.

10.60 The sieve box prices every tool: the classification completes as a theorem (T170)

T170 (`bilinear_sieve_probe.py`, contract `BILINEAR.SIEVE`, 40/40, 1.8 s, 70 frame-A windows $h = 142$–1445 on the T163–T169 union recipe, never re-tuned, 67 of them arithmetically complete ($h = 142$–1433; exponents quoted on both sets); $\kappa = 0.038821$ verified at every jump point up to $n = 4 \times 10^5$; the large sieve is *unconditional* — nothing assumes RH and nothing concludes it; verdict SIEVE-RESISTS) cashes in T169's standard shape: the bilinear form is written down exactly, and the toolbox is priced on it.

The explicit form, all theorems. $\hat{A} = B - \sum_{n \le X} (\Lambda(n)/\sqrt{n})\, X_n$ with X_n the two-point spline read of the T159 closed lag weights (T170-TH1, 3.5×10^{-15} against the direct $t_i^{\mathsf{T}} A_h t_j$), and the determinant splits exactly (T170-TH2/TH3):

$$\det \hat{A} = \det B - D(B, S) + \det S, \qquad \det S = \sum_{n,m \le X} \frac{\Lambda(n)\Lambda(m)}{\sqrt{nm}}\, K(n,m), \quad K = \tfrac{1}{2} D(X_n, X_m)$$

with $D(P,Q) = P_{11}Q_{22} + P_{22}Q_{11} - 2P_{12}Q_{12}$ the polarisation of det (Cauchy–Binet 1812/1815; Lagrange 1773; verified 1.3×10^{-10} and 1.5×10^{-14}). X_n is rank one to median defect 1.3×10^{-4}, so $K = \frac{1}{2}\varepsilon_n \varepsilon_m (x_n \wedge x_m)^2$ — a *wedge square* with mixed signs whose diagonal cancels. On the reference window ($h = 540$, $X = 2.465 \times 10^4$) the three pieces read $\det B = 6.18$, $-D(B,S) = -198.8$, $\det S = 192.6$ and sum to 4.22×10^{-3} against an absolute target of 1.10×10^{-5}: **a five-order cancellation between three ∼200-sized pieces** — any tool that bounds $\det S$ alone, however sharply, cannot see it.

The route table, every δ certified (target $\delta \ge 3.0$ on $1 - r_{12}^2 \le C\, h^{-\delta}$, 67 complete windows):

route	δ	status
truth: $1 - r_{12}^2$ itself	+2.747	measured
Cauchy–Schwarz on $\hat{A}$	−0.000	theorem
large sieve / duality on $\det S$	+0.669	theorem
Montgomery–Vaughan 1973, three linear sums	−0.384	theorem
exact split $\lvert\det B\rvert + \lvert D(B,S)\rvert + \lvert\det S\rvert$	+0.996	theorem (best)
Vaughan 1977, Type I + Type II	+0.669	theorem

The key question answered — the form collapses back, by two theorems. (TH4, *the rank-3 theorem*): D is the polarisation of det on 2×2 symmetric matrices, matrix $\left[\begin{smallmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -2 \end{smallmatrix}\right]$, rank 3, signature $(1,2)$ — hence $\operatorname{rank} K \le 3$ for *every* window, every h, every X (verified $\sigma_4/\sigma_1 = 2.0 \times 10^{-15}$;

effective rank 1.38–2.71 against $N \sim 10^3$ atoms): the bilinear form *is* the rank-3 polynomial $S_{11}S_{22} - S_{12}^2$ in *three* linear von Mangoldt sums — there is no bilinear content beyond three linear functionals, and every bilinear tool degenerates exactly to Cauchy–Schwarz on the object T161 already triaged. (TH5, *the Type II collapse*): the closed weights see n only through $\log n$; under $n = md$ that is $\log m + \log d$, and a bounded-frequency function of a sum is a finite-rank kernel — Vaughan Type II blocks have effective rank $O(1)$ for every U, V (2 singular values carry 99.9% against 153 generic; $U = 12$–70 scanned; the identity verified coefficient by coefficient, residual 0.0). The antisymmetry route: the ℓ^1 kernel mass exceeds the signed value $41\times$ — a triangle-inequality route loses before it starts. *The honest T161 comparison*: **gained** — 32 frequencies reduce to *three* linear functionals, by theorem; **not gained** — the precision, which binds: 5.3×10^{-5} relative per sum at $h = 285$, sharpening h^{-3}, against the RH yardstick $h^{-0.5}$ — 1.1×10^3 too coarse.

The chain and the stress. The chain becomes R4-free *for free* (T170-TH6): $1 - r_{12}^2 = \det \hat{A}/(\hat{a}_{11}\hat{a}_{22})$ is an identity — no A-positivity, no Rayleigh detour, the Weil fence never approached; sharper than T169-TH9 (-2.747 vs -2.771) and using strictly less. End to end the best unconditional bound carries $h^{-0.996}$ against the required $\leq h^{-2.50}$ — **missing by** 1.50 **in the exponent, and the shortfall is a theorem (TH4+TH5), not a measurement**. The scramble is the sharpest line: with scrambled positions the rank-3 collapse *and* the unconditional bounds are numerically almost unchanged, yet the truth moves by 2.76 in the exponent — **the arithmetic content sits entirely in the joint values of** (S_{11}, S_{22}, S_{12}) **against the arch block** B, not in the rank, the kernel, or the Type I/II split: a sieve bounds each coordinate, the phenomenon is the near-degeneracy of the triple. Flat-weight breaks ($h^{-0.116}$); L_P is exact ($\hat{a}_{12} = 8.7 \times 10^{-18}$, no decay); the ν-decoupling is reported unsmoothed (frames $\nu = 5/8/11$ fall at $-1.70/-1.27/-1.74$ — T169's $h^{-2.95}$ is a frame-A statement, but the *shortfall* is frame-independent: $+0.53/+0.34/+0.61$ certified on those same frames). All five tools were preregistered by the contract. The balance of T170: 6 theorem / 1 certified-uniform / 5 no-go. **R1, finally classified**: $\det \hat{A}/(|\text{row}_1||\text{row}_2|)$ is the *sine of the angle* between the two rows of $\hat{A}$, falling $h^{-2.711}$ — R1 asks for an unconditional certificate that two explicit finite Λ-sum vectors become collinear at rate h^{-3}: a *near-degeneracy*, not a size — which is exactly why no size-bounding tool, bilinear or linear, reaches it. The theorem-shaped cores of T168, T169 and T170 — the Lagrange/Wronskian anatomy, the exponent account with the Gershgorin certificate, the TH7 det-collapse, the exact bilinear form, the two rank theorems, and the R4-free chain with its discriminators — are promoted as v558 [`verification/v558_bilinear_rank_identities.py`] (Section 4.24).

After T170: the classification is final for the classical box

(R1, classified) an unconditional certificate of **near-collinearity**: the two rows of $\hat{A}$ — two explicit finite Λ-sum vectors — become collinear at rate $h^{-3+\varepsilon}$; equally, a joint near-degeneracy of the triple (S_{11}, S_{22}, S_{12}) against the archimedean block, at relative precision sharpening h^{-3}, 1.1×10^3 finer than the RH yardstick: *beyond-RH joint precision on three linear Λ-sums*, and not a size question at all. *(Housekeeping)* R2/R3 booked in T169; R4 entirely off the path (T170-TH6); nothing else is open. T171 (`final_map_probe.py`, contract `FINAL.MAP`) then assembled the capstone: the whole sixteen-link reduction chain end to end in one file — next subsection.

10.61 The capstone: the map is complete (T171)

T171 (`final_map_probe.py`, contract `FINAL.MAP`, 43/43, 0.6 s, 12 windows over *two* frames $\nu = 4/6$, $h = 142$–1445, lever arm $10.2\times$; $\kappa = 0.038821$ verified at every jump point up to $n = 4 \times 10^5$; RH_DELTA $= 0.5$ stays a yardstick and the quantifier over m stays *open* at link 16; verdict MAP-COMPLETE) reproduces the whole reduction chain — from the I5 floor to the final R1 shape — in *one connected run*, exhibits every classified no-go failing, and closes the precision ledger. The honest exclusion is said out loud: on the $\nu = 4$, $h = 1445$ window the low block is indefinite ($\lambda_{\min}(B_{LL}) = -343$), so the ladder links (6, 7, 12, 13) carry 11/12 while the identity links carry 12/12.

The chain, sixteen links, one check each (13 THEOREM: algebra valid for every h; 3 CERT: per

window, no quantifier in m):

#	link	type	sharpest number
1	T152 Schur two-block	theorem	sharp at 0.5λ vs 1.02λ; KMS 2.1×10^{-15}
2	T151 Sobolev $b_k \le C_S k$	theorem	worst ratio 0.483
3	T153 Ψ-collapse + Z-law	theorem	1.19×10^{-15}; $\sum w_d = 0$ to 10^{-16}
4	T154 Ritz ceiling	theorem	$g_{32}/s = 0.923$–0.986 from below
5	T155 12×12 floor	cert	$t = 7.8 \times 10^{-2}$–2.13×10^{-1}, sharp
6	T156 $F(P, r)$ chain	cert	$L_{16} = 5.0$–19.6
7	T158 Thomson/ladder	theorem	1.45×10^{-9}; 33/33 trials below
8	T160 sampling identity	theorem	1.05×10^{-15}
9	T163 TV floor / swap	theorem	overshoot $\times 7.3 \times 10^5$–6.2×10^8
10	T164 gauge / quantifier	theorem	$\Psi(x^*) = 1/g_{16}$ to 3.2×10^{-9}
11	T165 P_{pr} identity	theorem	2.2×10^{-16}; $= 1$ at x^*
12	T166 cascade $1/(1 - R^2)$	theorem	4.0×10^{-10}; diagonal-invariant
13	T167 $K = 2$ exactness	theorem	4.2×10^{-10}; mildest rung ($\times 1.36$–2.23)
14	T169 det collapse (R4-free)	theorem	$\le 2.2 \times 10^{-9}$ on all 12, positivity-free
15	T170 rank-3 collapse	theorem	$J = \{-2, -1, +1\}$; $\sigma_4/\sigma_1 = 1.6 \times 10^{-15}$
16	T170 the R1 shape	cert	$\sin\angle = 1.09 \times 10^{-7}$–$1.15 \times 10^{-4}$

At link 16 the *rate* is frame-dependent ($\nu = 4$: $h^{-1.86}$; $\nu = 6$: $h^{-2.87}$; pooled $h^{-2.33}$) while the *collapse* is not: on every frame the two rows become collinear at a rate steeper than h^{-2} — and no line of the file proves that rate for all m.

The no-go battery — all eight classified routes fail on instances exactly as classified: the size budget overshoots at $h^{+1.32}$ (the ℓ^1 majorant exceeds $|\det \hat{A}|$ by 8.4×10^3–6.5×10^5, growing); the symbol infimum is negative on 12/12 (-2.5×10^3 – -68); the sector/gauge change moves $1 - r_{12}^2$ by at most 2.1×10^{-9} (an invariance); perturbation theory addresses an object 2.0×10^4–1.9×10^7 below its scale (Kato 1949); the atom band outruns the archimedean band by $h^{+0.79}$; the sieve box certifies $\delta = +1.044$ against a target of 3.0 — short by $h^{1.96}$, and by the rank-3 theorem the kernel has rank ≤ 3, so the sieve cannot recover it; the scramble flips the *type* ($h^{-2.57} \to h^{+0.21}$, inflation $\times 11421$ median); and the L_P gain is 1.000000000000000 exactly.

The precision ledger — needed against available, in one display:

$$\underbrace{2.284 \times 10^{-7}}_{\text{needed at } h=1444,\ \text{rate } h^{-3.0}} \ll \underbrace{7.13 \times 10^{-4}}_{\text{best unconditional } h^{-0.996}} \ll \underbrace{2.63 \times 10^{-2}}_{\text{RH yardstick } h^{-0.5}}$$

short by $3.1 \times 10^3\times$ of the best unconditional exponent and by $1.2 \times 10^5\times$ of the RH-equivalent yardstick — and it is a *rate* gap: the demand sharpens with h, so the gap widens, which is why no constant and no range can close it. RH_DELTA is used in exactly one role, translating "how precise" into "which exponent"; nothing here assumes RH and nothing concludes it. Both self-similarity identities check (the T167 union to 4.2×10^{-10}; the T169 collapse at the exact stationary points to 9.9×10^{-10}, sign-agnostic including $\nu_2 < 0$ on 1/12). The balance of the five quotable parts (T162/T167/T168/T169/T170): 40 theorem / 5 cert-unif / 9 cert-window / 16 measured / 9 no-go — T126–T161 honestly left unaggregated. What T171 itself adds: 16 links end to end, 8 exhibited no-gos, 1 precision ledger, and **zero new uniform-in-m statements — the honest number, and exactly why it is promotable**. The capstone — the sixteen-link chain as one machine-checked pass, the no-go battery, and the precision ledger, recomputed on the declared small surface — is promoted as `v559` [`verification/v559_phase2_capstone.py`] (Section 4.25).

After T171: the map is complete

(R1, the only open phase-2 object) an m-free unconditional bound $|\det \hat{A}| \le C\, h^{-3+\varepsilon}\, \hat{a}_{11}\hat{a}_{22}$, equivalently $|S_{11}S_{22} - S_{12}^2 - D(B,S) + \det B| \le C\, h^{-3+\varepsilon}\, \hat{a}_{11}\hat{a}_{22}$ for the three explicit linear von Mangoldt sums — *classified* as a near-degeneracy, not a size. *(Inherited, not phase-2)* other frames and zones — the decay rate of link 16 is frame-dependent while the collapse is not — and the R-B‴ α/h surface. *(Housekeeping)* R2/R3 booked in T169; R4 removed by T170-TH6. Nothing else is open in phase 2. T172 (`frame_beyond_probe.py`, contract `FRAME.BEYOND`) then measured how far the map carries

beyond frame A — next subsection.

10.62 Beyond frame A: the map is portable, the numbers are not (T172)

T172 (`frame_beyond_probe.py`, contract `FRAME.BEYOND`, 44/44, 1.3 s, 54 windows on *four* surfaces plus 3 truncated-comb stress windows, $h = 128$–1444; 26152 prime-power atoms to $n = 3 \times 10^5$, $\kappa = 0.038821$ verified at every jump point — the one unconditional input; verdict PARTIALLY-PORTABLE) answers the capstone's inherited question — how far does the certified map carry beyond frame A? — with a recipe inventory and a transfer table.

The recipe inventory — four degrees of freedom, and only four. (1) The *frame rule* anchor → grid step D: frame A uses $D = g_k/(2\nu)$ from the *local* log-gap of the anchor sequence; frame B (new here) uses $D = \alpha/h$ for a declared h — gap-*blind*, so the (α, h) pairing is no longer dictated by arithmetic. (2) The subdivision ν: T171 used $\{4, 6\}$; the extremes $\{3, 8\}$ are new. (3) The *zone family* of the anchor sequence: prime powers, general non-prime-power integers, or a congruence class $p \equiv 1$ vs 3 mod 4. (4) The *atom set* — which is **not** a degree of freedom: Λ lives on prime powers, so "beyond frame A" means a different *anchor* arithmetic, never a different Λ. The surfaces: S0 the frame-A reference (12 windows), S1 frame B plus the ν-extremes (18), S2 non-prime-power anchors (12), S3 the two congruence classes (12); $hD = \alpha$ to 10^{-10} and $U_{\mathrm{ref}} = 2\alpha = \log X$ exact on every surface; all 54 complete-comb windows positive definite ($\lambda_{\min}(B_{LL}) = 0.092$–$0.293$).

The identity transfer — sixteen links, four surfaces. **Thirteen of the sixteen links transfer unchanged and none breaks**: the whole identity tower (links 1, 3, 7–15, re-derived on every window of every surface to the declared identity bars — Schur/KMS 3.2×10^{-13}, Ψ-collapse 3.9×10^{-14}, the sampling identity 1.6×10^{-14} (the most zone-sensitive link: the anchor arithmetic decides *where* the atom combs sit on the grid, and the identity is untouched), the det collapse stated absolutely at 5.2×10^{-15}, rank-3 with the polarisation at a 3.2×10^5:1 cancellation) plus both direction lemmas (the Sobolev ceiling 0.4826 *identical* on all four surfaces — it sees only h; interlacing on 54/54). The three that shift are exactly the three that carry a *number* rather than an equation: the two window certificates (link 5: t median 0.128–0.204 per surface — the one link that dies where the low block is indefinite; link 6: r_K median 0.25–0.52) and the rate of link 16. *The indefinite mechanism, isolated*: 3/3 truncated-comb stress windows have $\lambda_{\min}(B_{LL}) = -3.7 \times 10^5$ – -6.2×10^6 against 0/54 on the complete-comb surfaces — indefiniteness is located at the **declared sieve horizon** (the comb is cut at the sieve cap while the grid runs to $X = e^{2\alpha}$), *not* at a frame or a zone family, retiring T171's open worry about its one indefinite window; and the identity links keep holding there ($\sim 10^{-15}$), because none of them uses positivity. *The constants map*: of 11 carried constants, 8 stay inside frame A's own scatter band once the h-trend is removed (t, C_S, $1/g_{16}$, c_0^{ap}, μ_1^P, $\lambda_{\min}(B_{LL})$, cond, r_K); 3 wander, all on the congruence surface S3 (K^F ×2.3, U_{ref} ×1.2, s ×1.5). The honest consequence: **a certificate on a new surface is a new certificate** — t, r_K, L, $1/g_{16}$ must be recomputed there and the T163–T171 numbers may not be quoted for it; what transfers is the *procedure* and the *directions* (floor below, ceiling above, price ≥ 1, defect ≥ 0 — all held on 54/54).

R1 beyond frame A, and the placement finding (the key result). The near-degeneracy *persists on every surface*: the row angle is below 10^{-2} on all 54 windows, decays on all 9 legs ($1 - r_{12}^2 = 1.4 \times 10^{-7}$–$6.2 \times 10^{-4}$), and nowhere settles at $O(1)$ — the *classification* is the portable part, R1 is not a frame-A artefact. The *rate* is frame-bound, confirmed and extended (pooled per frame datum: $\nu = 3$: $h^{-2.19}$, $\nu = 4$: $h^{-2.46}$, $\nu = 6$: $h^{-2.55}$, $\nu = 8$: $h^{-1.83}$, gap-blind B: $h^{-2.76}$), with the zone arithmetic a secondary lever (spread 0.40 at fixed frame datum against 1.03 at fixed zone family — the frame datum dominates by 2.6×); the h^{-3} demand is met on none of the 9 legs ($C = (1 - r_{12}^2)h^3$ grows at $h^{0.22}$–$h^{0.76}$ per surface). And the sharpest new result: **scrambling the atom positions at a fixed Λ-value multiset raises the angle by a median 6.6×10^4 and removes the decay entirely** ($h^{+0.11}$ against $h^{-1.93}$ on the same windows) — the collapse is a property of the *actual prime-power placement*, not of the value multiset: it survives changing the anchor family and does not survive destroying the comb.

After T172: the map is portable, the numbers are not

(The three preregistered verdict inputs) identities universal = YES (13/16, 0 broken); all carried constants in band = NO (3/11 wander); R1 class persists = YES ⇒ PARTIALLY-PORTABLE. *(Candidates P1–P4, pending)* the transfer theorem, the indefinite mechanism, R1 class portability, the placement finding — bundling with T174 planned, nothing booked. *(Rest)* R1 unchanged (missing factor $h^{0.13}$–$h^{1.17}$ per leg); which frame datum maximises the rate (Q1); a frame with flat C (Q2); the m-quantifier (Q3). T173 (`frame_rate_probe.py`, contract `FRAME.RATE`) then asked whether the demanded rate is itself a frame datum — next subsection.

10.63 The demanded rate is a frame datum: the deficit map (T173)

T173 (`frame_rate_probe.py`, contract `FRAME.RATE`, 40/40, 2.2 s, 284 windows plus 109 scramble twins over *eleven* preregistered frame data — frame A at $\nu \in \{3, 4, 5, 6, 8, 10, 12, 16\}$, the gap-blind frame B, and the new smoothed frame C ($D = \bar{g}_k/(2\nu)$ with $\bar{g}$ the mean of eleven local gaps) at $\nu = 4, 6$ — $h = 128$–1447; 93192 prime-power atoms to $n = 1.2 \times 10^6$, $\kappa = 0.038821$ verified at every jump point; 0/284 windows indefinite; verdict DEFICIT-VARIES) puts the demand side itself on the measurement table.

The demand side, exact where it can be. $\mu_1^P = 4\sin^2(\pi/(2h+1))$ is a closed form (KMS 1953), identical on every frame; the floor t is where the frame enters. Which exponent the chain demands is not left to taste: writing $s = d\log\alpha/d\log h$, the identity $hD = \alpha$ per window gives

$$q \quad = \quad -\frac{d\log D}{d\log h} \quad = \quad 1 \;-\; s \quad \text{exactly (to } 1.9 \times 10^{-15}\text{)}, \qquad q \;=\; 0.667\text{–}0.777 \;<\; 1 \text{ on all eleven frame data}$$

so a $\Theta(D^3)$ law reads in h as h^{-3q}, and **T171's h^{-3} is the $q = 1$ idealisation — attained by no frame in the family** ($\alpha = \log n_{\text{zone}}$ grows with h along every frame). *The calibration*: of six preregistered gap functionals the *relative* I5 gap $t/\lambda_{\max}(B_{LL})$ reproduces the chain's 3 on frame A (median $E_h = 3.033$, legs 2.831–3.153) *and* is the only dimensionless candidate — matching the dimensionless delivered object $1 - r_{12}^2$: the dimensional argument and the numerical calibration pick the same functional independently. $(D/\alpha)^3$ is named a tautology (exactly h^{-3}, zero information — T172's convention, kept as reference). The demand table: REQ = 2.831–3.438 over the family (spread 0.607) — the demand itself moves with the frame, and the spread is the size of the correction T172 was missing.

The deficit map — one slope per frame datum. The deficit is read off the per-window ratio $R = \text{GAP}/(1 - r_{12}^2)$ as *one* slope with one error bar, never as a difference of two fits. The split-stability test first (the strongest evidence in the file): refit on the low/high half of each leg's lever — REQ drifts by a median 0.501, DEL by 0.453 (*neither* side is a clean power law: a single-sided exponent on this chain is reproducible only to about half an exponent, a declared limitation T171, T172 and this file share), while the deficit drifts by only 0.214.

frame	q	REQ	DEL	deficit ± 2 s.e.
A, $\nu = 3$	0.777	2.831	2.812	$+0.020 \pm 0.168$
A, $\nu = 4$	0.741	3.053	2.957	$+0.096 \pm 0.196$
A, $\nu = 5$	0.758	2.934	2.734	$+0.199 \pm 0.229$
A, $\nu = 6$	0.726	3.036	2.975	$+0.061 \pm 0.291$
A, $\nu = 8$	0.764	2.889	2.369	$+0.520 \pm 0.289$
A, $\nu = 10$	0.732	3.030	2.515	$+0.516 \pm 0.276$
A, $\nu = 12$	0.697	3.110	2.709	$+0.400 \pm 0.280$
A, $\nu = 16$	0.698	3.153	2.856	$+0.298 \pm 0.410$
B, $\nu = 4$	0.667	3.438	3.449	-0.011 ± 0.181
C, $\nu = 4$	0.765	2.929	2.796	$+0.133 \pm 0.198$
C, $\nu = 6$	0.722	3.065	2.935	$+0.130 \pm 0.167$

Pooled: **deficit** $= +0.155 \pm 0.102$ (error inflated $\times 1.54$ for over-scatter; unscaled ± 0.066, i.e. 4.7σ; inflated, still 3.0σ above zero). The cancellation is strong but not exact, both halves measured:

against a constant, REQ gives $\chi^2/\mathrm{dof} = 4.87$, DEL 4.17, their difference 2.38 — 1.8× flatter than either side, but 2.4σ from constant: constancy is rejected. Two exact internal supports: the μ_1^P-weighted convention shifts the deficit by exactly 2 (measured 1.9974, χ^2 identical — an exact-factor convention cannot change flatness: *the flatness, not the value, is the result*); and the fixed-h^{-3} convention has χ^2/dof equal to the delivery side's to 8×10^{-15} — a frame-independent demand subtracts a constant and cancels nothing. The driver scan: the residual tracks $\log \nu$ ($r = +0.76$, 2.8σ; confounded with the window's arithmetic content under the h-cap, declared) and *not* the frame's own regulator q (0.2σ); holding ν fixed and moving only the D-rule (local / blind / smoothed) the deficit does not move ($\nu = 4$: $\chi^2/\mathrm{dof} = 0.64$; $\nu = 6$: 0.16) — **the deficit is invariant under the anchor-to-grid rule**, the cleanest statement the surface supports and a theorem candidate for that half only. *The honest correction to T172*: in the fixed-h^{-3} convention this wider surface gives $+0.015 \pm 0.084$ — consistent with *zero*, and the worst-conditioned of the six conventions — so T172's C-growth of $h^{0.22}$–$h^{0.87}$ was neither sign- nor split-stable (3/5 shared frame data outside 2 s.e.); T172's PARTIALLY-PORTABLE verdict is untouched, the size and sign of that one number are not. The h-versus-D question is *not* resolved and declared (χ^2 2.38 vs 2.36; the fixed-α discriminator legs — $q = 1$ exactly, T171's R-B‴ surface, run as diagnostic only, outside the chain — give $+0.510/+0.299$ at ± 0.336 and do not settle it).

R1, stated frame-free (the key result). On every frame datum of the family, the delivered near-degeneracy $1 - r_{12}^2$ of the two explicit finite Λ-sum rows closes *slower* than the relative I5 gap it has to price, by $h^{+0.155 \pm 0.102}$ — one number across the three anchor-to-grid rules at fixed ν, with a residual drift in ν; R1 is the statement that this deficit is zero, and no choice of D-rule moves it. **No frame closes it**: 0/11 legs have a 2 s.e. upper edge ≤ 0 (best: B at -0.011 ± 0.181, upper edge $+0.170$); honestly the other way, 8/11 legs are individually consistent with zero (a single leg resolves only ± 0.23), which is exactly why the number is pooled *with* the over-scatter penalty. Frame shopping in T172's sense is over, by numbers: the fastest-delivering frame (B, 3.449) demands the most (3.438), in step. Stress: the scramble control removes the decay on all 4 declared legs (DEL $\to -0.09 - -0.13$ — the deficit is not a normalisation artefact); the lag-sum identity is exact ($\leq 1.9 \times 10^{-16}$); 0/284 windows touch the sieve horizon; anti-fitting three ways (family preregistered; jackknife ≤ 0.026; shuffle null -0.003 ± 0.279 with the measurement 3.7 null-sigmas away).

After T173: the deficit is the number

(The cancellation identity — the one thing that would close this half) show algebraically that the frame factors cancel between the relative I5 gap and the 2×2 near-degeneracy, in the style of T164's gauge machinery: that would turn the D-rule invariance into a theorem and make the deficit *the* number of the phase. T174 (`cancellation_identity_probe.py`, contract `CANCEL.IDENTITY`) then asked exactly that — next subsection. *(Rest)* the h-versus-D split (needs a frame whose α does not track $\log h$ while the comb still moves — hard under the cap; T171's R-B‴ surface); and the remaining rate: close $+0.155$ in the exponent unconditionally, for two explicit finite Λ sums — unchanged as the one open object, now stated frame-free. *(Candidates P5–P8, pending)* the frame-bound demand ($q = 1 - s$), the D-rule invariance, the T172 correction, the calibration — bundled with T174's cores into v560.

10.64 The cancellation identity: the gauge route exhausted, the deficit measured frame-free (T174)

T174 (`cancellation_identity_probe.py`, contract `CANCEL.IDENTITY`, 37/37, 6.5 s, 84 rectangle + 1020 power-check + 147 anchor-ladder + 178 leg windows; 93192 prime-power atoms to $n = 1.2 \times 10^6$, $\kappa = 0.038821$ verified at every jump point; the preregistered 12×7 (α, h) rectangle — 12 prime-power anchors $n = 41 \ldots 1087$ crossed with the declared geometric ladder $h = 160 \ldots 1280$, fixed *before* any window was built, comb complete on every cell, 84/84 positive definite; verdict PARTIAL-CANCEL) turns "do the frame factors cancel" into checkable algebra and then measures what does not cancel.

The decomposition — which factors is each functional blind to. Both sides of the T173 deficit are scale-free functionals of *one* raw Gram matrix $\hat{a}_{ij} = t_i^{\mathsf{T}} A_h t_j$: the demand is the relative gap

of its 16×16 μ-congruence, the delivery is the 2×2 corner of the very same matrix. Blindness on both sides is a cancellation in $R = \mathrm{GAP}/(1 - r_{12}^2)$ — and blindness is an algebraic property, checkable to machine precision, not a fit. The cancellation ledger:

factor	**status**	**size of the residue**
amplitude of c (κ)	THEOREM	≤ 0.028 of the round-off bar
scalar of the KMS ladder (μ_1^P)	THEOREM	≤ 0.009 — the whole h^{-2} channel
full positive diagonal group	THEOREM	delivery only, exact
shape of the KMS ladder ($\hat{S}$ vs k^2)	CERTIFIED	≤ 0.0075 in the deficit
arch/comb mixture in $\hat{A}$	*does not cancel*	additive; no factorisation
the frame rule itself	THEOREM	nothing to cancel: $R = R(\alpha, M)$

The load-bearing entry is the second: writing the KMS ladder as $S = \mu_1^P \hat{S}$ with $\hat{S}_k = \mu_k^P/\mu_1^P$, the scalar $\mu_1^P = 4\sin^2(\pi/(2h+1))$ — the one factor of the demand that is a pure function of the grid — cancels *exactly* on both sides: on the delivery because a correlation is invariant under the full positive diagonal group, on the demand because a relative gap kills any scalar congruence. **The entire h^{-2} channel is exactly absent from the deficit.** What survives is the *shape* $\hat{S}$, which the delivery is exactly blind to but the demand is not — and that one channel is bounded *unconditionally*: $|\log \mathrm{GAP}(k^2) - \log \mathrm{GAP}(\mathrm{KMS})| \leq 2\log((1+\varepsilon)/(1-\varepsilon))$ with $\varepsilon = (K\pi/(2h+1))^2/6$, worth at most 0.0075 in deficit units = 4.8% of T173's +0.155. The honest boundary: the frame enters the Gram *additively*, $\hat{A} = \hat{A}_{\mathrm{arch}} + \hat{A}_{\mathrm{comb}}$ exactly, and neither term alone has a positive Schur floor on a single one of the 84 cells — the 4%-arch term decides the positivity, no factorisation (frame) × (arithmetic) exists, and no gauge argument can absorb an additive term: *this is where the cancellation identity stops being exact.* Two side results settle T173's family: $G2$ is the *unique* doubly gauge-invariant and arithmetic member (the measured gauge degrees come out as integers), and the $G3$–$G2$ offset of exactly 2 is explained ($G2$ is μ_1^P-blind, so the offset is the μ_1^P exponent, measured $h^{-1.9975}$ — T173's 1.9974).

The frame rule is a sampling map, and the deficit is measured without it (the key result). The window builder takes $(n_{\mathrm{zone}}, \alpha, M)$ and nothing else, so $R = R(\alpha, M)$ identically: a frame rule is a *curve* on that surface, two frames reaching the same cell agree bit for bit, and pointwise there is nothing to cancel. On the preregistered rectangle the h-direction carries the deficit ($a = -0.281 \pm 0.066$, 4.3σ) while the anchor direction is flat; the exact between/within split of every leg deficit (the split is an identity, exact to 5.6×10^{-16}) shows the *grid response* carries it — 3.3× the anchor-selection part, whose sign is wrong on most legs: the cheap reading "the frame rule reads h off the local log-gap, so the deficit is a selection effect" is tested and rejected. The driver is then identified: on a leg, $\log \mathrm{dens} = \mathrm{const}(\alpha) - \log \nu$ identically (dens = comb atoms per lag cell), so no leg statistic could ever separate ν from comb content ($r = -0.921$ on the ten gap-reading legs — the same signal under two names); the rectangle separates them, and the deficit falls *monotonically* $+0.281 \to +0.172 \to +0.147 \to +0.062$ across the density cuts dens $\geq 0/0.5/1/3$ while the anchor slope stays flat. The leg table orders perfectly by density (A16/dens 0.38 → +1.19; A8/0.75 → +0.74; A4/2.5 → −0.10) — T173's 2.8σ $\log \nu$ driver is a named and discharged confounding. Exhausting the dense corner (1020 fresh power-check cells on 153 new anchors; pooled 1069 cells on 151 anchors with ≥ 4 rungs each, the cluster-robust anchor mean the honest estimator) gives

$$\mathrm{DEFICIT}(\text{frame-free, dens} \geq 3.0) = +0.1111 \pm 0.0222 \quad (5.0\sigma \text{ from zero; } 0.4\sigma \text{ from T173's } + 0.155 \pm 0.102)$$

— the deficit survives an independent measurement with no D-rule anywhere in it. Stress green: the comb scramble destroys the positive block on 24/24 test cells and moves R by 10^2–10^5; jackknives stable; every numerical horizon declared ($1 - r_{12}^2$ down to 9.3×10^{-8}, bars conditioned per cell).

After T174: the phases and the dense limit

(Verdict) PARTIAL-CANCEL — the gauge route is *exhausted*: multiplicative cancellation is theorem-form, the one shared channel is certified $\leq 4.8\%$, the additive mixture cannot cancel; any further cancellation claim needs a different pair of functionals. P6 does *not* become a theorem (only its multiplicative half) but is less load-bearing: the deficit needs no invariance argument — it is measured on a D-rule-free surface, and the fixed-ν invariance is explained (fixed ν = fixed density regime). The theorem cores of T172+T173+T174 are promoted as v560 [verification/v560_frame_deficit_identities.py] (Section 4.26). *(Rest)* the per-anchor rate is not universal ($\chi^2/\mathrm{dof} = 3.02$, no residual α-trend) — the untried candidate is the placement *phases* of the heavy low atoms $\mathrm{frac}(\log p/D)$; an admissibility question for the chain (T173 admits 0.11 atoms per lag cell; dens ≥ 3 against all is a factor 2.5 in the deficit); and the sparse corner (+0.51–0.80) — regime or spline artefact. T175 (`phase_placement_probe.py`, contract `PHASE.PLACEMENT`) then asked exactly that — next subsection.

10.65 The placement phases: real and causal, but non-smooth — and the heterogeneity dissolves into the error bar (T175)

T175 (`phase_placement_probe.py`, contract `PHASE.PLACEMENT`, 38/38, 20.2 s, 1504 cells on 188 anchors — *every* prime power in $[40, 1090]$ is an anchor, no selection, no frame rule anywhere in the plan, so no anchor can have been chosen for its phase — crossed with the T174 ladder $h = 160 \ldots 1280$ plus the declared floor $h = 128$; the two phase families, the density bins, the deformation grid, the intervention offsets and the split rule all declared before use; verdict PHASES-RESIST) measures the placement phases directly and takes the dense corner to its ceiling. The bridge first: the frame-free deficit *reproduces* on the wider anchor set, $+0.1069 \pm 0.0222$ (4.8σ, 158 anchor clusters, 0.13σ from T174) — and so does the problem it inherits, the per-anchor heterogeneity $\chi^2/\mathrm{dof} = 3.14$.

The phase construction — and the trap said out loud. For the atom at $\log p$ the reference assembly splits its mass between two lag cells with weight $\varphi_p = \mathrm{frac}(\log p/D)$: the placement phase is the exact interpolation weight of the assembly, not a nuisance parameter. The generating identity $\varphi_p(h) = \mathrm{frac}(\log p/D) = \mathrm{frac}(h\,\theta_p)$ with $\theta_p = \mathrm{frac}(\log p/\alpha)$ is certified at 1.1×10^{-13} on all 1316 cells — one real number per anchor generates the whole rung ladder of phases (Weyl 1916). But the preregistered anchor invariant θ_p is *unusable*, and the probe says so before fitting anything: θ_p is a smooth monotone function of α (Rayleigh $Z = 1296$ — as far from uniform as a circular sample gets), so a regression on it would be an α-regression in disguise, and T174 already reports no residual α-trend. Only the cell phase $\varphi_p(h)$ is free and equidistributed (largest $Z = 1.41$ on 1316 cells). On that honest regressor the phases are *in* $\log R$: the declared family $p = [5, 7, 11, 13, 17, 19]$ raises R^2 from 0.045 to 0.151 on the 1059 dense cells, $F = 10.80$ against a within-rung-scrambled null whose 95% point is 1.69; held out under a rank-parity split the amplitudes keep +10.3% of the residual variance, a composite placebo family scores $0.81/-0.0275$, and the amplitudes are monotone in p (0.018 at $p = 5$ to 0.141 at $p = 19$, 7.7σ).

The honest correction — the heterogeneity was substantially an error-bar artefact. The phases do *not* carry the per-anchor rate spread: χ^2/dof falls only $3.14 \to 2.52$ (target < 2 missed), the held-out excess scatter gets *worse* (-15.4%), and the deficit itself moves 0.18σ. The exploratory scan closes everything else: density fine structure, rung count, α, $\mathrm{frac}(\alpha)$, prime-power order, residues mod 3 and 5, per-anchor curvature — every block improves in sample and worsens held out. That leaves one suspect, which is not an anchor property at all: the error bar. The OLS slope error assumes independent rung residuals, and the phase modulation just proved they are not; with jackknife slope errors the excess scatter goes $0.1224 \to 0.0000$ and the robust pull spread $1.41 \to 1.22$, and the model-free interleaved half-split gives 0.2722 against $\mathrm{sd}(e_n) = 0.2789$ — consistent with the jackknife, not with OLS. T174's χ^2 was substantially an OLS independence assumption, not a physical spread; the last 22% of the pull spread is named, not swept. *The* 5.0σ *deficit is unaffected either way*: its bar comes from the anchor-to-anchor scatter itself, never from se_n.

The intervention (the key result). A regression can be confounded; an intervention cannot.

With an exact rebuild control ($\delta = 0$ reproduces R to 0.0 in $\log R$ on 40 cells), displacing the six family atoms inside their own lag cells — cell index fixed, mass conserved exactly, twelve lag entries change — gives

$$\frac{d \log R}{d\delta} = 879, \quad \text{linear to a factor 1.011 over the four declared decades } \delta = 10^{-7} \ldots 10^{-4}$$

— **the placement phase is causal for** R, the first interventional statement of the series. And it overturns the *shape* of the regression result while confirming its substance: the fitted first harmonic predicts a local sensitivity of only 2.89 — a factor ∼300 too small, with predicted-versus-measured shifts correlating at $r = +0.037$ — so the 10% the phases explain is the small smooth component of a response that is locally enormous and not first-harmonic. Positive definiteness dies for $\delta \geq 10^{-3}$ (displacing six of ∼24 000 atoms by 1% of one lag cell destroys the floor on 18% of cells), and GAP $= \lambda_{\min}/\lambda_{\max} = 1.6 \times 10^{-9}$–$4.8 \times 10^{-6}$: the Schur floor is a *near-degeneracy*, so $\log R$ is necessarily non-smooth in the phases, and no phase formula is certifiable. Consistently, mass-preserving alternative assemblies are *inadmissible* (lump 0%, wide 46%/0% PD survival), and the admissible continuous deformation gives sensitivities +0.384 (sparse) against −1.054 (dense), ratio 0.4 — the sparse corner is a density regime, *not* an assembly artefact.

The dense limit, measured to its ceiling. The corner is exhausted by a theorem about the caps (dens ≤ 361 from the sieve cap and $h \geq 128$; 359 reached). Over three decades of comb density, in six declared ratio-4 bins with cluster-robust bars, the deficit falls $+0.982 \pm 0.213 \to +0.576 \to +0.207 \to +0.194 \to +0.088 \to -0.057 \pm 0.132$ — monotone within 2σ at every step, density slope 6.1σ, and *the densest reachable bin is consistent with zero* (0.4σ at median density 167). The extrapolation fence is the most important sentence of the block: a true zero at finite density, a power-law approach to zero and a plateau below 0.132 are *all* consistent with these bars, and no probe under this sieve can separate them — the limit is undecided, not claimed. The sharpened confound test (only the rectangle separates $\log h$ from log dens: within one anchor they are collinear at −1.000) finds *both* channels significant ($\log h$ $+0.2435 \pm 0.0670$, log dens -0.0631 ± 0.0172, curvature $a_2 = -0.137 \pm 0.029$): T174's density driver survives but is no longer unique — the deficit is not a single exponent but a local slope of a curved surface. A rule proposal goes to the chain convention, not applied retroactively: dens ≥ 1 ($+0.722 \pm 0.094$ on unresolved cells against $+0.099 \pm 0.019$ on resolved ones, 6.5σ separation, 6% of cells); the comb scramble destroys 12/12.

After T175: the larger sieve, nothing else

(Verdict) PHASES-RESIST — the phases are real and causal, but no phase formula is certifiable (the response is provably non-smooth: the floor is a near-degeneracy), and the heterogeneity they were meant to explain dissolved substantially into the error bar — a correction to T174's reading, not to its number. *(Rest)* the shape of the phase response in closed form, $d\lambda_{\min} = v^{\mathsf{T}}\, d\hat{A}\, v$ — first-order perturbation theory of the near-degenerate low block, no fit anywhere (it turns the phase regression into an identity or kills it); **the dense-corner zero needs a larger sieve, nothing else** — a sieve at 10× the atom cap moves the ceiling from 361 to ∼3607 and is the only way to decide the limit; the curvature ($a_2 = -0.137 \pm 0.029$) means "the deficit" needs a declared reference rung or a curvature term; whether the positivity margin has an h floor at all; and the last 22% of the pull spread. *(Candidates P175-1–P175-8, pending)* the generating identity, the density curve, the two channels, the causality finding, the error-bar resolution, the spline-positivity finding, the dens ≥ 1 proposal, the exclusion — no promotion here. T176 (`dense_limit_probe.py`, contract `DENSE.LIMIT`) ran exactly that larger sieve — the next subsection (landed after the reverse-flow part T177; numbered by contract).

10.66 The bigger sieve: the density curve sits at zero, and the measurement programme closes (T176)

T176 (`dense_limit_probe.py`, contract `DENSE.LIMIT`, 24/24, 118.7 s, verdict SITS-AT-ZERO; landed after T177 — part numbering by contract) is the last decidable measurement of phase 2, and it is deliberately *one* change: the sieve cap rises by a factor 20.8 (`ATOM_MAX` $= 1.2 \times 10^6 \to 2.5 \times 10^7$; 1 566 727 prime-power atoms) while the ratio R, the cluster-robust estimator, the assembly, the

KMS basis and the Schur block are held *verbatim* at T175, so the extension is an extension and not a new measurement. The cost statement is the first result: the sieve builds in 0.08 s at 59.7 MB peak — *the sieve was never the binding constraint* (the 14 532-cell grid costs the 110 s) — and the honest resource fence follows: a further factor 5 in the cap would fit the budget, a factor 100 (the next decade of window, $\sim 2.5 \times 10^9$) would not. The density ceiling moves $361 \to 6120$ (6118 reached), 14 532 admissible cells on 692 anchors carry a positive definite low block, 0 indefinite.

The extended curve. Eight declared ratio-4 bins over 4.0 decades of comb density, cluster-robust over anchors:

dens bin	anchors	med. dens	deficit ± s.e.	
$[0.1, 0.4)$	11	0.27	$+0.8886 \pm 0.1027$	
$[0.4, 1.6)$	34	0.92	$+0.5148 \pm 0.0883$	
$[1.6, 6.4)$	69	3.5	$+0.1948 \pm 0.0861$	
$[6.4, 25.6)$	117	14.1	$+0.3922 \pm 0.0935$	(declared violation)
$[25.6, 102.4)$	218	56.8	$+0.1520 \pm 0.0573$	
$[102.4, 409.6)$	403	227.6	$+0.0579 \pm 0.0542$	
$[409.6, 1638.4)$	471	779.2	$+0.0376 \pm 0.0410$	**new** (0.9σ)
$[1638.4, 6553.6)$	272	2511.8	$+0.1046 \pm 0.0881$	**new** (1.2σ)

Pooled over the two new bins the deficit is $+0.0496 \pm 0.0372$ (1.3σ): *no sign change anywhere on the curve, no stabilisation above zero — the curve sits at zero from dens* $\sim$410 *upward.* The density slope is -0.0725 ± 0.0090 (8.0σ over 4.0 decades), and the bin-4 monotonicity violation is declared, not smoothed: the finer $2^{1/4}$ rung ladder samples the curvature of $\log R$ in $\log h$ differently from T175's $\sqrt{2}$ ladder (below). What the bigger sieve actually bought is a number: *any plateau of the deficit above* 0.074 *is excluded at* 2σ *for dens in* $[410, 6118]$*, against T175's* 0.264 — *a factor* 3.6 — and that is the whole content of the extra 48 MB of sieve. The consistency stress could have killed the file and did not: all six T175 bins, re-measured with T175's exact anchor set and rung ladder under the new sieve, reproduce at 0.0σ. The curvature map survives with T175's coefficients (both channels: $\log h$ $+0.1529 \pm 0.0258$ at 5.9σ, $\log \text{dens}$ -0.0434 ± 0.0060 at 7.3σ; per-anchor $a_2 = -0.0842 \pm 0.0085$; all $\leq 1.7\sigma$ from T175), and none of the six preregistered reparametrisations makes the surface one exponent — the fitted direction wants $\text{dens}^{+0.284} h$, and no candidate sits there.

R1 closed, and the 300× *anomaly explained.* The Feynman–Hellmann form

$$\frac{d \log R}{d\delta} \quad = \quad \frac{v^{\mathsf{T}} d\tilde{A}\, v}{\lambda_{\min}} \quad - \quad \frac{w^{\mathsf{T}} d\tilde{A}\, w}{\lambda_{\max}} \quad - \quad \frac{d\text{DEL}}{\text{DEL}}$$

reproduces the full rebuild to 3.37×10^{-6} at the minimum of a declared step ladder whose U-shape is exhibited (no floor $\Rightarrow$ the residual is the difference quotient, not the formula): T175's intervention regression is now an *identity* about the assembly, and the decomposition corrects the expected story — dDEL/DEL carries 84% of the response, $\lambda_{\min}$ only 7%. And the $\sim$300× gap between T175's first harmonic and its local derivative is *a pole*: positive definiteness survives only 2% of the phase period ($\delta_{\text{PD}} = 5.9 \times 10^{-3}$ median, $\delta_{\text{PD}} \cdot |d \log R/d\delta| = 5.8 = O(1)$), so a first-harmonic regression estimates a Fourier coefficient of a function with a pole in the interval — neither number is wrong. Measured against the going-in hypothesis: the two lowest modes of the normalised low block sit 13.4% apart *already at* $\delta = 0$ — the intervention does not create a degeneracy, it runs along one, which closes the loop with the decomposition: $\text{DEL} = 1 - r_{12}^2$ *is* the near-degeneracy of the bottom two directions, exactly the channel T174 proved does not cancel. Stress: the occupancy-preserving scramble kills positive definiteness on 194/194 cells in the first new bin; L_P exact at 1.9×10^{-15}; the anti-fitting battery is mechanical (bin edges ratio-4 exact, the first five bins bit-for-bit T175's).

The ladder caveat (the anti-promotion). The bin estimator is *ladder-dependent* at the 0.20 level: a ratio-4 bin holds $\sim$12 rungs of the $2^{1/4}$ ladder against $\sim$6 of T175's $\sqrt{2}$ ladder, and with $a_2 \neq 0$ they sample the curvature differently. Consequence, carried into the ledger row: any quoted bin deficit must cite its rung ladder, or the number is not reproducible.

After T176: the measurement programme closes

(Verdict) SITS-AT-ZERO — at the densest combs a 2.5×10^7 sieve can build, the deficit is consistent with zero, with no sign change and no stabilisation above zero; the plateau window is 3.6× narrower than T175's; and the three readings T175 could not separate — true zero at finite density, power-law approach, small plateau — are still three readings, confined to a smaller box. No finite sieve can decide the limit (the window shrinks like the square root of the anchor cap), so **the phase-2 measurement programme closes here, as planned**: no follow-up probe. What T176 leaves for the *papers* rather than for new probes: a ladder-robust estimator (integrate the curvature instead of sampling it), the dens$^{+0.284}h$ direction (derive it from the assembly or drop the word "exponent" from the chain), and the 2×2 effective model of the near-degenerate bottom — the cheapest structural question left, and the file's real contribution to it is that the phase response there is now closed in form. The exact cores are load-bearing: [`verification/v562_dense_limit_identities.py`] (`PRIME.DENSE.LIMIT.01`; Section 4.27). The work continues in the classification papers (Paper II) and the backflow lines (T177–T199, Sections 10.67–10.88).

10.67 The reverse flow, resumed: the CP mirror narrows four ledger rows (T177)

While T176 was running, the reverse flow resumed: T177 (`cp_invariant_probe.py`, contract `CP.INVARIANT`, 49/49, verdict DEGENERATE) carries the newest tool of the phase — gauge-degree bookkeeping under declared group actions (T173/T174) — back to the physics side and runs it on the four CP ledger rows (`REDTEAM.D.01`, `E8.CHAN.01`, `CP.MOD.01`, `DUAL.FRAME.01`). All four constructions are first rebuilt exactly from the compiler primitives (the magnitude bijection, the $248 = 120 + 128$ channel split, the frozen reading $\delta = \pi/3 + 3\lambda_C^2 = 1.198231638$ certified against the lattice, the oriented volume $\det(\mathbf{1}, d, n) = 21$, the classical Jarlskog anchor $J = 3.327 \times 10^{-5}$ with the CKM unitary to 10^{-41}), and one new exact identity falls out of the rebuild: on the hexagonal unit $\rho = e^{i\pi/3}$ the phase channel carries $\sum_k \rho^{d_k} = 4\rho$ (arg $= \pi/3$ exactly, sheet-*even* because the E_8 degrees are even) while the magnitude channel carries $\sum_k \rho^{m_k} = 4$ (arg $= 0$ exactly, sheet-*odd* because the exponents are odd) — the channel typing has an exact fiber realisation, not only a counting analogy, with the honest caveat said out loud (reality of the exponent sum alone is not hexagon-specific).

The mirror itself is a 6×4 table — six candidate functionals against the μ_3 branch group, the positive rescalings, the parity/sector involution and a *declared* D_4 deck model, with the character bookkeeping verified as a homomorphism in every non-trivial cell — and it returns the pre-registered kill verdict on both tiers. Tier 1 (strict invariance under all four groups) is *empty by algebra*: both complex kernels commute with conjugation, so every $\pi/3$-carrier is κ-odd, and invariant + odd forces $\operatorname{Im} F = 0$. On the Jarlskog-class criterion (Tier 2) *two* rows pass, in two inequivalent character classes — the channel angle (sheet-even) and the scale-free orientation $\operatorname{Im} \rho = \sin(\pi/3)$ (sheet-odd) — so the $\pi/3$ is invariant as a *value* but is not pinned to one functional, and *no* invariant carrier couples to the frame: what is frame data is the oriented volume 21 and the sign convention, not the angle (the raw orientation functional is homogeneous of degree +1 in the frame normalisation, and dividing by the oriented volume removes the frame identically, $\operatorname{Im}(\rho \det)/\det = \operatorname{Im} \rho$ exactly). The net effect is a verdict of *degeneracy* with four *narrowed* ledger readings and no marker moves: the magnitude data carry exactly zero phase ($\arg \prod c = 0$ on both sectors), the `REDTEAM.D.01` residual is a *fiber* datum, the phase-channel angle is sheet-blind, the full δ has no character (a CP-even correction on a CP-odd angle; only the leading term $\pi/3$ is character-covariant), and `DUAL.FRAME.01`'s $21 \sin(\pi/3)$ splits into an invariant fiber core and a non-invariant frame factor. Which of the two Tier-2 characters is physical stayed open at T177 — the sheet-odd reading reproduces `CP.MU6.01`'s quark/lepton split, a structural preference and not a derivation — together with the frame-coupling question; both are decided one part later (T178, Section 10.68). The exact cores are promoted to the physics side: [`verification/v561_cp_channel_identities.py`].

Where phase 2 now stands. The map is complete, its portability is measured, and its frame dependence is dissected: the reduction from the I5 uniformity question to a single object runs

end to end and reproduces in one file (T171), its identity tower carries unchanged onto every frame and anchor arithmetic tried while every carried number must be recomputed per surface (T172), the demand side itself is a frame datum whose difference against the delivery is the one surviving number (T173), and the gauge route on that number is exhausted by theorems (T174): everything multiplicative cancels exactly — including the entire h^{-2} channel — the one shared channel is certified unconditionally at under five percent, the additive arch/comb mixture admits no factorisation, and the deficit stands measured on a frame-rule-free surface at $+0.1111 \pm 0.0222$ (5.0σ), its driver the comb density per lag cell. The interior of that number is then opened (T175): the placement phases of the heavy deep atoms are real in $\log R$ and causal for it under an exact-rebuild intervention, but the response is provably non-smooth — the Schur floor is a near-degeneracy — so no phase formula is certifiable; the per-anchor heterogeneity dissolves substantially into the OLS error bar (a correction to T174's reading, not to its number); and the deficit falls monotonically with comb density over three decades to a densest reachable bin consistent with zero, with true zero, power-law approach and low plateau undecidable under this sieve. The instrument question is then answered by the instrument itself (T176): the twentyfold sieve raises the ceiling to 6120, both new bins are consistent with zero (pooled $+0.0496 \pm 0.0372$), the plateau window narrows by a factor 3.6 — SITS-AT-ZERO, the last decidable measurement of phase 2 — while the derivative identity closes in Feynman–Hellmann form and the $300\times$ harmonic anomaly resolves into a pole. *With that, the phase-2 measurement programme closes as planned*: what is left is one statement — an unconditional certificate that two explicit finite Λ-sum vectors become collinear at the demanded rate, now stated frame-free, with the true-zero/power-law/plateau reading of the dense limit explicitly open and no finite sieve able to decide it — and the work continues in the classification papers (Paper II) and the backflow lines (T177–T199), not in further probes of this instrument. The measurement surface is certified, the quantifier is open, R1 is classified; the capstone that certifies exactly this map is load-bearing [`verification/v559_phase2_capstone.py`], the frame/deficit identities of the closing arc are load-bearing [`verification/v560_frame_deficit_identities.py`], and the dense-limit identities of the endgame are load-bearing [`verification/v562_dense_limit_identities.py`].

10.68 The reverse flow, continued: the frame door of the CP mirror, decided (T178)

One part after the mirror, the backflow line answers T177's two open items — and they are pure finite algebra, so the answer is a decision, not a measurement: T178 (`cp_frame_carrier_probe.py`, contract `FRAME.CARRIER`, 31/31, verdict FIBER-PINNED). **R1 is decided.** The natural (metric-free) half is negative *by theorem*: the fiber acts on the frame by a scalar on the n-row, so every homogeneous frame block factorises exactly (checked on fully generic degree-1..4 polynomials with symbolic coefficients), and the metric-free invariants of $(\mathbf{1}, d, n)$ have *one* generator — the exact bracket enumeration gives six permutations $= \pm\Delta$ and 21 identically vanishing brackets (Weyl 1946 first fundamental theorem as the cited class boundary) — so every scale-free ratio is constant and no frame-coupled scale-free $\pi/3$-carrier exists without a metric: T177's obstruction sentence ("one determinant only") is now a theorem. The metric half couples but is *non-canonical*: declared metrics admit frame-coupled Tier-2 carriers, but they split into two character classes ($\Delta/\langle d, n\rangle$ and $\Delta/\langle \mathbf{1}, n\rangle$ at $(-1,-1,+1)$ against $\Delta^2/\langle n, n\rangle$ at $(-1,-1,-1)$) and the admissible normalisers contain a whole continuum $q_t = t\,n_1^2 + n_2^2 + n_3^2$ (deck-invariant for every t, values $441\sin(\pi/3)/(25t+117)$) — no canonical frame factor, and the exhibited factors 21/8, 21/2, 441/142 are normaliser-relative frame data with the anti-numerology fence wired as a check. **R2 is selected.** The classical anchor itself does the selecting: the Jarlskog invariant carries the full character tuple $(-1,-1,-1)$ under conjugation, the sheet flip and the declared deck reflection (each certified at 40 digits), matching the sheet-odd orientation class and not the sheet-even channel class — of T177's two degenerate classes, exactly one; the unique anchor-matching *canonical* carrier is the fiber-pure $\operatorname{Im}\rho = \sin(\pi/3)$ (selected, conditional on the anchor — not derived). **And the degeneracy is explained**: the single-sine lemma — on a μ_n fiber the nonzero carrier magnitudes $\{|\operatorname{Im}\rho_n^k|\}$ form a one-element set iff n is crystallographic, $n \in \{1,2,3,4,6\}$, by $\sin(4\pi/n) - \sin(2\pi/n) = 2\sin(\pi/n)\cos(3\pi/n)$ —

makes T177's DEGENERATE verdict *forced*: the compiler's two clocks μ_4 ($j = 1728$) and μ_6 ($j = 0$) are exactly the two maximal single-sine fibers, so value-based pinning of functionals is impossible there by theorem (on μ_5, values do separate carriers). No marker moves, the frozen reading is untouched, and the CP residual is not closed — it is *relocated with a proof*: the invariant CP core is the fiber datum $\sin(\pi/3)$ and provably cannot be made frame data without a non-canonical metric import. The exact cores are promoted to the physics side: [verification/v564_cp_fiber_pin.py].

10.69 The backflow, continued: the relative pencil answers the one-mode question (T179)

The backflow then runs in the *other* direction — from the physics-side operator algebra back onto this document's own open problem. The parabolic self-code module had proved, symbolically, that the Paper-II rank-3 polarisation is (2+1)-Lorentz geometry and that the low block carries a relative pencil $Z = B^{-1}S$ with three exact identities; the one measurable question that decides whether that lens buys anything is whether *one* pencil eigenvalue stays uniformly separated from 1 on the reachable surface. T179 (`lambda_pencil_probe.py`, contract `LAMBDA.PENCIL`, 4/4, verdict ONE-MODE) answers it on the declared T170 frame-A scan (all 70 windows, the v563 surface bit for bit): the identities are machine-tight at 2×10^{-14}; the *locking* mode is λ_2, hugging 1 at median 1.000021, while the spectator λ_1 sits far *above* 1 ($[4.53, 180.64]$, $\min|1 - \lambda_1| = 3.53$) — the label guess of the motivating review, reversed — so the two-dimensional cancellation of the classification paper reduces to the single scalar $1 - \lambda_2$ on the whole reachable surface (on the $h = 540$ reference window $1 - \lambda_2 = -2.27 \times 10^{-5}$ reproduces $\det \hat{A} = 4.22 \times 10^{-3}$ exactly), and the position scramble explodes $|1 - \lambda_2|$ by $\times 1.9 \times 10^6$ while the identities keep holding: the locking is arithmetic placement. No uniformity claim beyond the surface, no rate, no RH statement — Problem 7.1 is *typed*, not bounded. The exact core is load-bearing: [verification/v569_lambda_pencil_onemode.py] (Section 4.29).

10.70 The backflow, continued: the separation floor closes on the surface (T180)

One part later the open question T179 named — does the spectator stay separated? — is answered on the reachable surface by reduction rather than by more measurement: T180 (`separation_floor_probe.py`, contract `SEPARATION.FLOOR`, 9/9, verdict FLOOR-CERTIFIED). Two one-line theorems turn the separation into a closed per-window floor ($\lambda_1 \geq \sqrt{\det S / \det B}$ when the pencil determinant is positive, $\lambda_1 \geq \operatorname{tr} Z$ when it is not), and on all 70 declared windows the floor clears the margin, $\min F = 2.130 \geq 1.05$: the separation now *follows* from two closed scalars instead of being read off an eigenvalue routine. The dominance behind it — the comb determinant over the arch determinant — is large (median 30.1) and grows with depth ($h^{0.90}$ on the surface ladder). The honest negative travels with it: the cheap two-atom witness route is dead ($\det S$ is itself a pair-level cancellation, negative pair mass up to 13.7× the total), so the h-uniform dominance stays open — typed now as a pair-level size-vs-cancellation question, strictly weaker than Problem 7.1 but not free. The scramble control flips the trace sign (-78.28): arithmetic placement again. The exact core is load-bearing: [verification/v570_separation_floor.py] (Section 4.30).

10.71 The backflow, continued: where the dominance lives (T181)

The typed question T180 left — *where* does the dominance sit, so that a certificate knows what to aim at? — is answered by dissecting the pair kernel: T181 (`pair_band_structure_probe.py`, contract `PRIME.PAIRBAND`, 10/10, verdict LONG-RANGE). The Cauchy–Binet band decomposition is exact per window; on every selected window *every* near band (< 16 lag cells) and the diagonal drag *negative* — so near-diagonal Chebyshev budgets aim at the wrong sign — while the far band carries the whole positive excess with a stable net ($[1.55, 1.91] \times \det S$) even as its gross mass grows to $13 \times \det S$: the dominance is long-range coherence. Sharper still, the excess is scale-locked ($\geq h/2$ pairs carry it) and weight-concentrated — the top-10%-weight pairs alone net

0.85–$1.00 \times \det S$ on every window, essentially $\det S$ itself — while the lag-grid phase plays no role (flat, shrinking with h): the certificate target is *heavy-pair coherence at window-scale separation*. The scramble control cuts both ways: random placement at the same masses explodes $\det S$ by $\sim \times 49$ and collapses the far share — the real placement *suppresses* $\det S$ fifty-fold while organising its coherence. Declared surface and subset only; no rate, no RH statement. The exact core is load-bearing: [verification/v573_pair_band_structure.py] (Section 4.31).

10.72 The backflow, continued: the edge reads close in Chebyshev–Loewner form (T182)

A second external review proposed a closed formula for the polarized parity cross weights and an exact structure theorem at the reflection edge; T182 (`cheb_loewner_probe.py`, contract `PRIME.CHEBLOEWNER`, 9/9, verdict FORMULAS-EXACT) audits all of it against the corpus definitions — and everything holds *exactly*: the cross formula closes the whole $K = 2$ read block analytically; the edge read is a scaled Loewner matrix of U_{s-1} (rank $\leq s-1$; the first edge read is rank one *by theorem*); the leading profile's Lorentz image is the Pythagorean null triple $(5, -3, 4)$ — the same triple as $(g_{\mathrm{car}}, N_{\mathrm{fam}}, |\mu_4|)$, typed compression; the normalized edge defect is the exact coefficient $-\frac{3\pi^4}{175}(s^2-4)(11s^2+6)N^{-4}$ (one order better than the h^{-3} demand, locally); and the pair-kernel sign polynomial $P(s,z)$ (transition $s/z \approx 0.503/1.987$, $P(2,3) = 0$ exact) is the analytic prototype of T181's measured near-negative/far-positive anatomy. The three-regime split refines Problem 7.1 without touching its quantifier. The exact core is load-bearing: [verification/v576_cheb_loewner_edge.py] (Section 4.32).

10.73 The backflow, continued: the null-ray census (T183)

The review's sharpest prediction — the locking mode should align with the rank-one null ray $(2, -1)$ — is put to its preregistered test: T183 (`nullray_census_probe.py`, contract `PRIME.NULLRAY`, 5/5, verdict NULL-RAY-CORRELATED). The alignment is real on every one of the 70 windows (23.5–33.8°, all below random) and the scramble destroys it (+21°) — but the decline is slow ($h^{-0.10}$), so the clean success bar is honestly *not* met: the ray is a real *partial* carrier, and the macro regime bends the locking direction away from the pure edge ray, exactly as the three-regime split predicts. The follow-up is the macro-regime direction from $K_\infty(\sigma, \tau)$, not more angle fits. The exact core is load-bearing: [verification/v577_nullray_census.py] (Section 4.33).

10.74 The backflow, continued: the two-scale kernel signs match everywhere (T184)

The macro object T182 introduced is put to its global test: T184 (`macro_kernel_probe.py`, contract `PRIME.MACROKERNEL`, 6/6, verdict SIGNS-MATCH). The sign of $K_\infty(\sigma, \tau)$ predicts the sign of the real two-lag kernel on *every* resolved grid cell at $h = 300$ (332/332), the diagonal $\det W^\infty(\sigma)$ is negative on $\sim$72% of the range (same-scale pairs couple negative, analytically — the macro origin of T181's near drag), and the wrong polarisation breaks the match loudly. The *geometric* half of Regime B closes here; what remains is the *arithmetic* half — which (σ, τ) cells the primes actually occupy, with which Λ-weights: exactly the weighted summation that must beat the 8:9 cancellation. The exact core is load-bearing: [verification/v579_macro_kernel_signs.py] (Section 4.34).

10.75 The backflow, continued: the primes populate the sign map (T185)

The arithmetic half of the two-scale question gets its measured object: T185 (`occupation_map_probe.py`, contract `PRIME.OCCUPATION`, 5/5, verdict OCCUPATION-FOLLOWS-KERNEL). The lag-side polarization identity $\det S = \frac{1}{2}\sum c_{\mathrm{at}} c_{\mathrm{at}} D(W, W)$ is exact, and its binned occupation map shows the primes *populating* the geometric sign map: every significant cell sign-matches $K_\infty(\sigma_1, \sigma_2)$ on every tested window, with the mid-diagonal block net negative and the cross-scale cells net positive. The open

Regime-B content is now purely *quantitative*: why the cross-scale cells win by exactly $\det S$. The exact core is load-bearing: [verification/v580_occupation_map.py] (Section 4.35).

10.76 The backflow, continued: the transport lemma dies honestly (T186)

The second review's Priority-4 lemma — summable locking-direction increments $\sin\angle(v_h, v_{2h}) \le Ch^{-1-\delta}$ — is measured on 40 doubling pairs: T186 (`locking_transport_probe.py`, contract `PRIME.TRANSPORT`, 4/4, verdict TRANSPORT-KILLED). The drift is small (0.3–6.7° per octave, median 2.35°) but *grows* ($h^{+0.36}$): the increments are not summable, the review's own kill criterion fires, and the route is ended without romantic extension — any future transport must carry the window-dependent macro direction with it. The exact core is load-bearing: [verification/v581_locking_transport.py] (Section 4.36).

10.77 The backflow, concluded for this round: the dominance is density-driven (T187)

The distilled question — why do the cross-scale cells win by exactly $\det S$? — receives its first-level answer: T187 (`density_dominance_probe.py`, contract `PRIME.DENSITYDOM`, 5/5, verdict DENSITY-DRIVEN). The smoothed comb reproduces $\det S$ to 99.8–99.9% (2% smoothing) on every regular window, while on the scrambled window the smooth model captures only 70%: the real comb is anomalously smooth at the $\det S$-relevant scales, and the arithmetic content of the dominance is the deterministic density *profile* — which suppresses $\det S$ fifty-fold below random while carrying ∼99% of it. The v570 h-uniform question relocates to PNT-level regularity of the smooth functional (plus a ∼1% fluctuation correction needing only a crude bound); the anomalous $h = 1292$ boundary is named. The exact core is load-bearing: [verification/v582_density_dominance.py] (Section 4.37).

10.78 The prime-free closed form lands the same day (T188)

The named next step of T187 — a closed-form bound on the smooth functional — is executed immediately: T188 (`pnt_model_probe.py`, contract `PRIME.PNTMODEL`, 8/8, verdict PRIME-FREE-CLOSED-FORM). The two-term classical law $\mathrm{mass}(u) = 4e^{u/2} - 2(\zeta'/\zeta)(\frac{1}{2})$ — no zeros — reproduces $\det S$ on all 69 regular windows *parameter-free* to 1.016–1.143, and the anomalous $h = 1292$ window is typed: the prime-free model gives +1844 where reality gives −1465 — the sign flip at the sieve horizon is genuine prime-fluctuation content. The dominance question now splits cleanly into a deterministic prime-free inequality plus a zero-oscillation bound. The exact core is load-bearing: [verification/v583_pnt_model.py] (Section 4.38).

10.79 The two-layer split closes the day (T189)

The complementary question — does the density model also reproduce the *locking* cancellation? — is answered: no, by 2–4 orders of magnitude, and that is the result: T189 (`pnt_locking_split_probe.py`, contract `PRIME.LOCKSPLIT`, 6/6, verdict TWO-LAYER-SPLIT). The Paper-II defect splits into a deterministic density layer ($h^{-1.43}$, coarse lock, always positive) and an arithmetic layer ($h^{-1.08}$ extra, median 1350×, with sign overshoots at the deep windows): the one-mode geometry is density-level, the depth of the lock is the primes — the true arithmetic core of Problem 7.1 is now isolated as a single multiplicative layer. The exact core is load-bearing: [verification/v585_pnt_locking_split.py] (Section 4.39).

10.80 The direction is density-fixed (T190)

The direction side of the T189 split lands immediately: T190 (`pnt_lock_direction_probe.py`, contract `PRIME.LOCKDIR`, 7/7, verdict DIRECTION-DENSITY-FIXED). The real locking direction tracks the prime-free model's to a median 0.030° across the ladder (max 0.331°, decaying $h^{-1.16}$), while the depth stays 1350× apart: the primes are a pure depth amplifier along a density-fixed

direction, and the deterministic direction certifies the real defect as a witness within a factor ≤ 2.9. The v577 slow-decay puzzle dissolves as a comparator artifact. The exact core is load-bearing: [verification/v586_pnt_lock_direction.py] (Section 4.40).

10.81 The entry closed forms land (T191)

The classical program of T189/T190 takes its first executed step: T191 (`w_closed_form_probe.py`, contract `PRIME.WCLOSED`, 6/6, verdict CLOSED-FORM-EXACT). A *new exact formula* — the diagonal companion of the v576 cross formula — closes the corpus diagonal lag weights in a single expression, machine-exact for all lags; with it, the exact cell-sum reproduces the prime-free model's entries and determinant to 0.03%: the S-side of the deterministic layer is closed-form, and the δ_{PNT} expansion (the $h^{-1.43}$ layer) becomes an expansion of explicit elementary functions. The exact core is load-bearing: [verification/v587_w_closed_form.py] (Section 4.41).

10.82 The deterministic layer closes (T192)

The remaining two named steps of T191 land in one part: T192 (`closed_delta_probe.py`, contract `PRIME.CLOSEDDELTA`, 6/6, verdict CLOSED-DELTA-EXACT). The archimedean side is the *trivial-zero ladder* of one exact function G, the comb side its pole exponent; the closed defect matches the v585 density layer to 0.2% on all 70 windows and decays $h^{-1.40}$. Problem 7.1's deterministic layer is now the explicit formula's skeleton — pole vs Γ-factor — in closed form; only elementary asymptotics separate it from a theorem, and only the arithmetic depth layer (v585) belongs to the primes. The exact core is load-bearing: [verification/v588_closed_delta.py] (Section 4.42).

10.83 The zeta zeros enter the instrument (T193)

The arithmetic layer gets its object: T193 (`zero_comb_probe.py`, contract `PRIME.ZEROCOMB`, 4/4, verdict ZERO-COMB-IDENTIFIED). The residual mass oscillation of the atom table is the explicit-formula zero comb (correlation 0.82 with 200 damped zeros; pointwise at low u), and adding the comb to the density model recovers a factor 5.7 of the lock depth at the smallest window — while the deep windows demonstrate, honestly, that the 10^{-5}-level lock demands oscillation amplitudes far beyond a 200-zero sum. The zero-oscillation bound is now a quantified explicit-formula target. The exact core is load-bearing: [verification/v589_zero_comb.py] (Section 4.43).

10.84 The direction law falls out of the closed forms (T194)

The formal expansion of the closed entries begins and immediately delivers a theorem-grade identity: T194 (`pole_rank_one_probe.py`, contract `PRIME.POLERANKONE`, 5/5, verdict POLE-RANK-ONE). The pole term is *exactly* rank-one (the denominators factor, $\det \equiv 0$ symbolically), and its null direction gives the closed locking-direction law with limit exactly $-\frac{1}{2}$ — deriving the v577 null-ray conjecture and explaining the v586 drift as $-6\pi^2/(a^2+4\pi^2)$. The density layer of Problem 7.1 is now the correction theory of a rank-one matrix. The exact core is load-bearing: [verification/v591_pole_rank_one.py] (Section 4.44).

10.85 The determinant law and the certificates (T195)

The correction theory of the rank-one pole term takes its first step: T195 (`continuum_det_law_probe.py`, contract `PRIME.DETLAW`, 6/6, verdict DET-LAW-DERIVED). The closed determinant is one full exponential order suppressed ($\det S \sim 640\pi^2 e^a/a^3$, exact), the real dominance growth tracks it at correlation 0.9985 — deriving the v570 growth at the continuum level — and the entry-level arithmetic deviation is certified by $\sup|\mathrm{Osc}| \times \mathrm{TV}$ with a 47.6× margin, conditional on the measured oscillation sup. The exact core is load-bearing: [verification/v592_continuum_det_law.py] (Section 4.45).

10.86 The cutoff resolves the gap, and the first unconditional bound lands (T196–T197)

Two parts close the round: T196 (`cutoff_completion_probe.py`, 4/4, CUTOFF-CLOSES) shows the v592 finite-N gap was the integration cutoff — the density layer is closed end-to-end with an $O(1/N^2)$ residual; and T197 (`unconditional_cert_probe.py`, 5/5, UNCONDITIONAL-ENTRY-CERT) delivers the program's first unconditional statement via one published, verified-zero-backed bound. The exact cores are load-bearing: [`verification/v593_cutoff_completion.py`] and [`verification/v594_unconditional_cert.py`] (Sections 4.46–4.47).

10.87 The dictionary closes the books (T198)

The residual bookkeeping lands: T198 (`mapping_completion_probe.py`, contract `PRIME.MAPCLOSE`, 4/4, verdict MAPPING-CLOSES). The v593 residual was the σ mapping; with $a_{\text{eff}} = ND/2$ the closed forms match the deterministic model at the identity level (1.00000–1.00017), the kernel piece sits at 9×10^{-6}, and the density layer of Problem 7.1 is a finite set of exact elementary evaluations. The determinant law is formalized in Lean (`det_S_closed`). The exact core is load-bearing: [`verification/v595_mapping_completion.py`] (Section 4.48).

10.88 The arithmetic layer becomes one identity family (T199)

The 1D reduction lands: T199 (`lock_projection_probe.py`, contract `PRIME.LOCKPROJ`, 6/6, verdict NEAR-IDENTITY-1D). Along the closed v591 direction the zero functional cancels the closed density value to a median 7.7×10^{-4} (decay $h^{-1.01}$), collectively across the zeros — the arithmetic content of Problem 7.1 is one explicit identity family, equidistribution-flavored. The involution group of v590 is additionally formalized in Lean with its Klein-four closure $A \cdot B = S_0$. The exact core is load-bearing: [`verification/v596_lock_projection.py`] (Section 4.49).

Part VI
The closed-form theory, the surface theorems, and the detector

This part consolidates what the diary produced in closed form. Section 11 states the seven exact theorems of the parity read block with derivation sketches, and Section 11.2 poses the one remaining arithmetic statement self-contained, without TFPT vocabulary — together with the frame that accumulated around it: the W1 window form and its theorem-level closure at the measure level, the rank-3 surface theorems with the unconditional determinant positivity on the declared family, and the off-line-zero detector line. A mathematician can enter the document here.

11 The closed-form theory of the parity read block: statements

The backflow campaign left behind a compact body of exact statements that deserve one consolidated listing (proof sketches live in the cited modules; the algebraic cores of Theorems 2–4 are additionally formalized in Lean 4, `TfptCarrier/ParityWeightLaws.lean`, building against Mathlib).

The closed-form theory: seven statements

Theorem 1 (diagonal weight formula, v587). For the parity mode t_k on the window of half-length m ($N = 2m+1$, $\omega_k = 2\pi k/N$), the corpus diagonal lag weight is $W_{kk}(d) = \frac{2}{N}\big[(N{-}1{-}d)\cos(\omega_k d) + \sin(\omega_k(d{+}1))/\sin\omega_k\big]$ for $d \geq 1$, $W(0) = 1$; the apparent piecewise break at $d = m$ cancels (Lean: `branch_merge`), and the reflection-edge suppression is an exact two-term cancellation. Together with the v576 cross formula this closes the whole $K = 2$ read block.

Theorem 2 (one-function assembly, v588). With $G_{ij}(\beta) = \sum_{d\geq 1} e^{-\beta d} W_{ij}(d)$ (exact geometric-trig series), the prime-free model's comb side is the pole exponent ($\beta = -D/2$) and the archimedean side is the trivial-zero ladder ($c_{\mathrm{ar}}(d) = -Df(dD)$, $f(w) = \sum_n e^{-(2n+1/2)w}$): the deterministic defect is the explicit formula's skeleton, pole against Γ-factor, in one function.
Theorem 3 (rank-one pole term, v591). The pole part of the closed entries is exactly separable: $S^{\mathrm{pole}} = -32\pi^2 a e^a\, gg^T$, $g_k = k/(a^2 + 4\pi^2 k^2)$, $\det S^{\mathrm{pole}} \equiv 0$ (Lean: `pole_rank_one`).
Theorem 4 (locking-direction law, v591). $v_2/v_1 = -(a^2 + 16\pi^2)/(2(a^2 + 4\pi^2))$, with distance from $-\frac{1}{2}$ exactly $-6\pi^2/(a^2+4\pi^2)$ (Lean: `direction_law`, `direction_law_approach`); matches the measured directions to 0.04–1.4% and derives the v577 null ray as the limit.
Theorem 5 (determinant law, v592). $\det S(a)$ is closed with the e^{2a} coefficient identically zero and exact leading law $640\pi^2 e^a/a^3$; the measured dominance growth tracks it at correlation 0.9985.
Theorem 6 (cutoff completion, v593). With the $\sigma_0 = u_0/(2\alpha)$ cutoff the closed entries match the model to 0.7–1% and the determinant to 0.2–1%: the density layer is closed end-to-end; the residual is $O(1/N^2)$.
Theorem 7 (unconditional entry certificate, v594). Via partial summation and the published bound $|\psi(x) - x| < 0.94\sqrt{x}$ ($11 < x \leq 10^{19}$, Büthe 2018), the entry-level arithmetic deviation obeys the unconditional bound $\sup|\mathrm{Osc}| \leq \sim 17$ on the declared surface, and the certificate $|S^{\mathrm{real}} - S^{\mathrm{model}}| \leq \sup \times \mathrm{TV}$ holds with large margin — the first unconditional statement of the program. The determinant-level lock stays with the zero-oscillation theorem.

11.1 Derivation sketches

Theorem 1. For the sine mode $v_j = \frac{2}{\sqrt{N}} \sin(\omega_k(j+1))$ the autocorrelation and self-convolution are Dirichlet sums: $\mathrm{ac}_k(d) = \frac{2}{N}\left[(m-d)\cos(\omega_k d) + \frac{\sin(\omega_k(d+1)) + \sin(\omega_k d)}{2\sin\omega_k}\right]$ and the two convolution branches close analogously (all machine-exact, v587 E2.1). Substituting into the corpus assembly $W(d) = 2\,\mathrm{ac}(d) - \mathrm{cv}(2m-1-d)$ and using $\omega_k(2m+1) = 2\pi k$, the two branches produce the *same* expression — the cancellation is the linear-in-trig-values field identity formalized as `branch_merge` in Lean. At $d = N-1-s$ both terms are $O(s)$, which is the exact edge suppression.

Theorem 2. The cell masses $2(e^{(d+1)D/2} - e^{dD/2})$ are geometric in $e^{dD/2}$, and the Weil kernel is the geometric ladder $f(w) = e^{-w/2}/(1 - e^{-2w}) = \sum_{n\geq 0} e^{-(2n+1/2)w}$; hence both sides of the defect are values of the one finite geometric-trig sum $G_{ij}(\beta)$, evaluated in closed form via $\sum_{d=1}^{L-1} z^d$ and $\sum_{d=1}^{L-1} dz^d$ with $z = e^{-\beta+i\omega_k}$.

Theorems 3–5. The Laplace integrals $2a\int e^{a\sigma} g_{ij}(\sigma)\, d\sigma$ are elementary; their denominators factor as $D_{11} = (a^2+4\pi^2)^2$, $D_{22} = (a^2+16\pi^2)^2$, $D_{12} = (a^2+4\pi^2)(a^2+16\pi^2)$, so $D_{11}D_{22} = D_{12}^2$ and the exponential parts are exactly separable ($S^{\mathrm{pole}} = -32\pi^2 a e^a\, gg^T$, $g_k = k/(a^2+4\pi^2k^2)$; Lean: `pole_rank_one`). The null direction of gg^T gives the direction law with $-\frac{1}{2}$ limit (Lean: `direction_law`, `direction_law_approach`); the determinant follows by expanding $S_{11}S_{22} - S_{12}^2$ — the e^{2a} terms cancel by separability and the e^a coefficient is $128\pi^2 a^3(5a^2+32\pi^2)/D_{12}^2 \sim 640\pi^2/a^3$ (Lean: `det_S_closed`).

Theorem 6 and the exact dictionary. The lag variable is $\sigma = d/N$, so the continuum form of the cell sum carries $a_{\mathrm{eff}} = ND/2$ and the limits $[u_0/(ND),\ 2\alpha/(ND)]$; since $ND = 2\alpha + O(D)$ by the window construction, the naive α-form carries a first-order-in-D residual — which is exactly the v593 gap, closed at the identity level (1.00000–1.00017) by v595.

Theorem 7. Partial summation gives $\mathrm{mass}(u) = 4\sqrt{x} - 2 + 2E(x)/\sqrt{x} + \int_1^x E(t)t^{-3/2}dt$ with $E = \psi - t$ (machine-verified); inserting the published bound $|E| < 0.94\sqrt{t}$ ($t > 11$) and the exact small-range integral yields the unconditional sup, and integration by parts against the read profiles gives the certificate (v594).

11.2 The remaining arithmetic statement, self-contained

For external consumption, the one remaining arithmetic statement of the program can be posed without any TFPT context. Fix a window (an admissible pair (α, D) with $M = \lceil 2\alpha/D \rceil$ even, $N = M + 1$ odd, $h = M/2$) and let $W_{ij}(d)$ be the closed parity weights of Theorem 1 and $X_v(u)$ the read profile projected onto the closed direction of Theorem 4. Define, for each nontrivial zeta

zero $\frac{1}{2}+i\gamma$, the explicit coupling

$$c_\gamma \;=\; -2\int_{u_0}^{2\alpha}\frac{\sin(\gamma u)}{\gamma}\,dX_v(u) \qquad (u_0 = 2\log(-C_\infty/4),\; C_\infty = -2(\zeta'/\zeta)(\tfrac{1}{2})),$$

and let $K(\alpha, D)$ be the closed density value of the same projected form (an elementary function by Theorems 3–6). The machine-verified observation (v596) is

$$\sum_{\gamma>0} c_\gamma \;=\; -\,K(\alpha,D)\,(1+O(h^{-1}))$$

across the declared ladder, with median relative deviation 7.7×10^{-4}, and with the sum *collective* (no single zero contributes more than 6%; scrambling the underlying prime comb breaks the identity by six orders of magnitude). **Open problem (equidistribution form).** Prove, for the window family, that the zero couplings satisfy this cancellation law with a uniform rate — i.e. bound $|\sum_\gamma c_\gamma + K|/|K|$ by Ch^{-1} uniformly. No RH-type input is expected to be necessary for fixed windows (the couplings decay in γ and the identity is an explicit-formula statement); the *uniformity* in the window is the open content. The measured constant is now explicit and *frozen as a module* ([verification/v618_uniform_constant.py]): on the declared surface (69 floor-passed windows, $h = 142$–1445) the model value q_{model} keeps *one* sign on the whole ladder (0.039–0.112; no model zero crossing anywhere), and on every *lock-sign* window (67 of 69) the relative deviation times h stays below 1 (measured max 0.982), with tercile medians 0.61/0.45/0.39 (non-increasing) — so the conjecture is posed with the concrete constant $C = 1$. The sharpened typing: exactly *two* windows carry a q_{real} sign flip ($h = 1219$ at $9.2\cdot10^3$ and the edge window $h = 1445$ at 3.5), and the violator set of the $C = 1$ bound equals the flip set *exactly* — the sign predicts the bound violation window-sharp. (This corrects an earlier note that attributed the $h = 1219$ blow-up to a q_{model} zero crossing and missed the edge flip.) Scrambled combs break the bound by more than 10^4. The flips themselves are since *explained* ([verification/v619_flip_mechanics.py]): the two flip windows are exactly the two whose atom demand 2α exceeds the prime-power data cap $U_{\max} = 12.899$ — their combs are truncated — and truncation injection into complete windows reproduces the flips with matched sign and magnitude at both scales; on the complete-comb surface ($2\alpha \le U_{\max}$) the $C = 1$ bound holds with *zero* exceptions. The flip windows are data-boundary artifacts, not arithmetic phenomena.

Two further machine-checked facts frame this statement externally (2026-08-02). *The geometric frame:* through the exact Lorentz congruence $P^{\mathsf T} J_{\det} P = J_{\text{fix}}$ ([verification/v624_external_lattice_audit.py]) every complete window's vector (S_{11}, S_{22}, S_{12}) transports to the cover polarization lattice, and *all* 67 *complete windows land in the positive cone, on one sheet* ([verification/v627_hodge_chamber.py]; $\det S > 0$ everywhere; honest typing: chamber membership is a density-layer property, the fine $C = 1$ cancellation lives *inside* the chamber). *The operator frame:* the atom layer of the window form *is* the prime measure of Suzuki's screw function, literally (positions log prime powers, weights $\Lambda(n)/\sqrt{n}$, exact; [verification/v630_suzuki_contact.py]), and the smooth layer converts by a *derived* dictionary ([verification/v631_w1_dictionary.py]). *Convention erratum (2026-08-02, corrected the same day):* the first W1 rounds read Suzuki's eq. (1.3) with Lerch coefficient -1; the paper's own §2.2 data lock $+1/4$ ([verification/v643_w1_theorem.py], C0.1). All their identities hold for the chain kernel $\tilde g = g - \frac{5}{4}\,\text{Lerch}$ and every measured number transfers verbatim via $c^{\text{sm}}_{\text{gal}}(\tilde g) = -4\,c^{\text{sm}}_{\text{gal}}(g)$; on Suzuki's *true* g the smooth layer is $+\rho$ (the pole block — the rank-one piece the program tracks separately, v591 — unchanged), the dictionary is the *single* scalar $+1/D$ on both layers, and the origin constant vanishes, $\kappa = 0$ exactly. The boundary cells obey exact closed constants with the derived, window-independent $1 + 1/(6d^2)$ moment law ([verification/v640_w1_boundary.py], $\tilde g$-normalization), the frozen dictionary transports *unchanged* to three fresh windows ($h = 285/540/997$; atom constant D^2 at machine precision; preregistered kill criterion met nowhere, [verification/v641_w1_portability.py]), and the full quadratic form closes at the operator level ([verification/v642_w1_matrix.py]: pole block rank 1, two typed lattice residuals). **W1 is now THEOREM-closed at the measure level** ([verification/v643_w1_theorem.py]): the

projection lemma shows Suzuki's L_0^2 condition is automatic on the u-side and the window parity sector is a subspace of the form domain; $A_{\mathrm{arch}} = -g''_{\mathrm{smooth}}$ exactly at every lag (3.4×10^{-52}), full form equality 1.28×10^{-10}. W2 (form density) is *started* ([verification/v644_w2_form_density.py]: classical FEM-density slice at rate, Rayleigh–Ritz monotone from above on nested spaces, $\lambda_a = 0^+$ within $\sim 10^{-9}$ consistent with Suzuki Thm 1.3, no sign statement; Mosco/norm-resolvent remainder typed). W3 (uniform positivity) stays *open*, now with a tool diagnosis ([verification/v648_sign_uncertainty.py]): the sign-uncertainty/LP-ceiling toolbox carries a real 25-digit dictionary to the critical strip (phase slope $= L = \log \pi - \psi(\frac{1}{4})$, the W1 δ_0 weight) but its mass lever dies at $d = 1$ ($e^{-E^*} = 0.967$ where $< \frac{1}{2}$ is needed; $d \geq 21$ required) — empirically the surface is positive on all 67 complete windows ($\min \lambda_{\min} = +8.26 \times 10^{-4}$; the incomplete combs are exactly the flip set $\{1219, 1292, 1445\}$; $\mathrm{corr}(\log \lambda_{\min}, \log \varepsilon h) = +0.704$). On the geometric side the congruence matrix P itself is now canonical and constructed: the unique minimal-Frobenius operator-compatible class in the full census ([verification/v635_p_canonicity.py]) and, mod sign, the Frobenius-minimal integer congruence transporting the C_V null frame onto the canonical rank-one rays ([verification/v636_p_construction.py]); the honest negative [verification/v637_fine_c1_bridge.py] closes the direct fine-geometry route to the $C = 1$ margin (no independent information after h-control). The closing bundle of 2026-08-02 recalibrates the W3 reading against ground truth: on a proven RH analogue (the Ihara zeta of Ramanujan graphs) true positivity has *no uniform margin* — the margin goes to zero exactly beyond the support-resolution depth — so shrinking margins on deeper windows are the *expected* behavior, the alarm signal being only $\lambda_{\min} < -\mathrm{floor}$ (never observed); conversely the identical machinery breaks by $\sim$13 orders of magnitude the moment the Euler product is removed (the Epstein firewall, [verification/v668_ground_truth.py]), and the external Baez–Duarte frame shows the log-slow drift is intrinsic while the resonance needles are frame-side ([verification/v667_baez_duarte.py]). The fourth round of the same day closes the W2 *density plane*: the total window symbol is identified *exactly* as the Fejér-smoothed zero-counting density, $s_{\mathrm{tot}} = 2\pi(F_a \star dN)$ (Weil explicit formula on the tent test pair, verified 2.9×10^{-6} pointwise), and an unconditional Riemann–von Mangoldt + Trudgian chain certifies $\rho_{a,\delta}(t) \geq 0.306 \log(2 + t) - C_0$ a-uniformly with finite zero-free certificates ([verification/v669_fejer_density.py]); the honest floor $4\pi A_1 = 1.4074$ explains the Gårding drift (per-mode control below the floor is not certifiable this way), so the remaining W2 piece is the packet-to-point translation. The needle search narrows threefold: not low-rank ([verification/v670_w3_block_deflation.py], an honest FAIL of the block deflation), not zero-comb-driven ([verification/v671_lehmer_resonance.py], a clean Lehmer null), not in the Baez–Duarte frame ([verification/v667_baez_duarte.py]) — the mechanism search moves frame-/assembly-side. And the Li strand connects: the W1 window positivity *contains* 30/32 finite Li coefficients ([verification/v672_li_corollary.py]), and the E_8 shell data and the ζ comb agree in one Li sequence with residues $\pm 1/240$ ([verification/v673_li_e4.py]) — finite statements, not Li's criterion.

The great bundling (2026-08-03) lands the rank-3 chain as a *surface theorem* and extracts the SOS mechanism. *The rank-3 theorem:* the prime influence on the load-bearing 2×2 block is three exact linear functionals of the comb weights (assembly vs sieve 4.9×10^{-16}; [verification/v683_rank3_functionals.py], all entrywise absorption routes typed circular, the surviving det-level form at $\kappa_{\max} = 0.0895$); the one zero-side functional is treated by the explicit formula with budget-certified zeros (22 491 to $T = 2 \times 10^4$, residuum $\leq 6.9 \times 10^{-6}$) and yields the *unconditional* $\kappa_{\mathrm{unc}} = 0.039$–$0.190 < 1$ on all five declared windows, the pretentious escape blocked by the classical zero-free strip (damping $\times 634$; [verification/v684_rank3_zeroside.py]); and the uniformity quantifier closes on the whole declared surface — a symbolic envelope $K_{\mathrm{env}} \leq 0.9798 < 1$ for all 62 complete windows with $a \geq 3.434$, finite certificates $\kappa' \leq 0.108$ for the five below, hence $\det S > 0$ unconditionally on the entire declared family ([verification/v685_rank3_uniformity.py]; a *surface* statement, not uniform in all a; the absorption margin T-B stays open — no RH claim). *The kernel collapse:* the positive band-limited kernel class of the W2 capture threshold collapses exactly onto $\{\lambda \cdot \mathrm{Fejér}\}$ (pinning + Shannon), class supremum $0.0901684 < 0.10076$ — the kernel-optimization

escape is shut, W2 stays closed via the v680/v681 route ([verification/v687_extremal_kernel.py]). *The falsifier:* the in-kernel matched filter detects every off-line zero quadruple from $2\alpha\delta \geq 1.974$ on all mapped cells ($\times 2$ over the v677 mode map), with masking fully adjudicated 48/48 ([verification/v688_interpolation_detector.py]). *The mechanism:* the naive smooth $L + K$ split of the deployed window form dies structurally — the extremal pencil direction sits at $\gamma_1 = 14.13$, found from primes and digamma alone: the windows resolve individual zeros ([verification/v682_lk_split_theta.py]) — while the Hecke-SOS factorisation $A = B^*B + P$ exists *exactly* in the Ihara lab (B by Chebyshev recursion, $P \succeq 0 \iff$ Ramanujan), the deployed ζ form is exactly the sine/defect half of the canonical split, and the Euler mechanism is measurable on the resonance lattice — the missing piece is one named operator gap (Z1, Hilbert–Pólya type; [verification/v691_hecke_sos.py]); the circle-free trace source for such an operator is in place ($\Lambda(n)$ reconstructed exactly from E_8 shell counting, transport open; [verification/v686_geometric_sos.py]). All of it typed; no marker move, no RH statement.

External context (verification-side, complementary). A recent Nature Communications paper — Wei, Zhai, Lu et al., "The Riemann Hypothesis manifested in dynamical quantum phase transitions" (Nat. Commun. 2026, DOI 10.1038/s41467-026-74935-8) — engineers a "primon gas" whose energy levels are $\log n$ with weights $\Lambda(n)/\sqrt{n}$ and reads the completed-zeta amplitude as a dynamical observable, the zeta zeros appearing as the times of dynamical quantum phase transitions. Those energies and weights are *literally* the atom table of the window form used throughout this paper (positions log prime powers, weights $\Lambda(n)/\sqrt{n}$; v563, [verification/v630_suzuki_contact.py]), and the DQPT times are the zero comb of the diagnostic line. The work is *complementary* in type: it verifies zero locations dynamically (a verification instrument, like the comb certificates [verification/v666_turing_cert.py]), whereas the route pursued here is a positivity route through the localized Weil operator; it is *not* progress on RH for either side, and we type it honestly as an independent physical reading of the same arithmetic objects. Nothing in this paper or its companions claims progress on any conjecture beyond these statements.

Part VII
The Hilbert–Pólya candidate: the operator program and the keystone

This part is the frontier of the document, in diary order. The T-B absorption margin is typed and largely closed on the surface (Sections 12.1 and 12.72); the Z1 series constructs measure, canonical operator, and forced masses and positions (Section 12.2); the chain arc and the L1 montage assemble the continuum candidate (Sections 12.3 and 12.4); the moonshot arc records the Hilbert–Pólya candidate at measurement level, with its explicit no-proof fence (Section 12.5); and the keystone round states the wall in four equivalent languages (Section 12.6), followed by the review strikes of the morning after (Section 12.7), the incidence programme (Section 12.9), and the diagonal gram round — the frequency front closed exactly, the paired tail unconditional, and the wall relocated into q_f (Section 12.10); the qf offensive then adjudicates that wall and closes the diagonal route at a moving spectral edge, with the surviving theorem-shaped facts typed (Section 12.11). No marker moves anywhere in this part, and no RH claim is made.

12 The operator program and the keystone

The subsections follow the diary order of the frontier rounds; provenance is stated in place, exactly as in the rest of the document (ledger-carried modules are cited by their `verification/` tags, everything else is sandbox).

12.1 The T-B chain: the absorption margin typed and largely closed on the surface

The afternoon bundling (2026-08-03) resolves the one piece the rank-3 surface theorem left open: T-B, the razor-thin absorption margin of the deployed 2×2 lock block. *The typing* ([verification/v692_rank3_lockgram.py], 7/7): via the v677 master identity, polarized, the lock block decomposes exactly per entry as $\hat{A}_2 = G_Z + P + \text{tail}$ — a *positive zero-Gram* G_Z (each zero contributes the psd rank-≤ 2 matrix $w_\gamma \,\mathrm{Re}[\phi(\gamma)\phi(\gamma)^\dagger]$ with the two lock profiles $\phi_i(\gamma) = F_i(D\gamma)$; Cauchy–Schwarz is manifest per γ) plus a psd *rank-1 pole layer* $P = c_P s\, s^\mathsf{T}$ in closed form. The margin identity

$$\det(G_Z + P) \;=\; \det G_Z \;+\; c_P\,(s_\perp^\mathsf{T} G_Z s_\perp)$$

shows the T-B margin *is* the transverse zero mass — a sum of squares of the weighted mass the zeros place orthogonal to the pole direction in the 2D lock space (γ_1 alone carries 3–61%). On-line zeros can only *increase* the determinant (2×2 psd superadditivity), so the computed truncation is safe-side and the entire question compresses to off-line quadruples *above the cited verification height*: "E kippt nicht" is a quantified partial-RH tail statement — neither an envelope question of the 2c kind nor a renaming of W3 in two dimensions. *The closure* ([verification/v693_rank3_density_close.py], 5/5): replacing the worst-case envelope by the strongest published explicit bounds — Platt–Trudgian 3×10^{12}, the Hasanalizade–Shen–Wong count, functional-equation halving, and the explicit Ingham-form zero density (arXiv:2507.15184, valid from exactly 3×10^{12}) — inside a sharpened safe-side envelope (sinh difference pairing, $V(0) = 0$, on-line subtraction; the penalty drops $\times$6.5–14.4), 60 **of the** 70 **complete frame-A windows close unconditionally-modulo-citations**. The open remainder is exact: nine windows with required heights $T^* \in [1.0 \times 10^{13}, 3.0 \times 10^{13}]$ and the deep window $h = 5690$ at $T^* = 8.5 \times 10^{14}$; the extension beyond the declared family grows like $e^{3.74a}$. Honest scope: this closes the determinant of the 2×2 lock block; full window positivity (W3, all modes) remains the conjecture, and no RH claim is made.

12.2 The Z1 series: measure, canonical operator, forced masses and positions

Offensive 5 attacks the exactly-named gap of the Hecke-SOS mechanism (sec:theory-open; [verification/v691_hecke_sos.py]): construct a self-adjoint, geometrically defined operator — zeta-free and zero-free by an AST-enforced firewall — whose polynomial traces are the deployed window moments. Four modules record the series honestly; the contract `PRIME.Z1.OPERATOR.01` stays *open* (sharpened 2026-08-05 after the diagonal-Gram closure, Section 12.11). *(5) The measure comes from counting* ([verification/v695_z1_trace_operator.py], 25/25): the seam and orbifold candidates are dead as Z1 spectrum (null-calibrated: AP phases, $1/D$ drift, no tower structure beyond chance), but the E_8 counting route delivers the Weil measure *exactly* — atoms from the shell recursion at deviation 0.0, arch and pole layers via the closed Γ-duplication split, 7/8 target spikes at $q = 0.000$; the deployed window symbol is signed, yet the comb sequence $c +$ pole is *positive-feasible*: the signedness of the deployed sequence *is* the pole subtraction. *(5b) The canonical operator exists* ([verification/v696_z1_jacobi.py], 18/18): the trigonometric moment problem produces the CMV/Jacobi pair explicitly, reproducing all moments at 2×10^{-13}; no closed coefficient law is found, but the renaming suspicion is explicitly *refuted* — the coefficient features sit on the prime-power slots ($q = 0.000$), the amplitudes carry the masses ($r_{\log} = +0.76$), and positivity is load-bearing (smooth and scramble controls lose positive-definiteness entirely at $k \approx 170$, the true masses stay PD to 2865). *(5c) The masses* ([verification/v697_z1_uvarov.py], 14/14): the atoms are lag-side insertions, not orthogonality-measure point masses (duality proven); the transfer identity $\Delta\alpha = w_1/E$ is exact (5.6×10^{-17}), and the *inverted* stabilization law holds — the Γ flow predicts the counting masses to $\sim$10% (median ratio 1.026, correlation 0.86): the flow knows the masses, not vice versa. *(5d) The positions* ([verification/v698_z1_flow_recursion.py], 15/15): the prime-power positions are *forced* by the flow (slot windows 0.5–2 cells, jitter null 30/30, adversary deficit -516 lags), and a shooting bisection — too-light and too-heavy insertions diverge through opposite reflection channels — recovers the counting mass at the true position to 0.11%

median. The honest negatives: autonomous reconstruction fails at greedy saturation (a lookahead is missing), the residual beyond arch+pole+primes is noise-like, and the E-transport is recursive. *What remains for Z1:* the lookahead autonomy and the continuum reading of the operator. Two adjacent routes were killed and parked the same day, for the record: the review-7.3 dynamic-cone reading of the prime flow is dead (prime updates are translations, ∼99% of increments spacelike, no semigroup; [verification/v699_cone_dynamics.py]), and the orbit-60 cascade $60 \to 6$ carries no canonical structure (no involution witness in the line group of order 11520, Coxeter does not descend, no A_5 signature; parked per the review protocol, [verification/v700_orbit60_census.py]).

12.3 The chain arc S-A–S-G: an honest section narrative

The third daily bundling (2026-08-03) runs the question left by the Z1 series — *can the flow generate the comb, and if not, what exactly selects the masses?* — through a preregistered chain of seven sections plus two kill tests, each with its own bars. The narrative is deliberately honest: most sections return *typed* negatives, and the arc is reported as such. *The lookahead decision* ([verification/v702_z1_lookahead.py], 13/13, `FLOW-VERIFIER-NOT-GENERATOR`): the Γ flow *verifies* the full comb ($913 \geq 898$; every fake dies ≥ 184 lags earlier) but does not generate it — both prescribed lookahead routes (depth-2 beam search with horizon-3 deepening, and a global DP) reach 2–4 slots, and the per-slot information demand $g \sim 5$–12 bits is information-theoretic and parameter-robust. *S-A, the circularity decider* ([verification/v703_chain_tolerance_scaling.py], 10/10): the tolerance an autonomous induction would need is RH-grade (over-RH on fine windows) — the induction route is dead; positively, the Levinson recursion is an unstable *shooting* problem whose unique bounded trajectory is the true comb. *S-B* ([verification/v704_chain_mass_law.py], 5/5, `G1-OPEN`): the best closed mass law is C2 (the next Levinson order), median 0.9947 but max 0.3130 on soft slots — no closed formula reaches the bar. *S-C, the geometry anchor* ([verification/v705_chain_deck_sector.py], 7/7): the three digamma channels of the arch density *are* the three deck sectors of the 48-site NS lift — exact tower traces with the v628 twists $\{1/6, 1/2, 5/6\}$, deviation 3.4×10^{-16}, the scalar forced to 1, must-fail off by ×82. *S-D* ([verification/v706_chain_weyl_mass.py], 10/10, `WEYL-NULL`): the point-evaluation reading is falsified, but the flow α *are* the Schur parameters of the background operator (10^{-40}), the background breakdown sits exactly between the $n = 2$ and $n = 3$ cell, and the bare flow knows $\log 2$ to 0.1–0.6%. *S-E* ([verification/v707_chain_section_edge.py], 9/9): the section-positivity edge is an exact resolvent identity (3×10^{-13}); the true mass sits at corridor position median 0.534, IQR $[0.511, 0.559]$ — neither edge nor midpoint: "explain the ~ 0.53" is the named open question. *S-F* ([verification/v708_chain_position_functional.py], 7/7): the best selection functional is the Levinson energy extremum (median 0.9986, max 11.7%, scalar-free transport); the outliers sit exactly on the prime powers 16/64/81. *S-G, the diagnostic* ([verification/v709_chain_zero_layer.py], 10/10, `LAYERING-REJECTED`): behind a *temporal* SHA256 firewall (the residual is hashed before any zero is read), the S-F residual carries no zero signature (best corr 0.042, the scramble level); ∼64% is window geometry. *The two kill tests*: the composite two-stage law dies by its own bar ([verification/v710_chain_two_stage_hecke.py], 7/7; max 6.15% > 2%) — but its Hecke stage heals exactly the prime-power cells 11.7% → 0.53% with circle-free channel detection from the E8 counting; and the first-birth remainder *converges* under refinement ([verification/v711_chain_firstbirth_scaling.py], 7/7, promotion-run $\theta = 1.851$, $R^2 = 0.692$, bars $\theta \geq 0.5$ / $R^2 \geq 0.6$; a one-decade law, extrapolation typed [M]). The net result is the typed decomposition $G1$ = [E-edge] + [E-Hecke] + [M-rate] — exact edges, exact Hecke healing, a measured rate — with the selection principle (~ 0.53) as the one named open piece.

12.4 The L1 montage and the moonshot: the continuum candidate assembled, the atom layer groupoid-internal

Three modules of the same bundling move the Z1 program from "pieces" to "candidate". *The keystone* ([verification/v701_big_picture_hunt.py], 11/11): the deployed window form B is *exactly*

the sine-moment form of (c + pole) plus a rank-1 pole square — one identity that glues the v695 measure route to the v677 master identity; with it the three named gaps of the program collapse onto *one* (defect positivity modulo the operator question), and the triplication identity (the arch as the mean of three digamma channels on the ζ_{12} grid, μ_3-unique) gives the first exact bridge from the arch layer to the deck-sector geometry — closed *into* the geometry by [verification/v705_chain_deck_sector.py]; the V5 fixpoint numerology is dead (PSLQ empty). *The montage* ([verification/v713_l1_montage.py], 16/16, L1-ASSEMBLED-MEASURED): the continuum candidate of the Z1 operator is explicitly *assembled* from the corpus pieces (deck-sector arch + lag-insertion atoms + pole subtraction) — the cover limit converges at 6.2×10^{-8}, the point masses at the D^2 rate, exactly one UV slot remains, and the earlier 5b negative turns out to be a normalization artifact; the montage diverges exactly at $s \leq 1/2$, so the critical line appears as the natural *boundary* of the construction — typed honestly as a consistency signature, not a positivity statement; the missing theorems (self-adjointness, trace identification, positivity) are named, not claimed. *The moonshot, stage 1* ([verification/v714_moonshot_hecke_groupoid.py], 19/19, MOONSHOT-GO): the atom layer is *groupoid-internal* from the $\mathbb{Z}[i]$-E8 Hecke tower — the primitive degrees are exactly the Gaussian prime norms (no prime projector anywhere: the primes come *out* of the tower), the log generator and cell normalizer de-divisorize the traces at 4×10^{-13}, and the σ descent matches 100 positions, 100 masses and 368 moments at deviation 0.00. The fence is typed: the finite counting facts of this slice are *E8-unspecific*; the specificity question and the arch gluing belonged to stage 2 of the registered contract PRIME.Z1.MOONSHOT.01 — executed the same day (Section 12.5). On the adjacent geometry gate, the quasi-free CAR route makes the continuum limit quantitative ([verification/v715_qgeo_car_continuum.py], 22/22, QGEO-CAR-RATES-SUMMABLE: all 6+1 channels Schatten-norm Cauchy with summable rates, the FH renormalization lattice-derived via Barnes G at 8.8×10^{-7}, the RP cone stable) — the majorant lemma (L1)–(L3) is named with constants; GATE.QGEO does not move. [Dated note (2026-08-05): after the (L1) sector lemma ([verification/v745_qgeo_car_l1_sector_lemma.py]), its residue (a) is now *fully closed* in the corrected $(1 + \ln M)$ form ([verification/v778_qgeo_gridsup_logsum.py], 19/19 + 19/19): the literal v715 clause "one $B(\varepsilon)$ for all grid refinements" is false at $r_1 = 1$ and repaired to $\|D_N^{(M)}\|_1 \leq [B_0 + (J/2)\ln(M{+}3)]/N$, certified by the grid Fourier log-sum theorem with explicit constants; residues (b) and (c) stay open, and GATE.QGEO still does not move — the round-21 martingale route ([verification/v802_gnet_martingale.py]) bypasses them for the limit-*existence* step and relocates them into the typed normality step, a reallocation, not a proof.] PRIME.Z1.OPERATOR.01 stays formally *open*: the continuum theorems are missing.

12.5 The moonshot arc: the Hilbert–Pólya candidate, measured

The evening round of the same day executes the moonshot program end-to-end and turns "candidate" into a four-stage *measurement* — with the explicit fence, stated up front and repeated at the end: no theorem is claimed, no marker moves, and **no statement about the Riemann Hypothesis is made**.

Stage 2, the gluing ([verification/v716_moonshot_arch_glue.py], 18/18, MOONSHOT-STAGE2-GLUED): the archimedean place of the *same* $\mathbb{Z}[i]$-E8 commensurability groupoid delivers the arch tower of the Weil measure — one object, *one* normalization: the unified trace functional $T(t) = 2\cosh(t/2)\,dt - \rho(t)\,dt - \sum \Lambda$-orbit atoms is built with *no* per-sector scalar, and the free 3-scalar least-squares returns $\kappa = (1, 1, 1)$ to $< 10^{-12}$ on all five windows. Two normalizations stop being mysteries: the $1/4$ in $\mathrm{Re}\,\psi(1/4 + i\tau/2)$ is *derived* as the μ_4 fixed-sector offset of the 48-site NS lift (modes $m \equiv 1 \bmod 4$), and $\log \pi$ enters only through the *declared* self-dual (Tate) UV cell — the exact arch mirror of the declared $\sqrt{n}$ half-density at the finite places. E8 becomes forced *at the gluing*: the μ_2 tower (residual 0.326), the wrong μ_4 class (1.000) and the wrong deck twists (0.474) all rip.

Stage 3, the state ([verification/v717_moonshot_state.py], 21/21, STATE-ON-TRUNCATIONS): on every truncation the Weil functional *is* a state, constructively — Levinson positive definiteness

exhibits T as the GNS/CMV vector state of the truncated translation flow (min headroom 0.76), and the convolution binding identity $T(f * \tilde{f}) = 2V^{\mathsf{T}}\text{odd\_toeplitz}(c)V$ holds at 10^{-12} over the battery. The naive tower reading $\varphi_{\text{pole}} + \varphi_{\text{orb}}$ does *not* give T: the difference Δ carries $|\Delta|/\text{W3-margin} = 104\text{–}1563$ — the trace-formula boundary is exactly where the conspiracy sits, now *localized and measured*. The degree grading satisfies detailed balance at KMS $\beta = 1$ (the Bost–Connes critical temperature; half-density exponent $-1/2$ sharp), with $s = 1/2$ as the symmetric GNS splitting. Both control labs calibrate: the Ihara/Petersen world is an exact state; Epstein $x^2 + 5y^2$ is not (Levinson breakdown at depth 49/62).

Stage 4, the spectrum ([verification/v718_moonshot_spectral.py], 10/10, SPECTRUM-CONVERGES-MEASURED): the eigenvalues of the GNS truncations converge *measurably* onto the zero sequence. The diagnostics are hard-typed: the operator is zeta-free, and the node predictions of every window are SHA256-frozen *before* the 379 target zeros are loaded in the one declared function. The largest window ($h = 1433$) hits 100.0% of 377 zeros at tolerance 0.25 (97.1% at 0.10, 84.1% at 0.05); the h-ladder rate is -1.61 and the pure GNS K-ladder is monotone ($0.4678 \to 0.0003$) — the truncation eigenvalues move *onto* the zeros; the unpartnered nodes are exactly the expected Gauss quadrature filling. The scramble control (positions redrawn, masses kept) dies as a state (Levinson breakdown at depth 61); GUE light, as an observation only: matched-node $\langle r \rangle = 0.6178$ vs the zeros' 0.6189.

The Satz-1 slice, the trace formula ([verification/v719_moonshot_traceformula.py], 24/24, SATZ1-LEDGER-EXACT): on every truncation the finite trace formula is *exact* Gauss quadrature (max rel dev 1.7×10^{-11}), and the term dictionary to the classical Weil formula (1952) *closes*: the pole block is $\int \tilde{a}\, 2\cosh(u/2)\, du$ (1.7×10^{-13}), the atom block is the classical prime term (1.8×10^{-15}), the arch block is $-\int \rho\, \tilde{a}$ (3.4×10^{-16}), the UV cell is the stage-2 regularization (2.2×10^{-16}). The stage-3 difference term Δ is *classical*: it converges to the Γ'/Γ term plus the full prime term ($\sigma = 1.000161$, $c = -10^{-4}$; the one-sided sum honestly has no limit — UV drift exactly $\log(2)/2$ per D-halving). As a bonus census: among the dimension-8 unimodular $\mathbb{Z}$-lattices — classically $\{\mathbb{Z}^8, E_8\}$ (Mordell 1938, a declared classical input) — *only* the Gaussian E8 glues ($\kappa = 1$, residual 8.5×10^{-17}).

The K collapse and the capture lemma ([verification/v720_moonshot_k2k3.py], 14/14, K2K3-COLLAPSED-CLASSICAL; [verification/v721_k1_node_capture.py], 14/14, K1-LEMMA-CAPTURE-ONLY): of the named convergence theorems, K3 collapses onto *measured* tightness (compacta Cauchy; the normalization grows exactly like the smooth band density, $p_0 = 0.981 \log(1/D) - 1.306$ — a sigma-finite limit in the vague topology) and K2 is *classical analysis on* $s > 1/2$ (unconditional Chebyshev/PNT majorant with $c_{\text{RS}} = 1.03883$ machine-verified in range; tail law slope $= -(s-1/2)$ exact; discretization $\sim CD^2$), with the boundary *exactly* at $s = 1/2$ (Mertens slope 1.9994). The node-capture half of K1 is proof-near from classical quadrature theory (Markov–Stieltjes sandwich, symmetrized Christoffel bound, a closed capture condition, all machine-verified behind the same SHA256 typing) — but the certified radii sit $\sim$900$\times$ above the measured node errors: the classical lemma carries the *capture*, not the super-resolution; that precision is trace-formula content (the K1b question — resolved classically as atom pinning in the keystone round, sec:wall-four-languages).

The end ledger, stated honestly. What now exists at *measurement* level is structurally everything a Hilbert–Pólya candidate needs: a zeta-free glued object, a state on every truncation, an exact finite trace formula with a closed dictionary to the Weil formula, and a measured spectral limit onto the zeros. What does *not* exist is any continuum theorem. The remaining hard substance, ordered by PRIME.Z1.MOONSHOT.01: (L1) identification of the vague limit *at* the line $s = 1/2$ — no mass loss; the single place the zero fluctuation enters; (L2) node convergence as a theorem (capture is classical; the super-resolution is resolved as classical atom pinning in the keystone round, sec:wall-four-languages); (L3) routine lemmas, exhibited with constants. PRIME.Z1.OPERATOR.01 and PRIME.Z1.MOONSHOT.01 stay *open*; positivity — the Weil criterion — is untouched; **no RH claim**.

12.6 The keystone round: the wall in four equivalent languages

The closing round of the same day takes the end ledger of the moonshot arc and does the one thing left to do with a wall: walk its entire length and name it precisely, in every language it speaks. Eight further modules land ([verification/v727_l1_identification.py] through [verification/v734_s1_canonical.py]); the fence stays exactly where it was — **no RH claim, no marker moves** — but the remaining substance is now *maximally localized*.

The wall, maximally precise ([verification/v727_l1_identification.py], 11/11, `L1-DETERMINED-WALL-TYPED`). The identification slice first clears the classical ground: the raw τ-measure does *not* carry the limit (its naive moments grow one power under the density law; measured slopes 2.254/4.245) — the Weil object is the per-bandwidth lag reading, and in the Gauss-damped frame on the Weil axis the ladder moments are Cauchy and *identify* against the closed geometric side to 1.4×10^{-3} over $m_0..m_{16}$; the measured Gauss/Freud growth band $[0.638, 1.117]$ makes the Carleman sum diverge: the limit moment problem is *determined*, classically, with RH nowhere in the argument. The identification chain then reads: (a) the vague limit exists [measured tightness]; (b) the moment problem is determined [classical]; (c) the limit moments equal the geometric side [machine-exact per window, 4.7×10^{-14}]; (d) the geometric side equals the zero side [Weil 1952 — classical and *unconditional*, wherever the zeros lie]. The seeming paradox — "T_h is PD" plus "$T_h \to W$" would naively give the Weil criterion, i.e. RH — resolves honestly: *"T_h is PD" is a measurement per h, not a theorem* (the scramble control breaks Levinson at depth 61). What remains is one statement, the wall: **PD persistence** — infinitely many windows of the ladder are positive definite. That *suffices* for RH (PD on a subsequence, plus the (c) rates, plus (d), gives $W(g * \tilde{g}) \geq 0$ for all compactly supported g); each window is a *finite* Levinson computation, measured 9/9 (min headroom 0.718); the converse (RH $\Rightarrow$ every finite window PD) is *not* shown — an honest asymmetry. By Riesz–Haviland plus determinacy the same statement reads: *the geometric side defines a positive measure on* $\mathbb{R}$.

The four languages. On the largest window all readings are computed side by side, and with the canonical-system slice ([verification/v734_s1_canonical.py], 10/10, `S1-CANONICAL-SUZUKI-BRIDGED`) the count reaches four equivalent languages for the *same* wall, each finitely decidable per window, all measured 9/9:

- *Moments* (Hankel-PD): the damped moment matrices are PSD — min eigenvalue $1.37 \times 10^{-11} \geq 0$ at $h = 1433$;
- *State* (Levinson-PD): the lag vectors pass Levinson with headroom 0.830 — T_h is a GNS vector state;
- *Symbol* (Fejér ≥ 0): the window symbol is nonnegative — minimum 6.48×10^{-3};
- *Hamiltonian* (Krein $l_k > 0$): the Jacobi/Wheeler data define, via an exact Krein correspondence (Weyl identity to 1.6×10^{-15} over 4 complex $z \times 9$ windows), a canonical-system Hamiltonian H_h with rank-one steps; $H_h \geq 0$ iff all $l_k > 0$ — measured: min $l_k = 0.033..0.377$, all positive; the boundary phase carries the γ-side with scale *exactly* 1, and the scramble breaks the Wheeler/Krein chain at depth 31.

The h-universal quantifier — *all* (or infinitely many) windows — is the RH substance; everything below it is a finite computation.

The literature anchor, stated as the dual council reads it. The wall is not new; it has three names. *Weil 1952*: RH is equivalent to the positivity of the explicit-formula functional $W(g * \tilde{g}) \geq 0$ — our language (d) is his identity, unconditional. *Connes 1999* (Selecta Math. 5, the global trace formula) and *Connes–Consani 2021* (the Sonin-space zeta-cycles): the positivity route through the adèle class space, where the unconditional part lives on the atom-free regime $|u| < \log 2$ — exactly the regime our functional-equation symmetry buys, and no more ([verification/v729_strat2_rp_universality.py] below). *Suzuki 2021* (JFA 281, 109116): GRH is equivalent to the positivity/existence of an explicitly constructed canonical-system family, built unconditionally only for $\omega > 1$ — exactly the classical half-plane $s > 1/2$ where our K2 collapse already lives; our H_h is the first *computable, zeta-free*

truncation family with the same Weyl/phase data and a measurable $H \geq 0$ witness per window. And the proven boundary line is *Meyer 2005*: an unconditional spectral realization of the zeros *exists* — but on a space without positivity; the spectral realization alone is not the theorem, positivity is. What TFPT adds to this map is not a fourth name for the wall but a *computable, zeta-free truncation framework with emergent prime places*, in which all four languages become finite per-window decisions with measured convergence rates — the wall becomes an object one can walk up to and measure, which is exactly what these modules do, and nothing more.

The K1 diagram, closed classically. The super-resolution riddle of the moonshot arc (certified capture radii $\sim$900$\times$ above the measured node errors) is resolved — classically. The memo's locality/symmetry mechanism is *dead* ([verification/v728_k1b_superresolution.py], 11/13 with the two *preregistered* memo bars dying honestly, `K1B-ATOM-PINNING`): what carries the scale is *atom pinning* of the raw measure — Nevai-type mass-point attraction, lab rate $K^{-1.84}$, node-weight ratio $\to$ 1.03, real-data correlate 0.64. Behind it sits an explicit *pinning lemma* ([verification/v730_strat2_pinning_lemma.py], 12/12): a residual bound and its Kato–Temple sharpening, one-line proofs each, 0 violations at 598 symbol peaks and 535 matched band zeros including *all* 377 of the largest window — and the certificate *saturates at the window width*: sub-width certification requires raw atomicity beyond every finite window depth. The gap layer between the atoms supports *emerging* bulk universality at kernel-shape level ([verification/v729_strat2_rp_universality.py], 5/5) but the strong measured form is *killed* on the preregistered bar ([verification/v731_strat2_gap_universality.py], 5/5, `GAP-UNIV-KILLED`: the deviation plateaus at 0.33 and the uniform-in-h Szegő condition breaks like $h^{-0.64}$); what remains there is a *conditional* fixed-window theorem candidate on the exact literature route (Lubinsky 2009; Levin–Lubinsky 2008; Avila–Last–Simon 2010; Totik 2009). The diagram closes:

$$\underbrace{\text{capture theorem}}_{\text{v721, certified}} + \underbrace{\text{pinning lemma}}_{\text{v730, certified}} + \underbrace{\text{conditional gap rigidity}}_{\text{v731, conditional}} + \underbrace{\text{atom structure}}_{=\text{L1, open}} = \text{K1}.$$

Everything above the wall is classical; the single non-classical input left anywhere in the diagram is L1 — raw atomicity beyond window depth, which is the PD-persistence wall again.

Two more honest kills, and a weak functor. The naive Osterwalder–Schrader reflection-positivity route is dead on the truncations ([verification/v729_strat2_rp_universality.py]: the total window fails plain *and* KMS-twisted RP at full scale; the deployed positivity lives only on the Toeplitz/Bochner axis). The *direct* transfer of the Connes–Consani Sonin compression to the window discretization is dead ([verification/v732_strat3_sonin_prolate.py], all 21 construction checks pass and the preregistered kill fires: bad rank grows 19 $\to$ 86) — fenced cleanly: this kills the chosen one-cell cutoff on *these* truncations, not the CC mechanism itself, and retuning the cutoff after the result is forbidden. In the other direction a first exact bridge *into* Connes' framework does exist: on the finite periodic-orbit Laurent basis for $S = \{\infty, 2, 3, 5\}$, the σ-descended TFPT groupoid maps by an exact coefficient-one graded $*$-algebra functor candidate into the S-local periodic-orbit subalgebra ([verification/v733_strat3_gate0_census.py], 13/13, `GATE0-WEAK-FUNCTOR`: 6859 convolutions with deviation 0, $\kappa \equiv 1$ at all three places) — with the extension to the full adèle-class algebra typed as *open*. A closing slice completes the route portfolio honestly: the state-transport route also stagnates ([verification/v735_strat3_ucp_inductive.py], 10/10 construction checks, `UCP-STAGNATES`: all eight adjacent GNS-ladder maps exist *exactly* as UCP quantile couplings with linearly controlled Choi rank, slope 0.991, but the preregistered shift/grade covariance gate fails — mere state transport is not the missing L1 mechanism), so the L1 portfolio now reads: Sonin dead, UCP stagnates, Gate 0 weak functor, Suzuki bridge a measurement — what remains for L1 is atomicity/identification substance, not transport machinery.

The final fence. Nothing in this round moves a marker. The wall now has a maximally precise statement (PD persistence), four equivalent finite languages, a classical closure of everything around it, and honest graves for the routes that died. The h-quantifier remains unproven; **no statement about the Riemann Hypothesis is made**.

12.7 The morning after: four review strikes

The external review of the keystone round proposed four simplifications and sharpenings; all four were machine-tested the next morning, and the results split cleanly into two exact structures and two honest kills.

Strike 1 — the orbit packet lemma (exact). On the E8 side the review's compact number family closes as a lemma ([verification/v736_orbit_packet.py], 26/26, `ORBIT-PACKET-EXACT`): the σ orbit census on the 15 nonzero Gaussian classes gives Burnside $7 = (15+3+3)/3$; the 240 roots split canonically as $240 = 5 \times 48$ (one fixed block + four moved blocks of 12 Gaussian lines each, label-free canonical); the fixed-syndrome leader weights $(0,1,1,2)$ reproduce the $(1,1,2)$ anchor with leader pair e_6/e_7; and the $q = 16$ normal form reads $240 = 16 \cdot 15$, $248 = 240 + 8$. The circularity fence is part of the lemma: P1 and P2 come back as *exact internal reconstruction*, not derivation, and the packet law is σ-class-specific (free order-3 elements give Burnside 5).

Strike 2 — the seam-48 intertwiner (dead). The sharpest structural sharpening — a canonical intertwiner between the five 48-blocks and the 48 regular five-cycles — dies on its preregistered criterion ([verification/v737_seam48_intertwiner.py], 24/24 machinery, `SEAM48-DEAD`): the 48×5 incidence matrix is not identically 1 (entry distribution 0:96/1:64/2:64/3:16), representative-independent over all 2304 order-5 elements, with the must-fail control firing. The residues are themselves structure: the four moved blocks carry the v623 character table *exactly*, the fixed block deviates exactly by the 12 lifted marks, and $u^5 = \pm J$ holds exactly — but $240 = 48 \cdot 5$ remains a factorization *without* coordinatization.

Strike 3 — the ramified fiber and the seven channels (exact). The review's "local fiber at the ramified place" thesis holds at the level it was stated ([verification/v738_hecke_mod_ramified.py], 34/34, `HECKE-MOD-RAMIFIED-CHANNELS`): the Gaussian–E8 Hecke tower projects onto $V = L/(1+i)L$ exactly and σ-functorially over all 5907 submodules; every odd layer acts as degree $\cdot$ id (the fiber is Hecke-rigid), while the ramified layer *is* the 15 hyperplanes of V with the 2:1 deck. The channel structure resolves the review's 7-vs-8: the σ-orbits give 3 fixed + 4 moved = 7 nontrivial channels +1 trivial; NS/R is the σ-fixed parity character (the v722 grading re-derived), and ram-odd ($n = 2, 8, 32, \ldots$) is the unique negative-pressure channel. The honest boundary stands: the W3 margin is *cross-channel cancellation* — the projection alone yields no transfer-matrix reduction.

Strike 4 — the rank-3 jet simplification (dead). The review's reading of the rank-3 wall as a first-jet identity is killed by its own ε^0 test ([verification/v739_rank3_jet.py], 20/20, `RANK3-JET-DEAD`). What survives is documented as exact coordinates: the load-bearing 2×2 lock block *is* the first jet of one function $F(\gamma) = \sum_{n \le X} \Lambda(n) n^{i\gamma}/\sqrt{n}$ (to 4.6×10^{-16}, exact Loewner coordinates), and on the odd sector the strengthened claim holds with first survivor $[1 - r_{12}^2]_{\mathrm{odd}} = \frac{3}{8} S[\tilde{g}](1)\, \varepsilon^4$ (the Schwarzian form). But the ε^0 coefficient of $1 - r_{12}^2$ is identically 1 (not 0), the measured even sector exceeds the net entry (1.15–6×), and the pure jet block carries no collinearity (factor $\geq 1.9 \times 10^3$): the third power is even/odd *compensation* — T-B substance, not a jet identity; class (c) for the envelope route, and the section-8 Krein link is weakened (`KREIN-LINK-WEAK`, correlation 0.678 vs control 0.946).

Where this leaves the program. The morning's net effect is two exact structural lemmas (orbit packet, ramified channels), two graves with named residues (seam-48, rank-3 jet), and two new contracts registered in the contracts paper: the review master contract `PRIME.KMS.INDUCTIVE_STATE.01` (extend the Gate-0 functor to a positive global KMS state at $\beta = 1$; five separation steps; kill = complete positivity failing on a finite extension) and `PRIME.PD.PERSISTENCE.01` (the universal-theorem form of the wall with $\lambda_{\min} \geq 0$ instead of $\geq \delta$ — the Ihara lesson; cofinal PD windows via state/Gram/cone recursion, not eigenvalue bounds). No marker moves; **no statement about the Riemann Hypothesis is made**.

12.8 The simplification wave: the tower upgrade, two closed hopes, the KMS end-form, and the channel anatomy

The day's second wave ran every remaining simplification hypothesis to a machine verdict. The honest section: *one* structural upgrade, two closed hopes, the algebraic end-form of the KMS master contract, and the complete anatomy of the thin margin.

The tower upgrade (the structural gain). The window quantifier of the PD-persistence wall reduces *exactly* to the canonical nested (D, X) tower ([verification/v749_simpler_tower.py], 8/8, TOWER-NESTED-REDUCES): in X an ascending chain — X-nesting exact to 6.7×10^{-18}, so limit positivity is one limit object per chain — and in D dyadic downward inheritance, exact: PD fine *forces* PD coarse; every deployed window is a tower member at deviation 0.0. The historical window family was an *artifact* of selection: box step functions are dense, so tower-PD for $D \to 0$, $X \to \infty$ is equivalent to the Weil criterion (classical density step). The contracts paper restates PRIME.PD.PERSISTENCE.01 with the canonical truncation. The honest limits travel with the upgrade: incommensurable root-D chains connect only approximately, and the margins fall along the tower — the wall stands, in canonical nested form.

Two closed hopes. First, the gluing-rigidity hope: the gluing identity turns out to be an identity-level *off-line detector* — it sees $\delta = 0.01$, sharper than the PD break at $\delta^* = 0.02$, and its envelope slope measures δ — but the rigidity thesis is honestly *false*: the identity holds unconditionally, because both sides are constructed ([verification/v750_simpler_gluing.py], 5/5, GLUING-DETECTOR). What a proof lacks is not the identity but that the true lags can only be carried by an on-line spectrum — and that *is* positivity itself. Second, the Ward-monotonicity hope: the component running is deterministic in direction (100%, parameter-free ζ/L constants), the $\mp u/2$ drift anatomy is exact (from $p^2 \equiv 1 \bmod 4$), and the slope-Ward cancellation holds at 4×10^{-4} — the margin inherits *zero* deterministic drift — but the balance *sign* is zero content: the deterministic budget is $2.9 \times 10^4\times$ the margin, so the induction route for $\lambda_1 > 0$ is RH-circular at the balance level ([verification/v751_ward_monotone.py], 15/15, WARD-MONOTONE-MIXED). Shape provable, sign not.

The KMS end-form. The strong reading of the review master contract is algebraically dead, twice ([verification/v740_kms_extension_switch.py], 8/8, KMS-EXTENSION-DEAD): $\beta = 1$ KMS is impossible on Gate-0's nonzero-grade Laurent unitaries — $T(u_p u_p^*) = 1$ versus $T(u_p^* \sigma_i(u_p)) = 1/p$, exact defects $1/2, 2/3, 4/5$ — and exact covariant $*$-restrictions between adjacent windows do not exist. The Bost–Connes semigroup Toeplitz correction then repairs the algebraic KMS step *completely* (boundary-corrected identities exact, normalized bare error $10^{-2} \to 10^{-9}$), but the covariant inductive step stays dead: the preregistered UCP window covariance is nonmonotone ([verification/v741_kms_toeplitz_semigroup.py], 9/9, KMS-TOEPLITZ-DEAD). The end-form is therefore fixed: keep the proven BC-Toeplitz local KMS blocks, and state the single remaining global step as an operator-system compactness/subsequence problem — PRIME.KMS.INDUCTIVE_STATE.02 in the contracts paper. Local KMS existence is classical (Bost–Connes 1995); the TFPT burden is the compatibility of the glued window states.

The channel anatomy. The thin W3 margin is now anatomized end to end. It is *one* collective scalar balance — arch + ram-odd against split + inert — strictly monotone over all 35 deployed windows, with two exact Ward identities: the cross components cancel exactly on every window ($\sum c = 0$ to 1.8×10^{-15}), and the 8×8 rigid block equals $\dim_C \dim_{C'} m_0$ ([verification/v742_channel_interference.py], 14/14, INTERFERENCE-COLLECTIVE). There is no pair reduction (collectivity $k_{95} \in [4, 5]$): the channel basis is diagnostic, not reductive. No canonical column choice factorizes the window Gram either — the cos-half candidate is indefinite and the PSD-B^*B completion consumes 5–6 orders of magnitude beyond the margin; the Ihara *channel* route fails coherently (channels grade positions, positivity lives in frequencies — structural, not ζ-specific), and the finiteness measurement reads $\dim V = 16$ against 168–862 required frequencies ([verification/v744_hecke_channel_columns.py], 15 checks, HECKE-SOS-CHANNELS-PARTIAL). Finally the cone-preserving Schur recursion is retired by a one-liner: a layer map is cone-preserving iff the layer is positive, and no prime layer is ($c_0 = 0$ exactly; [verification/v743_schur_cone_recursion.py], 7/7, SCHUR-CONE-DEAD) — layer-wise recursion is the wrong granularity level. No marker moves;

no statement about the Riemann Hypothesis is made.

12.9 The incidence programme: five wins, one decorative seven, and the wall as the missing factor

The second external review proposed a seven-module incidence programme on the proven Gaussian quotient $V = L/(1+i)L = \mathbb{F}_2^4$; the day adjudicated all seven. The honest section: five exact wins, one decorative seven, and a keystone negative that names the wall as the missing left factor of the certificate.

The incidence lemma (the centerpiece). From the concrete lattice — $h = H/4$ is $\mathbb{Z}[i]$-valued with hermitian Gram determinant 1, the *unimodular* hermitian E8 lattice over $\mathbb{Z}[i]$ — the reduced form $\bar{h}$ is alternating (doubly-even) and nondegenerate: $(V, \bar{h})$ is symplectic over $\mathbb{F}_2$ with an explicit Darboux basis. On the 15 points of $P = V \setminus \{0\}$ the hyperplane incidence matrix $B_{xy} = [\bar{h}(x, y) = 0]$ satisfies the exact identity

$$B\,B^{\mathsf{T}} \;=\; 4\,I \;+\; 3\,J,$$

with the full design parameter set 2/3/4/7/15/16 on display: $|V| = 16$ classes over $\mathbb{F}_2$, 15 points, row sums 7, pair intersections 3, split $7 = 4+3$, and characteristic polynomial $(x-7)(x-2)^9(x+2)^5$ — the eigenvalue multiplicities $(1, 9, 5)$ are the *symplectic signature* of the lemma (the honest control finding: $BB^{\mathsf{T}} = 4I+3J$ alone holds for any nondegenerate form; what is symplectic is the projective part). The normalized channel $K = B/7$ satisfies $K^2 = (4/49)I + (45/49)\Pi_0$ exactly (K itself is honestly not psd), and the NS/R parity is a *point of the geometry*: $\chi_{\mathrm{NSR}} = \bar{h}(\cdot, y_0)$ for the unique σ-fixed point $y_0 =$ FSUM — the anchor bit — giving $112 + 128 = 240$ and, with the Cartan, $248 = 120+128$ ([verification/v752_projective_hamming_incidence.py], 36/36, `PROJ-HAMMING-EXACT`; companion Lean module `ProjectiveHamming.lean`, 32 theorems). *The v737 resolution:* the seam-48 incidence distribution 0:96/1:64/2:64/3:16 is now *derived* exactly — the 48×5 matrix is $16\times$ the 3×5 label table (five-cycles see $\bar{u}$-orbits, not σ packets) — and the 5×48 coordinatization *exists* via exactly one $\bar{u}$-invariant spread of the 56 (incidence identically 1 on the root level): the v737 kill is explained, not revised — the σ packets remain a non-spread.

The Arf compiler round (2026-08-05, three modules). The code layer now carries its canonical quadratic refinement: of the exactly 16 refinements of $\bar{h}$ the frozen selector picks a unique q^*, the parity lift $\iota\colon V \cong C_{\mathrm{even}}(5)$ carries $\beta = \bar{h}$ exactly, and $\mathrm{Stab}(q^*) \cong S_5$ makes $16 = 1 + \bar{5} + 10$ an *Arf geometry* — K_6 duads, $B = I + A_{KG(6,2)}$ with exact ovoid eigenvectors, the hypercharge table $X = (-2, -2, -2, 3, 3)$ and the master-moment chain 120/40/41/240/248 ([verification/v774_arf_spinor_compiler.py], 46/46, ARF-SPINOR-EXACT; Lean companion `ArfSpinorCompiler.lean`, 23 theorems). The preregistered physical kill *fired*: no $D_5 \oplus A_3/SU(5)$ convention makes even one full set of 15 classes pure — the code→matter reading, including the "empty mailbox = right-handed neutrino" reading, must not enter any main claim ([verification/v775_gaussian_class_d5_purity.py], 18/18, ROOTCLASS-MIXED). And the non-circular selection chain exists: boundary → code → E_8 → $\mathbb{F}_2^4$ → five slots with $g_{\mathrm{car}} = 5$ as *output* (census $135 \to 30 \to 14 \to 2 \to 1$; one deck-invariant $\mathbb{Z}_2$ flag bit; residuals R1–R3 typed then, all three moved in round 21: R1 → R1′ = three physical axioms via [verification/v799_seam_code_typeii.py], R2 placed and R3 named via [verification/v798_seam_clifford_modular_s.py]; no P2-removal claim) ([verification/v776_boundary_hamming_uniqueness.py], 31/31, BOUNDARY-CODE-UNIQUE).

The polarity, the two-step, the recursion, the extension (four more wins). The 15 ramified Hecke hyperplanes are *exactly* the $\bar{h}$-orthogonal spaces, via the canonical bijection $y(\varphi) = \bar{G}^{-1}\varphi$, σ-functorially, with deck and parity bit commuting; the canonicity census is sharp: of 28 nondegenerate alternating forms exactly *one* is σ-invariant, and it is $\bar{h}$ ([verification/v753_ramified_polarity.py], 25/25, `POLARITY-MATCH`). The real $n \to 4n$ down-up pass of the ramified odd layers *is* exactly K^2 — degree factor $196 = (2 \cdot 7)^2$ closed, $T = 4B^2$, KMS half-weight exponent-exact at 1/2 per pair, the same integer matrix on all nine contexts, cone-preserving — with the honest boundary that naive label chains do *not* carry K^2: the correspondence composition is mandatory ([verification/v754_ramodd_twostep.py], 19/19, `TWOSTEP-EXACT`). The ramified Schur recursion on

the canonical tower is *alive*: $S_k \succeq 0$ measured on all eight ram-odd stages, the Albert/Haynsworth induction frame stands, and the open proof need is exactly the structural Gram certificate per step — honestly, for ζ data this is equivalent to tower PD ([verification/v755_simpler_schur_recursion.py], 13/13, SCHUR-RECURSION-ALIVE). And step (3) of the KMS master contract is *solved on the ramified layer*: 105 explicit Kraus terms realize K as a CP unital map with all four compatibilities exact (σ, half-weight, $\beta = 1$ detailed balance, Gate-0), local candidate state $\tau_{\mathrm{inc}} \otimes \tau_{\mathrm{Toeplitz}}$ ([verification/v756_kms_incidence_stinespring.py], 7/7, KMS-INCIDENCE-CP-COVARIANT). [The compactness remainder named here acquires measured handoff evidence and an abstract Lean closure one round later — see Section 12.10; the 105-leg Kraus set later carries the full Stinespring carrier intertwiner (sec:descent-functor).]

The decorative seven. The preregistered Fano → discriminant(-7) bridge breaks exactly at its second condition: 2 *splits* in $\mathbb{Q}(\sqrt{-7})$ ($-7 \equiv 1 \bmod 8$), so every 2-adic ideal quotient has 2-rank $\leq 2 < 3$ — the code $\mathbb{F}_2$-structures are arithmetically unrealizable, the NS/R parity bit has no arithmetic carrier. The three-cycle half is real and exact (the Fano $\bar{\sigma}$ is the $\times 2$ difference-set multiplier; every equivariant lift sends the fixed point to the C corner, norm $14 = \dim G_2$), but the seven-correlation is permanently typed *decorative* ([verification/v757_fano_disc7_bridge.py], 26 checks, one preregistered break, FANO-DISC7-DECORATIVE).

The keystone: the wall as the missing factor. The review's assembled certificate [continuum root $\oplus$ Kraus] is *not mountable* ([verification/v758_simpler_certificate.py], 8/8, CERT-CONTINUUM-ROOT-DEAD): the arch+pol block is PSD on *no* stage of the ladder — the Levinson recursion breaks at cell 47, exactly between the $n = 2$ cell and the $n = 3$ cell, ladder-wide, and already W_0 is indefinite: each atom rescues the positivity "just in time" — the continuum Schur complement is no Haynsworth object, the additive split fails on every stage (the order-1 balance sits in the cross-term; the 15×15 incidence shape does *not* sit as a block in B_k), and the rescue cascade is strictly *cell-ordered*: no genuine channel subfamily carries. The wall is not hidden under a new name — *it is the missing left factor*. What remains real: the exact building blocks above, and the corrected target shape — a cell-wise cascade factorization $R = \prod$(cell factors), rescue order $n = 2, 3, 4, 5, \ldots$ across the channels — now the named open object of PRIME.PD.PERSISTENCE.01. No marker moves; **no statement about the Riemann Hypothesis is made**.

12.10 The diagonal gram round: the frequency front closes exactly, the paired tail is unconditional, and the wall moves into q_f

The global-handoff offensive asked whether the finite window states admit a controlled handoff to one limit functional — built from the source side alone, without target decompositions, zero data, or positivity tests. The round decided its five gates; the honest summary: the frequency front closes *exactly*, the paired boundary remainder is *unconditionally* bounded, the dense test family is canonical, the red team is green — and the wall relocates into a single bounded object, the battery near-kernel pairing q_f.

The offensive foundation. The ε-regulated resolvent $G^{\varepsilon} = (A_X + \varepsilon I)^{-1}$ is the admissible operator-system evaluation: fixed-observable Cauchy defects fall at rates $b = 0.174$–0.310 per X unit, robust over three decades of ε, while the bare A^{-1} does *not* stabilize (+45% over the reachable ladder — an unbounded observable, the named negative block), and the layer decomposition is a cancellation balance (the leading single channel exceeds the total defect by up to 1.9×10^{12}: no separate per-layer rate exists) ([verification/v766_handoff_bulk.py], 17/17, HANDOFF-BULK-CONVERGES). The growing-frequency PSD source Gram converges on the declared five-window ladder: Candidate A (the review count) missed its frozen rate gate honestly — its cell count sits below the anti-alias degree — and the *predeclared* iteration to Candidate B ($N_f = 2M+1$) delivers spectral errors $0.0855 \to 0.0377$ over $\dim Q$ $738 \to 5734$ at slope -0.369, PSD by construction with every Q byte-hashed before the target is built ([verification/v767_handoff_frequency_gram.py], 6/6, HANDOFF-GRAM-CONVERGES). Its three named remainders are decided in one merged module ([verification/v768_handoff_tail_weil.py], gates 11/13 + 4/4, TAIL-WEIL-PARTIAL+COMPAT-EPS3-CONVERGES): the quadrature tail *terminates algebraically* — the alias census $(M-1)$ + spread $\leq 2M$ holds on every window, max tail 2.4×10^{-13},

so *no quadrature tail exists* and every layer error is structural; the Weil identification on the frozen battery reaches $\kappa = 0.97768 \to 0.99028$ with total agreement 0.0485 (battery-limited, no claim beyond); and the two frozen $\varepsilon = 10^{-3}$ compatibility cells that part 1 honestly failed (atom-burst endpoint artifacts of the shorter ladder) pass the fresh preregistration on the deepest reachable 36-rung ladder under an oscillation-aware statistic ($b = 0.178/0.186$, med5 0.343/0.467, $b_2 = 0.069/0.096$) — with the first wall sign one decade below, at $\varepsilon = 3 \times 10^{-4}$ ($b_2 = -0.011$, reported, never gated).

The decision line. Gate 1: the handoff error is *not* a quadrature error — E_{total} is invariant under $N_f = q(2M{+}1)$, $q \in \{1, 2, 4, 8\}$, to $\sim 10^{-13}$ on all three fixed windows; the remainder is boundary/cutoff/object-gluing ([verification/v759_handoff_fixed_window_resolution.py], 8/8, `RESOLUTION-BOUNDARY`). The anti-alias claim is then an *exact theorem* of the M-cell space: Parseval, moment and Gram identities for *arbitrary* coefficient vectors ($2 \times 10^{-14}/3.5 \times 10^{-13}/5.9 \times 10^{-13}$), the exact alias formula $t_d = \sum_r (-1)^r \hat{s}(rN_f - d)$ matched at every ladder count, the designed must-fail witness at $N_f = 2M{-}2$ firing with error exactly 1 — and the minimality claim *corrected*: the sharp minimal count is $N_{f,\min} = 2M{-}1$ (spread-S version $M{+}S$); the deployed $2M{+}1$ stays exact, nothing is patched ([verification/v760_antialias_exact.py], 14/14, `ANTIALIAS-CORRECTED`; companion Lean module `AntiAliasExact.lean`, 19 general-N theorems). Gate 2: the paired atom+pole boundary remainder splits by Stieltjes/Abel $d\psi = dx + dE$ — the deterministic dx half kills the growing pole channel identically, E_{origin} closes via the W1/Suzuki bookkeeping (comb-independent), the matrix identity holds to 1.45×10^{-11} — and the arithmetic block admits the explicit *unconditional* bound $\eta_K = |\det_K| + |\Theta_K(0)E(1)| + B_1 I_K$ with $B_1 = \sup_{[1,24.29]} |\psi(x) - x| = 3.734$ certified by direct finite evaluation: all 24 kernels, all 5 rungs, pure $1/\log X$ decay, *no RH-strength input anywhere* (the circularity flag is machine-checked; kernels with window-growing support would need $|E(x)| = o(\sqrt{x})$ — RH strength — and are excluded by the frozen battery's fixed support). The companion probe derives the correct control standard from first principles (the symmetric zero-violation gate plus the envelope hypothesis $B_1^{\text{ctrl}} \le B_1$): Epstein breaks at $B_1^E = 2.95 \times B_1$ with 7/24 violations, scramble at 262×, the true comb passes 24/24 — while the parent probe's own frozen verdict `ABEL-DEAD` (its mis-aimed majority breaker missed by a hair) stays on record as the preregistered adjudication and the calibration lesson. Honest typing: the identity and the slope-1 pole normalization are measure-generic; the certified ζ-specificity is the envelope quality on the finite range plus the positivity of the measure ([verification/v761_atom_pole_abel.py], 24+10 checks, combined `ABEL-PAIRED-BOUND`). Gate 4: the test space is canonical — the countable dense family of dyadic boxes and hats of the (D, X) tower, with frozen enumeration, explicit stage map, exact $\mathbb{Q}$ correlation closure on all type pairs, admissibility located per class (hats directly; the bare box fails the r^{-2} criterion and enters only correlated/tent-read — the named caveat), quantitative density rates hat -1.566 / box -0.503 per level, and the 24-function battery typed as an initial diagnostic package — battery hat 8 *is* the family member hat$(2, 3)$, enumeration rank 35 ([verification/v762_dense_weil_core.py], 26/26, `DENSE-CORE-CANONICAL`; companion Lean module `DenseWeilCore.lean`, 42 theorems).

Gate 3, the new wall. The kernel-safe route treats $\varepsilon \to 0$ through the bounded Yosida transform $Y_\varepsilon = A(A + \varepsilon I)^{-1}$: the handoff observable is formulable entirely through Y_ε ($|b_Y - b_G| \le 10^{-13}$, sandwich $0 \le \text{diag} \le 1$ verified over all 360 evaluations) with *no* $1/\varepsilon$ bound and *no* positive-margin assumption anywhere — the preregistered circularity kill did not fire; the near-kernel converges to an identified stable object, and every fixed battery function has a monotone, contraction-certified ε-limit. But the pairing weight q_f of the battery with that near-kernel is *non-Cauchy at honest maximal depth* ($X = 15.1875$, the comb extended to 279849 atoms with all extension guards, 4× the parent rung density), and it *survives orthogonalization by structural subordination*: 89–99.9% of every battery function lies in the span of the six restricted near-null modes, so removing the pairing removes the observable content with it — Gate 3 closes *negatively* on the reachable surface ([verification/v763_yosida_handoff.py], gates 9/10 + 7/12 preregistered, `YOSIDA-PARTIAL`/`QF-DEAD`). **The q_f wall is the round's named wall object**: density and near-kernel avoidance collide on the same function class. Consequently the assembled diagonal certificate (the planned `v764`)

was *not executed* — its precondition failed via Gate 3; it is documented as the named open gate of `PRIME.GRAM.DIAGONAL.01`, not fabricated. *[2026-08-05: the q_f object is adjudicated by the qf offensive — Section 12.11: q_f is representation gauge, the corner-native Gate 3′ is dead, and the route closes.]*

The red team, green. The static firewall audit of the entire source-construction path is clean — zero blacklist hits over all 21 functions, module whitelist and target-field quarantine hold, source strictly before target — the baseline reproduces the audited run exactly, and all eight frozen corruptions break the certificate; the one auditor-side expectation miss (RT6) is decided by the companion: all three excess-mode deck corruptions break the *symbol gate* on every window (min symbol $-0.45..-1.72$) while all six other gates stay measurably blind, and the deficit mode moves the symbol *into* the interior and is caught by the rate/ratio/κ gates instead — the two corruption modes are covered by *complementary* gate families ([`verification/v765_handoff_redteam.py`], 16+8 checks, `REDTEAM-GREEN-COMBINED`). The binding promotion note: certificates must always carry *both* gate families — symbol/PSD *and* rate/ratio/identification.

The contract. The round's theorem form is registered as `PRIME.GRAM.DIAGONAL.01` (open at registration; closed 2026-08-05 per its own frozen stakes — Section 12.11): construct PSD source matrices G_n — no target decomposition, no zeros, no positivity tests — with $\max_{i,j\le n} |(G_n)_{ij} - W(\phi_i * \tilde{\phi}_j)| \le 2^{-n}$ on the canonical dense family; then every fixed m-corner of W is a PSD limit of PSD matrices — Weil positivity on the rational span, and the density step closes. Measured state: Gates 1/2/4 positive, Gate 3 negative on the canonical class (the q_f wall), Gate 5 not executed, red team green; the kill criteria that did *not* fire: no RH-strength input was needed, no spectral gap was assumed (exact zero modes stay admissible — the Ihara lesson). The $\varepsilon \to 0$ wall (PD persistence) and the battery-limited identification remain open exactly as quantified. No marker moves; **no statement about the Riemann Hypothesis is made**.

12.11 The qf offensive: the route closes at a moving spectral edge, and what survives

The follow-up round (2026-08-05; five modules from eight frozen preregistered probes, all first-run adjudications) decides the q_f wall — and with it the diagonal route — honestly and finally. *The audit pair* ([`verification/v769_qf_contract_necessity.py`], combined `QF-GAUGE/GATE3PRIME-DEAD`, 5/5 gates + 34/168 cells, 14/14 + 14/14 guards+controls): Gate 3's full-ladder Cauchy demand on q_f was provably *stronger* than the contract needs — a single deterministic ε-net subsequence (density 0.144) makes all 14 q_f profiles Cauchy at the exact parent bars the full ladder failed, and the contract's own E_Y corner objects are *blind* to q_f motion (corr +0.038; the $\mathrm{d}q = 0.34$ mode-entry jump at $M = 888$ leaves the corners at routine level; 370 cross-cluster pairs, median $\rho = 1.13$): q_f is *representation gauge*, owning the ε-scale distribution of the Yosida deficit, not the value (VAR_RATIO = 0.021); the double limit is non-uniform (the deficit X-slope grows 88× over four ε decades) but not certified singular. Yet the corner-native weakened Gate 3′ then dies on its own predeclared 1.6×10^7 comb ($X = 16.5625$, 202 rungs): the corner increments fall to $X \sim 12.9$ — exactly the falling regime every parent probe certified — and then *rise sustained* (second-half slopes +0.10..+0.34 per X unit at $\varepsilon = 10^{-3}$, 0/56 cells; broad-spectrum, not one bad channel): the `v764` precondition is refuted at the corner level. *The transport pair* ([`verification/v770_qf_spectral_bundle.py`], combined `QF-BUNDLE-PARTIAL/QF-SETTLES-POSITIVE`, 19/19 + 14/14): the Kato/polar parallel transport of the rank-6 near-kernel is well-defined with wide margins (rank stable, gap 6/7 = 0.34..0.56, angles $\le 2.72°$, TV flattening) and repairs the rigid frame for 14/14 functions (median med5 ratio 26.7 → 1.004) — and the residual drainage was a *transient*: on the 10^8 comb ($X = 18.375$) the band weights plateau at positive per-function constants ($R2$ family 0.225–0.358, $R1$ family 0.008–0.079; zero-bar depth $X^* \sim 227$, i.e. never), the 6/7 gap bottoms at 0.1008 and recovers, the 7th/8th threshold crossings are typed, and the holonomy exponent is ∼1 (step refinement closed). *The census* ([`verification/v771_qf_representation_census.py`], `QF-REPRESENTATION-OPEN`, 26/26 guards+controls): the $6 = 1+5$ hypothesis $E_{qf} \cong E_7 \oplus E_{-2}$ *fails* the frozen census — the deployed scalar comb realizes only two characters of V, no rank-6 lift exists,

and the near-kernel is a continuum-vs-odd-places balance — while all 27 wrong forms and 5 wrong labelings die precisely on σ-sharpness; the honest residue is *one* rung-stable contact: $\cos\theta_1 = 0.997$ with the ram-odd negative wavepacket, plus a second partial contact at $\cos \sim 0.64$. *The fixed-d pair* ([verification/v772_qf_feshbach_effective.py], combined `FESHBACH-PARTIAL/EDGE-KEEPS-MOVING`, 28/28 + 20/20; one declared guard-normalization correction, gates identical): the $d = 6$ Feshbach reduction is *exact* (reconstruction 8.8×10^{-13} against independent dense solves) and matrix-Herglotz certified, but its Cauchy tail flattens where mode 7 grazes the band; $d = 7$ converges strongly ($b_2 = +0.502$) but loses separation at the top rung; $d = 8$ has an internal 8/9 crossing — and on the 10^9 comb ($X = 20.6875$, 160 s) the 7/8 edge *closes into an avoided crossing* (gap 0.0039 at $M = 1240$, tail 0.0074 vs bar 0.10) while the transported $d = 7$ block stays Cauchy straight through; the 9th mode enters at $M = 1276$, and the measured entry cadence *widens*: $\Delta X = 1.625/1.8125/2.625$ (spread 0.381, not regular at the 0.25 bar). **The moving spectral edge**: every d gets a convergent transported block and then loses its edge to the next descending mode — convergence was never the problem, band ownership is; fixed-d Feshbach is closed permanently. *The closure decider* ([verification/v773_qf_cell_cocycle.py], `COCYCLE-DOMAIN-ONLY`, 2/8 gates, 14/14 guards+controls; one declared implementation correction — Hotelling refinement of the carried state, algebra and gates untouched): the exact Möbius/Redheffer cell cocycle — coefficients *are* the added source rows/columns — holds at 9.7×10^{-12} over 73 rungs $\times$ 5 z, every cell Schur block is PD (a monotone Loewner flow), and the Nevanlinna/Stieltjes domain is preserved with *zero* breaches through both typed mode-entry transitions — but no renormalized object converges at reachable depth (0/6 cells): *structure without limit.* **Per the frozen stakes, `PRIME.GRAM.DIAGONAL.01` is closed** — Gates 1/2/4 positive, Gate 3 dead at the corner level, Gate 5/`v764` never executed, cocycle domain-only; no further variants, the ten-item stop-list stays binding. *What survives as theorem-shaped facts*: the exact anti-alias theorem ([verification/v760_antialias_exact.py]), the unconditional paired Abel bound ([verification/v761_atom_pole_abel.py]), the canonical dense family ([verification/v762_dense_weil_core.py]), the Kato bundle frame and the settled coupling levels ([verification/v770_qf_spectral_bundle.py]), the Herglotz-certified exact Feshbach reduction ([verification/v772_qf_feshbach_effective.py]), and the domain-preserving cell cocycle through mode entries ([verification/v773_qf_cell_cocycle.py]). *The handover*: the sharpened `PRIME.Z1.OPERATOR.01` takes over — a self-adjoint geometric bulk operator with a canonical finite-dimensional boundary/threshold structure whose Weyl M-function delivers the qf block and whose relative trace formula reproduces the full Weil formula, constrained by the measured cadence (888/992/1108/1276, ΔX widening, the avoided crossing), the ram-odd contact, the settled levels, and the domain-preserving cell transport (the dated sharpening note in the contracts document). No marker moves; **no statement about the Riemann Hypothesis is made**.

12.12 The sharpened-Z1 trilogy: compactness carried, selection open

The handover is executed the next day as one module from three preregistered probes ([verification/v780_z1_compactness_trilogy.py]; gates 3/4 + 1/4+ legs 3/6 with *exactly* the preregistered fails; guards+controls 20/20 + 23/23 + 23/23). *The gate*: the N2 GNS/Jacobi truncation family — whose source identification is now explicit: the closed Gram route's window operator *is* the Toeplitz Gram of the N2 unified Weil measure — carries three of the four measured handover signatures natively in its self-adjoint frame: edge collisions at the source depth (min boundary-pair gap 0.0032 vs the source's 0.0039), the ram-odd contact direction in its threshold band (cos up to 0.9971, 5/5 on the last five rungs), and settled positive couplings (14/14 type P) — but *not* the widening cadence: its edge census is regular quadrature filling ($\Delta X \approx 0.4485$, the free-Jacobi type), so the cell structure must be *imported* as boundary data. *The import*: the finite boundary triple ($A = 2 - J_K$ bulk, imported Gram near-kernel band) is *exactly* Herglotz through all typed transitions ($\min \operatorname{eig} \operatorname{Im} H = +0.216$) — the frame survives — but the whitened $K = M/2$ import dies with the root cause measured: the Gram near-kernel packets are delocalized, cell-mass exactly 0.500 beyond the half-window on every gate rung, so the polar whitening turns truncation loss into quasi-generic boundary noise (delivery 5.88; still arithmetic-specific: 5.9 vs GOE 13.0 vs

random boundary 132.5; the full-depth GNS frame is provably out of reach by deepening — $e^{27.75}$ atoms). *The yield*: with the *raw* μ-weighted import (no whitening) the variable-edge Herglotz family is *bounded* (grid-sup ratio 0.936, falling) and *equicontinuous* (derivative ratio 0.807) through all three typed transitions; the entry mass is *summable* (c_{mass} max $0.032 \ll 1$ — the kill criterion is far away), and the entry residues are predominantly *rank-one* (ratios 0.796/0.869/0.804 — the 0.997 contact prediction confirmed in the raw metric); the delivery metric loses its discriminating power in this form (typed UNRESOLVED-BY-THIS-METRIC, disclosed), and the moment functionals do not settle (median tail oscillation 22.1). *The measured unification*: the weak-$*$ cluster demand of `PRIME.KMS.INDUCTIVE_STATE.02` and the operator demand of `PRIME.Z1.OPERATOR.01` split *exactly* along the measured line — the cluster-*existence* half is carried (Helly/Montel at finite level), and the *selection* half is the one shared open object: state selection *equals* import-faithful boundary coupling. The merged target is named Z1-COMPACTNESS (both contract rows carry the dated cross-referenced note); nothing beyond $X = 20.3125$ is claimed, and **no statement about the Riemann Hypothesis is made**. *[2026-08-06, round 23: the selection half is finite-level SOLVED — Mosco form convergence + Friedrichs minimality replace the moment selector on identical rungs, and the import diagnosis is completed at form level (nothing is whitened at the core); the* 22.1 *oscillation above stays on record as the measurement that forced the replacement. See* sec:floor-ratio *and* `[verification/v816_prime_mosco_selection.py]`*.]*

12.13 The positive protocol: the prime channel as a positive routing code, and what the descent still needs

The next promotion wave lands the 2-adic skeleton of the cusp channel at theorem grade, decides two preregistered structure questions negatively, and delivers the first finite-level version of the completely positive descent — seven modules, every claim behind an explicit fence, no marker moves, **no statement about the Riemann Hypothesis**.

The check32 theorem (`[verification/v785_hecke_check32.py]`, 20/20, CHECK32-THEOREM): $f_8 = \eta(2\tau)^4\eta(4\tau)^4 \equiv E_{\text{odd}} = \sum_{n\ \text{odd}} \sigma_3(n)\, q^n \pmod{32}$, verified coefficientwise to q^{100000} — 25000× the Sturm bound $(4/12)\,[\mathrm{SL}_2(\mathbb{Z}) : \Gamma_0(8)] = 4$, so the theorem-grade reading needs exactly *one* cited classical ingredient (Sturm 1987); the corollary $a_p \equiv 1 + p^3 \pmod{32}$ holds on all 9591 odd primes $p < 10^5$ with the explicit decoder $p \equiv (a_p - 1)^3 \pmod{32}$ (the cube map is a bijection on the 16 odd residues), mod 64 fails at q^3 (maximality), and the controls quantify the checksum reading (mutant failure fraction 91.7%; random growth-matched detection ∼31/32). The finite kernel is Lean-checked (`TfptCarrier/Check32.lean`, 13 kernel theorems). $32 = 2^{g_{\text{car}}}$ stays a typed *observation*.

The positive C_2 lift (`[verification/v788_positive_c2_lift.py]`, 42/42, C2LIFT-THEOREM): the signed congruence is upgraded to a *positive* decomposition — $A = (E_{\text{odd}} + f_8)/2 = \frac{1}{16}\theta_2(2\tau)^4\theta_3(4\tau)^4$ and $B = (E_{\text{odd}} - f_8)/2 = \frac{1}{16}\theta_2(2\tau)^4\theta_2(4\tau)^4$ are theta series with nonnegative integer coefficients, $B = 16R$ with the closed convolution $R(2m{+}3) = \sum_j \sigma_1(2m{-}4j{+}1)\,\sigma_1(2j{+}1)$ and the $\mathbb{N}^8$ microstate count over triangular numbers (degeneracies $1, 4, 10, 24, 43$), and the eta-free generator $a_n = \sigma_3(n) - 32R(n)$ reproduces *every* odd f_8 coefficient: check32 becomes a kernel *corollary* of the lift (Lean: `PositiveC2Lift.lean`, build green). On top sits the C_2 packet automaton ($M_n = \left(\begin{smallmatrix} A & B \\ B & A \end{smallmatrix}\right)$, Hadamard-diagonalized to $2\,\mathrm{diag}(\sigma_3, a_n)$, XOR composition censused on 41053 coprime pairs + 70 prime powers) and the five-condition packet validator: joint detection 1.000000 against growth-matched random packets, and the consistent-tamper blind spot is *exactly* the $16\mathbb{Z}$ lattice (15/16 detection) — the honest scope of the mod-16/mod-32 screens.

The multirate ladder and its proof (`[verification/v789_multirate_constdepth.py]`, 23/23 + 24/24, MULTIRATE-THEOREM/SPINLIFT-VERIFIED + CONSTDEPTH-THEOREM): the full 2-adic depth law $v_2(1{+}p^3{-}a_p) \geq 5 + 2[\chi_{-4}(p){=}1] + [\chi_8(p){=}1]$ is census-clean on all 78497 odd primes $p < 10^6$ with sharpness attained per class (witnesses 3/7/5/17; depths 5/6/7/8); the character sifts I0–I3 are Sturm-certified at level ≤ 512, the predeclared mod-512 must-fail fires at $n = 9$, and the constant depths of classes 3 and 5 mod 8 are *proven*: the multirate weights $(1, -3, 2)$ telescope to the both-odd convolution, class 3 is pinned by the essentially unique $x^2 + 2y^2$ representation and

class 5 by the ternary bridge $R_3(p)/8 \equiv \#\{x^2+4w^2=p\} = 1 \pmod 2$ — elementary except two named class-number-one ingredients (disc -8 and -4, both $h=1$), whose instances are machine-verified (the exact-depth mechanism of the remaining classes 7 and 1 follows in sec:descent-functor; per-class infinitude stays open). The Shimura lift of the `v537` witness acquires its ladder form $\mathrm{Sh}_{t=2}(g) = -8E_{\mathrm{odd}} + 256H$ (exact to 160), and the lift's Kronecker factor $(2/d)$ *is* χ_8 — the same character that rules the ladder's third rung. The $32 \to 256 \to 128$ semantics stays a [C neu] observation. *Round-22 fence (2026-08-06): the arithmetic theorem stands untouched, but the analytic Clifford-activation reading of the depth classes is dead — the class-sector positivity ordering does not track the proven depths (`[verification/v806_hecke_local_clifford.py]`, LOCAL-CLIFFORD-BREAKS; see sec:k5-edge-machine).*

The arrow ledger and the broadcast (`[verification/v787_hecke_arrow_broadcast.py]`, 48/48+21/21, ARROW-LEDGER-STRUCTURED/BROADCAST-EXACT): kept at arrow resolution instead of trace resolution, the ramified layer *is* the 15 polar labels (σ-equivariant, 7 NS +8 R), every ramified arrow meets the certified spread in census $(3,1,1,1,1)$ at its own polar label, the label-resolved two-step collapses to $T/196 = (4/49)I + (45/49)\Pi_0$ exactly with the 105 leg classes equal to the `v756` Kraus index set (weight $(7^{-1/2})^4$), and the trace collapse now derives $a_7 = +24$ *with sign* from the full 137600-line Kneser census. On the odd shells the joint (class × glue) census factorizes exactly as $\bar{T}_n = M_n \otimes I_{16}$ — place 2 as control plane, odd primes broadcasting $\sigma_3(n)$ full 16-state copies of the root packet, $B_n = 16R(n)$ arrow-exactly, with the new identities $b = 2A_p$ and $n_B = A_pB_p/2$; the "routing code" wording stays a [C neu] reading of an exact finite factorization, the entropy numbers are distribution measurements (no decoded message), and the honest negative is on record ($R(p)$ is *not* the type-B Weyl orbit count at $p=7$; completed by the full no-arrow-realization adjudication in sec:descent-functor).

Two preregistered adjudications: the packet-480 header (`[verification/v786_prime_packet480.py]`, 28/28, PACKET480-ADDRESS-ONLY) is exact 7-bit address arithmetic — Header(p) = (p mod 30, a_p mod 32), CRT-equivalent to p mod 480 with all $\varphi(480) = 128$ classes hit — but the sharp intertwiner question dies as preregistered: $(\mathbb{Z}/480)^\times$ has exponent 8, the canonical spinor torsor group $\mathbb{F}_2^7$ has exponent 2, only 3 of 7 bits are realizable, and the 128 *stays a fingerprint.* The SNF fingerprint and the μ_4 packet (`[verification/v790_snf_mu4_theta.py]`, 18/18 + 17/17, SNF-FINGERPRINT/MU4-PARTIAL): the correction theta's quadratic form has $256R(n)$ as its odd-vector representation numbers and its SNF $(1^4, 2^4)$ matches the ramified inclusion $(1+i)L \subset L$ exactly, but the isometry census is *empty* at every admissible level (kissing 240 vs 8; 0/20160 on the quotient, alternating vs odd form; the naive 4+4 split inadmissible by the replicated `v638` theorem) — a fingerprint, honestly typed; and the new exact identity $c_{\mathrm{sig}} = \frac{1}{15}E_4(q^2) - \frac{6}{5}E_4(q^4) + \frac{32}{15}E_4(q^8) - 8f_8$ places the cusp packet in the *quartic* μ_4 character with the exact normalization $-8 = 52 - 60 = -\mathrm{rank}(E_8)$, while the quadratic ($C_2$) channel is pure Eisenstein and the all-shell quartic vanishing is decidably false (closed series $\prod(1+q^{2n})^{-4}$).

The positive descent, and what it still needs (`[verification/v791_positive_descent.py]`, 30/30, DESCENT-PARTIAL): work in the positive algebra $\mathbb{N}[C_2] \otimes \mathbb{N}[\mathbb{F}_2^4] \otimes \mathbb{N}[\mu_4]$, build the state there, apply the sign characters only after. The packet GNS state is *manifestly* positive at every depth (verified in exact rationals; the naive linear pushforward is *not* a state — finding F1, quantified: signedness is observer projection); the GL1 sign sector of the packet correspondence operator is *bit-identical* to the deployed Weil window and is its *unique* PSD sector of all 24 (finding F2): every damped or twisted sector breaks because the register-trivial continuum cannot follow the atom leg — the breaking tensor factor is the *continuum/pole leg*, not C_2, $\mathbb{F}_2^4$ or μ_4. The contract `PRIME.POSITIVE_DESCENT.01` is registered *open* [O] with three demanded objects: (1) *sector-adapted continua* (the twisted-channel explicit formulas; the f_8-sector continuum is the finite-level falsifier), (2) *GL1-sector PSD persistence* in the Z1-COMPACTNESS frame of sec:z1-trilogy — Weil positivity retyped as "the sign sector of a manifestly positive packet object stays PSD" — and (3) *the carrier intertwiner* (population ↔ carrier register, at registration the one non-CP-trivial step). Stop conditions and finite-level falsifiers are named in the contract row. All three demands were delivered at finite level one round later (sec:descent-functor); the contract stays open for the infinite-level floor

— which round 23 reduced to the single ratio inequality of sec:floor-ratio (`PRIME.FLOOR.RATIO.01`), with selection and uniqueness both finite-level closed ([verification/v816_prime_mosco_selection.py], [verification/v817_positive_descent_master.py]), and which round 24 narrowed with the certified floor skeleton of sec:floor-skeleton (battery-relative and α-bounded, still open). **No statement about the Riemann Hypothesis is made.**

12.14 The descent functor completes at finite level, and the boundary code becomes three physical axioms

Round 21 (2026-08-06) executes the three demands of `PRIME.POSITIVE_DESCENT.01` at finite level, mechanizes the remaining 2-adic depth classes, and adjudicates two identification questions honestly. No marker moves; no RH/GRH claim anywhere.

Demand (1): the sector continua exist and are one functor ([verification/v793_f8sector_conductor.py], $15/15 + 16/16$, F8SECTOR-PSD/CONDUCTOR-FUNCTORIAL): the f_8 sector — broken at $-132..-151$ under the register-trivial GL1 continuum (finding F2 of [verification/v791_positive_descent.py]) — is PSD on the entire frozen ladder once the functor transports its *own* explicit-formula continuum (weight-4 archimedean kernel, conductor 8, no pole, Satake atoms): $\lambda_{\min} = +1.52 \times 10^{-4} \to +3.20 \times 10^{-5}$ over $X = 4..10$; the χ_4 sector likewise; the wrong continuum reproduces the parent breakage at -140.8. The continuum assignment is *one closed structure map* $\chi \mapsto q^{s/2} \prod_j \Gamma_{\mathbb{R}}(s+\mu_j)\, L_{\mathrm{an}}$ — one universal per-factor kernel plus $\ln q$ at lag 0, pole iff trivial — whose instances reproduce every deployed kernel (the weight-4 kernel through the duplication identity $\Gamma_{\mathbb{R}}(z)\Gamma_{\mathbb{R}}(z{+}1) = \Gamma_{\mathbb{C}}(z)$, two independent quadrature structures agreeing to 2.7×10^{-15}); the register-internal twist $f_8 \otimes \chi_{-4}$ receives conductor 16 from Atkin–Li plus the Fricke ward (6.7×10^{-16}; wrong N fail at ~ 0.5) and lands PSD with the rule continuum, nothing hand-tuned; the conductor datum is *load-bearing*: the $q = 8$ swap shifts $\lambda_{\min}$ by exactly $-\ln 2$, four orders of magnitude above the margins, while the PSD margins pin $q \geq 15.9996$ from below. The register mirror sector is typed structurally non-automorphic — the functor's domain is the automorphic register characters. Sector PSD stays finite-level *evidence*; the GRH-type statements are unproven.

Demand (2): the exact limit object, and the first measured sector floor ([verification/v794_cp_extension_gate.py] $17/17$, CPGATE-UNDECIDED-FALLING; two declared repairs carried): the finite windows form a restriction-*exact* net — an atom at u touches only nearby lags, so the sector Weil functional W_χ is well-defined on the union space V_∞ *without any limit* — with Cauchy interlacing and uniform bounds measured (the frozen $e^{X/2}$ pole-growth prediction was wrong: the comb cancels the pole lag by lag, the whole four-sector family is near-uniformly bounded). The demanded object is therefore *not a new state*: it is the positivity floor $\inf_X \lambda_{\min}(T_{\chi,X}) \geq 0$ per automorphic character — the Z1-COMPACTNESS demand of sec:z1-trilogy *sector-decorated* (the $\chi =$ GL1 instance reproduces the `v780` anchor 3.882×10^{-6} at $M = 1176$ on the same 10^8 comb). The deep trend decides at reachable depth: no sector crosses zero anywhere; GL1 falls power-law to 3.9×10^{-6} at $X = 18.375$ ($\beta = -1.69$), f_8 at the same rate ($\beta = -1.63$, certified-exact CRT coefficients to $X = 13$), the twist slightly faster ($\beta = -2.10$) — and χ_4 *saturates* at $\sim 1.5 \times 10^{-6}$, the first measured sector floor. *[2026-08-06, round 23: the floor demand is reduced to one ratio inequality $\rho = \tau/\tau_{\mathrm{pnt}} > 0$ with a measured $h^{-3/2}$ envelope — see sec:floor-ratio; round 24 adds the certified three-piece lower-bound skeleton of the lock-block floor on the deployed battery, closed at citation grade for all h up to $\alpha^* \approx 11$ — see sec:floor-skeleton.]*

Demand (3): the carrier intertwiner exists in Stinespring form ([verification/v801_prime_cp_intertwiner.py], $29/29$, CP-INTERTWINER-EXISTS): positive states must be transported by completely positive maps, and the probe constructs exactly that transport from the certified corpus machinery — $\Phi(a) = V^* \pi(a) V$ with V the 105-leg Kraus stack (unitality $\sum_e E_{x_e x_e} = 7I$ as an exact integer identity, re-deriving the `v756` anchor) and π the arrow representation of the *full* arrow $*$-algebra M_{15} (exact matrix-unit closure — the algebra, not its trace collapse); the Choi matrix is exact-rational with 105 eigenvalues each $1/7$; all covariances are exact. Its four automorphic character channels, applied to the one packet event stream, land on the four sector windows: the GL1 channel *is* the deployed Weil window (rel 6.0×10^{-16}) and χ_4/f_8/twist land on their $\Gamma_{\mathbb{R}}$-rule

objects ($\sim 10^{-15}$), all PSD — four positive sector outputs of *one* CP map, with the ramified conductor emptiness structural ($a(2^k) = 0$; glue-uniform μ_4 slot) and the (E, f_8) pair exhibited as one primitive element read by two characters. Finding F1 of the descent is retyped: the naive linear pushforward fails *as a negative Choi eigenvalue* (-6.6×10^{-2}; exact leg -4.50). What remains is typed as demands D1–D3 of `PRIME.CP.INTERTWINER.01`: the X-compatible dilation family, normality on the Z1-COMPACTNESS limit object, and the limit identification with the critical-line Weil functional — which contains RH and is not claimed.

The depth classes mechanized, and the $k = 5$ anomaly solved ([`verification/v795_halving_tails_k5.py`], 20/20 + 17/17, TAILS-IDENTIFIED/K5-MECHANISM-FOUND): the two classes the constant-depth theorem left census-level acquire exact identities — $v_2(D_p) = 6 + v_2(X_7(p))$ for $p \equiv 7$ and $8 + v_2(X_1(p))$ for $p \equiv 1 \pmod 8$, with termwise-integral divisor sums (composites included; 10^6 census, zero exceptions); the class-7 base closes on the $h = 1$ ternary form, $v_2(D_p) = 6 \iff R_3(p) \equiv 16 \pmod{32}$; the class-1 two-shape criterion is exact while its single-character bridge stays typed open. The measured $k = 5$ half-mass dip is then *derived*: k is deterministic on the residue-tower cells $(m, t) = (v_2(p-1), \text{depth of } 2)$ with zero exceptions at 10^6, and $k = 5$ lives only in cell $(5, 3)$ of density $1/64$, so $P(k{=}5) = 1/128$ — exactly the observed suppression; the class-7 mirror law $v_2(D_p) = 3 + v_2(p+1)$ through $k = 4$ explains the class-3 constancy from the other side. The governing-field derivation (Cohn–Lagarias frame) stays typed open; per-class infinitude is not claimed.

Two honest adjudications: the R-microstate identification dies ([`verification/v797_r_microstate_identification.`] 14/14, R-NO-ARROW-REALIZATION): $R(p)$ is an analytic packet count without an arrow-level realization at the tested places — the frozen dictionary is killed at $(1, 4, 13)$ vs $(1, 4, 10)$, the deployed extensions are ill-defined on the type-B set, and the exact count identity $16B_n = 256R(n)$ stays count-level only (SNF fingerprint shared, isometry excluded at 8 vs 240 minimum vectors); the packet and arrow layers share totals, not the R-grading. And the incidence programme's Kraus set becomes load-bearing: the `v756` 105-leg structure carries the Stinespring intertwiner above. **No statement about the Riemann Hypothesis is made.**

12.15 The K5 edge machine: five exact structures, two precise middles, and the honest kills

Round 22 (2026-08-06) promotes eleven modules from fifteen frozen probes (`v803`–`v813`, suite 795 → 806). On this front it delivers the Kraus context protocol, the Gray register chart of the intertwiner, one analytic honest kill with a positive mod-8 side-result, the Lorentz spinor reveal, and the P1 index-KMS identity. No marker moves; no RH/GRH claim anywhere.

The Kraus context protocol ([`verification/v811_prime_kraus_doily.py`], 21/21, KRAUS-DOILY-PROTOCOL; one declared report-text repair carried): the 105 Kraus legs of the sec:descent-functor intertwiner are a *complete context protocol* over the doily $W(3, 2)$ — 15 diagonal legs + 90 off-diagonal, with the canonical 45 + 60 split identified structurally: the 90 legs double-cover the 45 flags 2:1 (reversal quotient) and the $15 \times 4 = 60$ fiber expansion is exactly the `v783` stabilizer-ray census. The $Sp(4, 2)$ obstruction certifies that *no* leg-subset partition exists — the literal partition is spread-relative, matching the ledger T3 census — and the 45 transports are quadratic ($P_x P_y = \pm P_{x+y}$), spanning $\mathrm{End}(\mathbb{C}^4)$ at rank 16. The surprise is exact: $[B_{45}, B_{60}] = 0$, so $K = \frac{3}{7}E + \frac{4}{7}R$ is a *direct-sum protocol* with E a conditional expectation. The measured caveat is on record: the certified spread is *not* σ-invariant — the named next falsifier *[decided in round 23, sec:floor-ratio: zero fixed spreads, family-stable protocol, the commutant tower extends — [`verification/v815_kraus_spread_commutant.py`]].*

The Gray register chart ([`verification/v804_prime_carrier_gray.py`], 23/23, GRAY-INTERTWINER-EXISTS; one declared control repair carried): the affine Gray map encodes $C_2 \times \mu_4$ into the R-class torsor *exactly respecting* the `v786` Packet480 kill — a group isomorphism is impossible (exponent 4 vs 2) and is not demanded; the sharp intertwiner question died, the affine chart survives. The fiber family is deck-covariant exactly with *trivial* measured μ_4-cocycle (the residual is a pure $\mathbb{Z}_3$ line twist on the σ-fixed classes), and the extended Stinespring map's GL1 channel re-lands on the

deployed Weil window at rel 6.0×10^{-16}. The 128 stays a fingerprint; the intertwiner gains its register chart at the odd places.

The local-Clifford kill, and the mod-8 completion of the rule map ([verification/v806_hecke_local_clifford.py], 16/18 with the two FAILs the preregistered decider gates themselves, controls 4/4, LOCAL-CLIFFORD-BREAKS; one declared bar re-freeze carried): the analytic activation-ordering hypothesis dies — all four p mod 8 class sectors fail PSD, because signed class-indicator combinations carry no completed L-function (the trivial mod-8 component is imprimitive); the margin ordering does *not* track the proven activation depths (Spearman +0.40); class 1 is the unique positivity-completable class (Euler repair exact, $+1.4\times10^{-5}$). The arithmetic multirate theorem (constant depths of classes 3 and 5 mod 8, sec:descent-functor) is untouched — what dies is the *analytic* Clifford-activation reading, now decorative. The positive side-result: χ_8 and χ_{-8} are two *new* PSD instances of the $\Gamma_{\mathbb{R}}$ conductor rule (exact parities: χ_8 even, χ_{-8} odd) — the rule map of demand (1) now covers the *full* mod-8 dual group.

The Lorentz spinor reveal ([verification/v807_lorentz_nullselector.py], 22 checks with the three honest h-order FAILs encoded +14/14, LORENTZ-COORDS-REVEAL+NULLSELECTOR-ALGEBRAIC): the compiler triple is a rank-one spinor square — $M(5,-3,4) = 2\,(1,2)^T(1,2)$ exact, Euclid $(2,-1) \mapsto (5,-3,4)$ — and in spinor coordinates the locking slope is *strictly monotone* in the window width α (66/66 pairs, Kendall 1.000, cross-ratio contraction CV 0.004): the long-standing h-oscillation was α-jitter, a coordinate artifact. The approach $2 - r \sim \alpha^{-0.23}$ stays glacial and the three h-order gates FAIL honestly — the 1D monotonicity candidate is named, not claimed. The companion null selector forces $d = 4$ algebraically ($|v_d|^2 = \det M_d = d(4-d)$; bridge $g^2 - N^2 = 4d = |H|^2$) — a second, independent mechanism on the same invariant as the `v781` census.

The P1 index-KMS identity ([verification/v813_p1_index_kms.py], 13/13, P1-IDENTITY-CLOSED): the closed normalization identity $I \cdot \beta_{\text{angle}} \cdot c_3 = 4 \cdot 2\pi \cdot \frac{1}{8\pi} = 1$ is exact in formal arithmetic, with each factor independently sourced — the Jones index $I = 4 = |\mu_4|$ is a c_3-free theorem (zero occurrences gated in all four sources), the KMS angle $\beta_{\text{angle}} = 2\pi$ is independently measured (`v526`; the radian pin is the one shared convention and cancels), and c_3 *stays the P1 axiom that the identity explains* — it does not derive it. The finite evenness precursor is measured ($w_1 = w_3 = \frac{1}{4}$ exactly at every level, symmetric zero-sum offset $\pm 2^{-(l+1)}$ — the no-anomaly shape); the sea state does *not* equidistribute (dev 0.188), so the one open bridge of `P1.INDEX.KMS.01` [O] is the *even modular-response normalization* over the I deck sectors (kill: any asymmetric sector offset; control: index 2 gives $1/(4\pi)$, booked elsewhere by the corpus).

The carrier-side exact structures of the round — the jet-code normal form (non-splitness = the no-origin obstruction, [verification/v803_e8_jetcode_affine_nsr.py]), the Petersen carrier machine whose radial sixth power *is* the deployed transport spectrum ([verification/v808_petersen_sixthroot.py]), the doily incidence identity behind the recovery base rate 2/3 ([verification/v809_curve_code_doily.py]), the cut-code reading of the Arf split ([verification/v810_cutcode_moment.py]), and the syndrome honest kill ([verification/v805_e8_syndrome_algebra.py]) — are reported in the architecture and audit papers; the flavor-filtration fingerprint ([verification/v812_flavor_graph_filtration.py]) in the Standard-Model paper. **No statement about the Riemann Hypothesis is made.**

12.16 One inequality: the floor reduces to the prime comb against its own density

Round 23 (2026-08-06) promotes nine modules from thirteen frozen probes (`v814`–`v822`, suite 806 → 815). On this front it delivers three structural moves at once: the *selection* problem of the unified compactness contract is finite-level solved, the *uniqueness* of the limit state becomes an instance of one abstract master theorem, and the *sector floor* — the condensed RH-hard remainder — reduces to a single ratio inequality with a measured envelope, registered as the new open contract `PRIME.FLOOR.RATIO.01` [O]. No marker moves; no RH/GRH claim anywhere.

Selection solved by Mosco + Friedrichs ([verification/v816_prime_mosco_selection.py], 23/23 guards+controls, 6/6 legs, MOSCO-SELECTS): the moment-stability selector that oscillated at 22.08 in the sharpened-Z1 trilogy (sec:z1-trilogy) is replaced on the *same* rungs, the same battery and the same statistic by Mosco form convergence and Friedrichs minimality on the certified `v762` dense core — and the new route is Cauchy by four to five orders of magnitude under the bar: Friedrichs resolvent tracks oscillate at 0.0004, the variable-edge Weyl tracks at 0.0843 *modulo the typed summable entry-pole ledger*, and the raw-coupling μ-face resolvents at 0.0000, with the two interleaved cofinal subladders agreeing to 2.5×10^{-5} — one limit candidate. The recovery leg is exact (1.2×10^{-12}, the finite shadow of the certified union-space structure), the canonical weak-null family has strictly positive liminf margins (+2.63 minimum), and the moving-edge family is confirmed a *genuine boundary channel*, not an escaping sequence. The import diagnosis is thereby completed at form level: the raw μ-weighted coupling of the core is exactly rung-independent (Gauss exactness — nothing is whitened away at the core), while the whitened control reproduces the parent's operator-face delivery failure 5.8772 exactly. What an infinite-level theorem still needs is typed, not owned: Mosco-precompactness on V_∞, the limit form domain, and ledger summability at infinity — and *nothing* about positivity.

Uniqueness as one master theorem ([verification/v817_positive_descent_master.py], 14/14, MASTER-BOTH-INSTANCES; kernel-checked core in `TfptCarrier/PositiveDescentMaster.lean`): the abstract positive-descent theorem — summable state defects over character sectors force a unique positive limit state with sector commutation — applies to *both* continuum problems at finite level. The G_{net} instance re-verifies (Choi-PSD expectation, defects at 0.267/doubling), and the prime/packet instance delivers the round's new number: the packet GNS state defects on the 128-dimensional register are summable at 0.701/doubling $\approx$ the $\sqrt{}$-dilution law $2^{-1/2}$, with the per-event Eisenstein ratio $\hat{m}_2(p) = -1/15$ *exact* for every odd prime. The selection problem is eliminated on both sides; the control K4 shows positivity is a *separate* hypothesis (summable defects with one non-positivity-preserving step converge to a non-positive limit).

The floor as one ratio inequality ([verification/v818_sector_floor_attack.py], $9/13 + 13/15$ with exactly the preregistered honest FAILs, FLOOR-MECHANISM-PARTIAL + CAPTURE-BOUNDED / INGREDIENT-II-THEOREM-SHAPE / AMPLIFIER-MECHANISM): the lock-block margin factorizes symbolically, onem $= F(r)\,\varepsilon\,K$ with F bounded in $[4.20, 5.40]$ — the sign lives entirely in the vanishing weight $\varepsilon = \tau/\lambda$; by Cauchy interlacing the 2-mode lock block is an $O(1)$-*faithful witness* of the full sector floor (measured capture $\tau/\lambda_{\min}(A_h)$ median 1.53, range 1.04–4.36); the capture angle is frame-certified ($\cos\theta$ median 0.990, 0.7% from the ram-odd contact 0.9971, and *scramble-robust* — the typed control surprise: capture geometry is frame content, not arithmetic); the direction is owned by the density rotation law $dr/dt = -\kappa\,(w\cdot v_1)(w\cdot v_2)(1+r^2)/(\lambda_1 - \lambda_2)$, which closes symbolically and matches the real staircase sign-by-sign at 100% — r-monotonicity is now a calculus statement under a verified sign hypothesis; and the arithmetic depth amplifier is carried by the smallest prime powers (packet-imbalance correlation +0.964, partially trend-driven, typed) with the measured envelope $\rho\,h^{3/2} \in [4.85, 24.2]$ *non-decaying*. The reduced statement — registered as `PRIME.FLOOR.RATIO.01` [O] — is: $\inf_X \lambda_{\min}(T_{\mathrm{GL1},X}) \geq 0$ reduces, up to the measured $O(1)$ capture, to the single ratio inequality

$$\rho(X) \;=\; \frac{\tau(X)}{\tau_{\mathrm{pnt}}(X)} \;>\; 0,$$

the prime comb against its own smooth density, with the measured finite lower envelope $\rho \geq c\,h^{-3/2}$ ($c \approx 4.85$). Kill criteria: the envelope failing at depth, or the capture angle collapsing. Interlacing runs floor $\Rightarrow$ margin (necessary, not sufficient); nothing here bounds the floor from below by a positive constant; promotion of any ρ bound is fenced on the preregistered gates. *[2026-08-06, round 24: the lock-block floor $\lambda\tau$ itself now carries a complete certified three-piece lower-bound skeleton on the deployed battery — exact Lagrange sum of squares, analytic fixed-pair bound, certified family exhaustion, and the deep tail closed at citation grade for all h up to the validity horizon $\alpha^* \approx 11$ (sec:floor-skeleton); the contract stays [O],* narrowed*: battery-relative and α-bounded.]*

The Kraus commutant tower ([verification/v815_kraus_spread_commutant.py], 21/21 + 24/24, SPREAD-SIGMA-BROKEN + COMMUTANT-EXTENDS + AORB-NOT-MAXIMAL): the two falsifiers named by the round-22 Kraus protocol (sec:k5-edge-machine) are decided. σ acts on the six isotropic spreads as two 3-cycles with *zero* fixed spreads (the outer-S_6 pre-analysis confirmed), so the literal partition is outer-class-forced non-invariant and no nontrivial invariant coarsening survives — but the direct-sum protocol is σ-stable *as a family*, with the invariant witnesses $\sum_s B_{45}(s) = 4I + 2B$ and the per-orbit A_{orb} (integer spectrum $\{0^5, 1^2, 4^5, 7^2, 9^1\}$). The commutation $[E, K] = 0$ extends exactly to the channel level (all 225 matrix units), the KMS tower, the odd places $p = 3, 5, 7$ — where the unique nontrivial Hecke-commuting expectation is the C_2-*character* expectation $E_p(a) = (a + XaX)/2$ — and the assembled 600-dimensional register, integer-exactly. The full $\{K, \sigma\}$ commutant is 39-dimensional and nonabelian ($\mathbb{R} \oplus M_3(\mathbb{R}) \oplus M_3(\mathbb{C}) \oplus M_3(\mathbb{R}) \oplus M_1(\mathbb{C})$); A_{orb} generates its canonical abelian 5-dimensional subalgebra, identical for both spread orbits, and the refinement is *window-silent* beyond the K-spectrum: the exact scalar action $QK^kQ = (\mu_B/7)^k Q$ leaves only three scalar windows ($\mu_B = 7, 2, -2$), of which only the uniform one is PSD and it *is* the deployed GL1 window to 6.0×10^{-16}.

The vacuum route closes honestly ([verification/v822_prime_vacuum_dilation.py], 17/19 + 12/14 with the four FAILs being the preregistered PSD deciders themselves, UNSHORTEN-DEAD + DILATION-DEAD): the reading that the archimedean/pole continuum acts as the RM$(2, 4)$ *vacuum completion* of the 15-label window objects is dead in both transcriptions. The exact RM layer survives ($140 = 105 + 35$; the unshortening identity $A_{140}^T A_{140} = 28I + 7J$; the Weitzenböck bookkeeping integer-exact), but the one-slot bordering dies by the `v758` mechanism verbatim and on scale, with the shape theorem that one vacuum slot induces only a rank-one pure-J coupling — and the corrected target, the 35-row Stinespring dilation, fixes the shape exactly, buys $\times 13.1$, and *still* dies in both sectors under both frozen normalizations. The general obstruction is quantified as the frozen fence: PSD $\iff$ coupling $\le c^* = (2/21)\,\lambda_{\mathrm{pencil}}(T_{\mathrm{dep}}, T_{\mathrm{cont}}^2)$, with measured deficits $\times 3 \times 10^7$ (GL1) and $\times 2 \times 10^2$ (f8) — the reading dies with the calibration, not the geometry: the continuum's completion role is already exhausted inside T_{dep} (the explicit-formula balance), and the path-state construction is not buildable on this gating.

The K5 follow-up (the clock-only six-step with the canonical T_{10} proposal and the bit-exact SIXTH-ROOT correction, [verification/v814_k5_sixstep_transport.py]) and the RM cascade (the punctured-RM$(1, 4)^*$ identification with the $[[15, 1, 3]]$ CSS code, [verification/v819_prime_packet_rm14.py]; the canonical rule onto the 105 minimal dual checks with the plane-frame Hecke match, [verification/v820_prime_kraus_rm24.py]; the E_7 vacuum completion $112 + 21 = 133$ as a matrix identity, [verification/v821_prime_vacuum35.py]) are reported in the architecture, Standard-Model and audit papers. **No statement about the Riemann Hypothesis is made.**

12.17 The floor-theorem skeleton: the floor as an exact sum of squares, and the tail closed for all h

Round 24 (2026-08-06) promotes two modules from four frozen probes (`v823`–`v824`, suite 815 → 817) and gives the reduced floor contract of sec:floor-ratio its first *certified lower-bound skeleton*: on the deployed 14-rung battery the lock-block floor $\det \hat{A}_2 = \lambda\tau$ is now bounded below by an explicit, citation-grade chain — exact identity + analytic bound + certified family exhaustion — uniformly in h at every deployed window width α. `PRIME.FLOOR.RATIO.01` stays [O], *narrowed*, not closed: the bound is battery-relative and α-bounded, and it is necessary-side evidence only. No marker moves; no RH/GRH claim anywhere.

The floor as an exact sum of squares ([verification/v823_prime_lagrange_floor.py], 7/8 with the one preregistered-honest old-budget FAIL + 9/9, LAGRANGE-CONCENTRATED + PAIR-CERTIFIED; contract `PRIME.FLOOR.LAGRANGE.01`): every per-zero layer of the `v692` master identity is *exactly* rank-one — the odd-extension symmetry factors the common phase, so the layer at γ is $v_\gamma v_\gamma^T$ with $v_\gamma = 2\sqrt{w(\gamma)}\,(S_1, S_2)$, the real closed form $S_i(\phi) = \sum_j t_{ij} \sin\big((h - j - \tfrac{1}{2})\phi\big)$ and the alias weight

$w(\gamma) = D\,\mathrm{csinc}^2(\gamma D/2) \geq 0$ (layer deviation $\leq 1.2 \times 10^{-16}$) — and the Lagrange identity

$$\det(G_Z + P) \;=\; \sum_{\text{pairs } i<j} \left(a_i b_j - a_j b_i\right)^2$$

holds at machine precision on all 14 rungs (ward $\leq 1.4 \times 10^{-9}$): *the floor is an exact sum of squares over zero+pole rank-one carriers.* The census is sharp: the *pole* is the universal non-collinear leg (the pole family carries 1.0000 of the pair total; every top-100 pair is pole × zero), and the moving zero leg obeys the *alias-comb law* — the dominant carriers sit at $\gamma \approx 2\pi k/D$ (median fractional comb distance 0.0007–0.0061 against 0.25 for uniform placement). The certification of the fixed pair (pole $\times\gamma_1$) then closes by a *route change*, not a quantity change: the remainder $R = \hat{A}_2 - G_Z - P$ is an exact float 2×2 with $\lambda_{\min}(R) = 1.6 \times 10^{-7}$–$2.8 \times 10^{-6} \geq$ margin verified per rung, and the 2×2 psd fact $\det(B + R) \geq \det B$ eliminates the tail estimate from the chain entirely — the identity budget tightens from 0.14–7.1 to 2.4×10^{-12}–2.7×10^{-9}, a median of 9.8 *orders of magnitude*, and the certified interval $X^2(\mathrm{pole}, \gamma_1) - \mathrm{BUD}$ is strictly positive on 14/14 rungs (γ_1 pinned to `zetazero(1)` within 10^{-11}; a dps-40 recomputation lands inside every interval). The pair's share of the floor is 0.0063–0.5832 with slope $h^{+1.06}$ — a *growing* brick. Controls fire (the scramble explodes the identity residual ×45.8–49.0; the mass-matched Epstein comb changes the Lagrange floor at determinant scale ×1784–50029; the collinear synthetic family gives $\det = 9.2 \times 10^{-16}$ with a certified interval containing 0 — no false positivity).

The three-piece skeleton, and the tail closed for all h ([`verification/v824_prime_floor_skeleton.py`], 12/12 + 9/9, FLOOR-PARTIAL + TAIL-CLOSED-ALL-H; contract `PRIME.FLOOR.SKELETON.01`): the assembly grades its three pieces honestly. *Piece 2* (uniform-verified): the analytic $h \to \infty$ limit of the fixed pair,

$$X_\infty(\alpha) \;=\; 16\,\alpha\,\pi^2 \sin(\alpha\gamma_1) \sinh(\alpha/2)\,[\text{explicit bracket}],$$

with the Dirichlet closed form equal to the direct sum at 5.7×10^{-13}, zero sign flips over 14 deployed $\alpha \times 10$ synthetic h, and the explicit bound $L(h) = X^2_\infty(\alpha)\,(1 - C(\alpha)/h)$ valid for $h > h_0(\alpha) = C(\alpha) \leq 499.6$ — h-uniform at every deployed α; the honest limitation is typed: X^2_∞ has 25 α-*nodes* on $[2, 12]$ (the $\sin(\alpha\gamma_1)$ zeros, spacing $\pi/\gamma_1 \approx 0.2223$), so the fixed pair is h-uniform *per* α but not ladder-uniform in α — at the nodes the family must take over. *Piece 3* (uniform-verified): the certified top-100 (pole × zero) family carries a median 0.952 (range 0.931–0.974) of the floor with every carrier on-line by verified computation ($\leq 2 \times 10^4$); the residue is 0.015–0.049 with trend slope +0.010 — truncation depth, not a structural carrier — and the analytic family limit stays positive across *all* fixed-pair nodes (minimum 1.62×10^{-4} at $\alpha = 2.68$): the family never dies simultaneously. *Piece 1* (closed by the tail repair): the crude coherence-discarding deep-tail envelope closes only 4/14 rungs ($h \leq 434$) — the autopsy names the defect, taking $\sup|\mathrm{csinc}|^2 \times \sup|FF|$ independently multiplies two suprema attained at *opposite* points (measured band slack $\times 2.7 \times 10^3$–2.4×10^5) — and the repair is the *product sup* of the true per-zero envelope (an exact Dirichlet closed form, grid-converged with a doubling ward; its rigorous Lipschitz certificate is a *named elementary step*, typed, not claimed) combined with Abel summation against the Riemann–von Mangoldt counting function with explicit unconditional Trudgian-grade constants (factor 2.13; the Vinogradov–Korobov zero-free region is typed honestly *useless* at these scales, 0.985). The result: *all 14 rungs close at the fixed verified horizon* $T_{\mathrm{ver}} = 3 \times 10^{12}$ (Platt–Trudgian, cited) with margins 10^3–10^6, the secondary top-100 family gate closes 14/14 as well, and the *growth law changes*: the fitted $\log h$ coefficient drops from +1.98 to +0.13 — the $h\,e^\alpha$ growth of the crude envelope was an artifact of the coherence discard, and the sharpened chain is *h-free* at fixed α. The remaining growth is in α only ($q = +1.79$, the honest price of the worst-case off-line allowance $e^{2\alpha\delta}$ inside the critical strip), re-crossing X^2_∞ near the validity horizon $\alpha^* \approx 11.2$ — far beyond the deployed battery ($\alpha \leq 6.146$) but finite, and typed as the exact point where verification depth re-enters. Controls fire (synthetic *off-line* zero pairs break the psd chain 4/4 exactly at the off-line locus while respecting both envelopes; the scramble kills the family structure; the regression ward reproduces the old $\{184, 210, 218, 434\}$ closure boundary).

The net floor statement (verbatim honesty). On every rung of the deployed battery,

$$\lambda\tau \;=\; \det \hat{A}_2 \;\geq\; X^2(\mathrm{pole}, \gamma_1) - \mathrm{pert}(h) \;>\; 0$$

at *citation grade*: no per-rung eigencheck and no zero-location input beyond the critical strip are used — the inputs are the strip, the cited verified on-line depth T_{ver}, the cited explicit counting constants, the analytic pair limit, and the numerically certified product-sup envelope. What this is *not*: a positivity theorem. The bound is *battery-relative* (the deployed 14-rung ladder, frozen before the attack) and α*-bounded* (validity horizon $\alpha^* \approx 11$); the family version of the analytic bound at the $\sin(\alpha\gamma_1)$ nodes and the Lipschitz certificate of the sup are named open steps; and nothing here bounds $\inf_X \lambda_{\min}$ on V_∞ — the ratio demand of `PRIME.FLOOR.RATIO.01` at infinity remains the contract, narrowed. Floor positivity on the deployed battery is *necessary-side* evidence: it is what RH-compatibility demands of these windows, and its value is that it is now certified rather than merely observed. **No statement about the Riemann Hypothesis is made.**

12.18 The exclusion detector: the certified ladder becomes a window-verification instrument, and the floor survives its kill gates at full sieve depth

Round 25 (2026-08-07) promotes six modules from eleven frozen probes (`v825`–`v830`, suite 817 → 823), in two coherent strands. *Strand A* turns the verified PSD rungs of the GL1 prime-comb tower into a hash-preregistered *detector*: certified exclusion regions, an out-of-sample validated zero locator, and the capstone — a comb-native window verification whose census reconciles against the Riemann–von Mangoldt count (the new contract `PRIME.DETECTOR.WINDOW.01` [O]). *Strand B* plays the `PRIME.FLOOR.RATIO.01` kill gates at the certified exclusion-tower depths — they *survive*, growing — and removes the $\alpha^* \approx 11$ citation horizon of round 24 as a float-convention artifact. Every statement below carries the mandatory typing: the strand is *necessary-side* evidence, strictly weaker than classical verification on every axis; no marker moves; **no RH/GRH claim anywhere**.

The certified exclusion ladder ([`verification/v825_prime_exclusion_ladder.py`], 26/26 + 21/21, LADDER-EXTENDED + SATURATION-TYPED): each verified PSD rung T_X of the tower ($D = 1/64$, $X = MD$) carries a rigorous floating-point certificate — $\lambda_{\min}(T_X) \geq L_X > 0$ by Cholesky PD verification with the Higham backward-error bound plus a Rayleigh-residual enclosure — and is inverted, through the tent-read Guinand identity on the frozen rank-4 battery, into a quadruple-*exclusion* region: hypotheses $\frac{1}{2} \pm \delta \pm i\gamma$ are excluded where the subspace $\lambda_{\min}(T_X \pm Q) < -\mathrm{EXC}_{\mathrm{BUD}}$, every published boundary point re-proved by a rigorous witness-Rayleigh certificate. The ladder now stands at $X = 4$ / 10 / 18.375 / 20.72 / 22.09 / 23.48 / 24.81 (deep sieves to 6×10^{10}), and the census threshold $\Xi_{\mathrm{eff}} = X\,\delta_{\mathrm{mb}}$ falls $0.639 \to 0.457 \to 0.274 \to 0.218$ — then *plateaus* at $0.218 \pm 3\%$: saturation, typed support-active-but-X-stalled (the battery still converts u-support into width inside a rung, but added depth stopped helping; the rank-12 enrichment gain grows $19.3 \to 25.0\%$, below the 30% battery-limited bar). Honest reach: the γ-window is Nyquist-limited to $\pi/D = 201$ and every visible ordinate lies *inside* the Platt–Trudgian-verified strip — the region is an independent TFPT-data-only re-derivation, *not* new territory.

The preregistration discipline, and the plateau falls ([`verification/v826_prime_exclusion_battery2.py`], 16/16 + 13/13, FLOOR-LOWERED + RANGE-DECIDES-DECLINE): battery v2 was designed *a priori* from instrument diagnostics only and SHA-256-hashed (`fd39fb42..`) *before* any exclusion evaluation — no measured boundary, no exclusion map, and no zeta-zero datum entered the design; v1 is a subset of v2, so the decision content is slope and floor level, not the sign of the gain. On the same certified rungs the v2 floor drops to $\Xi_{v2} \leq 0.0979$ (old grid; the two deepest medians sat *exactly* on the grid floor 1/240 — censoring caveat, typed), and the measurement-range extension (one decade down at the frozen grid ratio, old grid an exact subset) *uncensors* the series: $\Xi_{v2} = 0.1066/0.0982/0.0960/0.0913/0.0816$, log-log slope -1.39 over the deepest three rungs — the v825 plateau was the *instrument*, not the tower. Side-finding (typed, hash-verified non-circular): the uncensored boundaries rank-correlate -0.872 with the distance to the nearest true ordinate.

The zero locator: the honest null, then the out-of-sample resolution ([`verification/v827_prime_zero_locator.py`], the one preregistered-honest C3 FAIL + 13/13, LOCATOR-NULL + LOCATOR-V2-RESOLVES):

near-ordinate rows have *larger* δ_{mb} — a real zero at γ makes the injected synthetic quadruple *redundant*, so the breakable margin shrinks and the boundary rises: the width profile $W(\gamma)$ peaks at true ordinates. The v1 locator honestly failed its frozen gates (the $\ln 1.5$ prominence bar admitted a 61% false-peak background) — on record. The post-hoc separation (matched prominence ≥ 0.82, false ≤ 0.73) froze the threshold $\ln 2.2$ for v2, preregistered and hashed (`f57a2e7f..`) *before* any test-window evaluation; on the disjoint, previously untouched window $\gamma \in [60, 120]$ the frozen rule scores $20/24 = 83\%$ detection, 0% false positives, precision 0.086, with all four controls (scramble and Epstein combs show no peaks; the shifted null collapses below the structural re-lock ceiling; grid refinement leaves every matched peak in place). The depth law is measured: detection $54 \to 62 \to 83\%$ at $X = 18.4/22.1/24.8$.

The capstone: a comb-native window verification ([`verification/v828_prime_comb_window.py`], 12/12, COMB-VERIFIES-WINDOW; contract `PRIME.DETECTOR.WINDOW.01` [O]): on the window $\gamma \in (60, 120]$ at the certified rung $X = 24.8125$, the *same* certified positivity object yields both legs — *location*: 21 of the 25 zeta ordinates, each within ± 0.25 (measured max error 0.242), zero unmatched peaks; *exclusion*: every quadruple hypothesis with $\delta \geq \delta_{\mathrm{mb}}(\gamma)$ excluded pointwise (uniform window bound 0.0496), witness-certified at the six frozen samples; *census*: $21 + 4$ typed misses $= 25 =$ the cached verified count $=$ the rounded RvM main term. The miss autopsy is complete (one pair-merge below the 0.25-grid resolution, three prominence-limited), and the depth-recovery prediction at the *new deepest certified rung* $X = 25.5$ (comb cap 1.2×10^{11}, $\lambda_{\min} = 8.07 \times 10^{-7}$, certificate $\geq 7.26 \times 10^{-7}$) was verified as typed. **Weaker than classical verification** (mandatory typing): Turing's method *proves* the count and $\delta = 0$ exactly; the comb reaches 21/25, leaves a strip of half-width δ_{mb} unresolved, and its measured depth law can never close the gap to $\delta = 0$ at exponential comb cost. The architectural reading, stated precisely: a tower assembled from the discrete compiler side with *no zero input* (hash-verified end to end) *carries* the low-lying zero positions — a necessary-side consistency demonstration of the explicit-formula reading; it does not constrain zeros beyond the certified exclusion regions and does not bear on RH.

Strand B: the kill gates survive at depth ([`verification/v829_prime_floor_depth.py`], 16/17 with the one preregistered-honest S5.HLAW FAIL + 12/12, ENVELOPE-HOLDS-DEEP + FAMILY-SCALES): the `PRIME.FLOOR.RATIO.01` gates — K1 envelope $\rho h^{3/2} \geq c = 4.85$ (no refit) and K2 capture $\cos^2\theta \geq \frac{1}{2}$ — were played at $X = 18.375$–25.5 on the uniform-grid tower frames, whose half-dimensions $h = 588$–816 sit *inside* the deployed h-range while the depth doubles: the test decouples depth from dimension, and if ρ were secretly a depth law the kill would fire. It does not: margins $\times 5.76/6.24/6.53/7.16/8.17/8.92$, growing monotonically, minimum $\cos^2\theta = 0.849$, non-decay trend $+0.060$ per log-unit — the h-law wins. The typed miss: the single fixed pair collapses at depth (gap $40\times \to 673\times$), and the certified top-100 pole × alias family takes over — 0.977–0.981 of the floor at every deep rung, the certified family bound within 2% of the measured ρ. The remaining caveat was the α-coverage: citation grade reached only $\alpha \in \{9.19, 10.36, 11.05\}$.

The budget, and the explicit envelope ([`verification/v830_prime_float_budget.py`], 10/10 + 19 checks with the five preregistered-honest asymptotic FAILs, BUDGET-LINEAR-CLOSES-ALL + ENVELOPE-ASYMPTOTIC-PARTIAL): the $\alpha^* \approx 11$ horizon was an *artifact of the frozen quadratic float convention* ($100\,\varepsilon\,\|A_2\|_F^2$, calibrated at $\|A_2\| \sim 300$, four orders above the true tail term at depth). The replacement is a fully explicit Higham-grade norm-*linear* chain (compensated banded accumulation with TwoSum folding, $N_{\mathrm{SUB}} = 2^{16}$; the irreducible atom-position term $\varepsilon X/D$; exact `fsum` dots; dps-40 spot anchors with $\times 10$ caps): budgets $7\times$–$171\times$ tighter, the citation-grade family gate closes 6/6 (old convention 3/6) — *citation grade now extends to* $\alpha = 12.75$*, full sieve depth.* Convention note: the deployed v818 convention stays frozen in the existing suite; the linear budget is registered for *new* modules only. On the analytic side the certified family sum acquires its exact closed form (reproducing the component sums on all 73 battery frames at 3.6×10^{-11}), the limit algebra is sympy-proven, phases cannot conspire (dip floor 0.170), and the envelope constant becomes *certified-explicit*:

$$\rho \;\geq\; \rho_{\mathrm{certfam}} \;\geq\; 4.335\, h^{-3/2} \quad \text{on } 73/73 \text{ points, non-decaying.}$$

The proof blocker is named honestly (the five expected FAILs): the alias *phase correlations* — the random-phase model predicts $\tau \sim h^{-1}$ where the $D = 1/64$ tower measures $h^{-2.5}$ — block the analytic uniformity; the envelope stands certified-explicit, not proven. `PRIME.FLOOR.RATIO.01` stays [O], depth-hardened. **No statement about the Riemann Hypothesis is made.**

12.19 The typed circularity boundary: the analytic-envelope route closes, and the floor–pair-correlation bridge is named

Round 26 (2026-08-07) promotes one module ([`verification/v831_prime_alias_second_moment.py`], suite 823 → 824; verdict ALIAS-CORRELATION-PAIRCORR, 15 checks with the *seven* preregistered-honest premise-overturning FAILs, pattern-gated) — the honest typed closure of the analytic-envelope route that round 25 left open as "the alias-phase proof blocker."

The exact bilinear form (sympy-proven, identity-warded): rotate each zero's component vector into the pole frame, with amplitude A^2 and alias phase ψ per zero; with $S_2 = \sum A^2$, $S_c = \sum A^2 \cos 2\psi$, $S_s = \sum A^2 \sin 2\psi$ the zeros+pole gram is *exactly* $M_2 = \left[\begin{smallmatrix} P_0+(S_2+S_c)/2 & S_s/2 \\ S_s/2 & (S_2-S_c)/2 \end{smallmatrix}\right]$, and the floor obeys

$$\tau \;=\; \frac{S_2 - S_c}{2} \;-\; \frac{S_s^2}{4P_0} \;+\; O(P_0^{-2}),$$

so the correlation combination multiplying the leading term is the coherent phase sum S_c (rotation-invariance ward at the conditioning-aware machine bar on all 73 battery frames; truncated-series deviation 3.8×10^{-7}).

The premise overturned. The round-25 blocker was typed "alias *phase* correlations" — random phases predict $\tau \sim h^{-1}$, the tower measures $h^{-2.5}$. The frozen wards encoding that premise *fail*, and the failure is the finding: the diagonal (random-phase) part $\tau_{\mathrm{diag}} = S_2/2$ does not slope -1 but -3.172 — the *amplitudes*, not the phases, carry the gap; the phase scramble and the Poissonized zeros land at $-3.05/-3.65$; and $q = S_c/S_2$ changes sign across the battery ($q \in [-0.324, 0.552]$, median 0.19; deep suppression only $\times 1.1$). The tower law decomposes *exactly*:

$$-2.505 \;=\; \underbrace{-3.172}_{\text{amplitude law}} \;+\; \underbrace{+0.668}_{\text{fading alignment}} \quad .$$

The Guinand split, and where the correlation lives. By the deployed identity the zero-side gram splits exactly as $G_Z = (B - S_p - P) + (S_p - S)$ — a geometric/smooth part and the prime-comb fluctuation. Per-frame medians: smooth share -994.15, fluctuation share $+995.15$ — each about a *thousand times larger* than their sum, cancelling to $|S_c^{\mathrm{fluc}}|/\sigma_{\mathrm{sqrt}} = 0.01$ of the Poisson-comb square-root scale (identity-route residue 0.019); the smooth surrogate does *not* carry it (residue 0.868); the comb scramble *explodes* the coherent sum by 2×10^6–1.7×10^7 and the Epstein $x^2 + 5y^2$ comb by 5×10^3–1.3×10^4. **The cancellation is the arithmetic**: the suppression exists because the prime-comb fluctuations cancel exactly as far as square-root cancellation allows.

The typed circularity boundary, and the closure. Proving the tower τ-law therefore requires bounding an oscillatory prime sum *at* its square-root scale — variance-type control, which by the explicit formula is pair-correlation-equivalent substance on the zero side (Montgomery-type, conjectural). And by Guinand the comb's self-cancellation *is* the zero-side floor statement itself: the required control is even deeper than pair-correlation input. The analytic-envelope route of `PRIME.FLOOR.RATIO.01` is therefore *closed honestly at this boundary* — its envelope constant is arithmetic substance equivalent to the target; no further analytic-envelope variants (a stop-list entry per house discipline). What survives is the named bridge, now a contract: `PRIME.FLOOR.PAIRCORR.01` [O] — the floor's envelope constant is Guinand-equivalent to the zero-side floor statement, with forward (unconditional variance bound, entry point declared) and backward (instrument calibration) demands and kill conditions typed. A boundary, not a theorem. **No statement about the Riemann Hypothesis is made.**

12.20 The two-route decision at the wall: the corner route's structural half closes, the SOS route dies by certificate

Round 27 (2026-08-07) plays the two independent corner-era routes against the bridge wall and *decides both* ([verification/v835_corner_hjelmslev_tower.py], 21/21+20/20, verdicts CORNER-IDENTITY-SYMBOLIC + HJELMSLEV-STRUCTURE-ONLY; [verification/v836_commutant_sos_closure.py], 29/29, verdict COMMUTANT-SOS-INFEASIBLE; plus the flag normal form [verification/v834_pg32_flag_completion.py], 26/26, PG32-FLAGS-EXACT + INCIDENCE-SPLIT; suite 824 → 829). The outcome is sharp: the corner route's *structural* half closes weight-generically at finite level, the SOS route closes definitively by exact certificates, and the arithmetic wall does not move — it *relocates into the identification step* and stays `PRIME.FLOOR.PAIRCORR.01`.

The corner identity is weight-generic — and comb-blind. The target identity $T_{\mathrm{GL1},X} = V_X^*\, e_{\mathrm{GL1}}\, \pi_X(P_X)\, e_{\mathrm{GL1}}\, V_X$ holds as *polynomial algebra* in free event weights w_j: exactly on a fully symbolic toy leg (sympy defect matrix $\equiv 0$, symbolic arch lags *and* weights) and on all three deployed α-smallest frame-A windows, comparing two *independent* deployed code paths (the `v563` spline-read lock against per-event Kraus-row quadratic forms). The entire equality step consumes *one* piece of arithmetic — the glue identity $\sum_m \Theta_m = 240\,\sigma_3$ plus the uniform H^* mass — which forces the corner coefficient $\hat{c}_j = -1$ *exactly* for every one of the 136 distinct events (exact group-algebra Fourier on the 128-dimensional register $C_2 \times \mathbb{F}_2^4 \times \mathbb{Z}_4$; e_{GL1} a rank-one projector, corner compression single-Kraus CP). The honest scope is measured, not asserted: the object carrying the *identity* (the packet window operator) is *not* positive (one of twelve sectors PSD); the manifestly positive object (the packet GNS corner) is *not* τ_X; and the scramble control proves identity and state corner *comb-blind* — they read no positions — while the deployed lock data moves ($\tau = +5.98 \times 10^{-4} \to -3.51$). Structural positivity can therefore never be the floor: the arithmetic lives *entirely* in the identification step — a positive P_{lock} whose corner *is* the deployed window form built from the actual prime placements.

The Hjelmslev tower is real geometry — and structure-only. The authorized next stage was executed, not speculated: the chain-ring ladder $R_m = \mathbb{Z}[i]/(1+i)^m$, $m = 1..5$, carries exact point/line/flag geometries with counts $15 \cdot 8^{m-1}$ / $35 \cdot 16^{m-1}$ / $105 \cdot 32^{m-1}$ and uniform reduction fibers 8/16/32; the flag incidences define unital CP channels of degree $7 \cdot 4^{m-1}$ whose $m = 1$ member *is* the deployed 105-leg Stinespring structure ($BB^{\mathsf{T}} = 4I + 3J$; `v756`/`v820` read-only); and the tower is *strictly projective* — the certified cb compatibility defect is identically *zero* (full exact steps $1 \to 2$, $2 \to 3$; sampled $3 \to 4$, $4 \to 5$), exceeding the conjectured $2^{-m/2}$ decay. The character corner is level-exact: $\hat{c}_j^{(m)} = -1$ at every level, with the circularity fence silent (only counting and normalization identities consumed). But the adjudicating stage decides *against* identification: the level-m state corner is comb-blind (the scrambled comb reproduces the series bit for bit) and its motion is the pure register-dilution law $\mathrm{value}_1 \cdot 16^{1-m}$ (ratio 0.0625 exactly at every step) — the tower, as a register/geometry lift, has *no channel* through which the prime placements enter. The companion trade-off probe sharpens the door: register-side Kraus dressings *provably cannot* carry the identification (`hjelmslev_position_kraus_probe.py`, POSITION-KRAUS-TRADEOFF, 13/13, probe-level) — the one live tower datum is *position-dependent* Kraus data on the level-m flags.

The flag normal form, and where the incidence obstruction sits. The 105+35 vacuum-completion combinatorics of round 24 acquires its normal form: $105 = 45+60$ is the $\mathrm{Sp}(4,2)$-*orbit* decomposition on both sides (flats and flags; $|\mathrm{Sp}(4,2)| = 720$ by brute force); on the 60 non-isotropic rungs the symplectic polarity $\Phi\colon (U,t) \mapsto ((t+U) \cap U^\omega,\ U^\omega)$ is a canonical bijective, equivariant, *incidence-preserving* dictionary; on the 45 doily rungs the obstruction is typed — $U^\omega = U$ and $(t+U) \cap U = \emptyset$ for 45/45, and the flag Gram is $18I + 3J \neq 22I + 6J$. So "105 checks = 105 flags" is exact as a deck-equivariant *bijection*, *not* as an incidence isomorphism — the obstruction sits exactly on the doily, the typed reason the `v822` continuum dilation died.

The SOS route dies by exact certificate. The second independent route — a rational sum-of-squares identity $T_{\mathrm{GL1}} = \sum_j V_j^* T_{\chi_j} V_j + \sum_k C_k^* C_k$ over the canonical five-dimensional abelian subalgebra $\mathcal{A} = \mathrm{span}\{\Pi_A(0), \Pi_A(1), \Pi_A(4), \Pi_A(7), \Pi_A(9)\}$ of the 39-dimensional commutant (`v815`) — is closed *definitively*, twice over. (a) The exact reduction lemma shows the abelian conjugation

reads only the Gram diagonals, so the SDP (four PSD blocks S^5_+, 60 unknowns, 85 equality constraints per reading) reduces *exactly* to a rational LP — solved exactly: every system consistent, *unique* (Vandermonde kernel $\{0\}$), and equal to the *trivial* diagonal rewriting — no cross-sector positive transfer exists. (b) The rank-3 determinant form has $\det S = t^2 - x^2 - y^2$ (signature $(1, 2)$; compiler null vector $q(5, -3, 4) = 0$), coefficient matching *forces* the unique $G = \mathrm{diag}(1, -1, -1)$ — not PSD — with the rational dual certificate $q(0, 1, 0) = -1 < 0$; and q-negative directions *are* achievable, even on the physical nonnegative cone (832 of 2799 truncation cuts carry $\det S < 0$). The fence audit types the boundary: FENCE-CLEAN on the algebra layer (mass-free), FENCE-HIT on the positivity branch — every nonnegative-cone certificate must use cross-event cancellation, i.e. an oscillatory prime-sum estimate. The nontrivial version of this route *is* pair-correlation substance: back at the typed boundary, a second stop-list entry.

The honest statement. Both routes end at the same wall from opposite sides: the corner route delivers everything structural (identity, projector, CP tower, level-exactness) and shows the structure is placement-blind; the SOS route shows no positive algebraic rearrangement can manufacture the floor. The identification wall — which self-adjoint operator the CP corner is applied to, and that its corner equals the deployed window form — stays exactly `PRIME.FLOOR.PAIRCORR.01` [O], with the demand unchanged and both closed routes stop-listed. **No statement about the Riemann Hypothesis is made.**

12.21 The wall gets coordinates: the corner closure theorems, the GUE saturation, the relation carrier, and the certified skeleton

Round 28 (2026-08-07) is the day's consolidation — four movements, six modules ([`verification/v837_corner_closure`
19/19+17/17+3, verdicts DOORS-CLOSED + LEVEL2-CLOSED; [`verification/v838_corner_expectation_position.py`],
13/13+13/13, EXPECTATION-CLOSED + POSITION-CARRIER-TRADEOFF; [`verification/v839_paircorr_bridge_satura`
9/9+7/7, BRIDGE-FULLY-BEYOND + GUE-SATURATING; [`verification/v840_gue_ablation_loopgain.py`],
10/10+8/8, SATURATION-STRUCTURAL + LOOP-SHORT; [`verification/v841_relation_carrier_ladder.py`],
18/18+13/13, RELATION-CARRIER-EXISTS + EXCESS-NONNEGATIVE; [`verification/v842_excess_certified_skeleton`
9/9, SKELETON-CERTIFIED; suite 829 → 835). The arc: the corner route's perimeter closes as a set of measured *compression-class theorems*; the zero side of the bridge is quantified and found *saturating*; the route then *reopens* with a different input class — the multiplicative relations between events — and the wall itself acquires certified corner coordinates.

First movement — the closure theorems. The corner program on the register group algebra $\mathbb{C}[C_2 \times \mathbb{F}_2^4 \times \mathbb{Z}_4]$ is closed as an identification device, with the quantifier *measured*, not asserted ([`verification/v837_corner_closure_quantifier.py`], [`verification/v838_corner_expectation_position.py`]). Door 1: the GL1 corner reads the full lock-pair *compression* — not a trace — so factor dressings are generically visible; but every visible dressing breaks the deployed-form identity by exactly its action on the lock pair (visibility ⇔ identity defect, on all 24 sweep draws; the commutant is invisible at 2.7×10^{-14}). Door 2: all 128 characters collapse exactly to 18 classes, and the generalized mass ward shows every character corner reads *one* position-blind Fourier mode of the carrier per event — the 90-cell class map has *no* identity∧visibility∧placement cell. The quantifier extends level-exactly through the Hjelmslev tower to $m \leq 3$ (2048 and 32768 characters; the jet characters read only $\{1, 0, -1/7\}$ — position-blind at every level). The compression toolkit closes with it: all 5276 subgroup conditional expectations (74 259 components; exactly two identity-retaining subgroups, matching the predicted pinning criterion $\mathrm{Ann}(H) \subseteq \{1, \psi_0\}$ subgroup by subgroup), the full pinching (whose only PSD summands are the two pinned sectors — the joint-use kill) and Stinespring-side compressions (the position data *is* in the dilation; the identity pins the dilated read to the constant 1). And the fourth coordinate is typed: a position-*dependent* carrier map can keep the state, but the identity is an *extremal* state condition — $\hat{c}_{\mathrm{GL1}} = -1$ locks the entire C_2 mass — so any identity-true carrier reads zero on *every* self-consistent comb, true or fake: the decisive null (the Epstein comb at its own log positions) kills all four frozen designs. The structural half of this closure is kernel-checked in Lean 4 (`TfptCarrier/SectorPositiveDescent.lean`, lake build green, no `sorry`: corner compression is CP with the corner-trace split, descent towers

transport compatible positive families, PSD survives limits — and the final theorem `sector_floor` carries `IdentificationPositivity` as the *single named hypothesis*, deliberately not a theorem: the kernel-checked logical geography of the wall).

Second movement — the zero side, quantified. The bridge demand of `PRIME.FLOOR.PAIRCORR.01` is written in Montgomery $F(\alpha)$-form on all 73 frames, with the scholarship correction carried throughout: Montgomery's $F(\alpha)$ asymptotic on $|\alpha| < 1$ is *RH-conditional*, so every supply row carries its conditionality tag ([`verification/v839_paircorr_bridge_saturation.py`]). The two-tier gap: unconditional per-realization input misses the gate by 52× at best (Selberg-rms by 12× — the triangle loss is the irreducible cost of one-level input); the RH-conditional window covers only 43–86% of the demand spectrum, and even the full GUE conjecture closes only 39/73 rungs at rms grade. The named minimal missing input: an *unconditional* form-factor bound $F(\alpha, T) \le C_F$ out to $\alpha \approx 3$, with $C_F = 0.33$–1.7 as tabled per rung. And the saturation: the demand/GUE ratio rises from 0.61 to the plateau $R_\infty = 1.11$ over the deepest twenty frames, inside the frozen band — the floor demand sits *at* the random-matrix boundary of the comb, consuming only a typical GUE realization ($\mathrm{rms}(z) = 0.72$); inverting the certified margins turns the ladder into a calibrated form-factor instrument, $\hat{F}(\mathrm{win}) = 0.44 \pm 0.14$ at $T \le 2 \times 10^4$ (Poisson control calibrates to 1; the scramble fires). The ablation battery proves the saturation *structural*: every source-native, Guinand-admissible tower variant lands at the plateau (1.058–1.199 against 1.154), with the tight band pinned at *unfolded* $\alpha \in 1..2$ under every lattice — a property of the tower class and of the unfolded zeros, not of the grid ([`verification/v840_gue_ablation_loopgain.py`]). The loop-gain analysis then quantifies why the ladder cannot bootstrap itself: $g = 1/(k^2 R^2 c_{\mathrm{sup}})$, headline $g(k{=}1) = 0.529$, and even the *ideal* supply gives $1/R^2 = 0.655 < 1$ — the wall's self-conservation *is* the saturation: the certified ladder supplies exactly the statistics of the zeros, never more, while the floor demands slightly more than those statistics guarantee (the Poisson circularity ward passes: the loop does not close for wrong statistics). The measured positivity everywhere is therefore genuinely finer-than-statistical information — consistent with the architecture reading, unprovable by the loop.

Third movement — the relation carrier. The closure theorems say what the identification carrier must *not* be; a carrier built on the *multiplicative relations between events* is the first structure in the program to pass all four gates ([`verification/v841_relation_carrier_ladder.py`]). The relational algebra is exact structure: the convolution frame carries $\mu * 1 = \delta$ and $\Lambda = \mu * \log$ as dict equalities on the log-p basis — every event's Λ-weight is derivable from the pair relations alone, and this input is the *Euler-product datum*, not a zero or position oracle. The four-comb discriminator is exact: the true comb reads 0 *exactly*; the self-consistent sums-of-two-squares null ($\zeta_{\mathbb{Q}(i)}$, $h = 1$) and the χ_4 fake read 0 — Selberg-class-*correct* blindness — while the genuine no-Euler-product Epstein comb $x^2 + 5y^2$ ($h = 2$) separates at 200.0, localized at the class-group obstruction sites ($21 = 3 \times 7$ represented, neither factor is). The relationally-routed carrier keeps the packet state exactly, degenerates to the deployed carrier at the true comb (the corner identity and τ_X re-derived by degeneration), moves under scramble and Epstein positions — and separates the self-consistent null through its *missing product relations* (14 of 70 events; excess -0.297 against exactly 0 at truth). The amplitude wiring closes both typed limits: the corner reads the Euler-product datum at amplitude grade (exact Fractions) and is L-function-safe *automatically*. Along the whole reachable ladder (67 rungs, $\alpha = 2.77$–6.30, masses to 3×10^5) the identified corner's *excess* over its comb-blind structural skeleton is positive on every rung (+2.29 to +3.70) while τ_X itself decays to 1.7×10^{-5}: the wall takes its sharpest form,

$$\tau_X \;=\; \lambda_{\min}(\text{structural}) \;+\; \text{EXCESS},$$

with the structural part certifiable and the excess the arithmetic substance. The route is registered as `PRIME.RELATION.MULT.01` [O].

Fourth movement — the certified skeleton. The certifiable part is certified ([`verification/v842_excess_certified_`] on *all* 67 reachable rungs the interval certificate encloses τ_X with width 5.4×10^{-11}–6.6×10^{-9} — three to five orders below the margin even at the deepest rung — and every enclosure is *strictly*

positive; the same discipline certifies the discriminator (the $h = 2$ Epstein fake fails its margin certificate on every anchor, routed-corner enclosure strictly negative, from the exact rational Λ_F recursion). The scaling law is typed honestly: certifiability degrades faster than the margin decays, so the coordinate system has a finite certification horizon — but the horizon $\alpha^* \approx 9.1$ (mass $\sim 7.5 \times 10^7$) lies far beyond the deployed ladder end: within every reachable rung, certifiability is not the binding constraint. Registered as `PRIME.RELATION.SKELETON.01` [O].

The honest statement. What remains is exactly the wall: the infinite quantifier over the sequence of strictly-positive certified enclosures. Any future route must supply a *uniform* lower bound on the excess margin τ_X — by the GUE-side findings a finer-than-statistical datum — and no finite table, certified or not, settles it. The bridge contract `PRIME.FLOOR.PAIRCORR.01` [O] is unchanged, now carrying the four closure coordinates, the saturation reading and the sharpened demand; the closed corner-era surfaces are stop-listed (`PRIME.CORNER.OPENDOORS.01`, `PRIME.CORNER.LEVEL2.01`, `PRIME.CORNER.EXPECTATION.01`, `PRIME.CARRIER.POSITION.01`). **No statement about the Riemann Hypothesis is made.**

12.22 The margin as a doubly-derived law, and the wall kernel-checked

Round 29 (2026-08-07) is the compact closing round: the margin itself — the certified series τ_X of Section 12.21 — becomes a typed mathematical object, and the wall's logical geography moves into the Lean kernel ([`verification/v843_margin_law_excess_lean.py`], 10 checks with the *one* preregistered-honest FAIL kept and pattern-gated, verdict MARGIN-RECURSES-THE-WALL, plus four mirror checks; Lean core `TfptCarrier/ExcessSkeleton.lean`, lake build green, no `sorry`; suite 835 → 836).

The law, doubly derived. Recomputed in the identified-corner coordinates ($\tau = \lambda_{\min}(S + C)$, agreeing with the deployed $\lambda_{\min}(A_h)$ at 2.1×10^{-12}), the margin series reproduces the floor-strand envelope of Section 12.18 *exactly*: $e_1 = (\tau/\tau_{\mathrm{pnt}})\, h^{3/2} \in [4.855, 24.209]$ on all 67 rungs, min ≥ 4.335 — the certified envelope constant, reproduced in a coordinate system derived independently of the one that certified it. Two laws, one object; the consistency itself is a nontrivial ward. The typed relationship is

$$\tau(\alpha, h) \;=\; e_1(\alpha)\, h^{-3/2}\, \tau_{\mathrm{pnt}}(\alpha),$$

with τ_{pnt} — the explicit zero-free PNT-density transversal — carrying the *entire* α-decay (segment slopes -0.23 to -0.30) and e_1 carrying the arithmetic. The honest miss is kept: the fit-free internal h-exponent (21 near-equal-alpha rung pairs, no global fit) measures median -1.341 against the frozen bar edge -1.35 — the $h^{-3/2}$ law confirmed at the $\sim$1% grade by an instrument whose preregistered bar was drawn 0.7% too tight; the 0.009 miss is typed and pattern-gated, not refit.

The negatives, typed. The tower gives *scale*, not recursion: the Hjelmslev reductions act as the identity on the GL1 corner reads, so the induced recursion lives on the dyadic mass subsequence 4^m ($m = 4..9$, exact at four levels) — and the ratio-defect series (0.6749, 0.5159, 0.0305, 0.3635) is non-decaying: no quantifier reduction from the tower at this depth. The excess is a *growing* cancellation: its per-event prime-power cells (ward $\sum_j \mathrm{cell}_j = v^{\mathsf{T}}(A - B)v$ at 10^{-8} on every rung) carry a cancellation index rising 5.65 → 29.46 along the ladder (median 15.6), with *no* per-prime positive control (22 287 of 351 122 (prime, rung) aggregates nonpositive; the deep $p = 2$ aggregate -0.623) — the excess is not a positive sum over primes: the wall is *self-similar at cell level*, and same-class margin-law re-attacks are stop-listed. The predeclared compiler-connection table is null (0/96 at 10^{-3}, Bonferroni-honest). The controls carry the arithmetic: the scramble turns the margin negative on every rung, and the $h = 2$ Epstein routed margin is negative *and* below its own unrouted baseline at all anchors ($-0.786/-1.100/-1.228$) — the Euler-product violation carries its sign through the exact rational Λ_F routing.

The wall, kernel-checked. The logical geography of the certified skeleton is now a Lean 4 module (`TfptCarrier/ExcessSkeleton.lean`, 18 declarations, no `sorry`, no `native_decide`): the two-giants decomposition (the excess carried *exactly* by the comb block — `formAt_add`, `excessAt_eq_comb`), the interval-certificate lemma (`pos_of_certified`: an enclosure clearing its

own budget forces strict positivity — the exact formal shape of every `v842` certificate), the certified-negative discriminator (`certified_negative_excludes` — the Epstein rejection logic), the finite-ladder quantifier (`ladder_pos_on_finset` — what the 67 certificates formally yield), and the bridge theorem `excess_floor`, which consumes `UniformMarginBound` as the *single named hypothesis* and returns the floor statement for all rungs together with the per-rung `IdentificationPositivity` instances of `SectorPositiveDescent` (`uniform_to_identification`, proven). The converse direction is the round's sharpest deliverable: `pointwise_pos_not_uniform` is a *kernel-checked counterexample* — margin sequences strictly positive at every rung with *no* uniform lower bound exist (witness $1/(m+1)$) — so no finite certificate table, certified or not, discharges the uniform statement. The finite-table limit, until now a typing discipline, is a theorem.

The honest statement. The infinite quantifier now sits on *one* scalar series: $e_1(m)$, certified ≥ 4.335 on every reachable rung, doubly derived, self-similar at cell level, with no tower recursion and no compiler-constant shortcut. What any future route must supply is a *uniform* lower bound on e_1 — by the GUE-side findings a finer-than-statistical datum — with its arithmetic entry point declared (`PRIME.FLOOR.PAIRCORR.01` discipline binding). Registered `PRIME.MARGIN.LAW.01`; the Lean layer `FORM.PRIME.EXCESS.SKELETON.01`. **No statement about the Riemann Hypothesis is made.**

12.23 The three-architecture decision, and the extraction chain complete

Round 30 (2026-08-07) consolidates the v5.4 strategy campaign: the three candidate *proof architectures* aimed at the wall of Section 12.21 are decided — all three honest negatives with typed positive cores — and the *continuum extraction* is separated from all arithmetic and closed as an implication chain ([`verification/v846_schur_spectral_mother.py`], $19+16$ checks, zero fails, verdicts SCHUR-INDEFINITE and SPECTRAL-TV-UNIVERSAL; [`verification/v847_wedge_cellcone_transport.py`], $13+9$ checks with the two frozen-honest FAILS kept and pattern-gated, verdicts WEDGE-PARTIAL and CONE-BROKEN + RAY-EDGE-CONFIRMED; [`verification/v848_extraction_chain.py`], 19/19, verdict EXTRACTION-CHAIN-COMPLETE; suite $836 \to 841$ with the two compiler compressions of the same round, [`verification/v844_message_doily_rank.py`] and [`verification/v845_cfin_normal_form.py`], cited in place in Paper 1).

Architecture (i): the relational Schur–Gram. The target was positivity by construction — a manifestly positive block operator whose GL1 Schur complement *is* the deployed window form. Three of its four gates pass: the construction is manifestly positive; the μ log descent routes every prime-power mass *exactly* (ward 10^{-16}) and refuses the $h=2$ Epstein comb at construction grade (bookkeeping defect 0.73–0.83 against a true-comb defect of 0 — the Euler-product gate fires before any spectrum is computed); and the multiplicative sectors ($\zeta_{\mathbb{Q}(i)}$, the χ_4 L-sector) pass through undamaged. What fails is the corner identity itself, and *structurally*: every relational cell that carries a cross term deposits its Cauchy–Schwarz diagonal alongside, so the constructed block is *forced* to $K + \mathrm{diag}(\text{price})$ with the price the row total variation of the deposits — measured at 3.1×10^4 to 4.5×10^4 times the margin even at the density-anchored fluctuation floor; the untouched-projection deviation *is* the price ($0 \leq 10^{-9}$), no predeclared projection removes it, and the trace fails by the same strictly positive diagonal object. The typed missing input — a non-diagonal spectral mother in which dilation acts unitarily and the mirror is an operator — was then *built*: on the deployed log-grid the mirror is the operator J with $J^2 = I$ (the deployed window form is verified to be the J-odd compression of a plain Toeplitz form at relative deviation 0), and the Euler product lives as the exact operator algebra $U_d U_m = U_{dm}$. The interference is real: at *symbol* grade the phase cancellation compresses the cross-term transport by measured gains of 20–33×. But the *Gram*-grade price does not move: every event-local manifest Gram pays the event-local total-variation floor (the modulus bound $a^2 + b^2 \geq 2|ab|$; best legal overhead 5.7×10^4 to 1.2×10^5 times the margin) — the TV price is *geometry-independent*. The closure typing of the whole Schur program, in its cleanest form: harvesting cross-event interference inside a manifest Gram is the Fejér–Riesz factorization of the *total* symbol, and the symbol's nonnegativity is the positivity being sought — the manifest-positivity route closes on itself.

Architecture (ii): the sign-register wedge. The exact part works: the naive signed Lagrange expansion is provably unrescuable (the wedge squares are linearly independent, so the pair weights are forced — symbolic), with $P^-/\det \sim 2\text{–}3 \times 10^4$ and the arch/pole block *negative definite* in this compression; yet the C_2 sign register turns the Möbius sign into difference geometry as *exact algebra*, and nonnegative wedge weights *exist* on every anchor (exact LP, residual $\leq 8.9 \times 10^{-16}$) — where the naive family provably cannot. What fails is the *law*: one frozen weight rule across frames reaches only ∼15% relative (design 0.170/0.151, held-out 0.153) against a floor need of 4.4×10^{-5} — four and a half orders — and even the per-frame class granularity cannot reach floor grade. This is the commutant no-go *transported*: in both routes the wall is uniformity in the window, not the sign. The one honest control miss is kept and pattern-gated (the scramble blows the residual ×7.5 against a frozen ×10 bar — a law that itself reaches only 15% has bounded structure to break; typed, not excused).

Architecture (iii): the completed-cell cone transport. The completed cells — each prime-power atom kick paired with its own continuum increment under two predeclared grouping rules — telescope *exactly* from the density transversal to the certified margin object (worst relative deviation 3.4×10^{-15}), and the congruence calculus is exact: $\Phi^{\mathsf{T}} J \Phi = c\, J$ with c the determinant ratio, at 2.2×10^{-16}. The measurement: *both* groupings exit the Lorentz cone on 67/67 rungs — at the $n = 2$ cell — and return only at the horizon end (the endpoint back inside the cone on 67/67 rungs *is* the certified $\tau > 0$). The monotone ray cascade is confirmed: the locking ratio rises $0.5700 \to 0.8018$ (Kendall +1.000) and the angle to the compiler null ray $(5, -3, 4) = (g_{\mathrm{car}}, -N_{\mathrm{fam}}, |\mu_4|)$ falls $46.26° \to 34.39°$ (Kendall −1.000) on both groupings. And the control surprise is kept as its frozen FAIL: the mass-matched Epstein comb does *not* break the transport (0/14) while the true von Mangoldt comb does — the cone dips are the true comb's own small-x fluctuation $\psi(x) - x$, which the smoother Epstein representation numbers do not reproduce. *The arithmetic sits in the violations.*

The extraction chain, complete. Work package E separates the continuum side from all RH semantics and closes it ([`verification/v848_extraction_chain.py`]): on the canonical (D, X) ladder, cofinal in the *mesh-refinement* order ($D_j = 2^{-j}$, $j = 4..11$, 68–8704 lags, the full pinned prime-power table) every element of the canonical dense family identifies its independently computed Weil value $Q_W = \mathrm{POLE} + \mathrm{ARCH} + \mathrm{ATOM}$ (exact piecewise-cubic autocorrelations in $\mathbb{Q}$ with fifth-point certificates), with measured rates $\sim 4^{-j}$ (median −1.818 per level against the dyadic prediction −2), a geometric ledger (median ratio 0.302), and the X-direction *exactly* stable once the window covers the element (44 cutoff pairs at float zero). The structural finding: the pure-box class carries the *exact* Weil value at every finite level (maximal error 5.3×10^{-14}, the float floor) — the sharpest dyadic-inheritance statement. The deliverable is the implication theorem with a *quantifier reduction*: under the named hypothesis (H) — finite positivity of the tower forms along a ladder cofinal in that same mesh-refinement order, *never evaluated here* — every ladder value is nonnegative, and a pointwise-convergent sequence of nonnegative reals has a nonnegative limit; *no* Mosco compactness, *no* uniform $\delta > 0$, *no* diagonal argument enters the implication — per-element convergence is the only analytic input, and the three infinite-level ingredients of round 23 migrate to the operator-level selection programme, where they stay typed. With the two classical legs cited (the density of the dyadic family and the continuity of the Weil functional at fixed support; Weil 1952, Bombieri 2000, Suzuki arXiv:2606.09096, Iwaniec–Kowalski Thm 5.12), the chain *cofinal finite positivity* ⇒ *Weil positivity* ⇒ (via Weil's criterion) *the target* is theorem-grade modulo the named citations. *Two later corrections to this paragraph, recorded here rather than silently applied:* the continuity leg as originally cited — C^0-continuity of Q_W at fixed support, in the pure sup norm — is *false*, and the per-element convergence leg is no longer measured but proven; both are settled in §12.39 below ([`verification/v912_form_convergence_theorem.py`]). Both controls fire (the density-only wrong limit stalls at ratio 1.6×10^6; the scrambled tower inflates the ledger by 5.5×10^5). Registered `PRIME.EXTRACTION.CHAIN.01` [O].

The honest statement. All three architectures die on the same substance: the total-variation price, the frame-uniformity failure and the unrenormalized $n = 2$ cell are three coordinate presentations

of the one finer-than-statistical datum that the GUE-side rounds typed — and the stop list extends accordingly (diagonal relational mothers; constant wedge laws; unrenormalized small-n cell transports). On the other side the extraction is now *complete*: the arithmetic wall is exactly hypothesis (H) — cofinal finite positivity, equivalently the uniform lower bound on the excess margin of `PRIME.RELATION.SKELETON.01` restricted to a ladder cofinal in the mesh-refinement order — *and nothing else.* No marker moves. **No statement about the Riemann Hypothesis is made.**

12.24 The invariance atlas: the wall is the same across compressions, geometries, criteria and input classes; the minimal hypothesis kernel-checked

Round 31 (2026-08-07, evening) consolidates the three-wave harvest that followed the architecture decision: two further route classes are closed *at class level*, the cone violations resolve into a graded kernel law, the relation discriminator becomes operator-level and geometric, and — the round's headline — the wall is mapped *invariantly* across every coordinate system tried, while its minimal hypothesis is formally minimized in the Lean kernel ([`verification/v850_completion_tax_flow.py`], 15+ 11, zero fails, verdicts TAX-GAP and FLOW-CROSSINGS; [`verification/v851_cluster_kernel_field.py`], $15 + 10 + 7$ with the six frozen-honest FAILS kept and pattern-gated, verdicts CLUSTERS-DIFFUSE, BOTH-PARTIAL, KERNEL-GRADED; [`verification/v854_relation_hodge_pfaffian.py`], 18/18, verdict CURVATURE-DISCRIMINATES-ONLY; [`verification/v855_invariance_atlas.py`], $19 + 8 + 7 + 3$, zero fails, verdicts SIEVE-SAME-GAP, ATLAS-SAME-WALL, PROFILE-DIVERGES, DIRECTION-TRANSIENT; [`verification/v853_bent_css_mangoldt.py`] part B, 30/30, verdict MANGOLDT-COMMUTATOR-EXACT; and the Lean mirror in [`verification/v849_cfin_unique_cofinal_lean.py`] part B; suite 841 → 848, with the compiler-side companions of the same round cited in place in Paper 1).

The tax theorem (the strongest closure of the round). Write the deployed comb operator as $\mathrm{Toep}(c) = R_X + R_X^{\mathsf{T}}$ with R_X its one-sided coupling half. Then *every* positive 2-sector dilation $[[P, R_X], [R_X^*, Q]] \succeq 0$ that routes the arithmetic through the fixed coupling block pays diagonal tax $\operatorname{tr} P + \operatorname{tr} Q \geq 2\|R_X\|_*$, with equality at the SVD pair — verified with *zero-duality-gap* certificate pairs on every anchor (primal $\lambda_{\min} \geq -8.9 \times 10^{-15}$, dual witness $\|W\|_{\mathrm{op}} = 1.000000000000$, $\langle W, R\rangle$ relative deviation $\leq 1.6 \times 10^{-16}$; the symbolic 2×2 closed form at 50 dps has gap 2.7×10^{-51}) — and the bound is *basis-invariant* (DFT/Mellin, the mirror J, seeded orthogonal: $\leq 1.1 \times 10^{-15}$), so the geometry-independence of the round-30 TV price is now a theorem-shaped corollary of nuclear-norm invariance. On the deployed ladder the per-cell tax exceeds the margin by ≥ 4057.6 on all 67 rungs (maximum 7.5×10^5), growing $\sim h^{0.24}$ against the margin's $h^{-3/2}$ decay; the measured event-local Schur designs paid only 13.7–20.4× more than the exact class optimum — *optimality could not have saved them.* The whole positive-dilation architecture class with this coupling is closed, in any basis; the one door the theorem does not close is typed: Fejér–Riesz of the total symbol, which *is* the positivity statement itself.

The flow decision. The homotopy from the pure-density endpoint to the deployed comb is not crossing-free — the premise "density endpoint PSD by construction" is *corrected by measurement*: $n_-(0) = 5..12$ on every rung, the comb supplies exactly that many upward crossings (634 total, all upward; the endpoint identity $\mathrm{SF} = n_-(0) - n_-(1)$ verified on 67/67 rungs), the last crossing hugs the deployed point (median lower gap 1.6×10^{-5}, shrinking faster than the margin), and the velocity test lands at ratio 1.00 (IQR 0.99..1.00): the "quantized amplitude gap" *is* the metric margin divided by the flow velocity — no independent topological protection exists on this path, and in finite dimension the index *value* is a reformulation of the endpoint pivot signs. The typed residue: eigenproblem-free pivot certification (Jacobi determinant recursions, verified against the eigensolver at both endpoints on all rungs) and the crossing-location law; the exclusion connection is stated — a crossing *at* the deployed point would be $\tau = 0$, which the certified margins exclude.

The cluster decision and the graded kernel. The connected multiplicative cluster expansion of the log-pivot exists and resums *exactly* (sympy telescoping on the Boolean lattice; the Möbius elimination — disconnected pair weights identically zero — exact; the 2^{16}-subset resummation ward at 0.0), but the frozen question fails honestly and is kept as its two FAILS: the order-2 enrichment

of multiplicatively connected pairs is 0.883 against a bar of 3.0, and the mass-fixed scramble does not collapse it — the cluster weights are *position-geometric* (the 2-dim corner compression destroys the cluster-arithmetic before the expansion can see it); the all-orders route through this pivot closes. On the cone side the strict violation census *wanders* with the horizon ($P_0 \sim X^{1.03}$ at $R^2 = 1.00$, maximum 244333 against a frozen cap of 100) — but the *depth-resolved* census does not: at relative depth 10^{-2} the G1 kernel is bounded by $n \le 73$ on all 67 rungs, collapsing to $\{2, 3\}$ above $\alpha \approx 3.9$, and the skin depth obeys $D(X) = 0.754\, X^{-0.548}$ ($R^2 = 0.99$, Kendall -1.00) with verified thresholds $X_0(\varepsilon)$ — *the wandering is vanishing-depth skin, not kernel growth.* The graded kernel+tail statement is registered as a measured theorem candidate (`PRIME.CELLCONE.GRADEDKERNEL.01` [O]; it bounds depth, not sign). The causal Lorentz tube field transports the state direction cleanly (0/355199 gated violations on all four rule×grouping combinations) but all four must-break controls do *not* fire — kept as their frozen FAILS: the field is self-calibrating, typed FIELD-STRUCTURAL-ONLY, consistent with the round-30 inverted Epstein signature.

The relation strand: operator level and geometric. The von Mangoldt commutator theorem ([`verification/v853_bent_css_mangoldt.py`], typed honestly as classical incidence algebra — the value is the packaging and the discipline transport): $\mathcal{L}_N = -M[D, Z] = -[D, \log Z] = T(\Lambda)$ *exactly* (three independent layers; $\log Z$ is *rational*, the BCH series truncates), powers enumerate ordered factorization chains (nilpotency $\lfloor \log_2 N \rfloor + 1$), and the four-comb discipline transports to operator level: the commutator machine always outputs $T(\Lambda_f)$, so Selberg consistency is a *support* statement — the three multiplicative combs are exactly prime-power-supported (including the ideal-level Dedekind Λ_K with norm aggregation $(1 + \chi_4)\Lambda$), while the $h = 2$ Epstein comb leaks at $[6, 14, 21, \dots]$ (first unramified site $21 = 3 \times 7$, values pinned exactly) and the leak is *repaired* by the class average ($a_A + a_B = 1 * \chi_{-20}$) — the class-group obstruction localized at operator level. The bridge: the deployed Möbius-pairing reconstruction *is* the first row of $\mathcal{L}_N$ (per-divisor term equality), rows are shifts, and $L^2 - [D, L] = T(\mu * \log^2)$ — the untapped structure is the Selberg hierarchy, which the supply audit below then measures. The same strand's geometric face ([`verification/v854_relation_hodge_pfaffian.py`]): the factorization complex is exact ($d_1 d_2 = 0$ at entry level, $b_0 = 1$, b_1 carried by the same-prime exponent chords), the true comb is *exactly flat* on every commuting square (multiplicativity *is* flatness, $\le 1.8 \times 10^{-15}$), the Epstein curvature localizes at the class-group products with a *proven* $\mathbb{F}_2$ Arf-lift obstruction (exact rank versus augmented rank) — and the honest boundary is typed: the window form is 0-chain-supported (coboundary sensitivity $\sim 3 \times 10^4$), *not* a function of the cohomology class, and the Pfaffian normalizer test fails anchor-independence. Any future cohomological route must supply a homotopy-invariant functional of the comb.

The invariance atlas (the headline). The supply audit rewrites the demand *exactly* in Selberg-hierarchy coordinates ($D_0 = d\mathrm{Head}_2 - d\mathrm{Chain}_2$, zero bookkeeping loss) and audits the elementary toolbox against it with on-range-verified explicit constants: the sieve rows are deterministic and *window-free*, so the $\alpha \in 1..2$ band that the zero-statistics map could not reach is reached in *scope* — but coverage in *grade* is zero (best factor 205× at the shallowest rung, up to 1.1×10^4×; the demand sits at 0.99–1.00 of τ_{pnt} on every rung and *exceeds* the factor-2 gate at truth, min = 2.28 > 1): no valid upper bound of *any* input class can close the gate — the wall is input-class-invariant, statistics-grade and sieve-grade alike. The demand concentrates in the bilinear *chain* fluctuation $d\mathrm{Chain}_2 = \sum \Lambda(a)\Lambda(b)\,\varphi_2(ab)$ − main — the sieve-coordinate name of the form-factor object. The criteria atlas aligns the three classical coordinate systems: the Li coefficients λ_n are computed unconditionally (no zeros) and positive to $n = 20$, the Nyman–Beurling distance d_N^2 is exact to $N = 64$ (decreasing $0.0783 \to 0.0465$ and staying above the Burnol liminf), and two cross-coordinate identifications land: the Baez–Duarte wall constant *is* $2\lambda_1$ (deviation 4.9×10^{-17}), and the NB span *is* the spectral-mother geometry term by term (one mother $\{e^v\}$, integer dilation shifts, Möbius-register weights, the $1/x$ mirror = the deployed J) — the machinery had been speaking Nyman–Beurling without the name. Yet the transfer fails by *cone geometry* (the window packets have a finite Li address, but their nonnegative-cone expansion residual is 60.6% against a 5% bar): each coordinate system owns the same *type* of unconditional partial data and lacks the same *type* of

uniform statement — no coordinate system holds hidden supply. At compression level the minimizer census closes the loop: the deployed lock block's minimizer converges to *neither* frozen closed-form candidate (the ray-locked reading belongs to the transport flow, not the deployed block), but an internal fixed direction exists (deep-third dispersion 0.0217 rad) and its one structural tracking law is measured: *the minimizer is the comb block's own soft direction* (0.0056 rad tail median; the drift is real — the Bonferroni demonstration passes 8 of 15 predeclared candidates, so no closed-form identification is claimed, and $-4/\pi$ is typed a coincidence). The one object in four languages: *the uniform e_1 envelope bound = the form-factor band $\alpha \in 1..2$ = the bilinear chain fluctuation $d\mathrm{Chain}_2$ = the comb block's soft direction.*

The minimal hypothesis, kernel-checked. Work package F1 formalizes the extraction chain's positivity input at its minimum (`TfptCarrier/CofinalWeil.lean`, 11 declarations, lake build green at 3415 jobs, axioms propext/Classical.choice/Quot.sound only; numeric mirror in [`verification/v849_cfin_unique_cofinal_lean.py`] part B): the *mathematical core* of the cofinal hypothesis (H_{cof}) is a strictly monotone ladder with PSD rungs, and the minimal implication is *proven*: form convergence plus (H_{cof}) force the limit functional nonnegative on the whole dense family, with *no* diagonal argument (one PSD certificate per rung covers every element simultaneously). *Cofinal in which order.* The ladder index is the mesh level, so (H_{cof}) asks for cofinality in the *mesh-refinement* order in which that form convergence holds — not for cofinality in the window or cap parameter alone: at a fixed mesh the deployed read is exactly cap-independent past the faithful cap, hence eventually constant, and its limit is the Weil value *shifted by the frozen quadrature defect* of that mesh. With the qualifier the hypothesis is exactly what the kernel proves, and cofinality then suffices for a single reason: were the limit negative, convergence would make $\{j : Q_j < 0\}$ a *tail* of the mesh order, and a cofinal set meets every tail. The hierarchy is strict and kernel-checked in both directions: uniform $\subsetneq$ pointwise $\subsetneq$ cofinal (witnesses $1/(m{+}1)$ and ± 1 on the even ladder). Consequently `UniformMarginBound` — the named hypothesis of the round-29 skeleton — is *formally demoted to an over-strong sufficient lemma*: the load-bearing chain of `PRIME.EXTRACTION.CHAIN.01` consumes exactly (H_{cof}), and the wall content has strictly decreased to cofinal PSD on one supplied mesh-refining ladder (`FORM.PRIME.COFINAL.WEIL.01` [F]). A subsequent noninterference hardening (`TfptCarrier/CofinalPredefinition.lean`) proves an explicit fixed-`idx` form, exhibits a sign-mined selector accepted by the old core, and adds the hardened `PredefinedCofinalHypothesis` API with a named abstract `Predefined(A,idx)` certificate. Ordinary binder or field order does *not* prove computational independence: the kernel checks the presence and type of that certificate, while the concrete source boundary excluding the matrix family and its sign outputs remains an external audit premise unless the construction language is formalized. (H_{cof}) itself stays a hypothesis everywhere; nothing in the round evaluates positivity of any actual tower form.

The honest statement. Round 31 closes three more route classes (positive-dilation completions with fixed comb coupling, in any basis; finite-dimensional index reformulations; corner cluster expansions), resolves the cone violations into a graded law that bounds *depth, not sign*, and proves the wall invariant across compressions, geometries, criteria and input classes — while shrinking the named hypothesis to its minimal kernel-checked form. The stop list extends accordingly; the demand is unchanged. No marker moves. **No statement about the Riemann Hypothesis is made.**

12.25 The grade law: the graveyard unified by one homogeneity theorem; the licensed elevator and its honest death; the phase-rescue family closed

Round 32 (2026-08-08, morning) consolidates two waves — the connected-current architecture and the grade theory — into the day's one conceptual statement ([`verification/v856_connected_current_lean.py`], $14/14 + 5/5$ plus four Lean-mirror checks; [`verification/v859_grade_no_go_elevator.py`], $11/11$ plus four Lean-mirror checks; [`verification/v860_phase_lever_closure.py`], $7/7$; suite $848 \to 853$, with the compiler-side audit companions of the same round cited in place in Papers 1 and 2).

The grade law (the headline, kernel-checked). Every member of the direct-positivity graveyard

on this front — the Schur–Gram tax, the covariance grade gap, the wedge uniformity failure, the phase-leverage failure — is an instance of *one* homogeneity law, now a kernel-checked class-closure theorem (`TfptCarrier/GradeNoGo.lean`, 13 declarations, 10 theorems, axioms propext/Classical.choice/Quot.sound only; numeric mirror in `[verification/v859_grade_no_go_elevator.py]` part A): a target that is *1-homogeneous* in the arithmetic weights can never be matched exactly by a Gram square of a linear map, which is *2-homogeneous* — evaluating on a single scaling ray at two scalars gives $2A(m) = 4A(m)$, so $A(m) = 0$ and the Gram factor vanishes with it. The proof is two lines; its brevity is the point: this is a *class* closure, not a construction failure. The affine escape is priced exactly: the representation error of an affine background against the pure cross term is *exactly* the sum of two PSD prices (constant + quadratic), so the tax is nonnegative — and the sharpened no-go kills the background too (exact affine matching forces target, background *and* factor to vanish). The measured exchange rate of the law is 10^4–10^5: the Schur–Gram diagonal price 3.1–4.5 $\times 10^4\,\tau$ with the class optimum $\geq 4057.6\,\tau$ (rounds 30–31), and now the covariance variance price 2.5–4.1 $\times 10^5\,\tau$ (below).

The connected covariance: the first design that evades the nuclear tax — and the grade gap that kills it. Package C builds the centered KMS correlator of the Möbius commutator current $\mathcal{L} = -[D, \log Z] = T(\Lambda)$ (exact at 7.2×10^{-15}; the Epstein h=2 comb leaks 76% of its mass off the prime powers at *relation* level, before any state): the states are PSD with no window input, the centered main term *is* the deployed smooth half-line subtraction (corr 0.99; the wrong half-weight loses the pair-weight carrier — the construction is anchored to the critical line by the state, as designed), and *the relapse kill does not fire*: the correlator does not collapse back onto the taxed coupling geometry (ratio ≈ 0.3 against the frozen band) — global centering is genuinely different bookkeeping from the event-local Schur completions, the first architecture to evade the round-31 tax theorem. It dies on the grade law instead: the connected covariance is the *pair-correlation* form — its stationary part carries the deployed pair weights $\lambda_m^2 = \Lambda(m)^2/m$, quadratic in the comb — while the deployed window form is *linear* in the comb, and routing the linear read through $\omega(B^*B)$ costs the variance itself: one global negative entry at 2.5–4.1 $\times 10^5\,\tau$ (`[verification/v856_connected_current_lean.py]`, CONNECTED-COVARIANCE-PARTIAL). What survives is typed: a zero-free, state-anchored pair-correlation *instrument* with the exact main subtraction built in — an instrument, not a floor certificate (`PRIME.RELATION.CONNECTED_COVARIANCE.01`).

The phase cells are empty; the phase-rescue family is closed by theorem. Package D splits the deployed form into a phase-parametrized head and a tail, exactly on all 67 rungs, with the head jointly affine per cell and the Kronecker input warded (log-prime independence by unique factorization, two lines): the census finds *zero* positive cells on the entire deployed torus (220–1760 cells per anchor, including $\{2,3,5\}$ and $\{2,3,5,7\}$; the phases control only $\sim$1% of the demand; the extension ladder kills harder, not softer), and the independent tail kill types the split itself: $\lambda_{\min}(\text{tail})/\tau \sim -3.4 \times 10^3 .. -4.8 \times 10^3$ — the split moves the demand onto an object *farther* from PSD (`PRIME.COFINAL.PHASE_CELL.01`, PHASE-CELLS-EMPTY). The phase-lever theorem then closes the whole finite-head family with certificates (`[verification/v860_phase_lever_closure.py]`, PHASE-LEVER-THEOREM): granting every window atom a *private* best phase, the budget never pays the arch deficit on any of the 67 rungs ($d_0 - \text{budget}(\{2,3\}) \geq 1.3894$; $d_0 - \text{budget}(\{2,3,5,7\}) \geq 0.6296$) — every such head is uniformly negative at every phase, no Kronecker sequence can rescue it; the D-frontier saturates at $0.73 < 1$ (structural, not a boundary effect), and beyond the budget's break-even the exact census remains the instrument — and has never found a cell (`PRIME.PHASE.LEVER.01`).

The conditional frame to (H_{cof}), kernel-checked. Package E wires the architecture into the extraction chain at kernel grade (`TfptCarrier/CofinalCurrent.lean`, 13 declarations, 7 theorems, axioms clean; numeric mirror in `[verification/v856_connected_current_lean.py]` part C): *ConnectedTail* + *PositiveHeadCell* + *PhaseRecurrence* $\Rightarrow (H_{\text{cof}})$, composed through the minimal H theorem to Weil nonnegativity on the whole dense family — no hidden wall can appear behind the two analytic lemmas; the $\delta = 0$ relaxation is typed (the chain consumes only PSD on the cell), and the counterexample `connectedTail_not_sufficient` is kernel-checked (the hypotheses are jointly necessary). The honest boundary is the round's architecture decision: hypothesis 1 is deliverable

by the covariance construction, hypothesis 3 is classical — and hypothesis 2 is *measured empty at the deployed frame* by package D. The frame stands as the named continuation contract awaiting a head with a nonempty cell, not as a discharge (`FORM.PRIME.COFINAL_CURRENT.01` [F]).

The licensed elevator and its honest death. The grade law leaves exactly one grade-1 positivity mechanism standing, and the round proves it abstractly and kills its deployed instantiation honestly ([`verification/v859_grade_no_go_elevator.py`], LOCAL-SIGN-FAILS). The elevator lemma (`tangent_psd_on_kernel_null`, kernel-checked): derivatives of PSD families at kernel-null directions are positively signed — *rates, not amplitudes* carry grade-1 content with a sign. The instantiation is real: the compound-Poisson local Euler factors are PSD with no eigenvalue input, the tangent is exact within the certified Taylor bound, the free null-sum positivity fires ($x^{\mathsf{T}}\Psi x = w|X(\theta_q)|^2$), and the rates are Euler-indexed exactly — the grade barrier is genuinely *passable*. Two independent kills fire anyway: the deployed per-event tent deposit has a *signed* Lévy density (negative-mass fraction exactly 0.500; first flip at $\theta^* = \pi D/2u_q$, matched to $\sim$1%) while any conditionally-positive tangent has a positive one (Lévy–Khinchine) — no positive jump mixture reproduces the deployed comb at any smearing; and the completed arch source is signed on the *digamma band*, ending at $\tau^* = 6.27$ where $\operatorname{Re}\psi(\frac{1}{4} + i\tau/2) = \log\pi$ — the closure's sharpest new number. The Lévy/conditional-positivity elevator class is closed on both deployed sides; the rates door itself stays licensed and typed (`PRIME.EULER.SCHUR.SEMIGROUP.01`).

The honest statement. Round 32 unifies the graveyard under one kernel-checked homogeneity law, prices every affine escape, licenses exactly one grade-1 mechanism and closes its deployed instantiation by measurement, closes the finite-head phase-rescue family by theorem, and adds two typed positive cores: the zero-free pair-correlation instrument and the conditional (H_{cof}) frame. The stop list extends accordingly (finite-head phase rescues; amplitude-Gram constructions without grade accounting; Lévy/conditional-positivity elevators on the deployed sides); the demand is unchanged. No marker moves. **No statement about the Riemann Hypothesis is made.**

12.26 The wall's operator form: the Krein coordinates, the SourceContractor formula, the Perron closure, and the quadrature reach

Round 33 (2026-08-08) consolidates the day's Krein/port/formula campaign — sixteen frozen probes in five strands — into the wall's *operator form* ([`verification/v861_krein_normalform_lean.py`], $37/37 + 8/8$ plus four Lean-mirror checks; [`verification/v862_defect_polar_weld.py`], $8/8 + 22/22$; [`verification/v863_redheffer_colligation.py`], $7/7$; [`verification/v864_softport_kappa_law.py`], $5/5 + 6/6$; [`verification/v865_weyl_port_readout.py`], $9 + 5/5$ with two frozen-honest FAILs; [`verification/v866_source_contractor_formula.py`], $10 + 8/8$ with one frozen-honest FAIL; [`verification/v867_radau_conditioning.py`], $7 + 8$ with two frozen-honest FAILs; suite 853 → 861, with the register-side canonicity audits of the same round ([`verification/v868_divisor210_audits.py`], $36/36 + 12/12 + 15/15$) cited in place in Papers 1 and 3).

The Krein coordinates. The deployed window form now has an *exact* Krein normal form, $Q_h(t) = \|B_+t\|^2 - \|B_-t\|^2$, machine-warded entrywise on every reachable rung for both canonical cuts, with the weights entering as $\sqrt{|\Lambda|}$ so the form is *linear* in Λ — the grade barrier of round 32 is passed by construction — and the Douglas pencil identity holds exactly: $1 - \|C_2\|^2$ equals the τ-margin in the positive-side metric (rel $\le 3.9 \times 10^{-9}$). "Which self-consistent comb is real" has become "which rate measure makes $B_-\,\mathrm{pinv}(B_+)$ a contraction", and the discriminator is the Douglas equivalence itself: truth contracts on every rung ($\|C\| \le 1$, razor-thin at depth) while the Epstein h=2 comb explodes to $\|C_E\| = 46.8$ and the scrambled comb to 3693 ([`verification/v861_krein_normalform_lean.py`], KREIN-CONTRACTOR-STABLE-NO-WORD). The register wiring behind the coordinates is typed in the same module: the μ-sign and the geometric deck are *two separate* C_2s (the deployed reading character separates them; the identification is one *faithful-*μ_4 character away, with three candidate constructions machine-verified — the diagonal characters, the Galois bookkeeping $\mu_K((p)) = \chi_4(p)$ exact on $609 + 619$ primes, and the Hall chain parity row $1 = \mu$), and the Redheffer/Mertens echo is exact: $\det R_n = M(n)$ with the vacuum column's *entire* Smith deviation concentrated in the single invariant factor $|M(n)|$ — vacuum completion as a rank-one origin

update, with the fit-free census finding *no* resolvable floor–Mertens correlation (Bonferroni ×4: min $4p = 0.378$). The defect theorem is kernel-checked (`TfptCarrier/KreinDefect.lean`, 14 declarations, 10 theorems, axioms propext/Classical.choice/Quot.sound only; numeric mirror in [`verification/v861_krein_normalform_lean.py`] part C): the defect representation, contraction ⇒ PSD via the exact factorization $B_+^{\mathsf{H}}(1 - C^{\mathsf{H}}C)B_+$, the full-rank Douglas converse, `SourceContractor` as the *named hypothesis* (contractor + factorization + independent certificate, with the circularity warning typed), and the composition `krein_cofinal_weil`: contractors on a cofinal ladder force Weil nonnegativity on the whole dense family — no hidden wall behind the contractor certificates (`FORM.PRIME.KREIN.DEFECT.01` [F]).

The rank of the wall is one; the class closure is a theorem. The defect operator $\Delta = I - C^*C$ has exactly *one* soft mode on every reachable rung, and λ_1 *is* the τ-margin by the exact pencil identity (separation 3.9–14.1×; Epstein carries 55 negative defect eigenvalues, the scramble 37) — the Redheffer single Smith invariant, the vacuum code's one-logical-qubit price and the Krein soft port are one rank-one structure ([`verification/v862_defect_polar_weld.py`], DEFECT-ONE). The word-census emptiness of the normal form becomes a theorem for *all* word lengths: every deployed source letter is a monomial matrix, the monomial class is product-closed, and the contractor's monomial mass fraction is 0.23–0.24, so every word of any length misses by residual ≥ 0.874 — typed with the honest surprise first: the clean two-band corridor picture is wrong at fine scale (the density's 408 sign flips interleave the bands), and the surviving closure is stronger (MONOMIAL-CLOSURE-THEOREM). The 2-torsion weld law supplies the missing operator's *type*: the deck's anticommutant on the deployed register is entirely faithful-μ_4 — the deck can enter a Clifford weld only as the quarter turn $J = MD$, never as a second involution; all four weld candidates close exactly on their forced domains, the unwelded control refuses exactly, and the contractor connection is null at the random-null value $\sqrt{2}$ (WELD-WITHOUT-CONTRACTOR).

The machine: contraction becomes a consequence. The 16-slot colligation architecture delivers what every earlier design had to assume: both source-built machines are exactly unitary, the vacuum-port Redheffer closure is well-posed ($\|D\| = 1/4$), and every closed transfer block is *automatically* contractive — never tuned ([`verification/v863_redheffer_colligation.py`], COLLIGATION-CONTRACTS-NOT-IDENTIFIES). The scalar limit reproduces the weighted lattice Redheffer determinant exactly — $\mathbb{F}_2^4$ *is* the divisor lattice of $210 = 2 \cdot 3 \cdot 5 \cdot 7$ with Walsh–Hadamard carrying the lattice Möbius function and $\det = \prod_p(1 - 1/p) = 8/35$ (the compiler-side discovery, booked with its canonicity guard in Paper 1) — and the machine is provably comb-blind (anti-circularity ward). The identification gate fails honestly and *localizes* the missing object: the Douglas contractor couples *different* $|\tau|$ density bands while the machine's only band mover (the J flip) couples $\pm j$ at equal $|\tau|$ — the label-side machinery is complete, what is missing is the grid-side band intertwiner: the explicit-formula kernel itself.

The scalar reduction: the port wave. The exact Feshbach/Schur reduction turns the matrix floor into one port impedance — $\tau > 0 \Leftrightarrow s > 0$ at any port carrying a stable share of the soft mode over a uniformly positive bulk — and four *source* ports pass, with the pole port carrying 84% of the soft mode at depth, the exact skeleton $\kappa^{-1} = \beta^2 + \lambda_1\rho$, the *closed-form* leading term $a = 1 - E_-/E_+$ (the Poisson/Cauchy average of the signed density at the pole point $s = 1/2$) and the measured law $\kappa - 1 \approx 6e^{-\alpha/2} \to 1$ (Pearson -0.99): the classical "main term minus boundable term" shape exists structurally, and the boundable term — ≤ 17 bulk-mode couplings cancelling 99.9% of the a-term — is exactly the arithmetic ([`verification/v864_softport_kappa_law.py`], KAPPA-LAW-PARTIAL). The arithmetic survives into the machine at *load* level: the divisor-tower Weyl load responds at $O(1)$ to mass doubling — the hull's comb-blindness is repaired — and the v1 extremal readout dies by a diagnosed mechanism (doubling is a near-reparametrization of the r-curve; extremal functionals read the curve as a set), which the repaired phase readout then removes: Epstein $\kappa = 42.7$ and scramble 77.6 land *outside* the 1.5× truth window — one to two orders of discrimination where every earlier functional of this chain had both inside — with the typed caveat carried verbatim: the load-bearing content is the discrimination, not the narrow-battery κ band ([`verification/v865_weyl_port_readout.py`], WEYL-PORT-BLIND + READOUT-REPAIRED).

The formula (the campaign's headline), and the Perron closure. The Douglas contractor factors *exactly* as a target-free closed-form source expression,

$$C \;=\; W_- \, F \, G_+^{-1} F^{\mathsf{H}} \, W_+$$

(max rel 1.4×10^{-14} on construction rungs *and* blind holdouts; density roots and window geometry only — no zeros, no τ, no fit), with *certified defect transfer*: at the measured budget k^* the reconstruction's contraction defect matches τ within the certified Weyl budget on all seven rungs — the `SourceContractor` of the Lean frame *exists as a formula* ([verification/v866_source_contractor_formula.py]; the frozen Sylvester-12 kill fires as designed and corrects the rank-12 reading to a $\sqrt{L}$ growth law, $k^* = 24..48$). Its norm statement is relocated, not reduced: $\|C\| \le 1 \Leftrightarrow G_- \preceq G_+$ exactly — better coordinates, the same wall (NORM-REFORMULATION). What the coordinates buy is a *proof about the tool space*: the Perron value $\rho(|C|)$ — the infimum over all absolute Schur tests — exceeds 1 everywhere ($2.36 \to 3.10$, growing with depth), so *no absolute-value Schur/Hilbert-type inequality can ever certify the bound*: the certification must use the phases of the kernel — cancellation provably carries the arithmetic. The named demand is registered (`PRIME.KREIN.CONTRACTOR.01` [O]): one *phase-aware* bound on the weighted frame kernel (oscillatory-integral / stationary-phase grade), strong enough to beat 1 by the measured $e^{\mathrm{slope}\,\alpha}$ margin — the single object that would discharge the `SourceContractor` hypothesis and, through `krein_cofinal_weil`, hypothesis (H_{cof}).

The quadrature reach, and the final typed wall. Gauss–Radau certificates for $s > 0$ *exist on every rung* — positivity is provable pointwise by quadrature alone on the whole reachable ladder, killing the Neumann frontier — but the needed depth grows with the certified law $m^* \sim \sqrt{\mathrm{cond}(G)}$ (Spearman +0.99; the $m \approx 17$ prediction honestly failed), and the preconditioned retry closes the escape: none of the three source preconditioners bounds the effective condition number — the conditioning is *intrinsic*: the soft end of the bulk is neither the smooth geometry nor the Krylov head but distributed arithmetic structure that any certificate must resolve mode by mode ([verification/v867_radau_conditioning.py], RADAU-DEPTH-GROWS + CONDITIONING-INTRINSIC; the limiting Jacobi chain is named — the free Chebyshev chain on the bulk support, Rakhmanov-generic typed). In quadrature coordinates the wall reads: *the certificate depth is the price of the cancellation, and no source-side change of metric waives it.*

The honest statement. Round 33 gives the wall exact operator coordinates (normal form, rank one, closed-form contractor, scalar port shadow), proves the absolute half of the classical toolbox closed (the Perron closure), proves the quadrature route reaches every rung at an intrinsic depth price, types the register canonical up to proven gauge, and registers the sharpest demand so far: one phase-aware kernel bound (`PRIME.KREIN.CONTRACTOR.01` [O]). The stop list extends accordingly (source-word searches at all lengths; absolute-value bounds on the frame kernel; fixed-depth quadrature families without a conditioning bound; chirality deciders through quadratic readouts); the demand is unchanged in substance. No marker moves. **No statement about the Riemann Hypothesis is made.**

12.27 The Christoffel–Darboux wave: the kernel named, the compensation exact, the anatomy of discrimination, and the preregistered next test

Round 34 (2026-08-08) consolidates the evening CD/Uvarov campaign — seven frozen probes in three waves — and commits the next test as a *frozen preregistration* ([verification/v869_cdcore_clarkphase.py], 10+8 checks, zero fails; [verification/v870_christoffel_gauss_frame.py], 17/17+9/9; [verification/v871_pole_uvarov
$7 + 11$; [verification/v872_damping_compensation.py], 10; suite $861 \to 865$).

The kernel is named. The reproducing kernel K_+ of the round-33 closed-form contractor is *exactly* the Christoffel–Darboux kernel of an explicit source measure,

$$\mu \;=\; \sum_{\text{bins},\, d_+ > 0} \frac{2\sin^2(\varphi/2)\, d_+(\varphi)}{L}\, \delta_x, \qquad x = 2\cos(D\tau)\,:$$

the displacement rank collapses from the growing τ-coordinate budget (12–14; the $\sqrt{L}$ law of round 33) to *exactly* 2 in the multiplication coordinate, with a hard σ_2/σ_3 gap of 6.6×10^{12}–4.8×10^{13} on all seven rungs *including the blind holdouts* kz 40/49/60, and the *firewalled* chain-only reconstruction — Stieltjes–Lanczos source Jacobi chain plus node/geometry data only, enforced by the reconstruction signature (no G_-, no target defect, no target eigenvectors) — rebuilds C entrywise at 8.1×10^{-14}–5.3×10^{-13} ([verification/v869_cdcore_clarkphase.py], CDCORE-IDENTIFIED). The whole contractor is now three named objects: the arm weights (density roots), the phase/geometry factors, and *one* source Jacobi chain — the "non-monomial band-spreading letter" of rounds 32–33 is named at kernel level, and the classical toolbox (Christoffel functions, chain asymptotics) applies. The honest boundary was typed *before* running and confirmed: the rank-2 collapse is *window geometry* — the CD derivation forces it for any even density, and Epstein/scramble collapse too — so the collapse names the geometry and cannot certify arithmetic; the arithmetic lives in the *measure*: the controls' Jacobi chains differ from truth at $O(1)$ (0.038/0.062 against the frozen 10^{-2} bar) and their bulk stays indefinite. In the same module the μ_4 quarter-turn Clark-phase hypothesis dies sharply: the coupled pairs obey the *equal-phase* coupling law (offset $-0.5..+0.6^\circ$ at $\bar{R}=0.97$–0.99, not the predicted 90°), the Clark rank leg fails (E-rank 85–99 against the ≤ 8 bar), and the phase coordinate itself is clean — source-only, with an exact circularity fence (CLARKPHASE-DEAD).

The factorization, and the frame decision. The contractor assembles exactly as $C = D_-UD_+$ ($\le 1.3\times10^{-14}$), with the two attenuations stably contractive — $\|D_+\| \le 0.9654$ *by the Christoffel variational theorem*, $\|D_-\| \le 0.8655$ measured — and U non-unitary *by theorem*: the deployed grid 2×-oversamples the folded pairs, so every non-null row of U has norm ≥ 1.158 — the $\sqrt{2}$ over-normalization is structural, and the classical unitary transition is exclusive to Gauss-tight node sets ([verification/v870_christoffel_gauss_frame.py], CHRISTOFFEL-PARTIAL). The Gauss-node frame then *decides* the repair: it is exact wherever algebra promises — $D_+^G = I$ at 4.7×10^{-11} (Gauss–Christoffel), the bridge lossless, the pole-port share *frame-invariant* (2.6×10^{-12}: the 84% port share of round 33 neither sharpens nor delocalizes in any frame) — yet the unitary core does *not* exist: the co-isometry defect stays $O(1)$ (0.93–1.30), concentrated at near-pole node pairs. The surviving asset is the *ladder-wide* damping bound $D_-^G \le 0.997041$ with 34×/2900× discrimination (Epstein/scramble), and the hypotheses a theorem would need are registered verbatim: (H-D) the uniform damping bound [measured 0.997041], (H-U) $\sigma_{\max}(U^G) \le 1+\varepsilon$ [measured 1.518 — the open one], (H-PORT) $s>0$ uniformly (GAUSS-STILL-DEFECTIVE).

The measure-relation closure. Two classical escapes close with certificates and typed residues ([verification/v871_pole_uvarov_crossdefect.py]). None of the three classical spectral transformations — Uvarov mass insertion, Christoffel multiplication, Geronimus division — relates the two channel measures exactly or at fixed small rank at the source pole $x_{\text{pole}} = 2\cosh(D/2)$ (the machinery itself warded exact on synthetics at 9.2×10^{-16}: the kills are findings); the typed residue: Geronimus with *small negative mass* is uniformly closest — Weyl residual 3.5–8.1% across construction and blind holdouts, non-growing, discriminating — but misses the frozen 5% bar and admits no $\lambda(D)$ law (POLE-TRANSFORM-DEAD). And the cross-measure co-isometry defect is *diffuse*: full-rank, growing linearly with h (Pearson +0.996), untouched by pole removal, with the closest-co-isometry bracket $O(1)$ at near-full rank — no finite-matrix compensation exists at this ingredient list (CROSSDEFECT-DIFFUSE). Both closures are stop-listed: classical pole transformations at x_{pole}, finite-rank cross-defect corrections, and (from the CD wave) μ_4-offset Clark kernels.

The compensation identity (the round's headline). The defect splits *exactly* into two source-built terms,

$$I - C^*C \;=\; T_1 + T_2, \qquad T_1 = I - U^*U, \quad T_2 = U^*(I - D_-^2)\,U,$$

warded at 10^{-12} on all 42 reachable rungs: T_1 is the *indefinite* cross-measure Jacobi geometry, T_2 the *PSD* arithmetic Christoffel damping ($D_-^G \le 0.997041$ makes $I - D_-^2 \succeq 0$ with zero negative entries) ([verification/v872_damping_compensation.py], COMPENSATION-EXACT). The anatomy is measured to all digits: the *exact global minimizer* of the directional compensation ratio — computed basis-free from the pencil — *is* the soft direction (overlap ≥ 0.9991 on every heavy rung; min ratio $= 1+\tau/|v^*T_1v|$ to all printed digits); the rank-one pole deflation gives $\Delta = \Delta_{\text{bulk}} + s\,pp^*$ with

the certified bulk floor at 98–100% of λ_2 and the isolated scalar $s = \kappa\tau$ reproducing the round-33 κ law. *The wall as two explicit source numbers*: at the soft direction the two terms read (kz 9) $-0.0328 + 0.0330 = 1.68 \times 10^{-4} = \tau$ — the open cofinal content is exactly the two-term inequality $v^{\mathsf{H}}T_2v > |v^{\mathsf{H}}T_1v|$ at one named direction, with both terms explicit source objects. The honesty line is typed: on these finite rungs the truth-side ratio ≥ 1 is *implied* by the upstream certified positivity — the identity buys the *form*, not new evidence. What it does measure fresh is the *anatomy of discrimination*: Epstein and scramble *break the damping's sign* ($\max D_- = 34.2/2917 > 1$: the Christoffel weights of a false comb overshoot 1) and under-compensate on their *own* soft directions (worst ratios $-175/-446513$) — what "arithmetic" means in these coordinates is the sign and alignment of the Christoffel damping against the Jacobi cross-geometry.

The preregistered next test (committed, not run). With the classical measure relations and the finite-matrix compensations closed, the named plan B — the Prüfer/Cotlar phase route — is committed with this round as the *frozen* preregistration `PRIME.PRUEFER.COMPENSATION.01` (`experiments/tfpt-discovery/pruefer_compensation_probe.py`): source-only Prüfer phases of the two arm chains (structural AST firewall), exactly 16 fixed phase cells (boundaries $k\pi/8$, entrywise pairing primary, danger families predeclared), Cotlar–Stein decision rules and all success/kill criteria frozen *before* any deployed-data run, deep holdouts kz 90/116/142/177/243 never used by the CD wave, and the decisive analysis locked behind an operational unfreeze flag (8/8 synthetic freeze wards green at freeze time). The frozen contract is committed byte-exact — SPEC SHA-256 `4621b899...7440811`, FILE SHA-256 `93208a1b...4825d9fd`; any modification invalidates the preregistration. No analysis mode was executed and no result was claimed at commit time. (The contract was executed in round 35 — Section 12.28 — and returned its own frozen verdict.)

The honest statement. Round 34 names the contractor's kernel (with the naming's own limit typed: the collapse is geometry, the arithmetic is the chain and the weights), proves the factorization's load distribution and the Gauss frame's verdict, closes the classical measure-relation and finite-rank escapes with typed residues, reduces the floor demand to one two-term inequality at one named direction with both terms source-built, and preregisters the next test before running it. The demand of `PRIME.KREIN.CONTRACTOR.01` [O] is unchanged in substance — now readable as: control one source Jacobi chain and its Christoffel weights against their free counterparts at the soft direction. No marker moves. **No statement about the Riemann Hypothesis is made.**

12.28 The Cotlar decision: the first fully preregistered uniformity test, executed

Round 35 (2026-08-08) executes the round-34 contract ([`verification/v873_pruefer_cotlar_decision.py`]; suite 865 → 866). This is the first test of the program run under *full* preregistration discipline, and the discipline is the point: every cell boundary, pairing rule, danger family, decision bar, success criterion, kill criterion and holdout was frozen and hashed *before* the first deployed-data run, and the frozen file was committed pre-execution — the hash chain, not the narrative, is the proof of precedence.

The contract and its execution discipline. The provenance chain is machine-warded inside the module's gate: the contract frozen and hashed (SPEC SHA-256 `4621b899...7440811`; 8/8 synthetic freeze wards; the decisive analysis locked behind an operational unfreeze flag, absent at freeze) → committed byte-exact *pre-execution* in the round-34 commit `526ca3eb` (FILE SHA-256 `93208a1b...4825d9fd`) → the unfreeze flag created *after* that commit, with its audit line committed → executed with exactly two documented *mechanical* fixes, both typed verbatim in the probe: the $h \leq 900$ battery filter that the frozen spec's own battery definition mandates (the committed code omitted the spec's own text), and a per-step joint positive renormalization against float overflow at near-breakdown chain tails — invariant for atan2 and the wrapped increments in exact arithmetic, validated at $\leq 6.6 \times 10^{-4}$ against a 60-digit `mpmath` reference. The `FROZEN_SPEC` literal is byte-identical throughout (SHA re-verified at every stage; executed FILE SHA-256 `15d35b26...5ffb4cc`), and the promotion module re-executes RUN 1 and RUN 2 verbatim from the embedded byte-exact source, pattern-gating the frozen census *including the failures* — nothing is refit.

The verdict (the contract's own rules). RUN 1, the anatomy: the predeclared danger geometry is *real* — the frozen cells $\{0,15\} \cup \{7,8\}$ capture 0.98/0.61/0.94/0.85/0.81 of $\lambda_{\min}(T_1)$ on the five anchors, so source-only Prüfer phase cells localize the negativity *without* the computed soft eigenvector (the would-be kill 3 does not fire). But the local compensation *fails* ($\sum_r \varepsilon_r = 6.88$–$10.08$ against the small-defect expectation), and the cross-block envelope does *not* decay: Spearman$(\text{env}, d) = -0.17..0.00$ against the frozen ≤ -0.8 bar, with the envelope *bimodal* — the second peak sits exactly at the predeclared π-resonance cells $d = 7$–8. RUN 2, the Cotlar decision: the 42-rung battery is *flat* (S_U 5.55–6.30, S_C 3.97–4.78) and the frozen deep holdouts *explode* (kz 142: S_U 32.07/S_C 58.78; kz 177: 14.29/30.76; kz 243: 13.94/44.85); the frozen rule declares both channels GROWING (log–log slopes $0.209/0.408 \geq 0.20$). The contrast bars pass — Epstein max rel 3927% with local-defect ratio 224×, scramble 314462% with 1.26×10^6: truth, Epstein and scramble do *not* share the block geometry. RUN 3 was correctly *not* executed (the contract gates it on BOUNDED). Of the five frozen kill criteria, kills 2 (the Cotlar sum grows as a power of h) and 5 (the compensation stays global) fire; kills 1, 3 and 4 do not. Composite verdict: COTLAR-GROWING — the blockwise Cotlar–Stein route on Prüfer phase cells is *dead by its own preregistered rules*, and is stop-listed.

The measured carry-forward law. The growth tracks kz-*depth* (α), *not* window size h: the holdouts kz 90/116 at $h = 1430/1433$ stay *inside* the battery band, while kz 177 at the *smaller* $h = 1219$ explodes. Any uniform certificate must therefore be α-aware, not h-aware — a typed constraint on every future candidate. (Retired in round 36: the law was the truncation's signature — see the dated correction below.)

What survives. The danger-geometry positive (source-only phase cells find the dangerous blocks) and the discrimination stand as carry-forward assets; the surviving demand of `PRIME.KREIN.CONTRACTOR.01` [O] is narrowed to a *non-stationary* phase bound on the explicit kernel, with the π-resonance as the known critical structure such a bound must handle (superseded in round 36: the demand now lives in embedding coordinates, `PRIME.CARLESON.PRIME.01` [O], and the phase-cell route itself reopens — see the dated correction below). And the honest note that is this section's real content: preregistration, frozen bars and committed hashes — the contract's verdict was not negotiable after the fact, and the program records the negative exactly as it would have recorded the positive. This is the standard the program now holds itself to. No marker moves. **No statement about the Riemann Hypothesis is made.**

Dated correction (2026-08-08, round 36). The RUN-2 deep-holdout data behind the verdict above was *comb-truncated*: the deployed atom table caps silently at `ATOM_MAX` $= 4 \times 10^5$ while the windows kz 142/177/243 demand comb support up to $e^{2\alpha} = 4.4 \times 10^5/8.0 \times 10^5/1.8 \times 10^6$ ([`verification/v877_complete_comb_reversal.py`]). The verdict stands as the record of the preregistered execution *on the data as measured* — the frozen rules were correctly applied — but the scientific conclusion reverses on complete combs: the identically frozen rule returns BOUNDED in both channels, the stop-list entry is amended (the class is *not* closed), and the α-not-h law is retired (it was the truncation's signature). See Section 12.29.

12.29 The complete-comb reversal: the artifact resolved, the Cotlar route reopened, and the Carleson chain made machine-exact

Round 36 (2026-08-08) is the consolidation of the evening campaign that followed the Cotlar decision — and it is the round in which the program's discipline machinery caught, typed and reversed its own freshest negative ([`verification/v876_carleson_polygon_wave.py`], [`verification/v877_complete_comb_reversal.py`], [`verification/v878_ledger_integrity.py`]; suite 866 → 871).

The artifact ([`verification/v877_complete_comb_reversal.py`], 8/8, DEEP-ALPHA-ARTIFACT). Two independent probes had reported *negative* Krein floors at the deep holdouts kz 142/177/243 ($\lambda_1 = -331/-135/-215$). The convention audit found the mechanism before any physics reading: `core.build_window` draws its atoms from a von Mangoldt table capped at `ATOM_MAX` $= 4 \times 10^5$ and *silently truncates* the tent assembly for windows demanding support beyond it — exactly those

three rungs, whose demand reaches 1.8×10^6. With the *complete* canonical comb (extended sieve, machinery verbatim) the certified corner enclosures are strictly positive ($+1.06 \times 10^{-5} / + 1.52 \times 10^{-5} / + 1.20 \times 10^{-5}$) and the Cholesky-certified full and Krein floors are positive on all three rungs: *no negative canonical window exists*; the certified skeleton ladder's end at kz 121 was the atom-table *data* boundary, not a structural α-wall; (H_{cof}) is unaffected. The silent truncation itself is typed as an infrastructure bug with a registered follow-up: the builder must fail loudly beyond its table.

The reversal (`[verification/v877_complete_comb_reversal.py]`, 9/9, COTLAR-BOUNDED-V2). The v2 protocol re-executes the round-35 contract with cells, pairing, danger geometry and decision bars frozen *identically* (the committed v1 spec SHA `4621b899...` re-verified at run time; v2 spec SHA `5fd6bf61...`); the single substantive change — the complete comb at the deep holdouts — is warded bit-identical everywhere else. The deep sums collapse into the battery band (kz 142: S_U 32.07 → 5.65, S_C 58.78 → 4.10; kz 177: 14.29 → 5.97; kz 243: 13.94 → 5.87), both channels are BOUNDED by the frozen v1 rule (holdout/battery ratios $0.95/0.94 \le 1.2$, log–log slopes $0.006/0.007 \le 0.10$) — *the v1 kill was entirely the artifact* — and RUN 3, gated on BOUNDED and correctly withheld in round 35, unlocks and *passes*: the analytic envelope candidates $K_1 = \max_d \mathrm{env}_U(d)(1+d)$ and $K_2 = \max_d \mathrm{env}_C(d)(1+d)^2$ are h-stable on all five complete-comb holdouts (8.36/8.14 and 40.35/40.47 against the 1.2× bar). Epstein and scramble still break the contrast bars. The truncation audit re-types every affected finding, dated: the v1 decision CONFOUNDED and re-decided; the residual horizon *strengthened* (its stalls detected the truncation correctly — a validated sign detector); the resonance α-law addendum void; the polygon holdout rows NUMERIC-VOID; the Radau, preconditioner, cdcore and Uvarov waves unaffected. The remaining gap is typed, not hidden: the Cotlar constant $B_C \sim 4$ against the needed 1, and the cell-local dominance defects $\sum_r \varepsilon_r \sim 7$–10. The route *reopens* (`PRIME.PRUEFER.COMPENSATION.02`); nothing about positivity is claimed.

The Carleson chain, machine-exact (`[verification/v876_carleson_polygon_wave.py]`, $6/8 + 7/8 + 5/5$ with the three frozen-honest FAILs pattern-gated). The wave that ran alongside the audit gives the wall its embedding coordinates. The ward: $\lambda_{\max}(M_-) = \|C\|^2 = 1 - \tau$ at relative $1.5 \times 10^{-13} / 1.3 \times 10^{-15}$ — arrows (i)–(iii) of the chain $\|C_h\| \le 1 \Leftrightarrow G_- \le G_+ \Leftrightarrow$ *the Carleson embedding* $\int |P|^2 \, d\tilde{\nu}_- \le \int |P|^2 \, d\tilde{\mu}_+$ on $\mathcal{P}_{h-1} \Leftrightarrow M(\tilde{\mu}_+ - \tilde{\nu}_-) \ge 0$ are machine-exact, with embedding constant $\|C_h\|^2 = 1 - \tau_h$; the surviving analytic target is registered as `PRIME.CARLESON.PRIME.01` [O]. The polygon compression $S_0(m) \ge |S_2(m)|$ *breaks* on all 47 rungs (critical prefixes at $2\theta \approx 1.5\pi$ — the π-resonance) and is typed the *wrong* compression of the Loewner order; the holonomy census is null with inverted contrast. The constructive side works at full strength: the residual quadrature realizes the positive representing measure ρ_h at *full degree* h on all 44 floor-positive rungs (k^*/h median 1.000, slope 0.974; mp escalation past the 53-bit noise wall), with a single atom just beyond the pole port and 7–9× truth/fake discrimination — and its three sub-h stalls were exactly the truncated windows. The resonance anatomy types the critical set *thick*: a constant ∼1/5 of the minus nodes, excision gains nothing, and the soft direction is disjoint from the resonant set — the wall is phase-distributed at this partition resolution.

The theorems next door. The same evening produced two exact compiler theorems and one census-to-theorem upgrade, booked in Papers 1 and 2 with kernel-checked mirrors: the feedback normal form of the winding line and the winding decoder of the clock alphabet (`[verification/v874_feedback_decoder_lean.py]`; `TfptCarrier/FlavorFeedback.lean`), and the global Gaussian shell theorem — $\hat{\Theta}_u / \Theta_L = -1/15$ for *all* odd shells, no census cap (`[verification/v875_shell_global_lean.py]`; `TfptCarrier/GaussianShells.lean`).

The honest statement. This round is the discipline working end to end: the residual horizon flagged the stalls, the artifact probe typed the silent truncation at certificate grade, the frozen v1 bars re-ran unchanged on complete data and reversed the closure, and the correction is booked *dated* on every surface while the round-35 record stands as what was measured. The same sweep found and fixed the ledger's first primary-key violation (a double-booked claim id, renamed with a dated QA note) and installed the integrity guard (`[verification/v878_ledger_integrity.py]`) that makes the whole class of bookkeeping errors impossible to reintroduce. No marker moves. **No**

statement about the Riemann Hypothesis is made.

12.30 The partition closure: the analytic core saved, the decomposition chapter closed, the certificate demand retyped non-decompositional

Round 37 (2026-08-08/09) is the compact closing round of the reopened Cotlar route: its two typed next steps are executed, and together they *close the whole partition chapter* of the certificate hunt while saving its analytic core ([`verification/v879_envelope_partition_closure.py`]; suite 871 → 872; the contract chain `4621b899...` → `5fd6bf61...` → `27d9f0a3...` → `2199e9b1...` machine-verified at run time).

The analytic core (the envelope derivation; 7 checks with the two frozen-honest FAILs T1.4/C.1 pattern-gated at exit 1, verdict ENVELOPE-EMPIRICAL-ONLY*).* Multiplication by $\cos\theta$ acts *exactly* tridiagonally on the odd frame: the mirror symmetry closes the top boundary inside the space ($A_{h-1,h-1} = -\frac{1}{2}$), the commutator $Q_- = [A, G_+^{-1}]$ carries a single rank-2 defect at $k = 0$, and the Christoffel–Darboux telescoping identity holds at max residual 1.3×10^{-13} on every rung — battery and all five complete-comb holdouts, $h = 168$ to 1445. From it, the *derived* pointwise bound

$$|U_{ij}| \;\le\; \min\Big(1,\; \frac{C_{\mathrm{int}} + \sqrt{(G_+^{-1})_{00}}\,(K_{-,i}^{-1/2} + K_{+,j}^{-1/2})}{|\cos\theta_i^- - \cos\theta_j^+|}\Big)$$

holds with zero entrywise violation and *h-uniform source constants*: $C_{\mathrm{int}} \in [1.232, 2.328]$ (max/min $1.89 \le 2.0$, log–log slope $+0.106 \le 0.15$) and $\sqrt{(G_+^{-1})_{00}} \in [0.661, 0.899]$ — **the first analytic uniformity statement of the route**, at entrywise grade. The two honest FAILs are the finding, not a defect: the analytic envelope certifies *no* cell decay (every Prüfer cell contains near-diagonal geometric pairs; $K_1^{\mathrm{an}}/K_2^{\mathrm{an}}$ blow up as h^2) — the measured cell-envelope decay is *cancellation-carried*, provably not entrywise-derivable — and the frozen discrimination bar was pointed at the wrong constant (C_{int} itself discriminates: 1.41 truth / 3.59 Epstein / 12.8 scramble). The gap decomposition types the rest: no predeclared accounting reaches below 3.568 (bar 1; the damped censuses C_1–C_4 and the cell variants $n = 8/16/32$), the diagonal share is 15–24%, and $\sum_r \varepsilon_r = 6.9$–$12.9$ is α-uniform on clean complete-comb data (deep holdouts *in/above* the battery band, slope $+0.237$) — the round-36 gap ($B_C \sim 4$ vs 1; local defects $O(10)$) is *structural to every cell frame*, not a constant to be tuned: no cell-partition Cotlar accounting of this family can certify.

The terminal statement (the wave-packet decision; 6/6, verdict PARTITION-CLASS-CLOSED*).* The typed next object — a non-cell partition where the diagonal carries the mass — is built honestly and killed by measurement. Both coherent-state designs (rank-Gabor packets on the node-rank phase space; chain packets through the exact discrete OP transform) are honest *tight* frames on every rung ($\|B\|_2 = \|C^G\|_2$ exact at 10^{-8} relative; the gauge identity $\sigma_{\max}(C^G)^2 = 1 - \lambda_1(\Delta)$ exact); the Gabor envelope *decays* in phase space ($0.529 \to 0.007$ at kz 243 — the first partition of the program to show decay); and the discrimination is massive (truth E_{Schur} 4.985 vs Epstein 128.7 vs scramble 9006.6). But the diagonal does *not* carry under any pairing (nearest-packet share ≤ 0.070, empirical-partner share ≤ 0.183 at injectivity ≤ 0.77 — diagonal mass $\le 18\%$) and the Schur constant *grows* ($4.985 \to 9.222$ along the ladder, slope $+0.295$; every functioning design ≥ 3.5 on every rung). The conclusion is the round's content: **the cancellation that holds $\|C\| \le 1$ is atomically global** — it lives in no partition of this family (cells at any resolution, wave packets on either natural phase space, any tight-frame decomposition tested or predeclared). The certificate demand of `PRIME.CARLESON.PRIME.01` is retyped *non-decompositional* (eigen-enclosure, Loewner-order or passivity arguments that treat the operator whole — all three measured territory); wave-packet partitions and entrywise envelope bounds enter the stop list (`PRIME.PARTITION.CLOSURE.01`, with the dated notes on `PRIME.CARLESON.PRIME.01` and `PRIME.PRUEFER.COMPENSATION.02`).

The honest state after round 37. The wall now reads: an atomically global norm statement — $\|C_h\| \le 1$, equivalently the Carleson embedding with constant $1 - \tau_h$ — carried by an operator whose *entrywise* structure is derived with h-uniform source constants but whose norm is held by cancellation that no decomposition localizes. The surviving assets are exactly three: the tridiagonal

multiplication identity with its rank-2 boundary defect, the h-uniform derived constants, and their source-law persistence as $h \to \infty$ — the named open object. No marker moves. **No statement about the Riemann Hypothesis is made.**

12.31 The operator geometry of the wall and its source law: four sides of one object, the port, and the prime edge

Rounds 38/39 (2026-08-09) take the non-decompositional demand of round 37 head-on — a sandbox campaign of 25 frozen probes, every spec amendment fail-first on record — and return with the wall's *exact operator geometry* ([`verification/v881_carleson_port_geometry.py`], 11/11 + 5/5 + 8/8 + 5/5; verdicts PICK-SCALARIZED, TESTING-IDENTIFIED, PORT-SCHUR-EXACT, INTEGRABLE-CONFIRMED) and with its *universal source law* ([`verification/v882_port_source_mellin.py`], 7/7+5/5+6/6; verdicts SQRT-UNIFORMIZED, MELLIN-SOURCE-CONFIRMED, BUDGET-CLOSED).

The operator geometry: four sides of one object ([`verification/v881_carleson_port_geometry.py`]). *Side one, the displacement.* The external review's literal rank-2 Pick target $I - C^*C = \Gamma^*\Pi\Gamma$ is *refuted as stated* (the defect is full-rank on every rung); what holds exactly is the rank-2 *Jacobi displacement*

$$[J, \Delta] \;=\; b_h(e_{h-1}r^{\mathsf{T}} - r\,e_{h-1}^{\mathsf{T}})$$

at 4.3×10^{-14}, source-only — on the construction rungs kz 9/12/13 *and* on the blind holdouts kz 26/40. At the Gauss nodes $\Delta = I - b_h^2\,\Phi CC^{\mathsf{T}}\Phi$ is a Cauchy–Gram of the *positive* measure ω (the $\tilde\nu$-weighted p_h^2); the scalar Herglotz carrier $v = b_h m_\omega$ exists exactly; and $\|C_h\| \le 1 \Leftrightarrow b_h^2\,C^{\mathsf{T}}\Phi^2 C \preceq I$ with $\lambda_{\max} = 1 - \tau$ exact (9.5×10^{-13}) — the wall scalarized through one Herglotz function [E] (the identities; the one-sidedness stays [O]). *Side two, the gauge identity.* $E = S(\sqrt{\tilde\nu}\,K_{\mathrm{CD}}\,\sqrt{\tilde\nu})S$ with $S = \mathrm{diag}(\mathrm{sign}\,p_h)$: the wall is *orthogonally equivalent to the plain Carleson Gram*; its diagonal is the *classical testing functional* $\tilde\nu_m\,K_h(y_m, y_m)$; the testing margin obeys $1 - T_h \sim h^{-0.70}$; and the port is *atom-carried* (arch share 0.016–0.178). One wording correction is carried from this identity through the whole document: where earlier sections say that the kernel phases carry the arithmetic cancellation, the gauge-exact reading now speaks more precisely — *the arithmetic sits in the node and weight distribution of the two measures; the critical cancellation is the coherent geometry of a Carleson Gram operator.* (Absolute bounds stay dead — they destroy the reproducing-kernel geometry; the Perron closure of `v866` is untouched.) *Side three, the port Schur reduction.* With the port P the lowest-τ decile of the testing diagonal, $I - E \succeq 0 \Leftrightarrow I - R \succeq 0 \;\wedge\; I - D_P \succeq 0$ with $D_P = P + X(I-R)^{-1}X^{\mathsf{T}}$; the Haynsworth inertia bookkeeping is integer-exact including *both* indefinite controls (Epstein $55 = 48 + 7$, scramble $37 = 30 + 7$); and $(1 - \lambda_{\max}(D_P))/\tau = 1.00$ on *all* rungs — **the port is the wall**: 13–54 nodes (≈15% of n) carry the entire margin while the bulk sits at 420–45000 $\times\,\tau$, cut-robust at 10^{-4} for cuts 1/20, 1/10, 1/5. This satisfies the round-37 demand exactly as typed: the reduction is *non-decompositional* (an exact Schur complement of the whole operator, not a partition accounting). *Side four, the integrable surprise.* $[Y, P]$ has rank 2 *exactly* and $[Y, D_P]$ has rank 2 *exactly* (10^{-14}): the Schur dressing *preserves the IIKS integrable class* — the dressed port operator is an integrable kernel with two explicit generators.

The source law: the port mass is the prime edge ([`verification/v882_port_source_mellin.py`]). Under the uniformization $r = \sqrt{n/X}$ the weighted prime measure $\eta_X = \frac{1}{4\sqrt{X}}\sum_n 2\Lambda(n)n^{-1/2}\,\delta_{r(n)}$ converges to Lebesgue dr (Kolmogorov distance 0.0148 → 0.0005 over $X = 10^4$–10^7, own SPF sieve; moments to 2×10^{-4}; the deployed rung tables *are* the true comb at 0.0033). The last-log-unit edge mass is $1 - e^{-1/2} = 0.393469$ (measured 0.3936): **the wall's ∼39% port mass is the classical PNT edge law** — ∼39% of the prime mass sits in the last log unit at the lag edge, exactly where the tent taper is $O(1)$. The universality is typed honestly: Epstein uniformizes too (the law is PNT-classical; the arithmetic lives in the *fluctuations*); scramble breaks hard. The port symbol is the Mellin–Cauchy kernel $1/(1 + 2i\tau)$ (0.0004 at 10^7), and the *fit-free* continuum prediction reproduces the deployed port numerators to 0.029 at the deepest rung (the deviation *is* the shrinking arithmetic fluctuation, 0.375 → 0.029). The port numerators are the *exact* deployed-window prime sums (1.5×10^{-13}), matching the untapered explicit-formula sum $-\sum_n 2\Lambda(n)n^{-1/2}\cos(\tau_j \log n)$ at

the alias frequencies $\tau_j = \theta_j/D$ to tent order. And the criticality budget *closes*: the α-slopes $\log|d|$ $(+1.405)$ + log geometry (-1.674) + $\log K$ $(+0.284)$ sum to $+0.015 = d\log T/d\alpha$ — $T \to 1$ is PNT growth exactly compensated by window geometry plus Christoffel: *the wall is an error-term statement about prime partial sums at the port frequencies* [E] (the identities and the budget; the limit [O]).

The hardness decision (`PRIME.ERRORTERM.SCALE.01`, decided). The demand-curve probe measures what the wall costs: the wall margin equals injected off-critical perturbation *energy* on the τ law exactly (ratios 1.00–1.10; $A^* \sim \sqrt{\tau}/\delta^2$ under the frozen convention $\Delta\lambda = O(A^2\delta^4)$), and the kill localizes through the port block. The decision is typed without cosmetics: **the reduction chain is an exact reformulation at RH strength** — there is no unconditional slack anywhere in it. Every identity above is exact; the one-sidedness $T_h \le 1$ is open at exactly that strength, and the program says so in plain words rather than hiding it in a constant.

The new open contracts. The campaign leaves the wall with named coordinates it never had: `PRIME.PORT.LIMIT.01` [O] (the fixed-space limit theorem; measured compressed operator-norm convergence slope -2.80 carrying the wall exactly, $\lambda_{\max} \uparrow 1$ from below, atom-at-edge Weyl-m class $\sigma \to -1$, $w_1 \to 0.72$); `PRIME.PORT.SCALAR.01` [O] (the wall as *one* scalar inequality $\sigma_h = [1 - \tilde{\nu}_{m^*} K_h(y^*, y^*)] - b^{\mathsf{T}}(I - E_{\text{rest}})^{-1} b \ge 0$, exactness 6.2×10^{-11} on 42 rungs, σ tracking τ; demanded: one-sidedness); `PRIME.PORT.COCYCLE.01` [O] (a fixed 12×12 window carries the wall exactly; demanded: the closed-form limit); `PRIME.CASE.SUMRULE.01` [O] (the deployed chains are measured Killip–Simon class; demanded: a Case C0/C1 sum rule $\Rightarrow$ unconditional $T_h \le 1$ — the closed budget as target); and `PRIME.PORT.RIEMANNHILBERT.01` [O] (the discrete 2×2 IIKS Riemann–Hilbert problem is *well-posed* — generators explicit, resolvent closure 1.2×10^{-12}; demanded: nonlinear steepest descent for $h \to \infty$; with the calibration finding that the smooth PNT continuum model *violates* the floor at finite depth (τ_{model} $-1.26 \to -0.11$) — the prime discreteness is load-bearing, the unperturbed problem must keep the lattice). All registered in the research-contracts companion.

The honest state after rounds 38/39. The wall now has exact coordinates on four sides — displacement, gauge, port, integrable class — and one source law: its margin is a prime error term at the port frequencies, its port mass is the PNT edge law, and its hardness is decided as RH-scale-equivalent (no unconditional slack in the exact chain). Identities [E]; the one-sidedness [O] at RH strength. No marker moves. **No statement about the Riemann Hypothesis is made.**

12.32 The flow corridor: the determinant chain to the tau function, the parametrix decision, the first certified positivity, and the complete-comb anatomy

Rounds 40/41 (2026-08-09) take the coordinates of rounds 38/39 into the flow direction — nine frozen probes, every spec amendment fail-first on record, embedded byte-exact in four promoted modules (suite 875 → 879) — and return with the wall's sign as an exact finite determinant chain (`[verification/v883_tau_chain_parametrix.py]`), a decided parametrix constraint, **the first rigorous positivity statement of the program** (`[verification/v884_certified_head_positivity.py]`), the complete flow/induction anatomy of the ladder (`[verification/v885_ladder_flow_anatomy.py]`), and one honest disenchantment on the register side (`[verification/v886_register_gapcode_null.py]`).

The determinant chain: the wall as a tau-function sign (`[verification/v883_tau_chain_parametrix.py]`, 5/5+3/3+3/3; verdicts TAU-CHAIN-EXACT, PARAMETRIX-MEASURED (FLUCTUATIONS-REQUIRED), ZONES-MEASURED (BOTH-REQUIRED)). The Schur determinant identities

$$\sigma_h \;=\; \frac{\det(I-E)}{\det(B)}, \qquad \det(I-E) \;=\; \det(I-R)\,\det(I-D_P)$$

are machine-exact (4.8×10^{-13} in log space) with positive bulk factors on every heavy rung, hence $\operatorname{sign}\tau_h = \operatorname{sign}\sigma_h =$ the sign of the wall with $\tau_h := \det(I - D_P)$ — the scalar wall of

PRIME.PORT.SCALAR.01 is the sign of a candidate tau function; the Jacobi variational identity $\frac{d}{ds}\log\det(I-sD_P) = -\operatorname{tr}[(I-sD_P)^{-1}D_P]$ holds on the true family (10^{-6}); and the determinant wall indicator is *false* on both indefinite controls. The κ/ρ readout is recorded: slope$(\log\sigma) = -2.93$ vs $2\times$ the Mellin fluctuation slope -2.53 — $\sigma \sim$ fluctuation^2 within 16%, the measured seed of the leading-sign contract below. The symbolic half — τ_h as the tau function of the discrete 2×2 IIKS Riemann–Hilbert problem of the v881 generators, for arbitrary h without numerics — is the demand (PRIME.PORT.TAU.01 [O]; since executed at the *finite-identity* level in round 224, verdict TAU-IIKS-EXACT, promoted as [verification/v955_tau_iiks_toda_dictionary.py] with the finite theorem set carved out as PRIME.PORT.TAU.FINITE.IIKS.01 [E] and the sign question refined to PRIME.PORT.TAU.NOPOLE.COFINAL.01 [O] — Section 12.57).

The parametrix decision (PRIME.PORT.LATTICE.PARAMETRIX.01, decided: FLUCTUATIONS-REQUIRED + BOTH-REQUIRED). Four worlds through the identical pencil pipeline on every heavy rung: the true comb holds the floor ($\tau = +1.7\times10^{-4}$ to $+6.7\times10^{-7}$); the prime-power lattice with fully smooth quadrature masses crashes to $\tau = -78$ to -1706; the lattice with *exact* local $\pm\frac{1}{2}$-log-unit mass sums crashes harder (-78 to -5537); the plain continuum sits at -0.11 to -1.26 — keeping the positions and smoothing the masses is *orders worse* than full smoothing. The zone refinement kills both ways: edge-only local-average smoothing ($u > U-1$, ∼40–44% of the mass) breaks the floor at $\tau = -3.7$ to -22.1; interior-only breaks it harder (-77 to -7551). **The Euler-product pairing is globally rigid**: the exact Λ-mass-position pairing is load-bearing in *both* zones, the unperturbed object of any RH analysis of the wall must carry the full von Mangoldt comb, every smoothing-based parametrix is dead, and the small parameters live exclusively in the flow/deformation direction (ladder order, isomonodromy/tau, s-deformation). Recorded as a decided design constraint.

The certified head: the first rigorous positivity statement ([verification/v884_certified_head_positivity.py], 5/5; verdict CERTIFIED-HEAD). $\sigma_h > 0$ is **proven** on kz 9 ($h = 184$), kz 12 (151) and kz 13 (168): an independent mpmath rebuild of the window lags (dps 60/100; conservative per-entry error model $\varepsilon_c = 10\times$ the dps-disagreement; cross-ward vs the deployed core at 1.1×10^{-14} — the ward caught a sign bug in the rebuilt Arch near-branch, which is what wards are for), a matrix shift by the total error budget ($\sim 2\times10^{-18}$, ∼14 orders below the margins), then *exact integer* Bareiss/LDL on the $Q = 10^{20}$ grid: all pivots positive (Sylvester) $\Rightarrow K \succ 0 \Rightarrow \sigma_h > 0$ via the exact v866 equivalence — no Lanczos, no floating point in the decisive step. Certified floors $\lambda_{\min} \geq 3.633\times10^{-4}$ / 3.327×10^{-4} / 3.842×10^{-4}; the Epstein control through the *identical* machinery is refused at pivot index 10. Typed honestly: proven *modulo* the declared conservative transcendental-evaluation error model (the validated-numerics standard); the ball-arithmetic closure and the 42-rung rollout were the named follow-ups — both since delivered (the rollout in [verification/v887_certified_ladder_complete.py], the rigorous interval closure in [verification/v897_certified_interval_ladder.py], Section 12.35). The head part of PRIME.PORT.TAIL.01 is delivered on three rungs.

The flow and induction anatomy ([verification/v885_ladder_flow_anatomy.py], 8/8 + 5/5 + 13/13 + 11/11; verdicts ISOFLOW-MEASURED (FLOW-OPEN), LADDER-MEASURED, PRIMELADDER-MEASURED, LAX2-MEASURED). The gauge-fixed IIKS generator flow over 41 consecutive ladder pairs closes in *degree 2* (residual medians 0.244/0.248 with $\{Y^2f, Y^2g\}$ added; degree 1 fails at 0.498/0.457) — an isomonodromy-compatible Lax structure, typed as a structural precondition (the controls close too; the arithmetic stays in the *value*). The window-ladder step is marginally perturbative (accuracy median 0.458) but non-monotone in raw h (only 52% of steps increase λ; $\|\Delta\|/\text{gap}$ = 594–2026) — raw h is not the flow parameter. The one-prime ladder decides the induction question: the pure-Arch start is catastrophically subcritical ($\tau(0) = -9.9\times10^{13}$ at kz 9; G_+ not even PD deeper), the last sign crossing sits at $u^*/U = 0.991$–0.999 in *every* insertion order (ascending needs 97–99% of the atoms, edge-first needs 100% — the ∼39% edge mass alone does not carry the wall), and the per-atom increments are only 54–64% positive: **positivity is a property of the complete comb** — the v877 truncation reversal sharpened to atom resolution; no monotone one-prime induction exists; induction must run in a flow coordinate, not in comb content. And the raw degree-2 coefficient basis is ill-conditioned noise

(roughness 1.365 vs 0.7; operator predictivity 0.350 vs 0.3) while the *integrated* tau-transfer identity is warded exact (5.6×10^{-11}) — the conditioned flow coordinate is the named open object (`PRIME.PORT.LAX2.01`; since adjudicated in round 225: the true minimal degree in the closing direction is *one*, and the closing direction is h, not s — LAX1-H-EXACT, promoted as [verification/v955_tau_iiks_toda_dictionary.py], Section 12.57).

The gapcode disenchantment ([verification/v886_register_gapcode_null.py], 24/24; verdict GAPCODE-MEASURED — an honest negative/limit row). The error-code reading of the 210-register is literally true on the prime side: the register bits $\{2, 3, 5, 7\}$ are the first four parity checks of the sieve (code rate 48/210, 16 syndrome labels, 0 bits of message capacity), the hard floor $g_k \geq w(p_k \bmod 210)$ holds with zero violations in 5.76×10^6 consecutive pairs, and $I(\text{state}; \text{gap}) = 1.14$ bits. *And the code is the sieve*: a zero-parameter wheel–Cramér null explains $R^2 = 0.99839$ of the state profile, the arithmetic excess is 4.02% and shrinking along the X ladder, there is no knee at four checks, 210 sits *on* the foreign-wheel trend ($z = -0.61\sigma$), and all eight anchor Walsh seats stay below 2.5σ exactly as CRT predicts. Everything the register knows about prime gaps, every 4-prime wheel knows; nothing moves with c_3 or φ_0. The register's uniqueness remains the arithmetic forcing $\prod(p-1) = 48$ of `v868` (`PRIME.DIVISOR210.GAPCODE.01`).

The open fronts after rounds 40/41. The parametrix constraint is now a standing design rule: the comb stays in the main problem, the small parameters in the flow. The named demands: `PRIME.PORT.TAU.01` [O] (the symbolic IIKS tau identification; the finite chain is delivered — and the finite-*identity* level has since been executed in full, `PRIME.PORT.TAU.FINITE.IIKS.01` [E] via [verification/v955_tau_iiks_toda_dictionary.py], with the remaining sign question `PRIME.PORT.TAU.NOPOLE.COFINAL.01` [O]); `PRIME.PORT.LEADING.SIGN.01` [O] (the moonshot: $\sigma_h = \kappa\rho_h + o(\rho_h)$ with closed-form $\kappa > 0$; the ρ candidate measured — $\sigma \sim$ fluctuation^2 within 16%); `PRIME.PORT.TAIL.01` [O] (head delivered on three rungs; the 42-rung rollout and the ball-arithmetic closure named — both since delivered, `v887`/`v897`; the tail half waits on the leading sign); `PRIME.PORT.LAX2.01` (the conditioned flow coordinate — the raw basis is measured noise-masked; since adjudicated LAX1-H-EXACT in round 225, `v955`); and the measured anatomy rows `PRIME.PORT.ISOFLOW.01`, `PRIME.PORT.LADDER.TRANSFER.01` and `PRIME.PORT.PRIMELADDER.01`, typed as structural preconditions and induction constraints. All registered in the research-contracts companion.

The honest state after rounds 40/41. The wall's sign is an exact finite determinant chain ending in a candidate tau function (since promoted to the exact IIKS Fredholm tau function at the finite-identity level, [verification/v955_tau_iiks_toda_dictionary.py], Section 12.57); its positivity is *proven* on the first three head rungs by exact integer certificates (modulo the declared conservative evaluation error model); its comb is globally rigid — no smoothing-based parametrix exists; and its induction, if any, runs in a flow coordinate over the complete comb. The determinant and tau-transfer identities and the integer Sylvester certificates [E]; the flow anatomy measured; the one-sidedness [O] at RH strength, exactly as decided in rounds 38/39. No marker moves. **No statement about the Riemann Hypothesis is made.**

12.33 The certified ladder complete: the four route decisions, the surviving positivity architecture, and the Möbius kills

Rounds 42–49 (2026-08-09) consolidate the flow corridor of rounds 40/41 — fourteen frozen probes, every spec amendment fail-first on record, embedded byte-exact in five promoted modules (suite 879 → 884) — and return with the completed certificate rollout ([verification/v887_certified_ladder_complete.py]), the four route decisions of the sum-rule program including one honest conditional retype ([verification/v889_route_decisions.py]), the surviving positivity architecture ([verification/v890_total_positivity_flow.py]), and the honest kill battery of the Möbius/carrier-invariance route ([verification/v891_moebius_firewall_kills.py]). (The finite Pfaffian/K_6 signature of the same rounds is a compiler statement and lives in Paper 1, [verification/v888_finite_pfaffian_signature.py].)

The complete certified head: $\sigma_h > 0$ proven on all 42 reachable rungs ([verification/v887_certified_ladder_compl

7/7*; verdict* CERTIFIED-LADDER-COMPLETE*).* The `v884` certificate machinery rolls out over the entire deployed `v563` window ladder: 12 rungs ($h \le 300$) by *exact-rational* integer Bareiss/LDL certificates and 30 rungs by *validated-precision* `mpmath` Cholesky at dps 120/200 (precision doubling; every pivot must exceed $10^6\times$ the accumulated per-pivot rounding bound at both precisions) — 0 refusals, 0 out-of-reach. The promoted module closes the two deepest rungs the parent rollout left at its frozen 45-minute deadline: $h = 859$ with certified floor $\ge 8.352 \times 10^{-6}$ and $h = 878$ with floor $\ge 9.187 \times 10^{-6}$, worst pivot-to-rounding-bound ratios $\sim 10^{115}$ (dps 120) and $\sim 10^{195}$ (dps 200) — nine orders above the frozen 10^6 bar; the 40-rung parent rollout stays sandbox evidence, runtime-excluded. The Epstein control is refused at pivot index 10 by the identical machinery. Typed honestly: the validated-precision tier is never labeled exact-rational, and everything is proven *modulo* the declared conservative lag-evaluation error model (the validated-numerics standard); the end-to-end ball arithmetic and the Lean composition through `krein_cofinal_weil` were the named remaining steps (`PRIME.PORT.CERTIFIED.LADDER.01`; the head half of `PRIME.PORT.TAIL.01` is now delivered in full — the asymptotic tail is untouched). *Update (rounds 54/55):* the ball-arithmetic half is delivered — [`verification/v897_certified_interval_ladder.py`], Section 12.35, re-proves all 42 reachable rungs with *rigorous* interval-arithmetic shifts (15 exact-rational + 27 validated-precision) and retires the informal lag-evaluation error model from the ladder; only the Lean composition remains named.

The four route decisions ([`verification/v889_route_decisions.py`]*,* $8/8 + 12/12 + 10/10 + 12/12$*; verdicts* DIAGSEP-MEASURED *(*DIAG-INTERMEDIATE*),* ZONESPLIT-MEASURED *(*ZONE-FROZEN*),* PNTGAMMA-MEASURED *(*PNT-INSUFFICIENT*),* HOMOTOPY-MEASURED *(*HOMOTOPY-INDEFINITE-ARITHMETIC*)).* **(1)** The wall dies 100% through off-diagonal coherence: under the frozen off-critical injection family the diagonal testing profile is statistically frozen at the kill point (max $T_m = 0.94$–0.99; the coherence share $\lambda_{\max} - \max_m T_m$ equals the entire baseline diagonal margin), with separation ratio 2.0–2.7 on the head rungs and > 10 deep — the $O(1)$ diagonal margin does not track the exponentially collapsing wall margin; typed honestly DIAG-INTERMEDIATE (`PRIME.PORT.DIAGSEP.01`). **(2)** The critical zone freezes at $\theta^* = 0.700$: outside $a > h^{2\theta^*}$ the testing margin is uniformly positive on all 42 rungs (deep-half infimum $+0.0214$; cut-stable; $\theta = 2/3$ fails), inside the margin law is $\sim 0.87/\log^2 h$ — the bar the arithmetic input must beat; the kz 13 outlier flag is on record (`PRIME.CASE.ZONESPLIT.01`). **(3) PNT is insufficient**: the PNT-smooth reference itself violates the port testing margin on *every* rung (-0.96 to -7.8×10^{-3}), pure-mask and pure-mass substitutions are strongly negative, and the truth's positivity is carried by the mask–mass *interaction* piece — no classical zero-free-region-strength input can close the diagonal theorem by smooth comparison (`PRIME.CASE.PNTGAMMA.01`). **(4)** The signed-homotopy classification: along the exact PNT-to-truth interpolation the envelope identity is exact calculus (4.3×10^{-14}), 66.3% of the samples have $J < 0$, the fixed-mask quadratic kernel is concavity-locked negative-semidefinite on both frozen rungs, and the truth endpoint wins only through the *signed first-order* alignment of the arithmetic residual with the extremal-polynomial weights — **the diagonal route's required input is pair-correlation class**, not any PSD kernel positivity (`PRIME.CASE.SIGNEDHOMOTOPY.01`). The honest consequence is a retype: `PRIME.CASE.SUMRULE.01` moves from "first unconditional theorem candidate" to a *conditional reduction theorem* — $T_h \le 1$ from a precisely statable weighted prime-pair correlation inequality.

The surviving positivity architecture ([`verification/v890_total_positivity_flow.py`]*,* $10/10 + 7/7 + 14/14$*; verdicts* HIROTA-MEASURED *(*MINORS-POSITIVE*),* SFLOW-MEASURED *(*CORRIDOR-CLEAR*),* TAUMOEBIUS-MEASURED *(*BRIDGE-DIFFUSE*)).* **Flag positivity (Sylvester):** on all 37 full-window rungs *every* nested leading principal minor $\tau_h^{(k)}$ of $M_h = I - C_h$ ($k = 1, \dots, 12$) is strictly positive — a strictly stronger finite statement than the wall scalar $\tau^{(12)} > 0$. (Typing corrected per the round-51 review: what is measured is the positivity of the twelve *leading* principal minors — Sylvester/flag positivity — not classical total positivity of *all* minors, which is neither claimed nor tested.) The Desnanot–Jacobi bilinear identity is exact on every rung, section size and s; the across-rung closure in raw h is honestly typed open (median residual 0.655; the bilinear/Toda equation must come from the IIKS generators). **The pole-free s-corridor:** no section minor

crosses zero on the whole corridor $s \in [0,1]$ from the trivial point to the wall on any rung, while the Epstein and scramble controls pull their pole inside; by Cauchy interlacing the first pole of any section sits at $s_h^* = 1/\lambda_{\max}(C_h)$, so $s_h^* - 1 = \tau_h/(1-\tau_h)$ — **the wall margin is the pole distance of the integrable s-flow** (measured ladder slope -2.90). **The multi-factor anatomy:** the exact factorization $Q_h = \det(W_{h+1})/\det(W_h) = \prod_i(1+\mu_i)$ shows the truth ladder keeping *every* factor off -1 on all 31 full-window steps over 24 orders of magnitude, while the frame-surviving smooth-mass world crosses factors on 22 of its 28 steps (first crossing already at $h\,210 \to 218$); the reviewer's one-scalar Möbius bridge is honestly closed (BRIDGE-DIFFUSE: ladder-wide bulk factor margin 0.005 vs the frozen 0.5 bar; the fitted Möbius datum $|\alpha| \sim 10^{-3}$ is determinant-inert) — **positivity inheritance, if it exists, is a multi-factor/total-positivity statement**, and its propagation theorem is the named open object (`PRIME.PORT.HIROTA.01` [O]; the anatomy recorded as `PRIME.PORT.TAU.MOEBIUS.01`).

The Möbius kill battery (`[verification/v891_moebius_firewall_kills.py]`, 10/10+13/13+7/7*; verdicts* CRFIREWALL-MEASURED *(*CR-DEAD*),* GAUGEFW-MEASURED *(*GAUGE-MADE*),* MOEBIUSCENSUS-MEASURED *(*UNDECIDED*) — the verdicts are kills and are recorded as such).* The earlier carrier-invariance finding (the $PSL(2,\mathbb{R})$-normalized port carrier as one fixed ladder function, residual 0.0015) was a **normalization artifact**, established fit-free three ways. The gauge-free cross-ratio battery over 8126 well-conditioned quadruples types CR-DEAD (pooled median defect 0.128 against bars $10^{-3}/10^{-2}$; the anchor triples disagree; the steps do not compose — 41 independent local fits, not a cocycle). None of the three source-frozen gauges is raw-invariant: two expose a collapsed near-constant carrier (invariance vacuous, caught by the degeneracy guard) and the determinant-phase frame shows real rung-to-rung motion; the discarded scalar cocycle does not track the wall (corr = 0.13 vs the 0.90 bar), and the Jacobi-transfer identification is dead ($d_P = 0.997$) — GAUGE-MADE. And the six-world control census finds *every* control world — including the smooth-mass worlds — losing window subcriticality (truth 41/41 alive; all controls 0), so the arithmetic separates *at the frame*, upstream of every Möbius discriminator, and on the bare cross-ratio and step-map bars the frame-dead scramble world even beats truth (0.0060/0.0011 vs 0.0218/0.0446) — the Möbius/cross-ratio structure is rank-2 window *kinematics*, not arithmetic dynamics. The surviving arithmetic remnant, localized by the battery itself: the deep-core 8-node cross-ratio coherence at 4.3×10^{-4} on truth vs 2.7×10^{-2} on the smooth-mass world — prime-mass-driven, the named residue (`PRIME.PORT.MOEBIUS.KILLS.01`).

The open fronts after rounds 42–49. The dead list is now explicit: the Möbius/carrier-invariance induction (`PRIME.PORT.MOEBIUS.KILLS.01` — closed by its own preregistered firewall battery), the one-scalar cocycle bridge (BRIDGE-DIFFUSE), the static Cauchy certificate (sandbox: dev/τ grows at slope +1.46 — the route can never certify the wall), and the PNT-smooth diagonal comparison (PNT-INSUFFICIENT). The alive list: the multi-factor total-positivity propagation (`PRIME.PORT.HIROTA.01` [O] — why does the truth keep every determinant factor off -1), the deep-core coherence remnant (prime-mass-driven), the pole-free s-flow with the wall margin as pole distance, the conditional pair-correlation contract (`PRIME.CASE.SUMRULE.01`, retyped: the weighted prime-pair correlation inequality is the named input), and the IIKS steepest-descent/tau program (`PRIME.PORT.RIEMANNHILBERT.01`, `PRIME.PORT.TAU.01` [O]; the tau identification since executed at the finite-identity level and the Hirota/Toda question since decided WALL-EQUIVALENT — `[verification/v955_tau_iiks_toda_dictionary.py]`, Section 12.57; the bordered formulation and its evaluation infrastructure since certified in `[verification/v958_bordered_tau_readout_dictionary.py]`, Section 12.58). All registered in the research-contracts companion.

The honest state after rounds 42–49. The wall's positivity is proven on every reachable rung of the deployed ladder (12 exact-rational, 30 validated-precision, modulo the declared conservative evaluation error model — since re-proven interval-rigorously in rounds 54/55: 15 exact-rational + 27 validated-precision with the informal error model retired, `[verification/v897_certified_interval_ladder.py]`, Section 12.35); its diagonal half is decided conditional (the input is pair-correlation class); its inheritance question is a multi-factor/total-positivity statement with the wall margin as the pole distance of an integrable flow; and its Möbius shortcut

is dead by its own preregistered firewall. The certificates, determinant factorizations and variational identities [**E**]; the route decisions measured; the one-sidedness [**O**] at RH strength, exactly as decided in rounds 38/39. No marker moves. **No statement about the Riemann Hypothesis is made.**

12.34 The closure architecture: the bounded-margin skeleton

Rounds 50–53 (2026-08-09) attack the named demands of rounds 42–49 head-on — eleven further frozen probes, every spec amendment fail-first on record, embedded byte-exact in four promoted modules (suite 885 → 889) — and return with the closure architecture of the wall ([`verification/v892_closure_architecture.py`]), the sources of its bounded margins and the printed theorem skeleton ([`verification/v893_relative_margins.py`]), the concrete conditional diagonal contract ([`verification/v894_diagonal_refinement.py`]), and the collectivity of the arithmetic ([`verification/v895_collective_comb.py`]). (The Wick functor arc of the same rounds is a compiler statement and lives in Paper 1, [`verification/v896_wick_block_functor.py`].)

The closure architecture ([`verification/v892_closure_architecture.py`], 16/16 + 15/15 + 14/14 + 16/16; verdicts RELCONG-MEASURED, PIVOTFACTOR-MEASURED, ROUCHE-MEASURED, DEEPCORESCHUR-MEASURED*).* **(1) The exact hermitian congruence.** The inheritance step is made exact and complete: $A_{h+1} = A_h^{1/2}(I + H_h)A_h^{1/2}$ with $H_h = A_h^{-1/2}\Delta_h A_h^{-1/2}$ hermitian (congruence warded at machine precision on every step), so by Sylvester's law of inertia PD inheritance *is* the margin sign $\eta_h := 1 + \lambda_{\min}(H_h) > 0$ — and the measured η ladder does **not** decay: min $\eta = 0.0050$, power-law slope $+0.108$ in h, while the wall scalar τ_h collapses at slope -2.74. The earlier spectral-radius reading $\rho(W^{-1}\Delta C)$ is exposed as the statistic of a strongly *non-normal* operator ($\|M\|_2/\rho$ up to 26, $\kappa(V)$ up to 134; the symmetric part can pass -1 with no crossing) — ρ was misleading, and the hermitian H is the honest carrier; the smooth-mass control does not even keep the congruence's PD premise (CONE-EXITED on 28/28 full-window steps). **(2) The flag/pivot chain.** On the 12×12 window the pivot factorization $d_{k,h+1} = d_{k,h}(1 + \mu_{k,h})$ is an exact identity and the content is the sign: *all* 12 unpivoted-LDL flag quotients are positive on all 31/31 truth steps — PD inheritance by pure algebra, no norm bound, no spectral gauge; the dangerous soft flag $k = 12$ carries the collapse scale (ladder min 2.059×10^{-14}) but its sign is *decidable* at $6.4 \times 10^9\times$ the eigenvalue noise floor against the frozen 10^3 bar, and the smooth world violates the chain already at the *bulk* flags $k = 1$ and $k = 3$, not at the soft flag. **(3) The honest Rouché kill.** The two-sided matrix-Rouché route is dead: the contour norm $M_h = \sup_\Gamma \|A_h(s)^{-1}\Delta_h(s)\|_2$ reaches $M \geq 1$ on 22 of 31 truth steps *including* the off-cap restriction $\mathrm{Re}\, s < 1$ and the exact cap-point lower bound — no one-norm contour bound can force the corridor zero count; the norm-bound theorem shape is the *wrong shape*: inheritance on the truth ladder is one-sided (the η/factor-avoidance direction), not two-sided contour-uniform. **(4) The wall is the fixed 8×8 Schur core.** Splitting the full wall operator along the fixed deep-core aliases $\{2, 4, \ldots, 16\}$, the Schur complement satisfies $\lambda_{\min}(S_h)\, w_{\mathrm{core}}/\tau_h = 1$ within 8.4×10^{-8} with core weight $w_{\mathrm{core}} \geq 0.999937$ on all 41 reachable rungs — the RH-critical object collapses to *one fixed* 8×8 *matrix family*; the exterior block stays uniformly PD with its *absolute* margin shrinking like $h^{-2.865}$ while the *relative* margin $\lambda_{\min}(R_h)/\tau_h$ is trendless in the band ∼210–2200 — the τ-relative conditioning that `v893` makes lawful; core inheritance has 0 crossings (η_{core} min 0.0315), and the per-prime grain is dead at fixed dimension too.

The margin sources and the printed skeleton ([`verification/v893_relative_margins.py`], 20/20 + 18/18; verdicts ETASOURCE-MEASURED (SCALE-MATCHED), RELFLAG-MEASURED*).* **The source of η:** the frozen hypothesis is confirmed and typed SCALE-MATCHED — in the dangerous direction the increment and the operator collapse *together* (power-law slopes -3.716 and -3.456 in h; residual correlation 0.974: 94% shared variance after detrending), so η is scale-free *because* its numerator and denominator are sections of one arithmetic object — the non-decay of the inheritance margin is *explained*, and what stays open is only the $O(1)$ scatter of the ratio (why 0.005 and not 0.5); the seat of the dangerous direction is the core, and the binding block of the full-vs-core comparison is

the 4-alias exterior. **The relative laws:** the soft pivot obeys the exact algebra $d_{12} = 1/(A_h^{-1})_{12,12}$ with the rung-to-rung factorization $r_{12} = (\tau'/\tau)(c'/c)$ warded at 2.1×10^{-16} — the soft-flag quotient is *exactly* the τ quotient times a conditioning quotient; the conditioning ratio $c_h = d_{12}/\tau_h$ is trendless (the kz 21 outlier is diagnosed in the round-53 sandbox as a *coordinate artifact* — the kz-19 mechanism repeats; corrected spread $124 \to 15$: the soft flag is *τ-tied*); the exterior relative margin is trendless with uniform constant $c_R = 210$ ($\lambda_{\min}(R_h) \geq c_R\,\tau_h$ as a measured relative law); and the two compressions are COMPRESSIONS-DISTINCT (corr 0.278: the port bulk floor is an *absolute* law while the core exterior margin is τ-tied — two mechanisms, never to be conflated). **The printed theorem skeleton:** (i) base certified (`v884`/`v887`; interval-rigorous since `v897`); (ii) $\lambda_{\min}(R_h) \geq 210\,\tau_h$ trendless; (iii) core $\leftrightarrow$ τ at seven digits; (iv) $\eta > 0$ non-decaying — **the one unconditionally open statement is the τ-sign inheritance**, reorganized from collapsing-margin form ($Q_{12} \sim 2 \times 10^{-14}$) into *bounded-margin* form (c, c'/c, c_R, η — all $O(1)$) (`PRIME.PORT.ETASOURCE.01`, `PRIME.PORT.RELFLAG.01`).

The diagonal refinement (`[verification/v894_diagonal_refinement.py]`, 11/11 + 13/13 + 12/12; verdicts PAIRCONTRACT-EXTRACTED (KERNEL-BANDLIMITED), ENDPOINT-MEASURED (ENDPOINT-PARTIAL), INNERLINEAR-MEASURED (INNER-EMPTY)). The route typing *stands*: per `v889` the diagonal route is conditional on a pair-correlation-class arithmetic input — nothing here moves that; the module makes the *condition* concrete. The weighted prime-pair correlation functional whose nonnegativity implies $T_h \leq 1$ on the critical zone is extracted *exactly*: the prime kernel $K_{h,m}$ regroups the alias sum at defect 4.5×10^{-15}, is band-limited (compact spectral support, printed), all 79 finite deficit margins on the frozen census are positive, and the LaTeX contract statement is frozen *inside* the probe — a precise registered statement with a printable formula, not prose (`PRIME.CASE.PAIRCORR.CONTRACT.01` [O]). The endpoint bracket: a one-sided lower bound on the endpoint derivative — a *linear* signed-Λ functional, much weaker than the full pair-correlation input — would suffice; the global linear collapse *fails* (142/227 cells, typed ENDPOINT-PARTIAL honestly), but the deep small-a cells pass with monotone margins — the linear route is real exactly where the theorem must live and dies at shallow rungs and the zone edge. The inner-zone accounting: under the frozen absolute rule the inner zone is empty (INNER-EMPTY), but the disclosed scale-normalized diagnostic is *flat* (slope -0.052; every scale of the problem itself shrinks like $\sim h^{-1}$), and the genuinely hard middle band retains $\sim$20% of the aliases carrying 38–56% of the mass, *non-shrinking* with depth, with a-graded pass/fail stability on a mid shell — the condition is localized, not removed.

The collectivity of the arithmetic (`[verification/v895_collective_comb.py]`, 18/18+12/12; verdicts FACTORAVOID-MEASURED (AVOIDANCE-STRUCTURAL), DEEPCORE-MEASURED). **The one-sided law:** the norm-bound shape does not exist on truth ($\rho \geq 1$ on steps of the ladder, always via the harmless side); the true law is one-sided — $\max_{\text{steps}}(-\min \mu) = 0.9950 < 1$ on all 31 truth steps, *which is PD inheritance itself*; the geometry/atom split of the increment is exactly degenerate (the atom cutoff is slaved to the tent edge; the moving block carries bitwise zero of the increment through its own tents) — there is *no separable atom channel*: the increment is the collective window re-test of the full comb; and the per-prime leave-one-out responses are non-decompositional (single primes lever the increment by factors up to 987×; a leverage census, not a decomposition) — the avoidance is a property of the *collective* comb, not of any per-prime cascade term (`PRIME.PORT.FACTORAVOID.01`). **The deep core is the port core:** the surviving fit-free gauge-free remnant of the cross-ratio firewall lives on the fixed even aliases $\{2, 4, \ldots, 16\}$ at the Bessel-normal coordinates $a_m = \pi^2 m^2$ (matched to 0.1%, coherence knee $k^* = 10$), and *all four* frozen surgeries kill the signal — edge-smoothing, interior smoothing, echo removal, and the wrong-Λ substitution — so the signal is the **entire multiplicative von Mangoldt comb**: no smaller or smoother sub-object carries it (`PRIME.PORT.DEEPCORE.01`). This is the measured ground on which `v892`'s fixed 8×8 Schur core stands.

The open fronts after rounds 50–53. The dead list grows: the two-sided matrix-Rouché/contour-norm route (`PRIME.PORT.EULER.ROUCHE.01` — an honest kill with the mechanism documented), every norm-bound shape on $W^{-1}\Delta C$ (ρ was the statistic of a non-normal operator), and every

per-prime decomposition of the increment (non-decompositional twice over, `v892`/`v895`). The alive list has *one* RH-side statement: **the τ-sign inheritance in bounded-margin form** — base certified, $\lambda_{\min}(R) \geq 210\,\tau$ trendless, core $\leftrightarrow$ τ at seven digits, and $\eta > 0$ with every constant (c, c'/c, $c_R = 210$, η) measured $O(1)$ — plus its two named feeders: the $O(1)$-ratio source (why 0.005 and not 0.5; `PRIME.PORT.ETASOURCE.01`, the round-53 sandbox continues with the mechanism decomposition) and the non-shrinking diagonal hard band of the extracted pair-correlation contract (`PRIME.CASE.PAIRCORR.CONTRACT.01` [O]). The multi-factor propagation demand of `PRIME.PORT.HIROTA.01` [O] is answered in bounded-margin form by the congruence and the flag chain; its demand narrows to the same $O(1)$ ratio bound. All registered in the research-contracts companion.

The honest state after rounds 50–53. The wall's inheritance question now has a shape: an exact hermitian congruence whose margin does not decay, a flag chain positive by pure algebra, a fixed 8×8 Schur core equal to the wall at seven digits, and relative laws that tie every collapsing scale to τ itself — one open statement, printed, in bounded-margin form. The congruence, pivot and kernel-regrouping identities [E]; the margin ladders, relative laws and surgery kills measured; the diagonal route stays conditional (pair-correlation class, exactly as `v889` typed it); the one-sidedness [O] at RH strength. No marker moves. **No statement about the Riemann Hypothesis is made.**

12.35 The certified interval ladder and the norm-square contract

Rounds 54–56 (2026-08-09) deliver the reviewer's "immediate" list — eight further frozen probes, every spec amendment fail-first on record, embedded byte-exact in four promoted modules (suite 889 $\rightarrow$ 893) — and return with the certificate rollout re-proven under *rigorous interval arithmetic* ([`verification/v897_certified_interval_ladder.py`]), the exact deflated-Christoffel pivot identity and the norm-square form of the conditional diagonal contract ([`verification/v899_christoffel_normsquare.py`]), and the exact update anatomy of the normalized deep core ([`verification/v900_core_update_anatomy.py`]). (The channel-mixing KMS/Schur candidate state of the same rounds is a compiler statement and lives in Paper 1, [`verification/v898_kms_schur_mixing.py`].)

The certified interval ladder: the informal error model is retired ([`verification/v897_certified_interval_ladder.p`
probe verdicts BALLHEAD-PROVEN, BALLLADDER-COMPLETE, 10/10, 3960 *s).* The `v884`/`v887` certificates were proven *modulo* a stated-but-informal conservative lag-evaluation error model ($\varepsilon_c = 10\times$ the dps-60/120 disagreement). That model is now **retired from the ladder**: every window lag is enclosed in a rigorous `mpmath.iv` interval — the GL-48 node-enclosure lemma (Newton seeds at dps 110, interval radius 10^{-90}, a definite P_{48} sign change on every node interval, all 48 intervals pairwise disjoint, 2 inside the interval weight sum), native outward-rounded interval `exp`/`expm1`/`log`/`sqrt` through the archimedean layer and the tent atoms (per-atom loop-range rigour), exact `fadd`/`fsub`/`ldexp` midpoint/radius extraction, and the rigorous spectral shift $2h\,\mathrm{rad}_{\max}$ ($+\,h/Q$ on the tier-1 integer grid) with no working-precision rounding anywhere in the tier-1 decision path. Under these shifts $\sigma_h > 0$ is certified on **all** 42 **reachable rungs** $h = 142$–878: 15 *exact-rational* integer Bareiss/Sylvester certificates ($h \leq 300$, floors 4.5×10^{-5}–2.1×10^{-4}) and 27 *validated-precision* Cholesky certificates (dps 120/200 with precision doubling, floors 5.2×10^{-6}–8.4×10^{-5}, pivot-to-bound ratios 2.3×10^{115}–9.9×10^{115}; the tier label is carried honestly and never upgraded). The Epstein $x^2 + 5y^2$ control is refused at pivot index 10 by the identical interval machinery. Suite economy, declared: the 66-minute rollout is not rerun — the promoted module embeds the frozen probe byte-exact (file and spec SHA-256 warded), re-certifies all 15 tier-1 rungs plus the 3 shallowest tier-2 rungs verbatim, and freezes the full 42-rung census as printed constants. $\sigma_h > 0$ now holds on the whole reachable ladder modulo nothing but IEEE-754/`mpmath` interval correctness (tier 1) resp. the stated validated-precision bars run from a rigorous interval shift (tier 2); the Lean composition through `krein_cofinal_weil` remains the one named step (`PRIME.PORT.BALLLADDER.01`; the asymptotic tail is untouched and stays with `PRIME.PORT.TAIL.01` / `PRIME.PORT.LEADING.SIGN.01`).

The Christoffel pivot identity and the norm-square contract ([verification/v899_christoffel_normsquare.py], 19/19 + 15/15 + 19/19; verdicts CHRISTRATIO-MEASURED *(*WINNER-CHRISTOFFEL / EXTSOURCE-DECOUPLED*),* KERNELSOS-MEASURED *(*INDEX-LARGE; RMIN=1; NOT-ABSORBED*),* EDGEDEFECT-MEASURED *(*NORMSQUARE-ACHIEVED; KILL=PERIODIC; CHAIN=CARRIED*)).* **(1) The exact identity.** The dangerous soft pivot obeys $1/d_{12} = 1 + v_* K_\sigma(y_*)$ *exactly*: d_{12} is a Christoffel/CD-kernel evaluation of the *deflated* signed functional $\sigma = \mu_+ - \nu_-$ (Woodbury, factor-Gram and block-inverse chain, verified through an independent $h \times h$ route at 2.0×10^{-11} on every full rung); of the three reviewer identifications the Christoffel form wins the boundedness exposure by *two orders of magnitude* (max piece-variance 2.96 vs 362/430). **The honesty fence, warded exactly:** $\lambda_{\min}(I - G) = \tau$ — identification (iii)'s positivity premise is the wall's own PD premise *in Christoffel coordinates*, a coordinate change, not an independent positivity source (the smooth control breaks both premises). Measured boundedness: $c \geq 1$ exact, c'/c sensitivity range $[0.223, 5.258]$ on 29 pairs (raw $[0.054, 23.6]$; the kz-21 pairs excluded per the round-53 coordinate-artifact rule), $r_{12} > 0$ on 31/31, and the exterior reserve is *decoupled* (corr +0.454 against the 0.5 bar). The printed ratio theorem — bounded c-ratio + positive pivot quotients + certified base ⇒ positive ladder — has both hypotheses measured, neither proved for all h. **(2) The negative index.** The constructional frequency weights of the `v894` pair kernel are $W_{f,m} \geq 0$ *exactly* on the whole frozen census (79 (rung, alias) pairs, eight rungs to $h = 1433$); every negative DCT coefficient is closed-form boundary leakage of the symmetric extension — *not* arithmetic — and in u-space the entire negative content collapses to **one top-edge tent** ($r_{\min} \leq 1$), typed honestly INDEX-LARGE / NOT-ABSORBED (worst $|N|$/margin = 1.381 at kz 9 alias 9). **(3) The kill.** That tent is an *extension-convention artifact* (the DCT-I halving $w_{M-1} = 1$ at the fold point) and dies under the **periodic full-weight fold**: the cosine weights become exactly the constructional $W_{f,m} \geq 0$ (negative index 0 everywhere), all 79/79 modified margins stay positive, the kz-9-alias-9 absorption failure disappears (ratio 1.381 → 0.000), and the boundary term $\beta = -(T_1/2)(E - E_0)$ is carried closed-form and comb-computable (max $|\beta|/\widetilde{\text{margin}} = 0.21$; the implication chain to $T_h \leq 1$ on the critical zone stays exact). The alternatives fail honestly (Dirichlet, taper, enlarge). **The conditional diagonal contract is now a norm square in the frequency weights** — case 1 of the reviewer trichotomy at the finite level — and the arithmetic hypothesis itself remains conditional (pair-correlation class, exactly as `v889` typed it) (`PRIME.PORT.CHRISTOFFEL.RATIO.01`, `PRIME.CASE.KERNEL.SOS.01`, `PRIME.CASE.EDGEDEFECT.01`).

The normalized core update anatomy ([verification/v900_core_update_anatomy.py], 20/20+23/23; verdicts NORMCORE-MEASURED *(*FAMILY-BOUNDED / UPDATE-EXACT / REGION-OPEN*),* GRAPHREGION-MEASURED *(*COUPLING-FREE / GRAPH-REGION-OPEN / SHADOW-LAWLESS*)).* The normalized deep core $X_h = S_h/\tau_h$ (the `v892` fixed 8×8 Schur core in wall units) obeys the *exact* update $X' = c\,(X + U)$ at max relative defect 8.3×10^{-16} with the scalar $c = \tau/\tau'$; the family is bounded (Frobenius diameter 2533, $\lambda_{\max}$ trendless) and the input is effectively *two-dimensional* — the scalar c plus one matrix mode carrying 97.5% of the U-energy. The honest negatives are registered as the **boundary conditions of any future invariant-region theorem**: no linear coupling law $u \sim F(X)$ exists (best two-feature R^2 0.158/0.163, both *below* the 200-shuffle null q_{95}), the rank-1 truncation fails — the dominant mode is PSD and the positivity is carried by the 2.5%-energy *off-mode* (the rank-1 surrogate is margin-negative at 12/39 points where the true update is positive) — and the naive product and graph-tube invariant sets are refused at every stage. The registered direction: any region construction must keep the exact $X + U$ with its sign-dependent off-mode structure — no mode may be truncated (`PRIME.PORT.NORMALIZED.CORE.01`, `PRIME.PORT.GRAPH.REGION.01`).

The open fronts after rounds 54–56. The certificate base of the bounded-margin skeleton is now interval-rigorous (`v897`); the one unconditionally open RH-side statement is unchanged — the τ-sign inheritance in bounded-margin form — with its diagonal feeder sharpened: the pair contract is a positive-weight *norm square* on the finite band with the boundary term carried exactly (`v899`), and the region program has exact coordinates and registered boundary conditions (`v900`). The named demands: the Lean composition of the certificate chain; the band inequality

of PRIME.CASE.PAIRCORR.CONTRACT.01 [O]; the $O(1)$-ratio source of PRIME.PORT.ETASOURCE.01; and the invariant-region construction under the v900 boundary conditions. All registered in the research-contracts companion.

The honest state after rounds 54–56. The wall's finite positivity is certified with rigorous interval arithmetic on every reachable rung; the soft pivot is an exact Christoffel evaluation whose positivity premise is the wall's own (said twice, in both coordinate systems); the conditional contract is a norm square with one carried boundary term; and the core update is exact, two-dimensional, and honestly hard (no coupling law, no truncation, no naive region). The interval certificates, the pivot identity, the fold identity and the update identity **[E]**; the ratio ladders, censuses and negatives measured; the diagonal route stays conditional; the one-sidedness **[O]** at RH strength. No marker moves. **No statement about the Riemann Hypothesis is made.**

12.36 The tangent-Schur coordinates and the relocation map

Rounds 57–59 (2026-08-10) are a *cartography* round: eleven further frozen probes, every spec amendment fail-first on record, embedded byte-exact in four promoted modules (suite 893 → 897) — the wall rewritten in tangent-Schur coordinates with the first substantive τ-screen pass and its honest certification negative ([verification/v901_tangent_schur_bfloor.py]), the pivot and moment-matrix relocation map typed explicitly as *mechanism, not progress* ([verification/v902_wall_relocation_map.py]), and the 12-bit pivot health word typed a pure diagnostic with zero RH content ([verification/v904_healthcode12_diagnostic.py]). (The seam RP/modular exclusion of the same rounds — strict reflection positivity and the v898 mixing floor are mutually exclusive on the deployed family, plus two registered dead readings — is a compiler statement and lives in the research-contracts companion, [verification/v903_seam_rp_exclusion.py].)

The tangent-Schur split ([verification/v901_tangent_schur_bfloor.py]; probe verdicts TANGENTSCHUR-MEASURED, BFLOOR-MEASURED, SIGNFREE-MEASURED, $26/26 + 24/24 + 18/18$*).* Along the soft direction v_h of S_h (backward spectral data only) the normalized update $M_h = X_h + U_h$ splits *exactly*: with the co-block $B > 0$,

$$M_h > 0 \iff n_h - q_h > 0, \qquad n = v^{\mathsf{T}} M v, \quad q = b^{\mathsf{T}} B^{-1} b,$$

all identities warded ($\det M = (n - q)\det B$ at 2.8×10^{-14}; the PD equivalence boolean-exact on 39/39 steps), and both scalars are *source-only* — no τ_{h+1}, no forward sign anywhere: the gap sign is output, never input. The co-block floor is the round's headline measurement: $\min \mathrm{eig}(B_h) = 0.679$ ladder-wide, the family $O(1)$ in $[0.679, 84.0]$ while τ falls by a factor 552; the τ-screen (frozen bands, pass $|s| \le 0.30$, relocation $s \ge 0.70$) reads slope -0.247 — the *first* screen pass with wall content: B-uniformity is a genuinely easier subgoal, and after the split the entire residual difficulty of the wall sits in the two scalars n, q. Coordinate honesty printed a priori: the unnormalized co-block tracks τ with slope $+0.753$ — the $O(1)$ floor *is* the statement that the raw co-block scales exactly like τ; the pass is substantive because the coordinate is purely backward-normalized and the PD question coordinate-invariant. The honest negative: CERTFLOOR-DEAD — all four classical certified bounds (Gershgorin raw/scaled, Brauer–Cassini, Weyl) are negative on all 39 steps (best maximum -88 against the true floor $+0.679$): the floor is *measured*, never certified; a certifiable floor needs a damping congruence, not better classical inequalities. The failure-seat anatomy refutes the plan diagnosis: at all 12 v900 rank-1 failure points the co-block B_s *itself* dies, the 2.5%-energy off-mode restores it linearly, and the frame-invariant failure seat is orthogonal to the critical direction. The sign-free normalization $\ell = \det(S)^{1/8}$ de-circularizes the v900 coefficient ($Y' = g\,(Y + V)$ with $g = \ell/\ell'$ source-only; the positivity margin coordinate-invariant at 0.0315) — *circularity removed, positivity not gained*, both said explicitly (PRIME.PORT.TANGENT.SCHUR.01, PRIME.PORT.BFLOOR.01, PRIME.PORT.SIGNFREE.NORMALIZATION.01).

The relocation map ([verification/v902_wall_relocation_map.py]; probe verdicts EDGECHRISTOFFEL-MEASURED, GRAMRADAU-MEASURED, $22/22 + 23/23$*).* Typed explicitly as a relocation/mechanism map, *not* progress: every identity is a coordinate change of the wall's own PD premise. The

pivot identity splits into the exact positive-weight form $1/d_{12} = 1 + \sum_m W_{h,m}\, Q(x_m)^2 - \beta$ with $W > 0$ exactly (min 9.7×10^{-14}, no tolerance) and $\beta \geq 0$ termwise — and the decisive ratio $r = \beta/(1 + S_{\text{pos}})$ stays below 1 on every deployed rung while $1 - r$ *tracks* τ (slope +1.008): the open uniform $q < 1$ target is the wall's PD premise *quantified*, not an independent positivity source (`PRIME.CASE.EDGE.CHRISTOFFEL.01`). On the operator side, $M_0 = I - H$ *is* the moment matrix of the signed comb measure $\nu = \mu_+ - \mu_-$ (two-route ward 4.9×10^{-13}; $\lambda_{\min}(M_0) = \tau$) — the wall statement is '$\nu \geq 0$ on squares of degree $\leq 2h - 2$' — but the Lukács/Gram completion with $W \succeq 0$ fails on every rung at the $x = +1$ edge where the comb nodes accumulate, with violation magnitude outrunning the wall margin ($|\text{floor}|/\tau$ to -152): no source-only Gram representation exists at matched degree (`PRIME.PORT.GRAM.COMPLETION.01`). The honest positive: the Gauss–Radau construction at the classical node is exact algebra and its anchored weight *is* the Christoffel function (med ratio 0.9978) — a pure μ_+ object; the P2 pivot is *not* a Radau weight (`PRIME.PORT.RADAU.WEIGHT.01`). Where the difficulty lives (the NEG support, the $x = +1$ edge, the τ-tracking ratio) is now registered; nothing here narrows it.

The health word (`[verification/v904_healthcode12_diagnostic.py]`; probe verdict HEALTHCODE12-MEASURED / DIAGNOSTIC-ONLY, 18/18). The sign word of the 12 leading LDL pivots of the `v899` 12-window compression is a per-rung *diagnostic*: the true word is all-(+) on 37/37 (the certified Sylvester census repackaged — no new content); three plan claims are corrected (the 12 factors live in the `v899` compression, not in `v897`; the softest raw pivot is $k = 2$, not $k = 12$; smooth tips flags $\{2, 1\}$ first and never flag 3); the one surviving claim is reproduced byte-exact (the `v897` Epstein control refused at pivot index 10); and the seeded syndrome table distinguishes four adversarial fingerprints (coherent cosh injection, scramble, mass rescaling both ways) — a classifier of manipulated combs, a tool, not a target; ledger-typed *diagnostic, zero RH content in either direction* (`PRIME.ARITH.HEALTHCODE12.01`).

The open fronts after rounds 57–59. Unchanged in kind, sharpened in place: the one unconditionally open RH-side statement remains the τ-sign inheritance in bounded-margin form, now localized to the two tangent-Schur scalars n, q over the measured (uncertified) $O(1)$ co-block floor; the named demands are a *certifiable* B-floor (a damping congruence — the classical route is dead by measurement), the band inequality of `PRIME.CASE.PAIRCORR.CONTRACT.01` [O], and the invariant-region construction under the `v900` boundary conditions. All registered in the research-contracts companion. *Dated note (2026-08-10/11, rounds 60–63):* the certifiable-B-floor demand is *delivered* — the missing piece was the certificate class, not a damping congruence (sec:bhalf-halfgap, `[verification/v905_bfloor_ideal_certificate.py]`).

The honest state after rounds 57–59. The wall now has a mechanism map: an exact two-scalar split over a measured $O(1)$ co-block, a pivot rewrite and a moment-matrix identity that relocate — provably never remove — the difficulty, one pure diagnostic, and (in the companion) a seam exclusion theorem with two registered dead readings. The split, rewrite, moment and Radau identities **[E]**; the floors, screens, censuses and negatives measured; nothing gained positivity anywhere, and every relocation is typed as such. No marker moves. **No statement about the Riemann Hypothesis is made.**

12.37 The certified B-half surface floor, the tail cartography and the registered half-gap target

Rounds 60–63 (2026-08-10/11) deliver the continuation of the cartography: fourteen further frozen probes, every spec amendment fail-first on record, twelve embedded byte-exact (two more cited as frame) in four promoted modules (suite 897 → 901) — the certified ideal-object surface floor of the B-half (`[verification/v905_bfloor_ideal_certificate.py]`), the tail mechanism map typed explicitly as *negative cartography, not progress* (`[verification/v906_tail_cartography.py]`), and the registered half-gap target with its first blind holdout passed (`[verification/v907_halfgap_registered_target.py]`). (The seam equilibrium wiring of round 60 — strict 2-cycle reflection positivity impossible on the whole covariant class, the strict-collar obstruction a two-seat linear law whose equilibrium kernel witnesses carry the full 1/200 mixing at zero entropy production — is a seam-side statement

and lives in the research-contracts companion, [verification/v908_seam_equilibrium_wiring.py]; its registered successor contract — is the deployed pure-I wiring compiler-forced? — is closed in rounds 69–71 as a *deployment choice*, not a compiler theorem, [verification/v911_wiring_freedom.py].)

The certified surface floor ([verification/v905_bfloor_ideal_certificate.py]; probe verdicts BFLOORPG-MEASURED *(*CERTIFIED-SURFACE-FLOOR-ACHIEVED*(MIN c_B=0.5914)),* BFLOORPGIVAL-MEASURED *(*IVAL-SURFACE-FLOOR-ACHIEVED*(MIN c_B=0.5523)),* $21/21 + 25/25$*).* The round 57–59 demand — a *certifiable* B-floor — is delivered, and the delivery mechanism is itself the lesson: after three dead certificate rounds (raw Gershgorin -88, Christoffel congruences $-140..-154$, the moving-node congruence unbuildable) the missing piece was not a damping congruence but the *certificate class*. The chain

$$B \succeq \tfrac{1}{2}P_G + c_{\mathrm{dom}} I, \qquad P_G \succeq c_G I, \qquad \Longrightarrow \qquad B \succeq c_B I,$$

with P_G the source-only CD-Gram co-block of the wall's own positive chain, is decided inequality-by-inequality by *exact-rational* LDL (Fraction arithmetic on the float64 entries, which *are* exact rationals — the `v897` certificate class) on all 39/39 reachable steps: c_B min/med 0.5914/5.92, efficiency $c_B/\lambda_{\min}(B) \geq 0.871$ (the chain pays at most 13% for its structure), τ-screen pass (slope -0.256). The honest surprise: P_G is Gershgorin-*positive* on its own (min $+0.33$, 39/39) — unlike B, the positive chain is classically certifiable; the single non-classical link is the dominance decision. The interval rollout (SPEC v3) then lifts the statement from the computed to the *ideal* source objects: the full pipeline (lags → FFT density → Lanczos chain → frame → Schur complements → B) carried as rigorous outward enclosures, fail-closed at every step (an interval too wide to decide *refuses* rather than passes), with an Ogita–Rump high-accuracy tier (error-free TwoSum block accumulation, radius factor ∼150–300) certifying exactly the six former width-refusals: $\min c_B = 0.5523$ on 39/39 ideal-object steps (93.4% of the float reference), 0 refused / 0 failed / 0 skipped, and the smooth world *refused* by the identical exact machine (0/35). Honest scope, said everywhere: this is a *surface* statement — certified on the reachable 39-step ladder over the ideal source objects with the frozen frame convention (Q, τ_1, $s = \frac{1}{2}$ exact-rational constants of the statement); the h-uniformity beyond $h \sim 900$ is open, every h beyond the surface is open, and the $n > q$ half of the end-form is untouched and RH-hard (`PRIME.PORT.BFLOOR.PG.01`, `PRIME.PORT.BFLOOR.PG.IVAL.01`).

The tail mechanism map ([verification/v906_tail_cartography.py]; probe verdicts TAILSIGN-MEASURED, TAILREPAIR-MEASURED, PGSCHUR-MEASURED, TAILABEL-MEASURED, SEGSPLIT-MEASURED, OSCPAIR-MEASURED, $14/14 + 10/10 + 20/20 + 22/22 + 24/24 + 22/22$*).* Typed explicitly as mechanism/negative cartography, *not* progress: five dead routes on the $n > q$ half, each with a named seat, and two structural positives. The positives first. HEADFLOOR-O1: the *entire* h-decay of the wall margin lives in the sign-measured atom tail — the head at the B-cover cut is $O(1)$ (τ-screen slope $+0.113$, med 0.388) while $\mathrm{tail}_B \leq 0$ holds net on 67/67: the open arithmetic statement collapses to the *one-sided* bound $\mathrm{tail}_B > -\mathrm{head}_B$ between two $O(1)$ objects whose gap is the margin itself. And the 12-bin reduction: the tail pairing decomposes over the half-range cosine frame with a *tiny* low-frequency window-harmonic carrier census (n_{90} med 3, harmonic index $j^* \in \{1, 2, 3\}$ on 67/67, measurably *not* γ-concentrated at these window depths — med $d_\gamma = 13.17$ against the uniform null ∼1, all carriers below $\gamma_1 = 14.13$; a result, not a celebration), and 12 frozen global bins carry share med $+1.006$ on 67/67 — the tail bound is now a statement about ∼12 windowed ψ-fluctuation Fourier coefficients $\widehat{F}(\omega)$, $\omega \lesssim 5$; honestly typed: the reduced margin carries the *full* decay (screen -1.042) — the reduction changes the *dimension* of the open statement, not its difficulty. The five dead routes: the pointwise sign (the net negativity is an arithmetic cancellation fact — $\sup q_v$ beyond *every* cut is positive on 67/67 and the terminal lobe is positive: MECH-NET-ONLY); the pointwise repair (no small named exception set — SET-DIFFUSE, k_{90} med 646; the repair support grows as $X^{0.89}$ and the enlarged head is anti-τ); the directional Löwner route (with $D = \frac{1}{2}P_G + c_{\mathrm{dom}} I$ the Löwner step $q \leq \bar{q}$ holds *exactly* on 39/39, but the sufficient form $n > \bar{q}$ dies on 38/39 with loss $\bar{q}/q$ med 91.3, max 408 — D is an excellent *floor* model of B and a catastrophic *inverse* model along b: D^{-1} inflates exactly the components B^{-1} suppresses); the cumulative/Abel route (the transport and the ψ/Riesz cumulant identities are

exact, but no order $r \le 3$ fixes the dual-kernel sign and the honest all-integer envelope closes nothing — gap law $e^{+1.744\alpha}$; the measured hole is the *oscillation*: the true remainder itself is $\sim 10^4\,\mu_1$ units); and the segment split (the telescoped segment identity is exact at 3.5×10^{-16}, the per-segment demand is *named* — a one-sided windowed ψ statement at deep anchors — but the segment route is *costlier* than the global one, gain < 1 on 67/67: the *anchor price* — every Chebyshev-class envelope $|\psi(x) - x| \le \kappa x$ pays the deep anchor at every point of the segment; the deficit carrier is the interior segment, not the terminal lobe; the named missing object is a *difference* envelope for $\psi(y) - \psi(x) - (y - x)$ on short deep windows — Brun–Titchmarsh/Selberg class instead of Chebyshev class) (`PRIME.PORT.TAILSIGN.01/.02`, `PRIME.PORT.PG.SCHUR.01`, `PRIME.PORT.TAIL.ABEL.01`, `PRIME.PORT.TAIL.SEGSPLIT.01`, `PRIME.PORT.TAIL.OSCPAIR.01`).

The registered half-gap target and its first blind holdout (`[verification/v907_halfgap_registered_target.py]`; probe verdicts HALFGAP-REGISTERED *(*REG-FROZEN*(1/2,* NO-ADJUST*)),* DEEPHOLDOUT-SCORED *(*HALFGAP-HOLDOUT-PASS*(28/28)),* $17/17 + 14/14$*).* A *falsification instrument*, explicitly **not evidence**: the target inequality

$$n_h - q_h \;\ge\; \tfrac{1}{2}\,\mu_1(h), \qquad \mu_1(h) = 4\sin^2\!\Big(\frac{\pi}{2h+1}\Big),$$

is *registered* on the 67-rung surface with the constant *exactly* $\frac{1}{2}$ frozen under an explicit NO-ADJUST clause that is part of the registered object (a later miss may not be repaired by 0.49, reweighting or rung exclusion — a fail is a first-class result): surface pass 67/67 with margins $+0.0025/+0.5273/+1.6845$ (min at kz 98/h 997; the second-tightest rung is *shallow* — the tightness is arithmetic scatter, not a depth trend), blind protocol declared (registry sha256 `ae292e55`…, scored verbatim, no refits, no constant moves), and the four candidate origins of the $\frac{1}{2}$ typed *open* — the registration demands an algebraic derivation of the constant before any claim upgrade. REPARAM-DECLARED in every print: the inequality is the wall margin *reparametrized* (the exact pivot collapse $n - q = m$ along v; the round-61 frame — the Schur–Ward envelope identities, where the hub is exactly the H-term and the demand exactly the $-s$-term with R a third object, and the μ_1 normalization band $[0.502, 2.185]$, inf-flat — is cited at docstring level); its value is falsifiability, not novelty. The first genuine blind holdout then *passes*: the 10× deeper table is bit-exact against the deployed one and opens 28 new faithful rungs outside the registry ($h = 1219$–2854); pass 28/28 with margins min/med $+0.2232/+0.6567$ — the holdout minimum sits ∼90× *above* the registered surface minimum: no depth erosion; HEADFLOOR-O1 persists (slope -0.049) with the honest structural drift disclosed (the cut ladders drift: $n_{\min B}$ med 47 vs the deployed 17); the B-floor/P_G chain persists at declared *float* level ($\min \lambda_{\min}(B) = 1.6610$, dominance 27/27; no interval rollout in the depth — said explicitly); and the margin law continues ($p = -1.925$ over 95 rungs). Controls discriminate 3/3 (every control world sits far below $\frac{1}{2}$). A pass is survival of a falsification attempt, never evidence (`PRIME.PORT.HALFGAP.01`, `PRIME.PORT.DEEP.HOLDOUT.01`).

The open fronts after rounds 60–63. The B-half of the end-form is closed as a certified surface theorem over the ideal source objects; what remains open is exactly what the map says: the h-uniformity of the chain beyond $h \sim 900$, the $n > q$ half — now localized to a one-sided bound between two $O(1)$ objects, reducible to ∼12 low-frequency windowed ψ-fluctuation coefficients, with every tested certificate language dead by measurement and the Selberg-class difference envelope the named missing object — and the algebraic origin of the registered $\frac{1}{2}$ (four candidates typed, none derived). All registered in the research-contracts companion. (Rounds 64–68 then close the *finite* composed surface from cited inputs and price its zero supply — the next subsection; the all-h objects stay open exactly as stated there.)

The honest state after rounds 60–63. The wall now has a certified anchor and a sharper map: an exact-rational ideal-object floor on the whole reachable surface (fail-closed, smooth-refusing), a tail cartography of five dead routes with named seats and two structural positives, and a registered, blind-holdout-tested falsification target with a frozen constant and a no-adjust clause. The chain, transport, segment and floor-exchange identities **[E]**; the floors, censuses, holdout scores and negatives measured; the target inequality is an instrument, not a claim; nothing here bounds τ, the

two scalars n, q for all h, or anything at RH strength. No marker moves. **No statement about the Riemann Hypothesis is made.**

12.38 The finite wall closure and the transfer law

Rounds 64–68 (2026-08-11) finish the *finite* surface of the wall end-form and then price it. Two promoted modules (suite 901 → 904 together with the seam companion of the research-contracts lane) carry the result: [verification/v909_finite_wall_closure.py] (the composed closure) and [verification/v910_finite_zero_transfer.py] (the transfer law).

The finite wall closure ([verification/v909_finite_wall_closure.py], PRIME.WALL.FINITE_CLOSURE.01; 28/28). On the deployed finite ladder every reachable wall face is certified from cited inputs, and the composed census

$$B \ \wedge \ \mathrm{W1} \ \wedge \ \mathrm{W2}$$

holds on 39/39 matched surface + 8/8 deep rungs (frozen experiments runs of rounds 64–67, cited with their spec hashes). The three faces: the *B-half* is the certified chain of the previous subsection ($\min c_B = 0.5523$ over the ideal source objects, 39/39 surface + 27/27 deep steps at declared float level). The *W1 face* is new twice over: the two composition gaps that rounds 55–59 priced at +8 to +9 dex ("concentration" and the "diagonal law") dissolve into *one exact measure inequality* — measure monotonicity of the Christoffel–Darboux dual form (Loewner step, composition price +0.29 dex, v-monotone so a floor substitutes losslessly) — and the remaining per-fold supply at the named $j = 16$ seat is paid by *verified zeros as exact data*: the truncated explicit formula with 7,000 mpmath-computed ordinates, every read tested against the independent prime side and the tail bracketed by the unconditional Rosser–Abel envelope, closes the v-floor on 39/39 surface rungs, composed 67/67 with the cited deep 28/28. The *W2 face* is the recomposed certificate $m_{\mathrm{cert}} = FC + \widehat{P}_{\mathrm{arith}} - \mathrm{TAILB} > 0$ along the measured critical direction: 16/67 at the 7,000-ordinate cache, $67/67 + 8/8$ once the 20,000,000-ordinate certified cache pays the priced tail (Odlyzko tables + LMFDB/Platt, every ordinate below the Platt–Trudgian height $T_0 = 3 \cdot 10^{12}$ and hence on the line unconditionally — EXTERNAL-CITED). The language discipline is frozen and non-negotiable: W1 and W2 are *algorithmically independent evaluations of the same localized Weil form* — a strong mutual crosscheck, *not* two independent proofs; every certified statement is per-rung, on a finite family of Galerkin sections *along the measured critical direction* (direction-conditional; nothing is uniform in h or in direction); the deep block is float-level. The promoted module re-verifies the load-bearing spine self-contained on a reduced frozen subset (its own committed 7,000-ordinate cache re-warded in-run: Rosser corridor per index on both sides, γ_1 identity, independent $|\zeta(\frac{1}{2} + i\gamma)|$ spot checks; heart ward on every read with slack +2.1..+4.8 dex; closure 6/6; the off-line-impostor control fires at $8.2 \cdot 10^4\times$ the genuine residual) and cites the full census rather than recomputing it. The promoted gate pins only that reduced subset (meta-audit CCCXXXVI): a false cited full census would not fail the module — the $39/39 + 8/8$ census rests on the frozen experiments runs and their spec hashes, not on the `v909` gate.

The transfer law ([verification/v910_finite_zero_transfer.py], PRIME.WALL.FINITE_ZERO_TRANSFER.01; 10/10). With the closure in hand the economics become measurable: per rung, what zero-cutoff height $T_{\mathrm{req}}(h)$ does the certificate actually consume? The frozen full-ladder answer (75 rungs, 42 declared prefix anchors, heart ward on all 582×42 cells): $\log_{10} T_{\mathrm{req}} = -3.045 + 1.228 \ln h$ ($R^2 = 0.882$), i.e. $T_{\mathrm{req}} \sim h^{2.8}$, and the ratio $T_{\mathrm{req}}/(\pi/D_h)$ against the window's own spectral reach *grows* at +0.897 dex/$\ln h$ — the verdict is RATIO-GROWING: the zero supply is an *external battery* (W-UNBOUNDED-PER-RUNG), not a local sampling law, and a bootstrap even less. The census compresses into one parametrized statement, the *transfer law*: $\mathrm{ZerosCertified}(T) \wedge \mathrm{TailEnvelope}(T) \Rightarrow M_h \succ 0$ for *all* deployed $h \le H(T)$, with the measured monotone envelope $H(\gamma_{7000}) = 254$, $H(\gamma_{2\mathrm{M}}) = 1256$, $H(\gamma_{2\cdot 10^7}) = 2806$. The W1 face is cheap ($T_{\mathrm{req}} \le 5350$, binding on only 3/75 rungs) — the wall curve is W2-priced almost everywhere — and the next h-decade would cost $T \sim 3.4 \cdot 10^9$ ($\sim 10^{10}$ ordinates): the finite engine does *not* scale by buying zeros, and $H(T)$ is the measuring rod any future analytic per-window bound has to beat (a bound with $\sim \pi/D$ reach per window beats

the measured h^2 battery by construction). The promoted module re-verifies the shallow end (12 rungs, h 142..254, anchors $K = 250..7000$: closure census 2/3/4/6/8/12, zero non-monotone, H-consistency 12/12 against the cited boundary 254); its gate pins only that shallow subset (meta-audit CCCXXXVI) — $H(\gamma_{2\mathrm{M}}/\gamma_{2\cdot 10^7})$ and the full-ladder fits are cited, not pinned by the `v910` gate.

The open fronts after rounds 64–68. The finite surface is closed and priced; what remains open is exactly the all-h mountain: the h-uniformity of every chain beyond the deployed ladder, the all-h/all-direction Weil-positivity object (the UNIF-PATH caveat: positivity is certified on a finite family of Galerkin sections along the measured critical direction, never uniformly), the algebraic origin of the registered $\frac{1}{2}$, and — the transfer law's own message — an analytic per-window bound in the $\sim\pi/D$ reach class that would replace the external zero battery. A finite verified-zero sum can never prove RH; the closure consumes published zero verifications as citations (Odlyzko, LMFDB/Platt, Platt–Trudgian 2021, Rosser 1941, Büthe 2018) plus exact window algebra and the exact composition steps. No marker moves. **No statement about the Riemann Hypothesis is made.**

12.39 The form-convergence theorem: the extraction chain's convergence premise stops being a measurement

Round CCCLXV (2026-08-13) closes the analytic edge that the extraction chain of §12.24 had been carrying as a *measurement* ([`verification/v912_form_convergence_theorem.py`], 35/35, verdict FORMCONV-PROVEN). Besides the cofinal positivity hypothesis, the chain — and its Lean statement `CofinalWeil.cofinal_weil` — consumes exactly one analytic input: per-element convergence of the finite Galerkin forms to the limit functional. Until now that input was typed from the corpus rates $-1.58..-1.84$ per level of [`verification/v848_extraction_chain.py`]. It is now a theorem, and an unconditional one.

The statement. For f real, supported in the deployed window and *affine on every grid cell*, with $K = f * \tilde{f}$, $G = f' * \tilde{f}'$, $b_K = \operatorname{diam}\operatorname{supp} K$, $B = b_K + D$, $\kappa_2 = \|f'\|_2^2 = G(0)$, $\kappa_3 = \mathrm{Lip}(G)$ and κ_K the supremum of $|K''|$ over cell interiors,

$$|Q_D(f) - Q_W(f)| \;\le\; D^2\Big[(8\sinh(B/2) + \mathfrak{m}(B))A_2 + \Theta_0 A_0 + (A_0 + A_2)\big(\log\tfrac{1}{D} + \log B + 4\big)$$

$$+2A_0 e^{-B/2}\big(\tfrac{1}{B} + 2\big) + A_1(1 + D)\Big] + (1 + D)\tfrac{D^3}{12}\kappa_3,$$

with $A_0 = \kappa_2/12$, $A_2 = \kappa_K/8 + \kappa_2/12$, $A_1 = \kappa_K$ and $\Theta_0 = 3\log 2 + \pi/2 + \gamma + \log\pi = 5.372183419225665\ldots$ — that is, rate $O(D^2\log(1/D)) = O(2^{-2j}j)$. For continuous f one has $\kappa_K = \kappa_2$ exactly and the log coefficient collapses to $7/24$ (supremum $5/24$).

Why it is exact. Three steps carry the proof, and the middle one is the load-bearing exactness. First, the deployed lag vector *is* the capped Weil functional on the even tent, $c_d = W_C[S_{dD,D}]$, so the finite form equals $W_C[\widetilde{K}_D]$ on the even piecewise-linear interpolant of $k_d = Da_d$ — an identity, not an approximation. Second, the exact defect lemma: $k_d - K(dD) = -(D^2/12)\,G(dD)$ for *every* deployed lag, because $u \mapsto f(u)f(u + dD)$ is a quadratic on each grid cell, the midpoint-rule error of a quadratic is exactly $-(D^3/24)q''$, and $\Pi_D G = G$ since G is piecewise linear with grid-node breakpoints; the gates verify this with *zero* deviation in exact rationals over 1474 lag identities and 780 strictly interior points. Third, the faithful cap $C \ge b_K + D$ makes the capped and uncapped combs agree on the defect, so the whole error is W_C applied to *one* explicit function, bounded layer by layer with elementary majorants.

Two typed corrections belong to the statement. The interpolation constant is the interior supremum of $|K''|$, not $\|f'\|_2^2$: the two agree exactly for continuous f, and differ in both directions for jump elements (the frozen elements E09 with $3 > 2$ and E10 with $5/3 < 2$). And the law is genuine rather than a loose envelope: measured against a closed-form leading term — no fit enters any gate — the ratio of true error to $|e_D(0)|\log(1/D)$ lies in $[0.575, 1.066]$ and drifts to 1 through

$j = 18$. Uniformity (V) holds on every class with a *two-sided* support window $b_- \le b_K \le b_+$, $b_- > 0$, and bounded $(\kappa_K, \kappa_2, \kappa_3)$; it fails on an L^2 ball, since κ_2 is an H^1 seminorm.

The correction the theorem forced. The chain as originally written cited C^0-continuity of Q_W at fixed support for the passage from the dense family to the admissible class. In the pure sup norm that citation is *false*, and the must-fire control C5 says so explicitly: the even Lipschitz family $e_n(w) = \frac{1}{n}\min(1, we^{n^2})(1 - w/2)_+$ has $e_n(0) = 0$ and $\|e_n\|_\infty = 1/n \to 0$ while $|A[e_n]| = 2.57, 4.28, 6.19, 8.14, 10.11, 12.09$ grows linearly in n — the same archimedean divergence already measured in §12.24's companion round, where neither an L^2 nor a sup-norm continuity constant was found. The correct hypothesis is uniform convergence *plus* an equi-Lipschitz (Dini) condition at the origin, which the admissible even compactly supported BV class supplies automatically ($K = f * \tilde{f}$ with f of bounded variation is Lipschitz). The module verifies the corrected modulus of W_C on a declared twelve-perturbation battery. This is a correction of a wrong cited statement, not a downgrade of a result: the edge is narrowed, and what stays cited is the density leg — density of the dyadic cellwise-affine family in the admissible class, in *that* topology.

What is cited and what is pinned. Eleven classical inputs are typed, and none of them is a zero-free region, the prime number theorem, a zeta zero or a fit. The only cited arithmetic input is Rosser–Schoenfeld's $\psi(x) < 1.03883\,x$, consumed *only* for the support-uniform constant and never for convergence at fixed f; it is independently sieve-verified in-run to 10^7 (worst ratio 1.038821), while the all-x statement remains the citation. Also cited and *not* pinned by these gates: the W1 measure dictionary, Weil's explicit formula and criterion with the admissible class, and the universal quantifier itself — the gates run ten frozen elements and a twelve-member translate family at $j = 4..11$ exactly or through the deployed pipeline and at $j = 12, 14, 16$ through the stable defect route, so the statement for *all* admissible f and all levels is the paper proof plus the frozen probe specification. On the Lean side the new module `TfptCarrier/CofinalEnvelope.lean` states the envelope as a named premise, derives the convergence hypothesis from it, and restates the cofinal extraction theorems with that hypothesis replaced by the instantiated envelope (kernel-checked, no `sorry`); the envelope itself is not discharged inside the kernel, so the formalisation gap is reduced rather than closed. The window conditions (H-grid), (H-cap) and (H-align) hold for fixed f and all large j, which is exactly what the `atTop` limit needs.

Registered `PRIME.FORM.CONVERGENCE.THEOREM.01`. The chain's open item is unchanged: the predefined cofinal positivity inequality — cofinal in the same mesh-refinement order $D_j = 2^{-j}$ in which this envelope holds — untouched and of RH strength. Closing this edge does not prove anything about the Riemann Hypothesis. No marker moves. **No statement about the Riemann Hypothesis is made.**

12.40 The remaining obstruction is signed and alignment-carrying (localization)

Round CCCLXXIII (2026-08-13) does not close anything. It *types* what is left, and the exact, cheap and self-contained core of that typing is now a no-go module ([`verification/v913_signed_alignment_localization.py`], 24/24, verdict NOGO-CORE-VERIFIED). The statement:

> For the deployed budget shape the missing inequality is not a magnitude statement. The conversion constant of any hypothesis of the form $|\psi(x) - x| \le f(x)\sqrt{x}$ is bounded below by $\|K_D'\|_1 = 4$ uniformly in D, while the geometric supply stays $O(1)$; Littlewood's unconditional Ω-theorem therefore makes the entire magnitude class empty, with floor $4 \log\log\log N_h$ against a slack that shrinks with depth. The residual requirement is instead odd under the comb sign flip and invariant under every congruence of the matrix stage, so no reformulation, window choice or preconditioner can convert it; the signed part is exactly the Weil prime term of an explicit even test function, and the remaining statement is a zero-sum inequality against a negative bar. Its unsigned companion is bounded unconditionally but its class floor, attained at an admissible datum, still exceeds the need. Consequently any future input must be signed and must

carry alignment information; neither a magnitude bound nor a world-blind grammatical identity can supply it.

What the module pins. The six kernel identities are exact in D — $\int K_D = 0$, $\|K_D\|_\infty = 1$, $\|K_D\|_1 = 4D/3$, $D^{-1}\int K_D^2 = 2/3$, $\int K_D e^{-t/2} = -(D/8)T_D^2$ and the load-bearing $\|K_D'\|_1 = 4$ *independent of* D: the conversion constant of a magnitude hypothesis does not get cheap as the mesh refines. Against it the geometric supply, recomputed by two independent routes agreeing to 1.6×10^{-50}, is negative and $O(1)$ (-1.2648057425013541016 at $h = 184$, -1.3147508968858830811 at $h = 388$), while the Littlewood floor is 2.1654551 at $N_h = 263$ and 4.1231791 at $N_h = 14\,634\,429$ — a factor 214.7 above the module's own conservative slack and 1527.1 above the deployed one. The three-term split $s = (n_0 - q_0) - n_c + 2\langle b_c, x_0\rangle - q_c$ is an exact identity whose terms are invariant under *every* congruence $T = \mathrm{diag}(1, M)$, with the outer terms even and the carrying term odd under $b_c \mapsto -b_c$, and the F_4 kernel $A_2^{\mathsf{T}} B^{-1} A_2$ is itself a congruence invariant: an unsigned hull is flip-invariant and therefore structurally unable to bound the carrier below. Selberg's symmetry formula is provably neutral ($\Lambda = \mu * \log$ and $\mu * \log^2 = \Lambda \cdot \log + \Lambda * \Lambda$ are formal identities in the free module on $\{\log p\}$; information gain exactly 1.000). And the Beurling–Nyman/Baez-Duarte target is empty in exact arithmetic: $\lambda_{\min}(G_N) \le (\log 2\pi - \gamma)/N$ gives cap 0.420220 against the needed $c_B = 0.5523$, so no N-uniform Gram floor exists and the deployed hypothesis is false for $N \ge 3$, while the explicit family $\widetilde{G} = AA^{\mathsf{T}} + (4/5)^2 I$ satisfies every hypothesis of the certificate class at every N with $\widetilde{d}^2 \ge 1/2$ forever — the class is non-implying.

Ten classes are empty, five are merely unexplored. For this budget shape the following are proven empty: E1 every magnitude hypothesis $|\psi(x) - x| \le f(x)\sqrt{x}$; E2 every zero-density input with $A > 2$ ($A = 2$ and Lindelöf land exactly critical, with zero slack); E3 Selberg's symmetry formula; E4 every congruence reformulation, window choice, Jacobi/Cholesky preconditioner and resolvent split; E5 every unsigned hull of the comb correction; E6 every bound consuming only $\{w \ge 0, \|w\|_\infty, \|w\|_2, \mathrm{supp}\, w\}$; E7 exact multiplicative transforms (Möbius/Dirichlet); E8 natural-grammar identities (de Branges/HB, Abel); E9 the Beurling–Nyman transport target; E10 thin/cofinal Li criteria, by explicit counterexample. *No* emptiness is claimed for U1 unconditional statements about ordinate positions, U2 alignment statements, U3 a different budget shape, U4 the four premises of the reverse implication, or U5 global constraints on the source profile.

What it does not say, named. (O1) Nothing is said against arguments that legitimately consume zero positions — the remaining statement *is* of that kind, which is not the same as being false. (O2) Only bounds inside the cone class are forbidden; alignment statements are untouched and are exactly what is needed. (O3) Every clause is proven for the deployed kernel, the deployed frame family and the predeclared rung family; another budget shape lies outside. (O4) The reverse implication (inequality ⇒ RH) rests on four named premises and is *not* established; the theorem prices only the missing input of the forward direction. (O5) Any new global inequality restricting the admissible source profiles beyond $\{w \ge 0, \mathrm{size}, \mathrm{support}\}$ changes the cone class and escapes the argument. The theorem is moreover *finite-rung* — verified at $h = 184/388/839$ with the ladder to $h = 12632$ — it makes no all-h claim and says nothing about rungs outside the built cells. It does *not* say that the Riemann Hypothesis is unprovable, that the wall route fails, or that no proof exists.

The counter-evidence, carried and not smoothed. The NO-WITNESS finding stands: the certified reads are positive at all nine depths, with zero negatives and zero straddles, and the deepest certified read is $2.79579794131272506 \times 10^{-15}$ at $h = 12632$; this module adds none. On the n_0-normalised reading the decay *flattens* with depth — upper-bound exponents -3.21092511 globally against -2.67942161 on the deepest step $5746 \to 12632$ — and the re-derived s/D^2 step $1393 \to 2015$ is *positive* ($+0.50302892$), so the normalised drift is not monotone, and no re-derivable exponent comes near the frozen collapse bar -8.0. A no-go about the *type* of the missing input is not evidence about its *existence*.

Registered `PRIME.NOGO.SIGNED.ONLY.01` as an [O] no-go/typing row. It supplies no positivity, no new interval and no new certificate; it closes no gate, narrows no gate and moves no marker, and it must not be counted as evidence for or against the Riemann Hypothesis nor cited in support

of any positivity claim. The deployed matrix-stage reads that carry the same conclusion under a float64 entry-slack premise and the cited ladder stay in `experiments/` until that premise is discharged. **No statement about the Riemann Hypothesis is made.**

12.41 The kill atlas, the complete candidate map, and the Obstruction Completeness Theorem (sandbox probes)

Four discovery freeze rounds of 2026-08-14 (notes CCCLXXVI and CCCLXXVIII–CCCLXXX, with the surface-audit note CCCLXXVII in between) consolidate the whole campaign into one falsifiable object and one named endpoint. Everything in this subsection is *sandbox*: four frozen probes in the experiments tree, no module, no ledger row, no marker moved, no gate closed or narrowed, and no claim about the Riemann Hypothesis in any direction. The suite stood at 906 modules at the freeze, `v913` then the newest promoted module (954 today, after the `v914`–`v961` promotions of Sections 12.46–12.47, 12.49, 12.50, 12.51, 12.52, 12.53, 12.54, 12.55 and 12.60).

The map as one machine-refutable object (round 79, ***`kill_atlas_dag_probe.py`****,* 43/43 *gates, verdict* ATLAS-DAG-CONSISTENT + ONE-OPEN-EDGE-CONFIRMED + FOUR-CLASSICAL-CITATIONS-CARRIED + NO-SIGN-SOURCE-PASSES + LOCALIZATION-DID-NOT-MOVE-THE-EDGE*).* The RH implication chain is reconstructed explicitly as an 11-node, 10-edge DAG (acyclic under Kahn on the conjunctive hypergraph), typed 3 PROVEN / 2 CERTIFIED-FINITE / 4 CITED-CLASSICAL / 1 OPEN, with exactly one open edge E4 — positivity at every rung of one sign-independently predeclared family, cofinal in the mesh-refinement order. The honest non-PROVEN list, in topological order, is E1/E2/E8/E4/E7/E9/E10: two certified-finite heads, the one open inequality, and four classical citations (the Suzuki/W1 identification, density plus the equi-Lipschitz/Dini continuity modulus, the faithful cap, Weil's criterion) — the qualifier of Section 12.40 carried as an executable gate. The 50-route kill atlas assigns every killed route of the campaign to exactly one edge (17 on E4, 21 on the finite scalar edge E2, 8 on E10, 2 on E1, 2 off-chain seam/compiler results), every verdict token grep-verified inside its own cited artifact; the τ-screen reads 15 DISGUISE / 1 ADVANCE / 34 not-applicable, and the single ADVANCE (the per-step B-floor) does *not* land on E4. Six coordinate systems — $\tau_h > 0$, $n_h - q_h > 0$, $\sigma_h < 1$, the anti-cancellation scalar, the P_{err} budget, and `v913`'s best-conditioned (L) — are pinned as one and the same edge: the twenty notes of localization work sharpened the edge's type, closed the second open edge (`v912`) and corrected its statement (mesh order), but did not move it.

The SAT adjudication (round 80, ***`sat_projection_alignment_probe.py`****,* 35/35 *gates, verdict* SAT-DISGUISE + SAT-LITTLEWOOD-EMPTIED + SELECTION-SIGN-MINED*).* An externally proposed "signed projection alignment" closure of E4 is adjudicated dead as submitted. Its projection identity is exact — verified with zero residual in exact `Fraction` arithmetic — and empty: the column block of A_2 is unit-triangular with determinant 1 as exact integers, so $P = I$ and the alignment-looking term $\|(I-P)y\|^2$ vanishes identically; the ellipse inequality is $s \geq 0$ verbatim, and the "optimal signed profile" w^* is B-independent, purely archimedean-combinatorial, carrying zero arithmetic. The θ-average is verbatim the already-priced mean route (HH-CLAIMS-WITHDRAWN, CLASSICAL-GAP); the proposed θ-selection rule is exactly the `signMinedIndex` shape whose non-noninterference is kernel-checked in Lean; and the small-effective-rank premise is refuted by measurement (stable rank grows $109.6 \to 1595.2$ from $h = 184$ to 2854). SAT is coordinate system number seven for E4, not an attack on it; as the atlas's nineteenth candidate sign source it fails the frozen gate (separates without independently orienting — its readout *is* the wall scalar).

The untested five adjudicated (round 81, ***`untested_sign_sources_probe.py`****,* 33/33 *gates, verdict* SIGNSOURCES-ADJUDICATED*).* The five candidate classes the atlas still carried as untested all fail with exact killing numbers: the Euler-grouping/G2 weight law separates all control worlds but its defect energy is zero on truth and its class floor cannot force the deployed inequality (G2-SEPARATES-NOT-ORIENTS); every source-side Krein contractor certificate has margin exactly $1 - \|C\|^2 = s/n$, the wall sign renamed (KREIN-CERT-IS-WALL); the named unconditional ordinate-position types are empty — the honest instrument on the full 2×10^7 certified ordinate cache prices the required ordinate precision at $\sim$1% of the mean zero spacing, Weyl equidistribution carries no

rate, rate-carrying density is the emptied E2 class, pair correlation is conditional (U1-NAMED-TYPES-EMPTY); the minimal alignment statement MIN-U2 is named exactly and is beyond-classical — restricted to the deployed windows it already excludes off-line zeros at $\delta = 0.209 \ldots 0.364$, below the classical zero-free widths ≈ 0.444 at those heights (U2-BEYOND-CLASSICAL); and every enumerated global source-profile inequality dies on the downward-closure lemma: the zero comb is admissible in every enumerated class, so no upper-bound-type inequality can supply the required lower bound (U5-ENUMERATED-DEAD). The candidate map is thereby complete over the corpus vocabulary: **24 candidate sign sources, 0 pass, 19 fail, no blanket-untested class left.** (The round-108 audit, Section 12.47, makes the vocabulary fence concrete: five classes outside the corpus vocabulary are named and *unpriced* — ergodic/measure-rigidity positivity transfer, model-theoretic/o-minimal counting, bigger-group representation-theoretic positivity, GMC/Fyodorov–Hiary–Keating with an enumeration gap, and higher-order Fourier uniformity of Möbius.)

The bird's-eye round and the endpoint (round 82, `birdseye_shape_freedom_probe.py`*,* 35/35 *gates, verdict* BIRDSEYE-OCT-CONSOLIDATED*).* The three degrees of freedom every route had held fixed are adjudicated. (a) SHAPE-INDEPENDENCE-MEASURED: the deployed family is RH-saturating with measured margin-decay exponent -3.379; five structurally different admissible alternative families (Fejér, Gauss-tent, Hann, the $K \star K$ convolution square, a T5.2-style perturbation — all literally in the `v912/v848` class), priced in the same Weil-square currency with an exact explicit-formula ward, decay far shallower (exponents $-0.46 \ldots -1.09$, minimal gap 2.29) — the maneuver exists but buys nothing: single-window ladders fail the E7 density leg (54.5% projection residue of an admissible target on the pooled span), any mesh-dense family inherits $\lambda_{\min}(\Omega_{\mathrm{EF}}) \le s$ by an exact Schur bound, and the localization is shape-invariant — square shapes degenerate the alignment (all 15/15 position-blind ordinate worlds stay positive) and put the whole unconditional content into off-line exclusion *below* every classical zero-free width ($\delta_{\min} = 0.055 \ldots 0.26$ against ≈ 0.42, 5/5 beyond-classical), while the deployed linear coordinate reproduces the $\sim 1\%$ sub-spacing bar. The hardness is RH's, not the program's. (b) EXISTENCE-IS-DISGUISE: the existence direction is closed permanently at the border-membership step, in exact `Fraction` arithmetic — some positive extension always exists (the softness is contentless), the arithmetic completion being positive definite is equivalent to $s \ge 0$ both ways ($\det M = s \det B$), the Krein-admissible ellipsoid contains the arithmetic border with margin exactly s, and the Carathéodory–Toeplitz one-step interval is nonempty with the true next autocorrelation admissible at margin equal to the next Schur scalar: the residual gap between "an extension exists" and "the arithmetic one is it" *is* E4. (c) BOOTSTRAP-STILL-DEAD: under the round-81 repricing the 2×10^7-ordinate cache certifies MIN-U2 at 2/5 built cells and the τ coordinate at 0/5; the growth law is $T_{\mathrm{req}} \sim h^{4.77}$ on the τ coordinate, the cache's extrapolated τ-coverage ends at $h \approx 36$, and precision is not binding — height is.

> **The Obstruction Completeness Theorem** (wall architecture; finite-rung; machine-asserted premises; no RH claim): (i) the implication DAG has exactly one open edge, E4; (ii) the candidate map is complete over the corpus vocabulary — 24 candidate sign sources, 0 pass, 19 fail, no blanket-untested class; (iii) the missing input is exactly characterized — signed, alignment-carrying, unconditional, an ordinate-position statement at $\sim 1\%$ of the mean zero spacing, whose minimal form MIN-U2 is already beyond the classical unconditional toolbox on the deployed windows; (iv) the characterization is shape-independent across the audited admissible families — with the dual/existence direction closed and the finite-input bootstrap dead.

Scope, in the same breath. The theorem is finite-rung (cells built to $h = 2854$, the certified ladder cited to 12632); atlas completeness is completeness over the routes named in the campaign notes (an editorial claim, typed as such, not a gate); E7–E10 remain classical citations; and the counter-evidence travels unsmoothed — NO-WITNESS stands: all certified reads are positive to $h = 12632$ (deepest $2.79579794131272506 \times 10^{-15}$), the normalised decay flattens, and no collapse bar is approached. The theorem turns "why does RH not fall to this program?" into a machine-checked statement with an exact domain of validity; it says nothing about the truth, provability or

falsity of RH and is not evidence in either direction. A no-go about the *type* of the missing input is not evidence about its *existence*.

12.42 The real-root architecture, the Parity Lemma, and the sign-position predictor (sandbox probes)

Six further discovery freeze rounds of 2026-08-14 (notes CCCLXXXII–CCCLXXXVII) then attacked the one construction direction the Obstruction Completeness Theorem of Section 12.41 leaves untyped — building the missing object rather than bounding it — and ended with the missing object in its most concrete form so far. Everything in this subsection is *sandbox*: ten frozen probes in the experiments tree, no module, no ledger row, no marker moved, no gate closed or narrowed, and no claim about the Riemann Hypothesis in any direction. The suite stood at 906 modules at the freeze (952 today). One convention governs the numbers below: an adversarial bughunt round (round 87) audited everything produced in rounds 79–85 and corrected three MAJOR misstatements in the round-83/84 records; per the standing N12 convention the frozen probes and notes are *not* retro-edited — the correction is the record — and this subsection carries only the corrected values.

The real-root architecture (round 83, three lanes: `semilocal_realroot_limit_probe.py` 16/16*, verdict* REALROOT-ARCHITECTURE-OPEN*;* `stc_sol_convergence_probe.py` 4/4*, verdict* STC-OPEN*;* `stc_opus_convergence_probe.py` 20/20*, verdict* STC-DISGUISE + STC-FALSE *for the mesh family).* The first route of the campaign to genuinely leave the kill atlas: instead of proving positivity of the Weil form (edge E4), build finite *self-adjoint* source-only operators — real spectra by construction, no wall positivity, no zero data consumed — and identify the limit with Ξ through the full trace formula on a determining even Paley–Wiener class; compactness, Hurwitz, Hadamard and determination then give RH. Lane one audits the skeleton as executable gates (Hurwitz sound; the determination lemma sound in two classical steps with *no convergence rate consumed*; two repairable gaps found and repaired, an origin-exclusion minorant derivable from R3 itself and the source-only $E''(0)$ pin $\sum_\rho 1/(\rho(1-\rho)) = 2 + \gamma - \log 4\pi$), and measures the Connes extremal family: census fully real (41/41 at $x = 13$), all six trace rows converge, world controls separate at median 1.26×10^6–1.40×10^6 against the pre-frozen bar 5 — at the honest price that the minimizer direction exists only at internal precision $\sim e^{-4\pi x}$ ($\lambda_{\min} = 2.5 \times 10^{-54}$ at $x = 13$). The corrected zero-tracking read (round 87, finding F1) is *stronger* than the one first recorded: the exact-mp census puts the first-zero deviation at -8.4×10^{-16} at $x = 8$ *and* $x = 13$ — exactly the cache's own float64 quantization of γ_1 — and the mean displacement over all 15 matched zeros at 0.0 in float64 resolution (the minimizer zeros sit on the cached ordinates below the comparator's own floor); the corrected ε finals move by up to 117% under the exact census (round 87, finding F6) with all six rows still CONVERGES. Lane two builds the Connes–van Suijlekom quotient source-only, hits 14.1347/21.0220/25.0109 at $x = 8$ with no zero read, and proves the structural point: **R3 alone implies Weil positivity** ($h = |\hat{g}|^2$ on a real spectrum), so the trace convergence carries full RH strength. Lane three proves the **Nyquist density obstruction** for its mesh-CF family: $\operatorname{spec} H_N$ has constant density $\ell/2\pi$ exactly and mesh-independently (rel. deviation 2.22×10^{-16} over a 16× mesh sweep) while Ξ's zeros have density $\log(\tau/2\pi)/2\pi$ — so R3 and R4 fail for every mesh at fixed window; the unweighted trace diverges on all four batteries to $x = 144$ while the weighted trace converges (honest finals 9.3×10^{-4} / 3.3×10^{-6} / 2.3×10^{-6} / 6.2×10^{-4}; round 87, finding F4), but the weights are the Christoffel function of the zero measure — the unknown; the honest node-vs-zero pairing at $x = 55$ is 18-vs-3 one-sided (36-vs-6 two-sided; round 87, finding F2). The three lanes ended in a recorded conflict, deliberately frozen unresolved for an arbiter round.

The arbiter (round 84, `realroot_arbiter_probe.py`*,* 17/17 *gates, independently re-run bit-identical, verdict* REALROOT-DEPTH-UNDECIDED*).* The conflict is resolved *by mechanism*: different operator family, not different sums, not a bug, not primarily depth. All three lanes computed honest unweighted sums against the identically normalized source side. The discriminator is the band census across depth, with a new deep rung $x = 21$ built at the honest price ($K = 80$, 128 dps against the declared floor $4\pi x/\ln 10 + 5 = 119.6$, $\lambda_{\min} = 1.25 \times 10^{-93}$): the mesh-CF family's $(0, 30)$ count grows $7 \to 22$ from $x = 5$ to 144, tracking the uniform Nyquist prediction (measured

deviations 10.2/10.6/10.9% at $x = 13/21/34$ against the frozen 30% tolerance; round 87, finding F3) — lane three's density theorem confirmed for its own family — while the extremal/quotient family stays *exactly* on the Ξ census at every measured depth: $(0, 30) = 3$, $(0, 60) = 13$, $(0, 100) = 29$ at $x = 21$, minimum zero = γ_1 to 10^{-4}, with all 50 Nyquist-excess zeros exiled to $(100, 163]$ where fixed test functions are blind. On one unified table (same h, same normalization, same unweighted sum) the mesh family diverges on all four batteries while the extremal family converges on the same four. The verdict is REALROOT-DEPTH-UNDECIDED — not false-for-family (the obstruction reaches the extremal object in no measured band), not quotient-escapes (the thinning is *measured*, not proven; Connes' section 6.6 remains open) — and the new **Z1-transfer identity** names why a proof is the wall itself: the trig-Galerkin matrix *is* the Gram matrix of zero evaluations, $M[j,k] = 2\sum_\gamma B_j(\gamma)B_k(\gamma)$ with $\lambda_{\min} = 2\sum_\gamma E(\gamma)^2$ (deviation 6.31×10^{-5} after the analytic cache-tail correction, 92% model share) — the minimizer minimizes its own transform *on the true ordinates*: source-only in input, zero-measure in content. Since R3 alone is RH-hard, no finite rung can verify the architecture — falsification rungs are priced ($x = 34$: $K = 150$ at ≥ 191 dps, ∼1.5 h; $x = 55$: $K = 276$ at ≥ 305 dps, ∼10 h) — and the surviving lemma carries full RH strength.

The TPL round (round 85, three independent lanes on the thinning-persistence lemma: ***tp_sol_probe.py*** *7/7, verdict TPL-OBSTRUCTED;* ***tp_opus_probe.py*** *32/32, verdict TPL-REDUCED + TPL-DICHOTOMY-MEASURED + TPL-OBSTRUCTED;* ***tp_da_probe.py*** *9/9, verdict TPL-CIRCULAR).* Lane Sol proves the off-line-quartet obstruction: off the line the Z1 sum is an indefinite bilinear form ($\Delta Q = -4\delta^2 E'(t)^2 < 0$ at simple real zeros) yet Rouché preserves the band count — **energy sign and zero census are locally independent observables** (measured: $\lambda_{\min}$ flips to -9.1×10^{-17} at $1.25\,\delta^*$ while the $(0, 30)$ count stays 3 and roots move 5.5×10^{-9}). Lane Opus supplies the strongest mathematics of the round: the ARCH block *is* the smooth zero measure exactly (rel. 1.1×10^{-6}), the density-level ground state is negative and $O(1)$ on every rung — the entire e^{-cx} positivity is supplied by the prime block alone, so no density-input argument can prove TPL(i); $\|\text{PRIME}\|/\text{gap} \sim 10^{4.36x}$, so any prolate comparison must be an exact cancellation, never perturbative; **the Parity Lemma**: the basis collapses to one secular function, and TPL(i) *is* — no analysis, a finite combinatorial rewriting — the statement that the ground eigenvector of one explicit matrix carries a prescribed sign pattern, verified exactly on three rungs; and **the counting law**: the Nyquist surplus peaks at $D_{\max} = x - 3/8$, the tracking band ends exactly at the peak, and the marginal cost $d(-\log \lambda_{\min})/dD_{\max} = 10.1 \ldots 11.3$ is constant to 11% over 23 decades — the e^{-cx} law of the whole architecture is a counting law, each forced skip (each sign change) costing one fixed factor e^{-11}. Lane Advocate convicts the route's measured support as circular: the recorded band counts *are* the smooth Riemann–von Mangoldt values to < 1 (29.0023 against 29); the instrument transcribes whatever measure it is fed (the prime-free smooth world gets its own clean census); outrunning classical zero verification would need $x \sim 3.8 \times 10^{11}$, i.e. dps $\sim 2.1 \times 10^{12}$; the only off-line-sensitive channel is the wall scalar itself (measured slope 2.03, independently confirming Sol's $-4\delta^2 E'^2$ law). The three lanes reconcile into one picture, and the round's one-sentence status: the *number* of ground-eigenvector sign changes is forced by counting — the arithmetic substance sits exclusively in the *positions* of the sign changes, which the prime block alone must supply.

The simplicity round (round 86, ***simple_principle_probe.py****, 22/22 gates, verdict SIMPLE-PRINCIPLE-IS-TRANSCRIPTION).* Four readings of "the simple principle must thread through everything," each with pre-registered consequences and frozen bars. The one-axiom reading is refuted in its strong form (five natural normalizations, zero hits on the declared constant $\log \pi$, gaps 0.226–1.341; the difference-level dictionary is exact, so the equality is exact as a gauge orbit and empty as a source). The transcription reading is *confirmed* 3/3 on the sharpest claimed arithmetic outputs: the register's Mertens head is exactly $\prod_{p|210}(1 - 1/p) = 48/210$, i.e. 2.1293 bit = $\log_2(35/8)$ — the "arithmetic output" is the compiler pair itself — and the $E''(0)$ pin is the Hadamard identity to 2.4×10^{-8}. The fixed-point reading is refuted (the compression transcribes every fed measure: back-distance exactly $1.0000\,\delta$ under jitter). The Gram-seat square law $\delta_* = \sqrt{\text{margin/curvature}}$ is measured numerically exact on the Gram channel (ratio 0.9997, slope 2.0000) with the order split

(position first-order, reality second-order; on-line slope 0.9669) — and by the frozen FOLDCOV rule it is typed PATTERN-ONLY-EVEN-IF-CONFIRMED. What survives is one principle: **TFPT's positivity is always Gram positivity of node evaluations — a theorem exactly where the nodes are compiler data — and the arithmetic seam is the single place whose nodes are not.** Two measured corollaries sharpen the OCT without moving it: the order split mechanically explains why all 24 candidate sign sources died (they sought first-order information in a channel that carries reality only at second order), and the $\sim$1% sub-spacing price is instrument-dependent — $\sqrt{\text{margin/curvature}}$, scaling like $h^{-1.69}$ along the deployed ladder. Tested simplicity adds nothing beyond the OCT; it explains why the OCT looks the way it does.

The bughunt round (round 87, ***`bughunt_r79_r85_probe.py`****, 24/24 gates, verdict* BUGHUNT-FINDINGS*(9, max* MAJOR*)).* An adversarial audit of everything produced in rounds 79–85: nine findings, 3 MAJOR / 6 MINOR / 0 FATAL, *no round verdict flips*, every finding pinned by an executable gate. The three majors are the corrections carried above (the first-zero/mean-displacement conflation, whose honest resolution is stronger than the published claim; the 36-vs-3 pairing, honestly 18-vs-3; the float64-refinement artifact in the published ε finals, with gated survival of every verdict). One systemic minor: nine gates across the corpus are hardcoded `check(..., True)` and can never fail while counted in the N/N tallies — including the very line that records the round-80 "candidate #19 gate FAIL" verdict (the verdict is right; the gate tests nothing). Checked clean: every round-79–82 headline number reproduces on fresh re-runs, all ten probes re-ran green, no firewall violations.

The sign-position round (round 88, ***`signpos_probe.py`****, 19/19 gates, independently re-run bit-identical, verdict* SIGNPOS-PREDICTOR-EXACT + SIGNPOS-TP-DEAD + SIGNPOS-RIGID + SIGNPOS-K-STABLE*).* The campaign's most concrete endpoint: the positions of the ground-eigenvector sign changes — the single open object left by the Parity Lemma — attacked with the oscillation/total-positivity/node-duality toolbox. A **source-only predictor of the sign-change positions exists and is exact**: the parity of $\text{round}(\theta(\omega_k)/\pi + 1 + S_x(\omega_k))$ at the Nyquist lattice, where S_x is the band-limited phase built from the prime-node comb, is exact at *all 79 lattice points* across $x = 3/5/8/13$, with measured first-order arithmetic content: the density alone fails at exactly four lattice points and the prime-node comb repairs precisely those four (arithmetic load 4) — the first arithmetically computable first-order-channel object of the campaign; a scramble control fails 6/11 with $\lambda_{\min} = -1.11$, so the exactness is explicit-formula-specific, not a pipeline transcription. The catch is named precisely: proving the predictor's exactness for *all* x is half-integer-margin control of the band-limited S at the lattice points (measured minimum margin shrinking to 0.0317), which is verbatim TPL(i) itself — the RH hardness relocates for the *third* time to the same seat, now as a rounding-margin statement about one explicit band-limited function of the primes. Gantmacher–Krein is structurally dead (the density blocks are indefinite, $\lambda_{\min}(\text{ARCH}) = -2.666$; M is neither totally nonnegative nor a Green's/one-pair matrix in any tested ordering; the minor census localizes nothing); the map {ordinate positions} $\to$ {sign pattern} is *rigid* with modulus equal to the ordinate-to-lattice distance (TPL(i) is exactly the statement that the prime block never carries a node across the lattice); and the sign-change positions are stable in K inside the deployed band (the round-85 well-posedness caveat is discharged for the positions; the joint (x, K) declaration remains for the margins).

Correction of record (round 97, 2026-08-15, ***`sp_expsum_pricing_probe.py`****, 13/13 gates, verdict* SP-OPERATIVE-FALSIFIED *— carried per the round-87 convention: the frozen probes and notes are not retro-edited, the correction is the record).* The predictor's exactness holds at $x \le 13$ only. Round 97 (Section 12.45) extended the census from the 79 lattice points ($x \le 13$) to 18798 points ($x \le 890$, the cache edge $\gamma_{7000} = 7264.75$) and measured the *operative* form of this predictor — $\text{round}(N_{\text{src}}(\omega_k)) = N_{\text{true}}(\omega_k)$, TPL(i)'s finite form — **false from** $x = 21$ **onward**, with 567 τ-screened counterexamples: the round-88 exactness was a small-x accident of the margin budget. The mechanism is arithmetic-free — the Fejér taper's irreducible tracking-variance floor (rms $\to 1/(2\pi) = 0.159$, measured 0.26) against the 0.5 budget, so no taper in this family survives — and the same round priced the budget as classically out of reach (best unconditional miss $\times 8.2$;

even the RH-conditional bound reads 1.015 at the deepest rung), with the prefix/exceptional-set salvage measured dead in-window. The fence travels with the correction: what died is the deployed *source-side approximant*, not TPL(i) itself — the eigenvector truth side is unmeasured beyond $x = 13$ — and the round-88 claim as published (exact at all 79 points, $x \le 13$) remains factually true; its operative extension is now measured dead.

> **Where the arc leaves the campaign** (sandbox; no RH claim): the wall route is closed as a theorem (the OCT of Section 12.41); the only architecture that genuinely escapes the kill atlas is adjudicated REALROOT-DEPTH-UNDECIDED with its measured support convicted circular; and the missing object now has its most concrete form ever — the eigenvector sign-change positions, equivalently the rounding margins of one explicit band-limited prime function — and carries full RH strength in every coordinate system in which it has been written.

Scope, in the same breath. All of this is exploration: finite measurements on reachable rungs ($x \le 21$ for the deep census, $x \le 13$ for the predictor), no all-x statement, no positivity claim, no promotion, and per the bughunt's own framing its 24/24 PASS means "the audit is consistent," not "no bugs." RH is neither proven nor disproven here; what is documented is the complete map of why, now including the object-construction direction. **No statement about the Riemann Hypothesis is made.**

12.43 The Stieltjes–Vitali pin route and the Krein screw-function carrier (sandbox probes)

Two further discovery freeze rounds of 2026-08-14 (notes CCCLXXXIX–CCCXC) then attacked the resolvent direction the arc of Section 12.42 leaves open: replace the full semilocal trace convergence by countably many positive scalars, and build the one carrier the corpus did not have. Everything in this subsection is *sandbox*: two frozen probes in the experiments tree, no module, no ledger row, no marker moved, no gate closed or narrowed, and no claim about the Riemann Hypothesis in any direction. The suite stood at 906 modules at the freeze (952 today).

The Stieltjes–Vitali pin route (round 89, externally proposed, contract PRIME.STIELTJES.VITALI.PIN.01*;* `stieltjes_vitali_pin_probe.py`*,* 17/17 *gates,* 249 *s, independently re-run bit-identical, verdict* SVPIN-ROUTE-OPEN*).* The proposal replaces the full semilocal trace convergence by countably many positive resolvent scalars in the absolutely convergent Euler half-plane: the pins $P_x(\sigma) = \sum_k 2\sigma/(\lambda_{x,k}^2 + \sigma^2)$ must converge to $\xi'/\xi(1/2+\sigma)$ at the countable family $\sigma_r = 1 + 1/r$, where the right side is source-only (Γ, π, $\Lambda(n)$, n^{-s} — absolutely convergent for $s > 1$); then Herglotz normal-family compactness (*one* pin suffices), Vitali on the pins accumulating at the interior point i, the identity theorem on $\Im z > 1/2$ (where Ξ is zero-free), analytic continuation on the connected $\mathbb{C}_+ \setminus Z(\Xi)$, and the pole contradiction give (SV) $\Rightarrow$ RH, with the functional equation closing $\beta < 1/2$. **The skeleton is sound, gate by gate**: the weight-comparability constant $C_D = 65.01$ (bar 2×10^3) is executed per rung (normal-family ratio 0.033 on all four); the sign-convention ambiguity of earlier rounds is *harmless* for the pins — $\Xi(z) = \xi(1/2 - iz)$ and $\xi(1/2 + iz)$ give the identical target via $\xi'/\xi(1-s) = -\xi'/\xi(s)$, with a numeric lock at $\sigma = 1$ (dev 2.3×10^{-8}); the spectrum honesty is measured, not assumed (census 4/10/20/41 all real, no zero mode, min spacing ≥ 1.3); the degenerate Herglotz limit is excluded by the pin bound. The *one* named condition: the all-x realness of the spectra is the CF simplicity-plus-evenness hypothesis, so the skeleton is conditional on it. The named non-gap, stated as bluntly as the probe states it: **the chain moves the entire burden into (SV) itself** — no skeleton step is the wall; the wall is the premise. The tail bookkeeping is computed explicitly and all three tails decrease (true zero tail beyond the band $2.85 \times 10^{-2} \to 1.18 \times 10^{-2}$, within 2% of the $\log T/T$ model; measured Nyquist-excess edge mass $\le 1.31 \times 10^{-3}$, the window definition corrected in the spec after a smoke caught excess zeros spilling past the band edge; exact lattice tail $1.35 \times 10^{-2} \to 8.0 \times 10^{-3}$). The finite pre-check **converges on all 16 frozen σ values** ($x = 3/5/8/13$; $x = 21$ skipped

and declared by the arbiter cost law): Δ negative everywhere (the missing tail dominates), slopes $-0.62\ldots-0.65$, measured drop 2.6 against the a-priori Riemann–von Mangoldt model 2.4 (bar 1.6), with *no* $\sigma \downarrow 1/2$ rate degradation on this ladder (the expected boundary effect lies beyond reachable depth, stated honestly), and the full divisor row converges *faster* (the lattice comb compensates $\sim 2/3$ of the missing tail). All four world controls separate — the mesh-CF family is **mesh-poisoned through the same pins** (median overshoot growing $0.94 \to 6.32$ like $\log x$: the wrong limit, measured); SCRPOS/EPSTEIN/SMOOTH/SCRARITH separate at medians 2.7×10^5–8.8×10^5 (band metric) and 12.4–45.9 (target metric) on 16/16 rows (SCRARITH throws one imaginary pair — the world breaks the SV realness hypothesis, reported); Christoffel weights are structurally excluded by an AST gate; and the τ-screen shows 0/16 relocations at max slope 0.009 over ~ 47 decades — **no disguise**: the pins measurably live away from the wall margin. And the honest weight, Z1-typed as the actual adjudication: on the extremal family the measured pin convergence *is* the transcription of cache partial sums (band pins against cache rel 7.7×10^{-12} / 9.4×10^{-12} / 2.1×10^{-5}) — instrument consistency, never source-side content — and the naive Suzuki spectral readout is *dead* (SUZPIN-FORM-SPECTRUM: the hat-compression eigenvalues are a form-spectrum ladder $\sqrt{\mu} = 1.02/2.12/3.17\ldots$, not the ordinates ($\gamma_1 = 14.13$); $P_{\mathrm{suz}}(1)$ grows $1.85 \to 2.47$ against the target 0.046). The route is the *second* architecture of the campaign to survive every gate, and its reduction — countably many safe scalars in place of a test-class quantification — is the corpus's logically sharpest open target; its burden of proof is unchanged, only sharper localized, and the priced remaining task is exactly one: the Krein inverse-spectral realization of Suzuki's screw function, which the corpus did not have.

The Krein screw-function carrier (round 90, contract PRIME.SUZUKI.KREIN.REALIZATION.01*;* `krein_screw_realization_probe.py`*,* 22/22 *gates,* 18 *s, independently re-run bit-identical, verdict* SUZKREIN-CARRIER-OPEN*(lemma: uniform Weyl-disk contraction)).* The priced missing ingredient, built and adjudicated. The screw function g is extracted corpus-exactly (Suzuki's eq. 1.3 with the v643 convention lock; the Lerch coefficient $+1/4$ is *gated* against the origin expansion $g \sim \frac{1}{2}t\log t + At$ with $A = \frac{1}{2}(\log 2\pi - \psi(2)) = 0.707546$ — a -1 misreading would fail by $O(1)$); the screw property is verified ($G(t,s) = g(t-s) - g(t) - g(-s)$ PSD on all four windows, $\lambda_{\min}/\lambda_{\max} \geq -8.1 \times 10^{-19}$); the invoked theorem is named with its hypothesis *certified*: Krein's accelerant/continual-Schur correspondence (Krein 1955; discrete transcription Verblunsky/Geronimus), whose hypothesis — strict positivity of every Toeplitz section of the accelerant $-g''$ — is certified by the completed Szegő run ($\max|\alpha| = 0.156$); and the normalization is *derived, not fitted* — the lag reads are exact second differences of g (two independent routes agreeing to 1.8×10^{-10}), a partition of unity gives the pin as $\frac{1}{2}\langle -g'', e^{-\sigma|\cdot|}\rangle$, and the Weil formula identifies the target $\xi'/\xi(1/2+\sigma)$ layer by layer (archimedean, pole and prime layers each gated). The inverse solve is classical: **the Levinson/Szegő recursion *is* the Toeplitz-structured discrete Krein/Gelfand–Levitan equation**; the Verblunsky coefficients are the realized Hamiltonian data and are *causal* (exact Toeplitz nesting: deeper windows only append); and **the realized Hamiltonian sees the primes** — $|\alpha|$-spikes at $r = \log q$ for $q = 2, 3, 4, 5, 7$, offset ≤ 1 bin, contrast 2.8–5.2. The conditioning is trivial (amplification ~ 23 under 10^{-25} perturbations — the feared e^{cx} explosion belongs to the trig-family spectral-resolution ladder, not to the screw-kernel Toeplitz sections), and the δ-rough pointwise profile is declared honestly: the δ-stable objects (pins, disks, spike positions) are what is gated. The decisive measurement: with the 16 frozen σ values taken verbatim from the SVPIN spec, **all 16 rows converge, 16/16 truth-tight** ($\sigma = 0.6$: $\Delta = +7.30 \times 10^{-5} \to +6.33 \times 10^{-7}$ over $L = \log 3 \ldots \log 13$), with the Weyl-disk contraction *measured* as $R \sim e^{-(\sigma+c)L}$, $c = 0.85\ldots1.76$, median 1.36 (limit-point behaviour); and the round's surprise: **the disk center resums the missing tail** ($|\hat{P} - S_{\mathrm{trunc}}|$ against the tail model, median ratio 1.000; at $\sigma = 0.6$ the center beats the truncated source sum by a factor ~ 600) — the residual is the arithmetic fluctuation beyond the window. The controls die *at the positivity level itself*, the strongest separation of the campaign: the prime-free SMOOTH accelerant **loses screw positivity** at window depth $r = 0.264$ (the same r at both mesh sizes — a continuum property, not a grid artifact), the golden-scrambled SCRARITH world at $r = 0.744$: **the true primes — positions and weights — are necessary for the screw positivity of these windows.**

And the screen that every previous pin carrier failed is passed for the first time: **no Z1 cache transcription** (the minimum over all $N \le 7000$ of the max-σ relative deviation is 8.5×10^{-3} against the bar 10^{-6}; AST firewall zero-free; SUZKREIN-EXTRAPOLATES(1.000)) — and the carrier is explicitly distinct from the dead de Branges route (DEBRANGES-COMB-BLIND) in input (screw lag data, not wall-block moments), direction (inverse-spectral construction, not HB re-coordinatization) and adjudication (quantitative convergence plus world controls, not structure existence); the dead SUZPIN form-spectrum readout is never used. The repricing, from measurements: the literal SVPIN trace-class norm is *measured flat* (16/16 non-decaying; the appended Verblunsky blocks grow — prime atoms arrive forever), so the correct currency is not trace-class but Weyl-disk contraction, and the minimal missing lemma is stated exactly: *let g be Suzuki's screw function with every finite section of $-g''$ positive (= localized Weil positivity); then the Weyl disks $D_a(i\sigma)$ of the Krein realization of $-g''$ contract with $R_a(\sigma) \le C(\varepsilon)\, e^{-(\sigma+c)\cdot 2a}$ uniformly on $\sigma \ge 1/2 + \varepsilon$, with limit point $\xi'/\xi(1/2+\sigma)$* — suppliers: Krein's accelerant theorem (existence and identification *given* all-a positivity) and de Branges chain monotonicity (nesting); open is the uniformity as $\sigma \downarrow 1/2$ — **and the honest price, unchanged: the all-a section-positivity hypothesis *is* localized Weil positivity itself.** The carrier localizes the remaining task (one disk-contraction rate for one explicit canonical system) and replaces the wrong trace-class currency by the right one; it does not remove the positivity input. Disclosures travel with the record: pre-freeze float64 calibration, two smokes (six fixes, including a normalization slip caught by the $\Delta = -\text{tgt}$ pattern of the table itself) and a first-full-run abort at a circular liveness filter (repaired and disclosed in the spec, SPEC-hash change documented) — no bar, grid, ladder or verdict-bearing rule moved in any shakeout.

> **Where the day's arc ends** (sandbox; no RH claim): the wall route is closed as a theorem (the OCT of Section 12.41); the real-root architecture is adjudicated with its measured support convicted circular, and the missing object reduced to eigenvector sign-change positions with an exact source-only predictor (Section 12.42); the pin route is the sharpest reduction — countably many safe scalars in the absolutely convergent half-plane; and the Krein carrier is the first host whose convergence is *not* a transcription of cached zeros — and at the bottom of every route, machine-verified, sits the same single input: Weil positivity, in one currency or another. The program's map of why RH does not fall to it is now complete through the object-construction and resolvent directions, with the two open lemmata precisely stated: Weyl-disk contraction of one explicit canonical system (stated by round 90 with a $\sigma \downarrow 1/2$ uniformity that round 92 later showed not load-bearing — pointwise at the pins suffices; the adjective is retired of record with round 109, Section 12.47), and the sign-change positions of one explicit ground eigenvector.

Scope, in the same breath. All of this is exploration: finite measurements on reachable windows ($x \le 13$; $x \sim 21$ continuations priced and not claimed), no all-x and no all-a statement, no positivity claim, no promotion. The SV skeleton is conditional on the CF realness hypothesis; the carrier's convergence is a finite measurement of a lemma, not its proof; and the lemma's hypothesis is the very positivity the program has not supplied. RH is neither proven nor disproven here. **No statement about the Riemann Hypothesis is made.**

12.44 What any cohomology for $\operatorname{Spec}\mathbb{Z}$ must supply: a machine-grounded specification (sandbox probes)

Three closing discovery freeze rounds of 2026-08-15 (notes CCCXCII, CCCXCIII and CCCXCV) then turned the campaign's machine-checked characterization of the missing input into a literature-facing deliverable. Everything in this subsection is *sandbox*: four frozen probes in the experiments tree, no module, no ledger row, no marker moved, no gate closed or narrowed, and no claim about the Riemann Hypothesis in any direction. The suite stood at 906 modules at the freeze (952 today).

The patterns round (round 91, two probes, both verdicts honest negatives, both informative). On the mandate "find new patterns, proofs, predictions," the program's first magnetar giant-flare QPO

bed (`magnetar_qpo_ladder_probe.py`, 8/8 gates, grep-verified unclaimed) tested SGR 1806−20 (18/26/30/92.5/150/625/1840 Hz) and SGR 1900+14 (28/53.5/84/155 Hz) against the geometric (3/2) relaxation ladder with a 20000-draw log-uniform MC null, a 17-base look-elsewhere battery, a power gate (planted ladder $p = 5 \times 10^{-5}$) and a specificity gate (integer harmonics $p = 0.078$ not misread as a ladder): base 1.5 gives $p = 0.3163$ and $p = 0.3558$, Fisher-joint $p = 0.3584$ — verdict NULL, S15-typed, the standing favorite remaining torsional crustal/magneto-elastic modes (on record and not escalated: placebo base 1.80 is best in both sources, an LEE-loaded base-battery artifact candidate; SGR 0526−66 DATA-LIMITED by the frozen $\geq$ 3-centroid rule). The LEE-controlled scan for an unclaimed dimensionless observable hit by the frozen compiler vocabulary (`unclaimed_observable_scan_probe.py`, 5/5 gates after the disclosed amendment A1 — gate G02 was mis-specified as a minimality bar and amended to a membership bar, no verdict bar moved) returned SCAN-UNDECIDABLE with the structural finding as the actual content: the vocabulary is *saturated* at 2σ tolerance — even the two positive controls have cheaper accidental hits than the corpus expressions (median placebo $C_{\min}$ 3.66 $\leq$ control $C_{\max}$ 5.46) — so, read together with the round-86 transcription verdict, new *predictions* provably cannot come from numeric scanning and must be derived theory-side; new pattern *beds* remain legitimate, but this one is honestly NULL-typed.

The Verblunsky round (round 92, contract PRIME.SCREW.VERBLUNSKY.INVARIANT.01; ***vbk_invariant_probe.py****,* 23/23 *gates, deterministic* 14.1/14.0 *s, verdict* VERBLUNSKY-NO-INVARIANT*(mechanism exhibited)).* The invariance attack on the well-conditioned screw coordinate, with three foundational corrections pinned first: (V0a) $\sigma_r = 1 + 1/r$ lies in $(1, 2]$ and accumulates at the *interior* point 1, so pointwise Weyl-disk contraction on exactly that countable sequence plus the one-pin normal-family bound suffices for (SV) $\Rightarrow$ RH — the $\sigma \downarrow 1/2$ uniformity of round 90 is not load-bearing; (V0b) the exact δ_1 no-go — pin convergence cannot prove strict section positivity backwards, the carrier cannot supply its own existence premise; (V0c) **the Euler–Pick criterion, audited and pinned**: with $P(z) = \xi'/\xi(1/2 + z)$ and $\sigma_j = 1 + 1/j$, RH $\Leftrightarrow$ $P_N = [(P(\sigma_j) + P(\sigma_k))/(\sigma_j + \sigma_k)] \succeq 0$ for every N — forward, each zero contributes symbolically exactly the sum of two rank-one Grams; backward, the classical Pick theorem (Pick 1916; countable case by normal families), interior accumulation at 1, the identity theorem and the logarithmic pole residue give exactly the SVPIN pole contradiction, hypotheses gated one by one and finite N never sold as "all N." The source-only falsification ladder from Γ, π, n^{-s} and an own Λ-sieve (cap 4×10^6, 283146 primes, ψ ward 3.7×10^{-10}) reads $\lambda_{\min}(P_N) = 4.59 \times 10^{-2}$ down to 2.44×10^{-62} for $N = 1..12$, all positive (a certified negative eigenvalue would disprove RH), condition number 2.27×10^{61} at $N = 12$. And the **(AC) identity** $c^T P_N c = \langle -g'', f_c \star \tilde{f}_c \rangle$ is gated symbolically exactly and independently numerically against the corpus-exact screw function g ($8.581810362 \times 10^{-2}$ vs $8.578893345 \times 10^{-2}$, rel 3.4×10^{-4} at the declared $L = 9$ tail/taper): **Euler–Pick positivity *is* Weil positivity on exponential autocorrelations, by identity.** The kill itself: the complete p-packet update law is exact (Toeplitz, causal, inverse-update residuals $\leq 3.7 \times 10^{-15}$), but a complete packet is *not* rank one — it is a high-rank indefinite Toeplitz update of displacement rank two ($p = 2$ at dimensions 128/160/180: ranks 26/90/130, inertias $(13, 102, 13)/(45, 70, 45)/(65, 50, 65)$), the induced Verblunsky step is nonlinear and prior-state dependent (measured state effect $\times 316$), and sharpest: the true complete prime-packet sequence itself *exits* the Schur disk after $p = 2/3/5/7$ ($r = 1.122/1.620/1.950/2.400$) and only re-enters through later packets (after $p = 11$: $\max|\alpha| = 0.313$; full: 0.184) — no region inside the disk is preserved by every packet, the rescue is collective across packets, not multiplicative-local. The unit-disk control wards reproduce SMOOTH $r = 0.264$ and SCRARITH $r = 0.744$ exactly, with the $r = 0.264$ event *diagnosed*: not multiplicativity but a **support error** — the true prime measure has exactly zero mass before $\log 2 = 0.693$ while the PNT density inserts false mass from $u = 0$ (0.282 by depth r); past $\log 2$ the scramble shows the second condition — position kept, weight $\times 1.451$, death at 0.744: exact $\log p/p^{m/2}$ weights are necessary, but no local conservation law exists. The only exact survival condition consumes the inverted prior section and is WALL-EQUIVALENT. Clean kill; the remaining all-depth principle is Weil positivity in yet another currency.

The specification sheet (contract PRIME.COHOMSPEC.SHEET.*01;* `cohomspec_probe.py`*,* 20/20 *gates,* 8.8 *s, SPEC* `423d934e`*, independently re-run deterministic, verdict* COHOMSPEC-COMPILED*(7 properties: 6 machine-grounded, 1 citation-typed)).* The program's most valuable literature-facing artifact is the *measured shape of the missing theorem*, and this round compiles it as a requirements document: seven numbered properties that any candidate positivity mechanism — concretely, any cohomology/intersection theory for $\operatorname{Spec}\mathbb{Z}$ supplying "a Weil pairing with structural positivity," in the sense of Deninger's program or the Connes–Consani scaling site — must have, each machine-grounded by grep ward or independent recomputation in its owning frozen artifact. (P1) It must produce the **signed, alignment-carrying, unconditional, sub-spacing** input of the one open edge E4 (`v913`; the kill-atlas DAG; the 24/0/19 candidate tally; the ∼1% sub-spacing price), whose sharpest concrete corpus forms travel with it (the Parity-Lemma counting law $D_{\max} = x - 3/8$ at marginal cost e^{-11} per forced skip; the sign-position predictor at minimum half-integer margin 0.0317, arithmetic load 4 — corrected of record by round 97, Section 12.45: exact at $x \le 13$ only, the operative extension measured false from $x = 21$ onward with 567 τ-screened violations to the cache edge $x = 890$, a small-x accident of the margin budget; TPL(i) itself untouched, the eigenvector truth side unmeasured beyond $x = 13$). (P2) It must be **shape-independent** in the measured sense (deployed decay exponent -3.379, recomputed by OLS from the frozen five-cell ladder as -3.3795; the five admissible alternatives shallower by a gap ≥ 2.2 and losing the E7 density leg). (P3) It cannot be a **magnitude/density/existence** statement (Littlewood floor/slack 214.7 own-conservative, 1527.1 deployed; zero-density empty for $A > 2$; the existence direction closed at the border-membership step, $\det M = s \det B$ with ellipsoid margin exactly s, recomputed in exact `Fractions`). (P4) Its **Gram form on exponential autocorrelations must *be* the Euler–Pick matrices** — the concrete interface any cohomological trace formula must match, recomputed symbolically exactly and numerically (floor ladder $N = 1..12$ reproduced at max rel dev 3.8×10^{-9}; the (AC) weld at rel devs $\le 4.3 \times 10^{-11}$): a proposed trace formula can be tested against this interface *today*, at any finite N, by machine. (P5) Its positivity must **fail on the control worlds exactly as measured** — the pairing must consume the exact prime support (SMOOTH dies at $r = 0.264$ by the sub-$\log 2$ support error, recomputed) and the exact $\log p / p^{m/2}$ weights (SCRARITH dies at $r = 0.744$, weight $\times 1.451$ at kept positions, recomputed); a "structural positivity" that survives either control world is structurally incapable of being the missing input. (P6) It must **survive the τ-screen** (frozen bands $|s| \le 0.30$ / $s \ge 0.70$; fifteen campaign routes died as the wall scalar renamed; the one ADVANCE does not land on the open edge). (P7, citation-typed) **The function-field precedent, mapped to where it breaks**: in Weil's proof the pairing is the intersection pairing on $C \times C$, the structural positivity is the Castelnuovo–Severi/Hodge-index inequality, and Frobenius supplies the special cycle; for $\operatorname{Spec}\mathbb{Z}$ the sought analogues are positivity ↔ P1–P6, Frobenius ↔ the arithmetic flow (Deninger's foliated dynamical system; the Connes–Consani scaling action with periodic orbits of lengths $\log p$), and the surface ↔ the missing absolute-geometry substrate ("$\operatorname{Spec}\mathbb{Z} \times_{\mathbb{F}_1} \operatorname{Spec}\mathbb{Z}$," the square of the scaling site). The break points, verified against the current literature: Deninger's program has the shape of the cohomology and reads the explicit formula as a dynamical Lefschetz trace formula (Doc. Math. ICM 1998; arXiv:math/0505354), and the conjectured trace formula is now *proved* for smooth foliated flows with the required simple structure (Álvarez López–Kordyukov, arXiv:2402.06671) — but no arithmetic dynamical system realizing $\operatorname{Spec}\mathbb{Z}$ has been constructed, so the trace formula has no arithmetic host yet; Connes–Consani have Riemann–Roch on the periodic orbits C_p of the scaling site and for the ring $\mathbb{Z}$ (Selecta Math. 2017 and sequels) but *not* on the square of the scaling site — exactly the surface substitute whose Hodge-index inequality would be the structural positivity — while their semilocal/prolate line supplies the archimedean-place positivity (arXiv:2006.13771, Selecta Math. 2021) and reduces RH to an Euler-factor property $P(n)$ (arXiv:2310.18423) — Weil positivity in the corpus's measured currency, with the prime places exactly where P5's control deaths sit — and Connes' 2026 extremal construction (arXiv:2602.04022; zero reality by Connes–van Suijlekom Thm 6.1) leaves open precisely the convergence the corpus adjudicated REALROOT-DEPTH-UNDECIDED. The corpus-checkable content of P7 is that the cited extremal construction is carried

and was executed here, and that the interface such a theory must match is exactly the (AC) object of P4; the external literature claims are citations, never gates.

Scope, in the same breath. The specification sheet is a *requirements document derived from machine-checked no-gos*: it proves nothing about Deninger's program or Connes–Consani, asserts no defect and no progress of either, claims no progress on RH in any direction, and explicitly types as **open** whether any cohomological theory can meet the specification — the standing counter-evidence (NO-WITNESS: all certified reads positive to $h = 12632$) and the finite-rung scope of the Obstruction Completeness Theorem travel with every property. What the sheet adds over the published state of the art is precision, not evidence: the missing theorem's interface is now stated sharply enough to be tested against — at finite N, by machine — before anyone proves anything. **No statement about the Riemann Hypothesis is made.**

12.45 The three lemma forms, the certified instruments, and a correction of record (sandbox probes)

Nine further discovery freeze rounds of 2026-08-15 (notes CCCXCIV and CCCXCVI–CDIII; several run as concurrent lanes) then finished the campaign's instrument tier and, in one case, corrected its own published record. Everything in this subsection was frozen as *sandbox*: eight frozen probes plus a Lean formalization round in the experiments tree, no module, no ledger row, no marker moved, no gate closed or narrowed, and no claim about the Riemann Hypothesis in any direction; the suite stood at 906 modules at the freeze. **Promotion of record (2026-08-15):** the certified Euler–Pick floors of rounds 95 and 100 have since been promoted to the load-bearing suite as `[verification/v915_eulerpick_certified_floors.py]` (15/15; the $N \le 3$ floors and every cited-bound ward recomputed in-run, the cap-10^{13} $N = 4$ intervals pinned from the run-of-record with the split disclosed, and the reach cap of Section 12.47 carried verbatim: the channel is disproof-sound but reach-capped at $\gamma \sim 5$–8, with zero live falsification power in the classically certified range). Everything else in this subsection remains sandbox; the suite stands at 952 modules after this, the companion `[verification/v914_pascal_region_theorem.py]` promotion, and the later Epstein detection-arc, prime-front theorem-arc, spectral-balance/edge arc, endgame-arc, triple-cofinal-arc, post-bughunt-arc, mechanism-arc, Bughunt-X-arc, integrable-dictionary-arc and bordered-dictionary-arc promotions (Sections 12.49, 12.50, 12.51, 12.52 and 12.54). One correction of record travels with this subsection and is carried where the corrected claim lives (Section 12.42): round 97, below.

The collective-rescue round (round 93, `collective_rescue_probe.py`*,* 10/10 *wards* + 7/7 *law gates, deterministic* 24.0/23.8 *s, verdict* COLLECTIVE-IS-SUPPORT*).* Round 92's exit/re-entry phenomenon is *fully explained* — no new structural law, no prime conspiracy, no Lyapunov functional. Half of the law is a theorem: the tent moment c_d reads only atoms within one mesh of $d\delta$, so truncating the stream above u_{miss} leaves the Verblunsky prefix bit-identical (verified spread exactly 0) and the exit can never come before the exact causal onset $\lfloor u_{\mathrm{miss}}/\delta \rfloor - 1$; the only empirical content is the upper offset, measured at **onset** + $\sim$2 **lags** across all truncations on the extended window (the round-92 re-entry demystified: P5 differs from the full stream in exactly one coefficient, nothing is rescued, the window just gets covered). The completed stream's cumulative $\max|\alpha|$ is *constant* at 0.183932 from $r = 2.568$ to 4.788 with band maxima falling ($0.184 \to 0.120$) — strictly a finite-prefix measurement, never quantifier evidence (mesh dependence disclosed; the ladder corrected of record by the round-109 audit (Section 12.47): 0.222/0.184/0.156 at $\delta = 0.012/0.006/0.003$ — the earlier 0.207 was the $\delta = 0.008$ value; MESH-DAMPED stands). All Lyapunov candidates are killed with numbers — by causality the fixed-prefix candidate class is *empty* — and the mass-restored control ($\{2..7\}$ plus a density tail) dies at the *identical* lag 400 as the truncation it imitates: **mass-without-atoms does not rescue** — the exit law is support bookkeeping, but in-window survival requires the exact arithmetic weights.

The Euler–Pick certified ladder (round 95, `eulerpick_ladder_probe.py`*,* 31/31 *gates, deterministic* 101.3/101.3 *s, verdict* EULERPICK-CERTIFIED + EULERPICK-GENERIC-DECAY + EULERPICK-DETECTION-LAW*).* The round-92 falsification ladder rebuilt to *certified* standard: outward-

rounded interval floors end to end — $N = 1$: $[4.5917135 \times 10^{-2}, 4.5917136 \times 10^{-2}]$; $N = 2$: $[9.0288701 \times 10^{-6}, 9.0289075 \times 10^{-6}]$; $N = 3$ certified positive at cap 1.6×10^{7}: $[2.3643695 \times 10^{-10}, 1.1497752 \times 10^{-9}]$ — the falsification channel implemented, self-tested on a planted negative family, and **empty** (a certified hi < 0 at any N would disprove RH by the exact forward direction alone). The honest wall is the tail, never precision (enclosure width scales exactly as $X^{-(1+1/j)}$; the float ladder runs to $N = 24$ while certification dies at $N = 3$). The decay is *generic*: a control decomposition on the same nodes (Cauchy → RvM density → gap → quantile-atom discreteness) lands within 1.2× of the Euler value across $N = 4..24$ with the true γ_1 appearing nowhere — the decay carries no arithmetic information. The detection law from planted off-line quadruples reads $N^*(\delta) = 6.1 + 1.95 \log_{10}(1/\delta)$ (rms 0.46), with measured γ-blindness: $N^*(0.1) = 8/12/22$ at $\gamma = 14/20/30$, $\gamma = 50$ undetected through $N = 36$ — reach costs $O(\gamma)$ rungs. Promotion was honestly declined at the freeze — the certified slice sees off-line zeros only at $\gamma \sim 5$–8, territory classical verification closed decades ago — and later executed with exactly that reach cap typed into the module and the ledger row: [`verification/v915_eulerpick_certified_floors.py`] recomputes the cap-1.6×10^7 floors, all three cited ψ-bound wards and the detection subset at every suite run.

The rigidity round (round 96, ***`rigidity_inverse_probe.py`****, 9/9 wards, deterministic 12.2/12.3 s, verdict* RIGIDITY-TIGHTENING*).* The inverse problem answered: screw positivity to depth r pins the measure onto the prime comb *exponentially tightly*, and the pinning rate is exactly the conditioning price. Survival moduli shrink with depth (median factor 3.78 over 22 channels); **all 17 adversarial impostor candidates die** — the shifted-prime comb $p \to p + 2$, the golden quasi-prime comb at exact prime density with full power/weight structure, equal-spaced crystals, mass-matched crystal tails (atomicity plus correct mass is *not* enough), sub-mesh splits at every scale (even the sub-mesh tent profile is pinned), moment-matched jitters, a single deep weight swap — and all 15 single-atom deletions are fatal within $J \le 8$ lags of causal onset. The moduli law $\ln \eta^*(r) = -1.073\, r - 7.08$ gives $c_{\mathrm{mod}} = 1.073$ against the $\lambda_{\min}$ ladder slope $c_\lambda = 0.814$, within the frozen band and sharper via the rigorous eigenvalue survival floor (reached by the measured atom-2 modulus to factor 1.1–2.8): **rigidity is the conditioning price up to an $O(1)$ alignment factor**, not a new arithmetic-free law. The theorem candidate is typed honestly (anchors machine-measured, law fitted, all-depth OPEN), and its hypothesis remains localized Weil positivity — unchanged currency.

The SP-pricing round (round 97, ***`sp_expsum_pricing_probe.py`****, 13/13 gates, deterministic 0.8 s twice, verdict* SP-OPERATIVE-FALSIFIED + SP-PRICED + SP-CLASSICALLY-OUT-OF-REACH*(×8.2)* + SP-CONDITIONAL-LADDER + SP-PREFIX-DEAD-IN-WINDOW + SP-LITERAL-NONVANISHING-IN-RANGE *— the correction of record carried in Section 12.42).* The round-88 rounding-margin statement priced against the full classical exponential-sum arsenal — and the measurement overtook the pricing. The pricing first, all constants generous, deepest rung $x = 890$ / $t_{\mathrm{top}} = 6990$ against the need 0.5: trivial ℓ^1 5.53, van der Corput second/third derivative 5.23/5.53, the Bourgain exponent pair 5.53, the Vaughan+Vinogradov ceiling via the explicit $S(t) = O(\log t)$ bound 4.11 — **the best unconditional miss is** ×8.2, Lindelöf/density give no pointwise constant, and even RH itself (Carneiro–Chandee–Milinovich, plus Montgomery's Ω) stays above the budget at 1.015 on the deepest rung: the budget is unreachable *and* measured exceeded. The measurement: the census extended from 79 points ($x \le 13$) to 18798 points ($x \le 890$) shows the operative predictor form **false from $x = 21$ onward with 567 τ-screened counterexamples** — the round-88 exactness a small-x accident of the margin budget, by an arithmetic-free mechanism (the Fejér taper's tracking-variance floor, rms $\to 1/(2\pi) = 0.159$, measured 0.26, against budget 0.5 — no taper in this family survives); the prefix/exceptional-set salvage is dead in-window (the lowest violation drops to $k/K = 0.010$ and the prefix depth stalls at ≤ 131); the literal non-vanishing form survives trivially and is contentless without a repulsion law. The fence: what died is the deployed source-side approximant, not TPL(i) itself — the eigenvector truth side is unmeasured beyond $x = 13$.

The CCM round (round 98, contract on arXiv:2310.18423; ***`ccm_pn_adjudication_probe.py`****, 22/22 gates, 204 s, re-run bit-identical, verdict* CCM-CHECKED + CCM-SAME-CURRENCY*).* The one

external *published* RH reduction adjudicated: Connes–Consani–Moscovici's $P(n)$ property ($P(n)$ for all $n \Leftrightarrow$ RH; open for every prime-bearing $n > 2$, proven only at the archimedean place, and quantifying over an infinite-dimensional cone per fixed n — not finitely checkable in full). Two results. First, a **new finite instrument**: Galerkin sections of Q_n itself (24 tents, both pole constraints projected exactly, the prime side exactly finite) — **all six prime-bearing sections positive** ($\lambda_{\min} = 2.30 \times 10^{-3}$ at $n = 2.5$ down to 1.01×10^{-5} at $n = 13$, certified budgets $\leq 5.8 \times 10^{-60}$; the pole constraint is load-bearing — the unconstrained form is -0.093 at $n = 3$); a certified negative anywhere would have refuted RH, and the channel stays empty (honest limit: margins for $n \geq 4$ are resolution-limited). Second, the interface test: CCM's published local terms reproduce $\xi'/\xi(1/2+\sigma)$ to rel 2.3×10^{-50}, and their quadratic form on the exponential-autocorrelation family **reproduces the corpus Euler–Pick matrices entrywise** — specification-sheet property P4 of Section 12.44 executed on the external reduction itself; the τ-screen types it RELOCATION: the wall renamed. The controls die through the same construction (SMOOTH between $n = 3$ and 6, SCRARITH at $n = 6$, EPSTEIN at $n = 6/13$, separations 2.9×10^3–7.6×10^3 × margin) — $P(n)$ is arithmetic-sensitive at the positivity level exactly like the screw carrier, and the honest comparison is an identity: $P(n)$ ***is*** **localized Weil positivity**, n-localized but infinite-dimensional per instance. What the corpus adds to CCM's own gap: the (AC) interface makes any semilocal prolate candidate finitely testable today, the control deaths are consumption requirements now reproduced in the $P(n)$ currency, and the certified Euler–Pick ladder prices certified $P(n)$-type checks.

The Lean round (round 99, four new modules under ***`TfptCarrier/`****, full incremental lake build green, audit 7/7; 27 kernel-checked headline theorems with axiom footprint exactly* [***`propext`***, ***`Classical.choice`***, ***`Quot.sound`***]*, no* ***`sorry`****, no* ***`native_decide`****).* The reduction chain's best pieces are now kernel-checked. `EulerPick.lean`: the exact per-zero identity, the Gram-sum form, and `pickMatrix_posSemidef` at the SV nodes $\sigma_j = 1 + 1/(j+1)$ for every N — **the forward direction of the Euler–Pick criterion, kernel-checked for finite ordinate families** (the identification with ξ'/ξ and the backward Pick direction stay cited classical input, docstring-typed). `DeltaOneNoGo.lean`: the δ_1 counterexample package — PSD but never positive definite for $n \geq 2$ while the Carathéodory value stays positive. `ParityLemma.lean`, the largest piece: blow-up laws, existence directions, the combinatorial core, and the **full round-85 law** `zero_count_even_iff_opposite_residues` — under *one named simplicity hypothesis* (alternating signs at canonical gap midpoints; exactly where zero simplicity enters, docstring-typed), with boundary signs and gap-sign constancy proven, not assumed. `SVSkeleton.lean`, partial by design: the pin geometry proven (accumulation at the interior point 1), the hypothesis package with three cited fields, the composition `sv_implies_rh` — and **the honesty lock** `skeleton_not_unconditional`, an instance with RH = False: the packaging alone yields nothing.

The $N = 4$ certification round (round 100, ***`sieve4_eulerpick_n4_probe.py`****, 22/22 gates, 2682 s; segmented byte sieve to cap $X = 10^{13}$, the compiled helper's source SHA pinned in the probe, run-of-record* ***`sieve4_run1.log`****, and the hard external ward $\pi(10^{13}) = 346{,}065{,}536{,}839$ reproduced exactly by the instrument).* $\lambda_{\min}(\mathcal{P}_4) \in [8.278338 \times 10^{-15},\ 1.3840906 \times 10^{-14}]$ **is certified positive** (margin 0.749), the $N \leq 3$ floors tightened massively ($\log_3$ width $9.1 \times 10^{-10} \to 1.3 \times 10^{-16}$), and the falsification channel stays open and empty. These four certified intervals are now load-bearing: [`verification/v915_eulerpick_certified_floors.py`] pins them from this round's run-of-record (the 45-minute compiled sieve is not re-runnable in-suite; the split is disclosed in the module and the ledger row), recomputes the $N \leq 3$ floors at cap 1.6×10^7, re-derives the summation-error model from the pinned $\pi(10^{13})$ without the sieve, and re-proves the Nair floor necessity and the corrected budget at every suite run. The round also corrects round 95's upgrade-path model, load-bearingly: its X_{req} model omitted the tail floor beyond the Büthe range $x > 10^{19}$ — with $\psi \geq 0$ alone that floor exceeds λ_4 *at any cap*, so the named path was not worst-case executable; one airtight elementary input unblocks it — **Nair 1982:** $\operatorname{lcm}(1..n) \geq 2^{n-1}$**, hence** $\psi(x) \geq (x-2)\ln 2$ (warded against the round's own sieve data) — giving the corrected $X_{\text{req}}(4) = 10^{11.52}$ (the realized crude assembly also certified at 10^{13}; Nair makes the certificate a-priori guaranteed and triples the margin). And the $N = 5$ wall is corrected: it is *not* sieve-walled — the beyond-10^{19} floor exceeds λ_5 by a factor

4.7×10^5, so no sieve of any size helps — **the knowledge wall starts at** $N = 5$, not $N = 6$ as round 95 estimated.

The α-induction round (round 101, contract PRIME.SCREW.INDUCTION.01; `screwind_induction_probe.py`, 10/10 *wards* + 5/8 *law gates — the three fails are findings, typed; companion* `screwind_meshdrift_check.py`; *verdict* SCREWIND-KICK-LAW*(exact)* + SCREWIND-STEP-IS-WALL + SCREWIND-DICHOTOMY*(descriptive-only)).* The one inductive attack never attempted, in the best-conditioned coordinate — with a **depth record**: all sections positive to $r = 9.0$ at both meshes (the corpus record was 4.788), $\sup|\rho| = 0.183932$ with margin 0.816 attained at $k = 0$ — the stream never exceeds its own first coefficient. The flatness is carried by a cancellation of giants (band medians $\rho_{\text{arch}} = +288.5$ vs $\rho_{\text{prime}} = -288.5$ at $u = 9$ against net RMS 0.023), and **the exact kick law** holds at every atom arrival (affine-exact, deviation $\le 2.8 \times 10^{-16}$, numerator pure arithmetic in (p, m, δ)); the informative gate fail: net arrival α is *positive* on 33/39 and the atom-bin median $|\rho|$ sits below background (0.62×) — **prime arrivals stabilize**; the naive atoms-push-down picture is wrong. The Szegő dichotomy is typed DESCRIPTIVE-ONLY ("positivity + Szegő violation ⇔ pure point" is false as a theorem candidate). The sharpest true statement is an induction whose side condition is wall-equivalent (literally the round-92 object) — **the wall relocates into the certificate** — and the named open piece is stated exactly: a source-certified two-sided energy floor $e(u) \ge Ce^{-\kappa_+ u}$ with $\kappa_+ < 1/2$ would make the kicks self-improving (kick exponent $\kappa - 1/2 = -0.322 < 0$ measured everywhere), but that floor *is* localized positivity: the cleanest statement yet of where the wall sits in this coordinate.

The Hausdorff safe-point round (round 102, externally proposed, contract PRIME.HAUSDORFF.SAFEPOINT.01; `hausdorff_safepoint_probe.py`, 40/40 *gates,* 143.6 *s, independently re-run green, composite verdict* HAUSDORFF-EQUIVALENCE-SOUND + HAUSDORFF-REPRODUCED + HAUSDORFF-CERTIFIED-DEPTH*(86; γ* 189/563)* + HAUSDORFF-REPLICATION + HAUSDORFF-DETECTOR + HAUSDORFF-Z1-ZEROMEASURE + NOT-WORLD-NEW*).* An externally proposed moment-field criterion built, audited and adjudicated: with $\Phi(z) = \xi(1/2 + \sqrt{z})$, $F = \Phi'/\Phi$, $b_n(a) = a^{n+1}((-1)^n/n!)\, F^{(n)}(a)$ and the Pascal field $C_{n,k} = (-1)^k \Delta^k b_n$ at the safe point $a = 256$ in the Euler half-plane, the claim is RH ⇔ $C_{n,k}(a) \ge 0$ for all n, k. **The equivalence skeleton is sound**: the Hausdorff/Widder hypotheses need no extra boundedness ($0 \le b_n \le b_0$ follows from $C_{n,0}, C_{n,1} \ge 0$, executed on the certified range), no-atom-at-0 is gated, and the one sharp continuation step closes (shared Taylor data at a, the identity theorem on the slit plane minus the discrete off-axis pole set, removability at any putative off-axis pole — contradiction), with the companions symbolically exact (the Li binomial transform, the Möbius flow, the Hankel–Vandermonde SOS, Pascal conservation). The proposer's spec is numerically faithful in every reproducible detail (561/561 cells positive, weakest $C_{16,16}$ ratio 1.000220 to claim, pins to 6.452×10^{-37}). The headline: **certified depth 86 (**5290 **cells) with** γ^***-reach** 189 **at** $a = 256$ **and** 563 **on the** $a = 1024$ **arm — versus Euler–Pick's certified** $N = 3$ **(**$\gamma \sim 5$–8**) at** $\kappa = 2.3 \times 10^{61}$**: the cheapest certified RH-consequence instrument in the corpus by a wide margin** (143 s total; the min-cell margin decays by the spectral law, slope −0.3099 vs prediction −0.304, not Cauchy-catastrophically). Three-carrier replication: signpos ($x = 8$, max deviation 0.0829 over $j \le 5$), the round-90 Krein carrier (max $|\Delta P| = 2.93 \times 10^{-4}$, exactly the documented bias), and the SV pins (6.45×10^{-37}) agree — **three source-only carriers with no zero cache anywhere in construction**. The detector, honestly priced: the matched-ray phase is $O(\delta^3)$-stationary (derived exactly), so detection runs only through the $k = 0$ channel and the measured law $d^* \sim 1/\delta$ holds below γ_1 (78/104 vs predictions 79/105) — **above** γ_1 **the Pascal field is provably blind at the safe point**, made concrete by the Epstein control: an off-line witness zero *located* at $\rho = 0.69692705 + 36.37406369\,i$ ($|\xi_Q| = 1.5 \times 10^{-40}$, RH-for-$Q$ provably false) while the Q Pascal field stays positive to depth 100 exactly as the phase model predicts (the fires-iff-fires gate against a planted replica passed); the smooth world dies immediately (first negative $C_{7,1} = -2.3 \times 10^{-3}$ at depth 8). Correction of record (round 123, Section 12.49): that witness was never γ-minimal — the located off-line census of ξ_Q holds *four* pairs below height 45, and the γ-minimal *driver* $0.9330 + 15.6682i$ carries 26× its excess ([`verification/v916_epstein_weil_violation.py`]); the blindness demonstration above is unaffected (it

consumes only that an off-line zero exists at $\gamma \approx 36$). Z1 typing: b_n = cache partial sums + RvM tail — source-only in *input*, zero measure in *content*, like Krein. And the honest novelty fence: NOT-WORLD-NEW — Zhang (arXiv:2303.09396, Thm 2) is the identical criterion at the central point; the novelty is the safe-point shift into the Euler half-plane, the source-only decoder, and the certified conditioning. The naked remaining task: positivity of *all* $C_{n,k}(a)$ — Weil positivity in Hausdorff currency, **the third lemma form** beside the standing two (Weyl-disk contraction; ground-eigenvector sign positions) and, per the rounds-108/109 correction of record (Section 12.47), the *only* one of the three that is a proven RH-equivalence; the certificate extends the finite no-witness region far beyond Euler–Pick but touches the all-(n, k) quantifier nowhere.

> **Where the campaign now stands** (sandbox; no RH claim; wording corrected of record by the round-108 logic audit and the round-109 bughunt, Section 12.47): the one open input now holds **three RH-sufficient lemma forms** — Weyl-disk contraction of one explicit canonical system (pointwise at the pins, per the round-92 correction; the stale adjective "uniform" is retired with round 109), the sign positions of one explicit ground eigenvector, and Hausdorff cell positivity of one explicit Pascal field. Of the three, **the Hausdorff form is a proven RH-equivalence**; the Weyl-disk and sign-position forms are **one-sided pendants** — form ⇒ RH established, RH ⇒ form open (their converses are, respectively, the round-90 open lemma and the CF-realness/Connes-§6.6 hypotheses) — every mutual equivalence that holds routes *through* RH (no direct form-to-form edge exists), **transfer is none** (the round-108 counterexample world: an off-line quadruple at $\gamma = 20$ keeps every Hausdorff cell positive at $n + k \leq 50$ while the Euler–Pick section fires at $N = 12$), and **certified regions are form-local**. Around them: three agreeing source-only carriers, two certified falsification instruments (the Euler–Pick ladder at $N \leq 4$, since promoted as [verification/v915_eulerpick_certified_floors.py]; the Hausdorff field at depth 86, still sandbox — both disproof-sound but reach-capped, Section 12.47), the rigidity law explaining quantitatively why every control had to die, and 27 kernel-checked theorems around the reduction chain. At the bottom, unchanged: localized Weil positivity.

Scope, in the same breath. All of this is exploration: finite measurements and finite certificates on reachable rungs, no all-depth statement, no positivity claim; the certified Euler–Pick floors were subsequently promoted ([verification/v915_eulerpick_certified_floors.py]) as certified-finite results with the reach cap typed in — the certified instruments are falsification channels, not evidence — their certified slices sit in territory classical zero verification closed long ago, and both instruments' blindness laws are measured and carried. The round-97 correction of record is carried in Section 12.42 per the round-87 convention (frozen probes and notes are not retro-edited — the correction is the record). RH is neither proven nor disproven here; what is documented is the complete map of why, now instrument-grade in three currencies. **No statement about the Riemann Hypothesis is made.**

12.46 The road from here: a machine-gated path to a complete proof (sandbox probes)

Three discovery freeze rounds of 2026-08-15 (rounds 103–105; notes CDV–CDVI) then produced the one artifact the campaign never had: a staged, milestone-gated research roadmap from today's frozen state to a complete RH proof, in which every stage is falsifiable against the corpus's own instruments. Everything in this subsection was frozen as *sandbox*: three frozen probes in the experiments tree, no module, no ledger row, no marker moved, no gate closed or narrowed, and no claim about the Riemann Hypothesis in any direction; the suite stood at 906 modules at the freeze. **Promotion of record (2026-08-15):** the round-103 Form-A theorem (extended by round 106) has since been promoted as [verification/v914_pascal_region_theorem.py], and the certified Euler–Pick floors it consumes as Stage-0/1b acceptance values as [verification/v915_eulerpick_certified_floors.py]; the

suite stands at 952 modules (after the later Epstein detection-arc, prime-front theorem-arc, spectral-balance/edge arc, endgame-arc, triple-cofinal-arc, post-bughunt-arc, mechanism-arc, Bughunt-X-arc, integrable-dictionary-arc and bordered-dictionary-arc promotions, Sections 12.49, 12.50, 12.51, 12.52, 12.53 and 12.54). The roadmap itself remains sandbox. The honesty fences travel first: **a roadmap is not progress on RH; no stage is claimed started; the mathematics of Stages 1–2 does not yet exist** — the artifact's value is that every step is now falsifiable and interface-defined, so any candidate, from anyone, can be adjudicated against frozen gates in minutes.

The Form-A round (round 103, `moonshot_sol_probe.py`*, verdict* MOONSHOT-PARTIAL*(Form A)).* From cited external inputs only — Platt–Trudgian 2021 (all zeros to height $T = 3{,}000{,}175{,}332{,}800$ on the line), Rosser's 1941 counting envelope, and the classical γ_1 isolation; no zero table, no zeta evaluator — the round proved the **largest positivity region of the campaign**: at the safe point $a = 256$ the full Pascal field satisfies $C_{n,k}(256) > 0$ for *every* $n \geq 0$ and every $0 \leq k \leq 10^{19}$ (extended 530-fold to $k \leq 7.4 \times 10^{21}$ by round 106, Section 12.47) — an infinite half-strip in n and ten quintillion complete k-columns — with the scope stated inside the theorem: it touches the $k \to \infty$ quantifier nowhere and proves nothing about RH. The theorem is now load-bearing: `[verification/v914_pascal_region_theorem.py]` recomputes the complete four-region partition (all decisive ratios anchored to the run-of-record) at every suite run, together with the round-106 extension per the disclosed pinning split. The same round killed its three suggested proof routes with named mechanisms: the exact prime-side transmutation turns the positive Beta spectral window into a sign-changing generalized-Laguerre source window (the classical Laguerre zero theorem — no termwise-positive prime comparison exists); the Killip–Simon/Szegő direction dies on pure-point entropy (the zero comb has $w = 0$ a.e., entropy $-\infty$, floor $e \geq 0$ only); and the asymptotic-a transport is nonuniform in (n, k).

The class round (round 104, note CDV, `levinson_class_probe.py`*, 23/23, verdict* LEVINSON-ADJUDICATED*(*LEVINSON-FAIL*; tally* $24 \to 25$*,* 0 *PASS)).* The last formally unexamined classical theorem type about zero positions — the positive-proportion-on-the-line class (Selberg 1942; Levinson 1974, $\kappa \geq 1/3$; Conrey 1989, $> 2/5$; the standing record $\kappa > 0.417293$ of Pratt–Robles–Zaharescu–Zeindler 2020) — is priced against the wall budget: the literal (liminf) class constrains *nothing* at any finite height (an exact finite-modification argument), and even the granted-effective reading breaches every deployed cell by 2–3 orders of magnitude (breach ratios 382/1149/1059 at $h = 184/388/839$); a proportion theorem must come within $1 - \kappa^* < 4.8 \times 10^{-3}$ of *all* zeros ($\kappa^*_{\text{ledger}} = 0.9952/0.9983/0.9988$, each a lower bound) before it can feed the wall. **The atlas tally moves** $24 \to 25$ **candidates,** 0 **PASS** — and with it every named unconditional classical theorem type about zero positions is formally priced: zero-free regions, zero density, Weyl equidistribution, pair correlation, positive proportion.

The roadmap round (round 105, note CDVI, `roadmap_probe.py`*, 29/29 gates, SPEC* `79092ebf`*, deterministic re-run identical, verdict* ROADMAP-COMPILED*(stages; gates* $29/29$*)).* Every machine-groundable claim of the roadmap is asserted: the acceptance-gate values re-computed from cited constants or grep-warded verbatim in their owning frozen artifacts, the stage DAG checked acyclic with exactly the named open stages, the downstream chain's artifacts present with verdicts grep-verified. **Stage 0 (done, frozen):** the candidate atlas complete (25 candidates, 0 pass), exactly one open edge E4, the three lemma forms (each RH-sufficient; only the Hausdorff form a proven equivalence — the rounds-108/109 correction of record, Section 12.47), the two certified falsification instruments with open-and-empty channels (Euler–Pick floors $N = 1$: $[4.5917135 \times 10^{-2}, 4.5917136 \times 10^{-2}]$ down to $N = 4$: $[8.278338 \times 10^{-15}, 1.3840906 \times 10^{-14}]$ at cap 10^{13}, now carried load-bearing by `[verification/v915_eulerpick_certified_floors.py]`; Hausdorff certified depth 86, 5290 cells, γ^* 189/563), the $k \leq 10^{19}$ theorem (re-derived inside the probe: decisive ratios 1.176577/335.8/1416.8, the $n = 0$ mechanism wall re-located between 1.40×10^{19} and 1.41×10^{19}; now carried load-bearing by `[verification/v914_pascal_region_theorem.py]`), the rigidity law ($\ln \eta^*(r) = -1.073r - 7.08$), the seven-property specification sheet, and 27 kernel-checked theorems. **Stage 1 (open; new mathematics):** structural positivity of Hodge-index type for $\operatorname{Spec} \mathbb{Z}$ — the verified literature state: Deninger's dynamical Lefschetz trace formula is *proved*

for smooth foliated flows (Álvarez López–Kordyukov, arXiv:2402.06671) but has NO arithmetic host; Connes–Consani have Riemann–Roch on the scaling-site orbits and for $\mathbb{Z}$ but NOT on the square of the scaling site. Milestone (1a): an arithmetic dynamical system hosting the proven trace formula, periodic orbits $\log p$ with weights $\log p/p^{m/2}$ — acceptance gate: its trace formula must reproduce the (AC)/Euler–Pick interface $c^T P_N c = \langle -g'', f_c \star \tilde{f}_c \rangle$ at the 16 frozen σ of the SVPIN spec (minutes-scale, by machine). Milestone (1b): OR Riemann–Roch on the square of the scaling site — acceptance gate: the intersection pairing on exponential autocorrelations must reproduce the certified $\mathcal{P}_N$ ladder ($N \leq 4$ floors above as frozen acceptance values). **Stage 2 (open; new mathematics):** the Hodge-index/Castelnuovo analog on the Stage-1 substrate — acceptance gates from the corpus: it must fail on the surrogate worlds exactly at the measured depths ($r = 0.264$ smooth by support, $r = 0.744$ scramble by weights); it must be signed and alignment-carrying (the v913 class); it must survive the τ-screen (bands 0.30/0.70); its finite sections must reproduce the certified Hausdorff cells (depth 86) and strictly extend the $k \leq 10^{19}$ theorem's reach. **Stage 3 (conditional-complete; assembly, machine-held):** from the Stage-2 inequality to localized Weil positivity in any one of the three lemma forms, then the extraction the machine already holds — [`verification/v848_extraction_chain.py`] (EXTRACTION-CHAIN-COMPLETE, hypothesis (H) isolated), [`verification/v912_form_convergence_theorem.py`] (unconditional rate $O(D^2 \log(1/D))$), the Lean kernel tier (`CofinalWeil`/`CofinalPredefinition`, `pickMatrix_posSemidef`, `sv_implies_rh` with the honesty lock), the SVPIN skeleton, the Hausdorff equivalence, and the CCM same-currency interface — with the round-108 fence (Section 12.47): the only kernel-checked end-to-end extraction (`v848` + `CofinalWeil.lean`) accepts *only* the wall form H_{cof}, which is not among the three entry forms and is not producible from them without RH; the per-form descents are probe-gated plus classical, and the weakest link is the Stage-2→3 *interface* itself — finite acceptance gates (cells $n + k \leq 86$) against infinite-quantifier entry forms, a jump no corpus machinery bridges (`v912` bridges convergence, not positivity). **The falsification spine (parallel, permanent):** the two certified instruments keep running as standing disproof channels, both open and empty (a single certified negative would be an RH-disproof candidate) — with the round-108 reach cap (Section 12.47): both instruments are disproof-sound (the disproof direction consumes only the proven forward implication) but reach-capped far below Platt–Trudgian's 3.0×10^{12} (Euler–Pick's certified $N \leq 4$ — now [`verification/v915_eulerpick_certified_floors.py`], with the reach cap typed into the module — sees off-line zeros only at $\gamma \approx 5$–8; the Hausdorff field is provably blind above $\gamma_1 = 14.13$ at the safe point), so "open and empty" is a theorem given the cited inputs, not an observation — the certified ranges carry zero live falsification power; the extension walls are knowledge walls, not compute walls ($N = 5$ needs ψ-knowledge beyond 10^{19}, floor/$\lambda_5 = 4.7 \times 10^5$ at any cap; the Form-A $n = 0$ mechanism exhausts at $k \approx 1.405 \times 10^{19}$, relocated by round 106's stronger mechanism to $k \approx 7.44 \times 10^{21}$, Section 12.47).

The honest pricing, and the scope. Stage 1 and Stage 2 require mathematics that does not exist today — the roadmap proves interfaces and obstructions, never existence, and the people closest to the Stage-1 milestones are the named authors of the cited programs; Stage 3 is assembly, already machine-held. The corpus's contribution to every stage is the same: acceptance gates that turn each milestone into a minutes-scale machine check, so any candidate, from anyone, can be adjudicated immediately. No timeline is promised, no stage is claimed started, and nothing in this subsection is progress on RH. **No statement about the Riemann Hypothesis is made.**

12.47 The frontier extension, the code reading, and the campaign audited: the corrections of record (sandbox probes)

Four closing discovery freeze rounds of 2026-08-15 (rounds 106–109; notes CDVII–CDIX; run as concurrent lanes) then extended the roadmap's Stage-0 frontier, adjudicated the program's founding intuition under its own name, and turned the audit instruments on the campaign's own headlines. Everything in this subsection was frozen as *sandbox*: four frozen probes in the experiments tree, no module, no ledger row, no marker moved, no gate closed or narrowed, and no claim about the Riemann Hypothesis in any direction; the suite stood at 906 modules at the

freeze. **Promotion of record (2026-08-15):** the round-106 frontier extension below — with the round-103 base theorem and the HSW attribution resolved from the paper itself — has since been promoted as [verification/v914_pascal_region_theorem.py], and the certified Euler–Pick floors as [verification/v915_eulerpick_certified_floors.py]; the suite stands at 952 modules. The audits and the code reading remain sandbox. This is, first of all, a *corrections subsection*: the round-108 big-picture logic audit (`bigpicture_logic_probe.py`, 31/31 gates, 2.5 s, SPEC `889edf4a`, verdict BIGPICTURE-FINDINGS(3)) and the round-109 bughunt — the first adversarial audit of rounds 86–105 (`bughunt2_r86_r105_probe.py`, 30/30 gates, 11.6 s, SPEC `759d0209`, verdict BUGHUNT2-FINDINGS(7, max MAJOR): 1 MAJOR / 6 MINOR / 0 FATAL) — independently convicted the published "three equivalent lemma forms" language and two related headlines, and the corrected wordings are carried in Sections 12.45–12.46 per the round-87/97 convention: frozen probes and notes are not retro-edited — the correction is the record. No number was wrong; no round verdict flips.

The corrections of record, stated once in full. The honest form × direction table:

Form	form ⇒ RH	RH ⇒ form	net status
A — Hausdorff cell positivity (all n, k at $a = 256$)	audited-sound (H1c chain, independently re-derived gap-free in round 109) + literature (Zhang arXiv:2303.09396, Thm 2)	elementary + gated	**proven RH-equivalence**
B — Weyl-disk contraction (pointwise at the pins; "uniform" retired per rounds 92/109)	skeleton-sound (SVPIN + V0a + Lean `sv_implies_rh`, citation-typed hypotheses)	**open** — literally the round-90 lemma (measured 16/16 truth-tight, never proven)	RH-sufficient pendant
C — ground-eigenvector sign positions (TPL(i))	ladder ⇒ RH via `v848` modulo (H); C ⇔ ladder under the named simplicity hypothesis	via the Z1 transfer (measured 6.3×10^{-5}), conditional on CF-realness/Connes §6.6	RH-sufficient pendant

The genuine two-sided equivalences of the corpus are Form A, screw positivity of *all* sections at *all* depths (Suzuki — cited, never assumed), and Euler–Pick for *all* N (Pick cited forward, Lean kernel backward) — and every mutual equivalence that holds routes *through* RH: no direct form-to-form edge exists (gate J1.G5); the only direct bridge is the (AC) weld, stopping at the exponential-autocorrelation cone. **Transfer: none**, demonstrated by an explicit counterexample world — an off-line quadruple at $\gamma = 20$ (above γ_1) keeps *every* Hausdorff cell positive ($n+k \le 50$, b_n to $n = 240$) while the Euler–Pick section *fires* at $N = 12$ (certified negative pivot at 200 dps); the on-line control world is positive in both; a below-γ_1 quadruple fires the Hausdorff n-channel at $n = 90$. Consequence: **the $k \le 10^{19}/7.4\times10^{21}$ Form-A theorems ([verification/v914_pascal_region_theorem.py]) imply nothing about screw sections or Weyl disks — certified regions are form-local** — and their zero-location content sits below $\gamma_1 = 14.13$, where Platt–Trudgian is infinitely stronger. The two further fundamental findings are carried as fences in Section 12.46: the Stage-3 entry-form mismatch (the kernel-checked extraction accepts only H_{cof}) and the falsifier reach cap ("open and empty" is a theorem given the cited inputs). The honest OCT boundary is made concrete (J2, absence-gated at 0 atlas hits each): five classes outside the corpus vocabulary are named and *unpriced* — ergodic/measure-rigidity positivity transfer, model-theoretic/o-minimal counting, bigger-group representation-theoretic positivity, GMC/Fyodorov–Hiary–Keating (reduces-in-spirit to magnitude, with an enumeration gap), and higher-order Fourier uniformity of Möbius — so "no blanket-untested class left" is vocabulary-relative, exactly as the OCT's own editorial fence says. The localization lattice is typed (J3): "localized Weil positivity" is a *family* of six inequivalent localizations with different quantifier structures, each with its own completion leg, and mesh-monotonicity is false in general (explicit aliasing counterexample: coarse sections all PSD, fine section $\lambda_{\min} = -14.96$) — the corpus's mesh-cofinal correction was load-bearing. The round-109 minors, all on record: one hardcoded-true gate in the round-104 tally (the only live one across all 17 probes — the round-87 F5 class reproduced after its own warning); two decorative tautological conjuncts in the round-92 probe (the countable Nevanlinna–Pick step is asserted-with-citation,

honestly typed everywhere); the screwind docstring's mesh misattribution (0.207 is the $\delta = 0.008$ round-93 value; the true $\delta = 0.012$ sup is 0.2219; MESH-DAMPED stands, corrected in Section 12.45); the stale "uniform" adjective in Form B's name (contradicting the surfaces' own round-92 correction — retired); the roadmap probe's stale r95 $N = 3$ acceptance value ($2.3643695 \times 10^{-10}$, though round 100 tightened lo_3 to $6.9310239 \times 10^{-10}$ — frozen probe, finding on record); and two header slips in `sieve4_helper.c` (the certificate unaffected: true worst per-term $13.33u \ll K_{\mathrm{TERM}} = 32u$). Against all that, **the load-bearing re-derivations are clean**: the round-103 theorem re-proved end to end with an own implementation ($B_0 = 7.575655711 \times 10^{-10}$ exact match, all four partition ratios to 10^{-9}, the wall root $k^* = 1.40544 \times 10^{19}$ consistent), the $N = 4$ certification re-derived (Nair exact for $n \le 60$, far-floor necessity confirmed, the Cholesky certificate logic sound), the Hausdorff H1c continuation *has no gap*, the Lean theorems say exactly what round 99 claims (axioms exactly the standard three), all 16 in-scope probes re-ran green during the audit, and no round verdict flips.

The frontier extension (round 106, ***`moonshot_o3_probe.py`****, 16/16 gates, ∼104 s, independently re-run green, verdict* MOONSHOT3-EXTENDED + MOONSHOT3-PARTIAL + MOONSHOT3-OBSTRUCTED*).* The round-103 theorem extends 530-fold: $C_{n,k}(256) > 0$ **for every** $n \ge 0$ **and every** $0 \le k \le 7{,}444{,}682{,}106{,}464{,}286{,}365{,}865$ ($\approx 7.44 \times 10^{21}$), proven from Platt–Trudgian 2021 plus the *corrected* Hasanalizade–Shen–Wong zero-counting bound (J. Number Theory 235 (2022), Cor. 1.2: $|N(t) - M(t)| \le 0.1038 \log t + 0.2573 \log\log t + 9.3675$) — the honest source correction load-bearing: the commissioned $(0.110, 0.290, 2.290)$ constants bound $S(T)$, not $N(T)$ directly, and their Cheng–Graham input was later identified as invalid — replaced, not patched. (The attribution flag raised by a later audit is resolved from the paper itself, at promotion: Cor. 1.2 *is* the $N(t)$ bound with exactly these constants; Cor. 1.4 is the companion $S(t)$ bound and is not used — the probe docstring's "Cor. 1.4" sentence was the slip, the numbers were always right.) The theorem, with its base region and this extension, is now load-bearing as [`verification/v914_pascal_region_theorem.py`]: the complete round-103 partition and the law constant are recomputed at every suite run, the 33-cell frontier partition is pinned from this round's run-of-record with a six-cell full-packet recompute including the worst cell, and the form-locality typing below travels in the module and the ledger row. The mechanism retains the previously discarded q^k factor ($|1 - y| \le (\gamma^2 + 1/4)/(\gamma^2 + a - 1/4)$), counts verified zeros in 16384 RvM packets below $T-1$, encloses the unknown tail by a Gaussian series above T, and closes a 33-cell endpoint partition; the worst gate is the last cell (lower/tail ratio 1.000001535, minimum packet count 25,397,045.4), and the next 2×10^{-6} relative step fails this mechanism — the frontier is mechanism-exact. **The general law:** $k_{\mathrm{front}}(T) = (\lambda_\infty + O(1/\log T))\, T^2/(a - \frac{1}{2})$ with $\lambda_\infty = 0.226987054723979955$, defined by $\mathrm{erfc}(\sqrt{a/(a-\frac{1}{2})}\sqrt{\lambda}) = \sqrt{a/(a-\frac{1}{2})}\,\mathrm{erf}(\sqrt{\lambda})$ — *verification height qT buys* q^2 *columns* — and the formal geometric ceiling $k_{\mathrm{geo}} = (T^2 + 1/4)/(a - \frac{1}{2})$ is intrinsically unreachable by this mechanism (the route stops at $0.227\, k_{\mathrm{geo}}$). New exact structure travels with it: the two-variable generating function in closed form, $(X + Y - XY)\, G = X\, a\, F(a(1-X)) + Y\, \frac{a}{1-Y}\, F\!\left(\frac{a}{1-Y}\right)$, with the pole-divisor *equivalence* (all joint pole divisors positive-real and > 1 iff every $y_\rho \in (0, 1)$ iff RH — an equivalence, not an RH proof); under RH the array is totally sign-regular in the *reversed-k* orientation (the standard Pólya-frequency orientation is wrong — the 2×2 minor is manifestly negative); the important correction: **Hankel positivity alone is not Form A** — the uniform measure on $[2, 3]$ has every Hankel matrix positive definite yet $C_{0,1} = -3/2$; the localizers H^+ and $H - H^+$ are load-bearing; the fixed-N partial theorem $(b_m(a) = (\sqrt{a}\log a/8)\,(1/2)_m/m! + O_N(\sqrt{a})$, so every fixed Hankel and localizer determinant is *eventually* positive as $a \to \infty$); and the exact downward-transport obstruction: the flow is not sign-closed — for two atoms $\frac{d}{dt}\log\det H_2 = 4 - 3(y_1 + y_2)$, either sign possible — so eventual fixed-N positivity cannot be propagated down to $a = 256$ without controlling the full mixed/localizer hierarchy.

The code-synthesis adjudication (round 107, contract PRIME.CODE.SYNTHESIS.01*;* ***`code_synthesis_probe.py`****, 18/18 gates + wards, 2.2 s, deterministic re-run bit-identical, verdict* CODE-READING-TRUE-DECODING-DIRECTION*).* The program owner's founding intuition — *the primes as an error-correcting code in TFPT* — received its first formal adjudication

under that name, and it is **vindicated as measured structure**: the primes *are* the unique codeword of an error-correcting code in the compiler's own Gram grammar — ambient space the nonnegative atomic measures on $(0, r]$ at tent mesh $\delta = 0.006$; codeword the prime-power comb; check family the nested Toeplitz sections $T_k(-g'')$, one check per depth; **minimum distance the round-96 rigidity law, re-measured independently and identical to four digits** ($\ln \eta^*(r) = -1.0730\, r - 7.0783$); decoding radius the exact round-96 impostor exits (6/6); rate $\to 0$. It is the **same grammar as the compiler's literal E8 code layer** ($[8, 4, 4]$ self-dual, weight enumerator $\{0{:}1, 4{:}14, 8{:}1\}$, Construction A $\to$ 240 roots — rebuilt exactly in-probe): both membership tests are nonnegativity of a Gram pairing family, so TFPT's one axiom *instantiates* at the arithmetic seam as this code — structure, not progress. But it is a **different dual type**, and the decisive invariant is now a measured number: the E8 layer has dual rank 4 (finite, fixed distance), while **the arithmetic code's dual rank grows with slope** 1.0000 (the binding check index always sits at the frontier and never stabilizes; the syndrome rank grows $5 \to 12$; a non-codeword passes every bounded-depth check family bit-exactly below its defect), and the adversarial local-testability gap $\to 0$ beyond any bounded family's support — precisely the error class the RH quantifier cares about (local errors, by contrast, are locally testable and depth-improving). The honest close: the code runs in the **decoding direction** (finite checks $\Rightarrow$ identification of the measure) while RH needs the **encoding direction** (codeword $\Rightarrow$ all checks pass) — the quantifier does not compress, and the reason is now a measured number; both surprise channels (a stabilizing binding set; a depth-independent adversarial gap) were carried as live gates and are empty.

Scope, in the same breath. All of this is exploration: finite theorems from cited inputs, measured structure, and prose-level corrections; no all-depth statement, no marker moved. The two certified-finite results (the Pascal-region theorem; the Euler–Pick floors) were subsequently promoted as `[verification/v914_pascal_region_theorem.py]` and `[verification/v915_eulerpick_certified_floors.py]` — certified finite theorems, explicitly *not* RH evidence, form-local, with the reach caps typed in. The corrections change no number and flip no verdict — they grade the record's own language against the machine state, per the standing convention. RH is neither proven nor disproven here. **No statement about the Riemann Hypothesis is made.**

12.48 The vocabulary boundary priced, and the resolvent closure adjudicated: the CCM operator is the corpus's own record family (sandbox probes)

Two further discovery freeze rounds of 2026-08-15 (rounds 111–112; notes CDXI–CDXII; run as concurrent lanes) then closed the two open flanks the audits of sec:frontier-audits had left: the five argument classes the round-108 audit named as genuinely *outside* the atlas enumeration, and the owner's constructive resolvent-closure architecture. Everything in this subsection is *sandbox*: two frozen probes in the experiments tree, no module, no ledger row, no marker moved, no gate closed or narrowed, and no claim about the Riemann Hypothesis in any direction. The suite stood at 906 modules at the freeze (954 today, after the `v914`–`v961` promotions of Sections 12.46–12.47, 12.49, 12.50, 12.51, 12.52, 12.53 and 12.54).

The unpriced classes adjudicated (round 111, contract PRIME.UNPRICED.CLASSES.01, `unpriced_classes_probe.py`, 26/26 gates, 88 s, SPEC `ec787fc5`, deterministic re-run bit-identical, verdict UNPRICED-ADJUDICATED *— tally* 25 → 30 *candidates,* 0 PASS*).* All five classes outside the original enumeration are priced and killed; none survives as an open lever. The headline is U-A, the priority instrument: **Zagier's published horocycle RH-equivalence** (Zagier 1979/81: the horocycle average obeys $m_y(f) = m(f) + o(y^{1/2})$ unconditionally, and RH $\iff$ error $O(y^{3/4-\varepsilon})$; Sarnak CPAM 34 (1981); Verjovsky 1994) — a published RH-equivalence never adjudicated until now, confirmed absent from every atlas artifact. The instrument built: for the incomplete Eisenstein series $\psi(t) = t^2 e^{-t}$ the horocycle integral equals $3/\pi$ plus a scattering contour integral, warded two-route (dev $\le 1.5 \times 10^{-8}$; $|\varphi(1/2 + it)| = 1$ to 9.9×10^{-32} — the unconditional $1/2$ exponent is *unitary*, no spectral gap; the $3/\pi$ residue two-route). Measured on the ladder $y = 2^{-k}$, $k = 2..20$: **deep-window exponent** 0.7451 (the RH-range 3/4 in the

classically verified region), and planted-zero synthetic scattering worlds at $\beta = 0.6/0.75/0.9$ measure exponents $0.6971/0.6222/0.5473$ against the law $1 - \beta/2$ ($|\text{dev}| \leq 0.0029$) — **the horocycle rate is exactly the affine reparametrization $\theta = 1 - \Theta/2$ of $\Theta = \sup \operatorname{Re} \rho$**: the emptied zero-free/density currency (E2), with the alignment content linear $\log y$ phases — the round-97 class ($\times 8.2$). Rigidity technology is rate-free (Ratner) or non-horospherical (Lindenstrauss–Mohammadi–Wang CRAS 361 (2023): unspecified exponents, explicitly beyond the horospherical case — while the modular horocycle *is* horospherical, where the classical spectral $1/2$ has stood since 1981): verdict UA-RATE-IS-ZERO-CURRENCY, a disguise by rule R1. The other four: **U-B o-minimal/Pila–Wilkie is inapplicable by type** (the zero set of Hardy's Z is infinite — 29 sign changes on $[0, 100]$ measured, equal to the Riemann–von Mangoldt count — while o-minimal definable sets have finitely many connected components, so ζ on the line is definable in *no* o-minimal structure; the serious adjacent result verified and typed: Masser JNT 131 (2011) counts rational points on the *graph* of ζ, $\leq C(\log D/\log\log D)^2$ — count currency, position-free, unsigned); **U-C representation-theoretic positivity has its host gap confirmed** (the mechanism verified exactly: $\lambda \geq 1/4$ with positive multiplicity forces the line — the Selberg-zeta precedent — but $\lambda = 1/5$ with positive multiplicity yields the real pair $1/2 \pm \sqrt{5}/10$, so positivity alone does not force the line: it needs the identity *and* the $1/4$ gap, which at the arithmetic quotients is Selberg's own open conjecture — Kim–Sarnak floor $975/4096$, deficit $49/4096$ exact; for ζ the host identity is the mountain, Connes' trace formula being *equivalent* to RH — Stage 1 of the roadmap in another language, no new lever); **U-D GMC/Fyodorov–Hiary–Keating is magnitude-law-only** (every proven FHK-type deliverable is a $|\zeta|$-magnitude statement *in law*, the emptied E1 class; the exception check closes exactly: iid uniform phases have zero signed expectation and a flip-invariant law — the randomized Euler product carries *no* signed alignment observable); and **U-E higher-order Fourier uniformity is cancellation-only and ineffective** (Green–Tao $\log^{-A}$ savings are long-interval with Siegel–Walfisz-ineffective constants; MRTTZ 2023 short-interval higher uniformity is qualitative $o(1)$ on average; the charitable $C = 1$ envelope table shows the entire content sitting in the constant, and no effective constant exists for Λ-nilsequence correlations; the sharpest signed corollary of $|X| \leq \text{env}$ is $X \geq -\text{env}$ — orientation exactly at envelope strength, $\times 8.2$ short at the best explicit constants). The closing sentence: **the atlas stands at 30 candidates, 0 pass, with the vocabulary boundary itself now adjudicated — every named argument class inside and outside the original enumeration is formally priced.**

The resolvent closure adjudicated (round 112, contracts PRIME.SAFEPOINT.RESOLVENT.CLOSURE.01 + PRIME.CONNES.GROUNDSPACE.BLOCK.01, `resolvent_closure_probe.py`*, 36/36 gates, 262 s, SPEC* `ea7863f2`*, deterministic re-run identical, two disclosed X1 tail-cut amendments; composite* RESOLVENT-TRANSCRIPTION + CROSSCARRIER-BOUNDS + JENSEN-NO-DICTIONARY*).* The constructive resolvent-closure architecture was built and adjudicated, and it begins with **the identification**: arXiv:2511.22755's operator — CCM's rank-one perturbation $D_{\log}^{(\lambda,N)} = D_{\log} - |D_{\log}\xi\rangle\langle\delta_N|$ with ξ the lowest eigenvector of the Weil form, and its $\det_{\text{reg}}$ construction — is, **mode for mode, the corpus's own round-89 record extremal family** (the lattice $\pi k/a$ at $a = \frac{1}{2}\log x$; the round-89 arbiter quotient *is* the literal CCM operator). Built explicitly at $x = 3, 5$ and certified: kernel $M\xi = 0$ to 10^{-68}, spectrum equal to the record zeros ($10/10$ at $x = 5$, rel 2.7×10^{-8}), and their $\det_{\text{reg}}$ formula executable as a rank-one determinant identity to 10^{-75} — *the corpus had independently built CCM's operator before their paper.* The groundspace-block variant then **eliminates CCM's simplicity and evenness hypotheses at construction level**: the biorthogonal block projector onto the full minimal eigenspace gives $M_{\text{blk}} = D(I - P)$, basis-rotation-invariant to 1.5×10^{-16} and *stable* where the simple selection swings $O(1)$ under 10^{-10} perturbations, with the exterior/matrix-Wronskian determinant identity $\det(M_{\text{blk}} - z) = \det(D - z)\det(I_2 - W(z))$ to 5.2×10^{-15} and a PD quotient metric; the remaining realness step on E/K is the named unproven block-CCM lemma. The kill: the resolvent pins converge on $16/16$ rows (slope -0.65, τ-screen $0/16$) — **but the Z1 screen fires**: band pins equal cache partial sums to rel $7.7 \times 10^{-12}/9.4 \times 10^{-12}$ — the exact round-89 conviction, now

inherited by the literal CCM carrier; all controls through the same construction are self-adjoint with real spectra, so discrimination sits only in the pin limits, tail-dominated as round 89 predicted. Side by side at matched depth: the Krein benchmark extrapolates genuinely (max dev 2.9×10^{-4}) while the CCM carrier band-transcribes (1.3×10^{-1}) — **the Krein carrier remains the only non-transcribing attack surface in the corpus**. The cross-carrier test lands CROSSCARRIER-BOUNDS: $\Delta_m = \mathrm{Tr}[A_x^{m+1}]$ against the round-102 Hausdorff $b_m(256)$ gives **17/17 m-rows as monotone lower bounds from below**, the deficit explained by the RvM tail beyond the band edge (median ratio 0.786; 11/41 roots beyond the band kept in all sums — the operator mirror of the Nyquist excess): the Hausdorff field and the CCM operator are *one object only in the limit* — at finite depth the operator is a band-truncated basis of the field from below (all 45 operator cells $n + k \leq 8$ nonnegative, positivity structurally free; a third carrier confirms via Krein σ-derivative moments to 8.7×10^{-4}–2.7×10^{-3} within the bias model). The Jensen hyperbolicity wedge (arXiv:2608.08682: $n^3 \log^2(n+2) \geq Kd^5 \Rightarrow$ hyperbolic, polynomially improving GORTW) is verified verbatim but typed NO-DICTIONARY — the only corpus bridge, the Li transform at $a = 1/4$, is alternating-signed, so by the round-108 form-locality the wedge adds no unconditional Hausdorff cells. And the architecture's selling point is verified *as logic* and priced: one Stieltjes representation would generate all three positivity families simultaneously (F1), while finite certificates are form-local (F2: the Pick section fires at $N = 5$ where the Hausdorff field stays positive to depth 40) — **the logic is correct, and precisely therefore worthless without a non-transcribing carrier**. The remaining analytic theorem is doubly named: CCM's own §7–8 prolate approximation — equivalently the corpus's round-90 Weyl-disk contraction, pointwise at the pins.

Scope, in the same breath. All of this is exploration: adjudication instruments over cited literature, finite certificates, and measured structure; no all-depth statement, no promotion, no marker moved. The identification is a finite mode-for-mode certification at the built rungs, never a claim about the limit; the transcription conviction types the finite ladder, not CCM's theorem. RH is neither proven nor disproven here. **No statement about the Riemann Hypothesis is made.**

12.49 The Epstein detection arc: the certified Weil-positivity violation and the certified localized rate detection of the driver (rounds 123 and 125, notes CDXXIV and CDXXVI, 2026-08-15; promoted 2026-08-16)

Thirteen further discovery freeze rounds (notes CDXIII–CDXXVI, 2026-08-15, several run as concurrent lanes) remain sandbox except for the two durable certified results of the Epstein detection arc, promoted to the load-bearing suite on 2026-08-16 as [verification/v916_epstein_weil_violation.py] and [verification/v917_driver_rate_certification.py] (suite 908 → 910). The honesty fences travel first: **both results concern the Epstein zeta function of $Q = x^2 + 5y^2$, whose off-line zeros are classical (Potter–Titchmarsh 1935 computed the first ones; Davenport–Heilbronn 1936) — the promoted claims are the certified explicit finite certificates from arithmetic source data, never the existence; both rows are typed NOT-RH-EVIDENCE in either direction; the min-cut is unchanged (flows 4/4): a detector is a falsifier, not a prover, and for zeta the same instruments at any finite window or depth prove nothing (the all-m/dense-a/all-L omega absorbs every finite certificate).**

The certified Weil-positivity violation (round 123, `epstein_collapse_probe.py`, frozen SPEC `bb51d1a3299f4f62`, 27/30 *with exactly the three pre-declared falsified predictions failing,* 968.8 *s; re-run green-as-typed at promotion; promoted as [verification/v916_epstein_weil_violation.py],* 20/20*).* The Q-world, built *purely* from the Λ_Q sieve ($r_Q(n)$ exact lattice counts → log-derivative recursion, exactly warded; no Euler product exists, Λ_Q lives on all n; zero cache unused, witness constants firewalled out of every builder), dies by in-window Szegő collapse at $t_{\mathrm{death}} = 2.988$ (round-120 log-law point prediction 2.81), while true zeta *and* the conductor-matched null $\zeta_K = \zeta L_{-20}$ complete positivity through $L = 8.0$ — the round-120 background caveat is resolved: conductor alone does not collapse. The promoted object is the explicit certificate: a sha-pinned unit test vector (len 1038,

sha `b01436d53d2cbbee`, reproduced at every suite run together with the full quadratic form) with $V = x^{\mathsf{T}}Tx = -1.9697 \times 10^{-2} \pm 4.3 \times 10^{-68}$ and two-precision agreement 9.8×10^{-79}: **windowed Weil positivity for ξ_Q fails at $t \le 3.114$**, an upper bound on the true violation window, mesh-free. The value channel is typed CERT-NUM (source-side, unconditional-numeric); the step "$V < 0 \Rightarrow$ off-line zero" is COND-CLASSICAL (the Weil/Guinand explicit formula for ξ_Q, cited not re-proven). The three pre-declared falsified predictions are the record, carried verbatim: the attribution is COLLECTIVE (removing the round-117 witness moves the death by only 0.018; removing all four *located* off-line pairs survives to 4.428 — the located spectrum carries the collapse), the transferred death law misses by $0.505 > 0.5$, and the frozen census counts are falsified: **four off-line pairs live below height 45** ($0.9330 + 15.6682i$ — the *driver*, with 26× the witness excess and violation window $(232.6, 259.7) \ni 256$; $0.9377 + 29.9834i$; the round-117 witness $0.6969 + 36.3741i$, never γ-minimal; $0.8232 + 44.0001i$). What is unconditional: a validated falsification instrument with measured cost $t_{\text{death}} \sim 1.87 + 0.37 \ln(1/\delta)$ — any world whose sieve data collapse is *certified* non-RH-like.

The certified localized rate detection (round 125, ***`driver_cert_probe.py`****, frozen SPEC* ***`bcc64cb371dff9c8`****,* 32/33 *with exactly the single pre-declared G11 fail,* 876.9 *s; re-run green-as-typed at promotion; promoted as* *`[verification/v917_driver_rate_certification.py]`,* 13/13*).* The driver, source-located from the Λ_Q sieve alone ($\Delta\gamma = 6.6 \times 10^{-5}$ against the locator's own box, then audit-polished, typed WARD), fires in the rate channel at depth $m^* = 14$ with certified $\text{LB} = 1.000137 > 1$ and is *localized* from $m^*_{\text{loc}} = 15$ (the violating window provably contains the matched pin $a^* = 245.6815$ — it is the driver). The mechanism is the band peel: subtracting the resonant on-line band $\gamma \in (7.90, 31.40)$ (winding-count-complete 15=15/2=2, every peel $|\xi_Q|$-verified — phantom peel, the one false-fire mechanism, doubly excluded) caps the bulk at $v_{\text{edge}} = 0.639$ and collapses the unpeeled depth price $m_2 = 908$ by ×65; the sieve needed only $N = 2 \times 10^5$ atoms. The new exact algebra is recomputed symbolically at every suite run: at the matched pin the driver's weight is exactly 1 and its rate $v^* = 1 + \varepsilon$ is *exactly real* ($y^* = (\gamma + i\delta)/(2\gamma)$; LB converges to v^* to six digits by $m = 25$; certified depth 32). What emerges is a **quantified detection-cost law** (given-audit): fire at $m^* \approx \ln(B_{\text{band}}/\varepsilon)/\ln(1/v_{\text{edge}})$ with sieve price $\ln N \approx (m^* \ln Q + \ln(a_0/\text{tol}))/(\sigma_{\text{edge}} - \theta)$ — polynomial in $1/\varepsilon$ via the band, not the naive $\ln 2/\varepsilon$ depth; measured 3.19 certified depths per unit $\ln N$. The grade travels verbatim and is never upgraded: CERT-GIVEN-AUDIT (rigorous Cauchy enclosure given the incomplete-gamma audit comparator plus the *declared* tail/peel models) and DISGUISE-ADJACENT (the deep read is an Euler-computable truncated windowed Weil transform; the Krein carrier cannot exist on Q past the collapse, which is exactly why the deep route is direct). The honest caveat is carried: the pre-declared G11 gate (locator relocates 4/4) *failed* — the weak round-117 witness (excess 2.9×10^{-5}) sits below the locator's measured threshold at $N = 2 \times 10^5$; the locator is a 3/4 measured instrument, and no 4/4 claim is promoted.

Scope, in the same breath. Everything else in notes CDXIII–CDXXVI remains sandbox: no other module, no marker moved, no gate closed or narrowed. Both promoted rows say nothing about zeta's Riemann Hypothesis in either direction — known falsehoods detected by new instruments, orthogonal to proving positivity forever. **No statement about the Riemann Hypothesis is made.**

12.50 The prime-front theorem arc: the double-limit reduction, the secular/Guinand–Weil pinning, the loop/equivalence layer and the counting-class closures promoted (rounds 128–151, notes CDXXX–CDLV, 2026-08-16/17; promoted 2026-08-18)

Twenty-four further discovery freeze rounds (notes CDXXVII–CDLV, 2026-08-16/17, several run as concurrent lanes, audited adversarially by Bughunt III at round 130 and Bughunt IV at round 149 — the latter with 0 MAJOR findings and independent reproductions of the two central round-131 objects) remain sandbox except for the four durable certified theorem sets of the proof arc, promoted to the load-bearing suite on 2026-08-18 as `[verification/v918_doublelimit_reduction_theorem.py]`,

[verification/v919_secular_gw_pinning.py], [verification/v920_loop_equivalence_theorems.py] and [verification/v921_counting_dominance_closures.py] (suite 910 → 914, ledger 1061 → 1065). The honesty fences travel first: **all four rows are reductions, identities and counting closures — they say nothing about the truth of the Riemann Hypothesis in either direction; they close no omega, narrow no gate and move no other marker; the omega census stays at cardinality 4, the residue at {TOPROOT, ONSETCAP(=TLAWCAP), SUSCAP2R} + DELTA1FLOOR + the a-walls; every conditional or measured leg is typed and carried verbatim.**

Theorem R — the reduction (round 128, `doublelimit_proof_probe.py`, frozen SPEC `0c77ef7ce43fd62e`, 28/28*,* 405.3 *s run-of-record; re-run green-as-typed at promotion,* 398.5 *s; independently re-derived in Bughunt III; promoted as [verification/v918_doublelimit_reduction_theorem.py],* 13/13*).* Proven with elementary inputs: L1 + WPD ⇒ (H-conv) on the full Euler interval + (H-trace) with the *explicit rate* $\max(1, C)\,(-4\log(1-t/4))\,d_1$, via the exact series identity $\log(R_{\mathrm{fin}}/R_a)(t) = \sum_k t^k d_k/k$ — with the round-122 NF-closure theorem: RH ⇐ [NF-closure] + [Theorem R] + {L1, WPD} on dense a. Lemma N (L1 is necessary — the refinement is provably not weaker), Lemma S (subset defect ⇒ PD exactly, sharp at the band edge), Lemma X (moment transfer is not free: $d_1 = 0$ yet $d_2 = -1/128$ exactly) and the exact $C_{0k} \leftrightarrow$ jet dictionary travel with it. The typing is the theorem's own honesty: WPD's k-tail *is* window positivity (exact witness ladder $k^* = 29959/76013/145094$ from $4w^* = 10001/10000$), L1 stays obstructed at the node-law wall — **proven is the reduction and the coordinate, not the convergence**.

The secular identity and the Guinand–Weil pinning (round 131, `l1_weyllaw_probe.py`, frozen SPEC `7d8cc2e5ca8108fb`, 31/31*,* 220.2 *s run-of-record; re-run green-as-typed at promotion,* 217.4 *s; independently reproduced in Bughunt IV from the classical explicit formula,* $|\Delta \log_{10}| = 2.7 \times 10^{-7}$*; promoted as [verification/v919_secular_gw_pinning.py],* 11/11*).* The rank-one secular identity $\det(M_\eta - z) = -z\,C(z^2)/A_0$ holds *exactly* (recomputed on an own exact-rational instance): the operator nodes *are* the census-polynomial zeros, the node count is exactly $K-1$, and the η-degeneration is the boundary decay $A_0 \to 0$. On top of it the Guinand–Weil pinning: $\tau = 2\sum_\rho |E_N(\gamma_\rho)|^2$ within certified allowances — the variational value *is* the ℓ^2 norm of E_N on the true spectrum — with the certified smallness law and the certified gap law in the resolvability zone ($x = 13$: 21/21 zone zeros pinned, max gap $2.9 \times 10^{-17} \le 5.6 \times 10^{-8}$, γ_1 at 1.9×10^{-50}; edge layer measured-only). L1's tail half closes in closed form (HSW22 Cor. 1.2 + PT21), and the crossover theorem turns a frozen corpus constant into a theorem: the mode density crosses the RvM density exactly at $T = 2\pi x$, so the resolvable band fraction is exactly $1/\mathrm{KFAC} = 0.8 = \mathrm{INBAND\_F}$. The band half of L1 (H-pin) stays open, named exactly.

The loop/equivalence layer (rounds 140 and 142, `jetlock_bandmass_probe.py`, frozen SPEC `85e7ba691b3eef58`, 33/33*,* 3824.7 *s run-of-record + deterministic re-run, both logs kept,* pinned *at promotion per the multi-hour convention with its loop algebra re-gated in Bughunt IV, and `tlawcap_suscap2r_probe.py`, frozen SPEC `85971e173cd910f8`,* 26/26*,* 1941.7 *s run-of-record; re-run green-as-typed at promotion,* 1904.7 *s; promoted as [verification/v920_loop_equivalence_theorems.py],* 17/17*).* The anti-double-counting layer, all recomputed exactly in the module: **BAND-MASS ⇔ TLAWCAP ⇔ EPS-LOCK** modulo {JETLOCK, TAILVIS} (Theorem B2 — band-mass is EPS-LOCK-complete: every band-mass proof contains an EPS-LOCK proof), the onset *is* the jet ratio and the jet ratio *is* escaped census-root mass (J2/J3, so JET-LOCK(poly) ⇔ TOPROOT; J4: the hardness is cancellation alignment, not size), $s \times \mathrm{gap} \equiv 1$ is *not* an identity but the exact pinch $1 - g/\delta_1 \le sg \le 1$ (W1), **QSUBGAP ⇔ SUSCAP2R ∧ DELTA1FLOOR** (W2, both directions exact — the chain had looped on this leg), and DELTA1FLOOR ⇐ FULLGAP by Cauchy interlacing (W3, source-pure; measured tight, $\delta_1/\mathrm{FULLGAP} = 1.0000$ at all five rungs). The exact witness that DELTA1FLOOR is *required* ($s \equiv 1$ while the gap $\to 0$ — the Bughunt-IV F1 correction of record) is carried in the module. These are reshape theorems: no omega closes, but no future argument can ever count the looped coordinates as separate progress.

The counting-class closures (rounds 133 and 143, `dominance_proof_probe.py`, frozen

SPEC `00b98f5eebf9c5fb`, 40/40*,* 339.0 *s run-of-record; re-run green-as-typed at promotion,* 339.3 *s, and* `toproot_tailvis_probe.py`*, frozen SPEC `4fcd70beb7cf4f17`,* 27/27*,* 4016.1 *s run-of-record + deterministic re-run, both logs kept,* pinned *at promotion per the multi-hour convention with its T1 re-derived independently in Bughunt IV; promoted as [`verification/v921_counting_dominance_closures.py`],* 9/9*).* The pure classical-citation theorems: ball-counting dominance with no sign reads (Theorem M), edge counting bounds (Theorem E), **unconditional counting dominance** $N_{\text{fin}}(T) \le N_{\text{true}}(T)$ for all $T \ge T^*(x)$ (Theorem T, consuming only the exact node count [v919] + HSW22; ladder 63.55/79.17/102.80/147.13 recomputed against the classical γ_{K-1} anchors), the exact chain sharpening (Theorem C: MRB is strictly weaker than per-node one-sidedness — DOM is neither needed nor claimed), the conditional assemblage (Theorem A on H-pin, closed forms: $x_0 = 121$; the Q-swamp strip $D < 0$ on $x \in \{13..89\}$ carried, not smoothed), and **TAILVIS eliminated as a standalone arithmetic omega** (Theorem T1: window existence is classical — every window of length $L^* = O(20)$ carries a true zero unconditionally, from HSW22/RvM alone; Theorem T3 resolves the round-140 horizon limitation at $x = 24/28$ without new zeros). The window carrier ($x \lesssim 2.3 \times 10^5$) is disclosed — a carrier class like the census carrier, not a new omega.

Scope, in the same breath. Everything else in notes CDXXVII–CDLV remains sandbox: no other module, no marker moved, no gate closed or narrowed — in particular the round-147 adjugate elimination and the round-150/151 collapse- rate/multiblock coordinates stay experiments-side (their coordinates are still in active motion; the round-147 cell lemma is modulo simplicity, an open leg). All four promoted rows say nothing about zeta's Riemann Hypothesis in either direction — reductions, identities and counting closures with every open omega typed and standing. **No statement about the Riemann Hypothesis is made.**

12.51 The spectral-balance/edge theorem arc: the spacing/jet sum rules, the razor enclosure, the moment-Laurent ladder and the edge/cleanup closures promoted (rounds 152–161, notes CDLVI–CDLXV, 2026-08-18/19; promoted 2026-08-19)

Ten further discovery freeze rounds (notes CDLVI–CDLXV, 2026-08-18/19, several run as concurrent lanes; the family-wide exact algebra Lean-hardened in the second Lean round, round 158 note CDLXII — five new Lean 4 modules, 28 fully proven theorems on the three standard axioms, build green) remain sandbox except for the four durable certified theorem sets of the arc, promoted to the load-bearing suite on 2026-08-19 as [`verification/v922_spacing_jet_sumrule_theorems.py`], [`verification/v923_spectral_balance_razor_theorems.py`], [`verification/v924_moment_laurent_rootladder.py`] and [`verification/v925_edge_cleanup_closures.py`] (suite 914 → 918, ledger 1065 → 1069). The honesty fences travel first: **all four rows are identities, enclosures and finite/classical closures — they say nothing about the truth of the Riemann Hypothesis in either direction; they close no omega, narrow no gate and move no other marker; the omega census stays at cardinality 4, the residue at {TOPROOT, TLAWCAP-block, QSUBGAP-floor(= the pair {SUSCAP2R, DELTA1FLOOR})} + the a-walls; every conditional or measured leg is typed and carried verbatim.**

The HPIN/spacing theorem family (rounds 135 and 154, `hpin_floor_probe.py`*, frozen SPEC `80366b4e62779398`,* 33/33*,* 1837.6 *s run-of-record,* pinned *at promotion with three deterministic logs kept and its D1/D2 algebra re-gated by round 154 and Lean round 158, and* `nearalign_probe.py`*, frozen SPEC `f92b3fb59b142254`,* 31/31*; re-run green-as-typed at promotion,* 1572.7 *s; promoted as [`verification/v922_spacing_jet_sumrule_theorems.py`],* 13/13*).* The exact-algebra backbone every later spectral round consumes: the derivative identity $E_N'(\mu_j) == 4\sin(A\mu_j)F'(\mu_j^2)$ with the spacing-product form (the H-pin floor is a *zero-spacing product*), the jet sum rules of both orders (the boundary-jet telescope *is* the reciprocal-floor moment data), the A_0-cancellation (the demand ratio is A_0-*free* — the Connes scale cancels exactly), the certified lattice tail, and the sign-uniform corollary carried as an obstruction (census MIXED at every rung: the moment/harmonic route cannot see the zone floor). Lean round 158 proves the spacing product for *generic finite K over an*

arbitrary field — a genuine strengthening over the probe's per-block gates.

The pinch/enclosure/razor family (rounds 157 and 161, `spectral_balance_probe.py`*, frozen SPEC* `3ed388698a138e31`*,* 26/26*,* 3993.5 *s run-of-record + deterministic re-run identical; re-run green-as-typed at promotion,* 4077.8 *s, non-timing diff empty, and* `gfloor_probe.py`*, frozen SPEC* `cc7837138d41add7`*,* 26/26*,* 4068.2 *s run-of-record + deterministic re-run identical, both logs kept,* pinned *at promotion per the multi-hour convention with its GF algebra recomputed in the module; promoted as [verification/v923_spectral_balance_razor_theorems.py],* 20/20*).* The complete exact spectral-balance algebra: the trace loop $\tau\,\mathrm{TrH} == \mathrm{tf/FULLGAP}$ with the tail closed *unconditionally* ($1 \le \mathrm{tf} \le K-1$: DELTA1FLOOR $\Leftrightarrow$ TRACEFLOOR two-sided), the chi-cap, the enclosure $\rho^2\delta_1 \le g \le \delta_1$, the OVG and share identities (SB1–SB5), and on top **the razor enclosure** $1/(s+1/\delta_1) \le g < 1/s$ (GF1) with *two-level equality at the lower end* — the exact witness $\rho^2 = 1/(1+P\delta_1)$ realizes $s == P = 10^6$ and $g ==$ the lower end exactly, so no algebra-only improvement exists: **the g-floor is the s-cap given the δ_1-floor, nothing in between** (measured width $1.51\times10^{-4} \to 8.45\times10^{-8}$). The pinch law, the one-sided share pricing, the Newton value law $N_1 == 1/(H_t+s)$ (the bordered value *carries* ρ^2 — the round-153 self-reference pinned), the jet-ratio chase $J_g t_g == 1+g$ (the zero-jet mechanism's third rung) and the S1 absorption + twin squeeze (both final residue cores are *one statement-form at two weightings*; the value-merge refuted — the J-family is blind to g) travel with it (GF2–GF5). W1/W2/W3 stay cited from v920 — no double count.

The moment-Laurent/root-ladder family (round 156, `rootladder_probe.py`*, frozen SPEC* `02755d6b7ad0cfcb`*,* 29/29*,* 1804.8 *s run-of-record + deterministic re-run identical, zero amendments; re-run green-as-typed at promotion,* 1873.5 *s; promoted as [verification/v924_moment_laurent_rootladder.py],* 12/12*).* The escaped census-root ladder exactly solved: it is the zero set of the moment-Laurent function $\Phi(z) = z - 1 + \sum_m J_{m+1} z^{-m}$ (self-similarity iff moment flatness), **the quarter-cap is proven** ($J_2 \le 1/4 + z_0|\rho|$ with measured $z|\rho| \le 0.0102$: the saturation 0.1506 has *proven* headroom), the sum rules are exact (the ladder power sums *are* the jets), the sign-pattern law bounds the escape count, and the cascade certifies $T > 0$ *pointwise, source-pure* down to $Y^* = 1.02\text{–}2.1\,b_{\mathrm{top}}$ — the escaped-root cap is certified, not measured. The wall in final form travels in-module: the 2-mode witness inflates $J_2 \times 10^6$ with all identities intact — **J-boundedness is arithmetic-pinned**, refused only by the census-minimizer.

The edge/cleanup closures (round 159, `edge_cleanup_probe.py`*, frozen SPEC* `f6c0318841bb3942`*,* 34/34*,* 1759.1 *s run-of-record + deterministic re-run identical (run1 =* 31/34 *kept — its fails* are *the depth finding); re-run green-as-typed at promotion,* 1830.8 *s; promoted as [verification/v925_edge_cleanup_closures.py],* 14/14*).* Three finite/classical legs closed or sharpened. **Simplicity falls** (census row Z8 eliminated, 10 → 9 rows): the LM split forces $\delta_1 \ge 1/(B-1) > 0$ at every selected point — the selected points are *automatically* simple, Z8 was double-counted inside Z4 — and degeneracies are dodged at cost exactly zero. **The Q-swamp closes wholesale**, and the module recomputes it *at every suite run* from the committed PT21-warranted cache: $D_{\mathrm{cs}} > 0$ at all 357 Turing-class local band certificates (integer $x = 3..121 \times$ battery; worst margin 0.346 at $(12,1)$; heights $\le 7264.75 \ll 3\times10^{12}$), with $x_0(\mathrm{HSW}) = 121$ and $x_0(\mathrm{BW25}) = 112$ recomputed — the bridge is gap-free, so **Theorem A's conditionality sharpens to every integer** $x \ge 3$ (H-pin itself untouched). The sliver closes below horizon per-zero (pinned; the certificate price rides $e^{-\mathrm{depth}}$ — machine-pinned obstruction), and the band-edge separation demand is typed exactly (zero repulsion of 0.22–0.43 RvM mean spacings, arithmetic-signed-input width).

Scope, in the same breath. Everything else in notes CDLVI–CDLXV remains sandbox: no other module, no marker moved, no gate closed or narrowed — in particular the round-160 J_2 prime-force adjudication (note CDLXIV, verdict INFORMATIVE-NEGATIVE: no single prime property forces the J_2-window, the single-lever search closed, the window carried irreducibly collectively by the signed low-band spectral structure) stays experiments-side, deliberately: its record runs are multi-hour world-builds with no exact-algebra layer a module could recompute, and its residue reading is still in active motion (round 161 and the round-162 growth-law closure — since promoted in the endgame arc, 12.52 — work exactly the DELTA1FLOOR family it points at); likewise the

round-153 anchor coordinates and the round-155 census stay experiments-side. All four promoted rows say nothing about zeta's Riemann Hypothesis in either direction — identities, enclosures and finite/classical closures with every open omega typed and standing. **No statement about the Riemann Hypothesis is made.**

12.52 The endgame arc: the quartic/growth law, the cofinal block-average substrate, the window-instrument mechanism, the depth-block transfer and the sigma-floor terminal factorization promoted (rounds 162–169, notes CDLXVII–CDLXXXIV, 2026-08-19; promoted 2026-08-19)

Eight further discovery freeze rounds (notes CDLXVII–CDLXXXIV, 2026-08-19, several run as concurrent lanes) remain sandbox except for the five durable certified theorem sets of the endgame arc, promoted to the load-bearing suite on 2026-08-19 as [`verification/v926_fullgap_growthlaw_theorems.py`], [`verification/v927_blockaverage_substrate_theorems.py`], [`verification/v928_depthblock_transfer_theorems.py`], [`verification/v929_sigmafloor_factorization.py`] and [`verification/v930_window_instrument_mechanism.py`] (suite 918 → 923, ledger 1069 → 1074). The honesty fences travel first: **all five rows are identities, recipe-derived laws, finite/classical closures and one honest mechanism record — they say nothing about the truth of the Riemann Hypothesis in either direction; they close no omega, narrow no gate and move no other marker; the omega census stays at cardinality 4, the residue at {TOPROOT, TLAWCAP-block, QSUBGAP-floor} + the a-walls — now carried *with* the round-168/169 terminal coordinates: the λ-uniform budget content factorizes as [census schedule, classical per k, loop if for-all-k] × [SIGMA-FLOOR == DC-leg (classical per census) × jet-mass-floor (the terminal λ-uniform residue, typed open)]; the quartic law is typed MEASURED and not claimed proven; every measured leg is typed verbatim.**

The quartic/growth-law family (round 162, `fullgap_growthlaw_probe.py`, frozen SPEC `26bdb5a87f63c519`, 28/28, 4061.6 *s run-of-record + deterministic re-run* 4237.5 *s with timing-normalized diff empty, both logs kept;* pinned *at promotion per the multi-hour convention, its GL algebra re-gated in Bughunt V (round 164, note CDLXIX, G19/G20) and recomputed in the module; promoted as [`verification/v926_fullgap_growthlaw_theorems.py`],* 12/12*).* The proven layer: the exact leave-one-out instrument (GL1 pinch + GL2 arrowhead compression with the dual row), the one-row price equivalence with razor vacuity (GL3: **D3 collapses to the razor** — algebra-only per-row/per-zero price bounds refuted), the rate bookkeeping (GL4) and the ladder/quartic chases with the razor composition (GL5: **the QSUBGAP-floor flows from the Θ-window through the razor**). The measured law, honestly typed: $J == \Theta\, T_z^4$ with Θ flat over 24 dex of jet collapse — the round-146 open-classical candidate's closed form, *not* claimed proven (its open kernel is the Θ-window; the round-162 red team shows only arithmetic-consuming windows may pin Θ); the independent Bughunt-V free-exponent/AIC adjudication is *recomputed in-run* ($p = 3.9072 \pm 0.1136$, fixed 4 preferred, 3.5/4.5 rejected at $3.6/5.2\,\sigma$). The loo battery refutes per-zero deflation measured (ρ_j == 1.0000 for all 189 left-out zone zeros: jet-collective), and the finite-P moment model undershoots by 1.4–4.5 dex — the Y3/R4 rate-blind trace-cap vacuity class, one level higher.

The cofinal block-average substrate (round 166, `tau_blockaverage_probe.py`, frozen SPEC `d86f42a04c69a4e2`, 27/27, 2019.0 *s run-of-record + deterministic re-run identical; re-run green-as-typed at promotion,* 2372.4 *s, timing-normalized diff empty; promoted as [`verification/v927_blockaverage_substrate_theorems.py`],* 12/12*).* The substrate of the whole endgame: BA1 (a certified block sum > 0 forces ≥ 1 positive rung — the cofinal extraction), BA2 (**the determinant chain is the τ sign**, Jacobi/Sylvester), BA3 (**the budget floor** $\tau \geq$ zsum − OFF exactly, proven-mod-cited-inputs below the verified horizon: PT21 + HSW22 + the round-131 OFF recipe + ward-class cache — no-loop machine-gated), and the Weyl-split obstruction (the source-only additive route is vacuous by 10.9–54.6 dex: the cancellation *is* the arithmetic). The certified block enclosures land **positive-rung-per-block below the horizon** on both complete dyadic blocks; the wall dictionary is instantiated at all 25 rungs

(sign(τ) == sign(wall) == +1); the tlaw ladder gains 19 new corpus-depth rungs. The λ-uniform demand beyond the horizon is recoordinated to the AVG-BUDGET-WINDOW — one one-sided weighted-average inequality per deep block, strictly weaker than any pointwise window law — and stays open.

The depth-block transfer family (round 168, `depthblock_transfer_probe.py`*, frozen SPEC* `a4cd07144e0c5222`*,* 36/36*,* 1971.3 *s run-of-record + deterministic re-run (result lines bit-identical), zero amendments; re-run green-as-typed at promotion,* 2371.6 *s, timing-normalized diff empty; promoted as* `[verification/v928_depthblock_transfer_theorems.py]`*,* 11/11*).* The endgame's structural core. DT1 is **proven** (the transfer step is exact algebra on top of BA3); DT2 — the round's discovery — is **proven-by-recipe**: the demand law $\varepsilon == \sqrt{h}\,(1+\eta)^2\,G(T_{\mathrm{PT}})/G(2\pi h)$ is *derived*, with the dyadic demand factor falling monotonically onto $2^{3/2}$; **the census 3/2-law** is sympy-exact ($T_{\mathrm{req}}(h) \to (3\pi/\sigma_0)\,h^{3/2}$: census consumption grows strictly faster than the block — *the budget route can never become self-supporting*); the PT21 budget horizon is recomputed in-run ($h^* = 1.2566\times10^7$, $k^* = 23.58$: ∼22 complete dyadic blocks); the Bertrand entry interface is sieve-verified to 10^6 (the dyadic gap is never empty); and **universalizing the census is machine-detected as the RH loop** (the per-k schedule is acyclic and classical). The step is instantiated end-to-end (STEP(B2→B3) 9/9, STEP(B3→B4-partial) 11/11, substrate-direct: each block consumes fresh primes and fresh census — no self-supporting induction), the entry batteries confirm ENTRY-NEW 4/4 (cone entry always at the largest new *prime-power* atom of the dyadic gap, excluding trailing 2-powers — the note-CDLXXXIII formulation, per the Bughunt-VI F1 correction of record; fakes die by no-entry), and **the final coordinate is named**: the AVG-BUDGET-WINDOW factorizes into the classical census schedule times SIGMA-FLOOR — the entire remaining λ-uniform arithmetic content in one one-sided block-averaged inequality.

The sigma-floor terminal factorization (round 169, `sigmafloor_probe.py`*, frozen SPEC* `a16a4db1055488d2`*,* 35/35*,* 1971.4 *s run-of-record + deterministic re-run identical, amendment 1 disclosed (the all-rung onset-rate fit typed cache-top-limited); re-run green-as-typed at promotion,* 2375.2 *s, timing-normalized diff empty; promoted as* `[verification/v929_sigmafloor_factorization.py]`*,* 12/12*).* The terminal coordinate's official record. **The supply factorizes exactly** (SF1): $\sigma == (1-\mathrm{slop})\,\delta\,\mathrm{DC}$ — jet mass times equidistribution mass, identity devs $\le 2\times10^{-60}$ at all 25 rungs — so **the round-168 σ-flatness is explained as two trends** (δ $O(1)$-flat × DC rising toward 1/2): the last unexplained measured law of round 168 is decomposed. The DC leg is **DC-classical** (proven-mod-cited per census: Landau 1912/Gonek 1993 as form, the Gonek constant unpriced); the onset-excess cap is *refuted* as a floor (SF2); *any* explicit polynomial rate is census-absorbable (SF4, sympy-exact — the measured flatness is not needed); the pinning-supply route is machine-flagged as the RH loop's costume (SF5); and the sharpest controls finding travels verbatim: the raw floor *holds* in every fake world while the BA3 bridge is *false* — the arithmetic content is the {floor, bridge} pair. **The jet-mass-floor is named and typed open**: $\delta \ge r(h)$ at an explicit rate — the terminal λ-uniform residue, arithmetic-pinned, per-rung classical, *not* claimed.

The window-instrument mechanism record (round 167, `window_instrument_probe.py`*, frozen SPEC* `bfdcf374504006e7`*,* 46/46*,* 4138.5 *s run-of-record + deterministic re-run* 4047.0 *s, both logs kept, the mid-run owner-directive re-freeze disclosed;* pinned *at promotion per the multi-hour convention, its WI algebra recomputed in the module and the entry law re-gated by the round-168 batteries re-run green at this promotion; promoted as* `[verification/v930_window_instrument_mechanism.py]`*,* 10/10*).* The proven instrument layer: WI1/WI2 (**band positivity is exactly the carrier of the PSD premise**, at τ-priced thresholds — premise transport proven both directions, value transport refuted-in-class), WI3 (first-order symmetric), WI4 (the budget-moment dictionary onto the v924 moment-Laurent Φ), WI5 ($\Theta == c_1^2$ and the theta chases), WI6 (**BANDPOS(unbounded ladder) $\iff$ RH** — the loop identification: a landed value transport would close the program onto its own hypothesis), and the Stieltjes-moment refutation of the dyadic slot ladder recomputed from the pinned slots. The mechanism finding is pinned at its honest grade (typed measured, the v916 precedent): *no partial comb is positive* — the cone entry happens only in the last atom's tail

with the tangent criterion holding from the prime source, all fakes fail by no-entry; independently re-confirmed at true dyadic pairs by round 168 (4/4); chain-not-assembled typed. The honest demotion travels: the plant/ghost asymmetry *law* is generic-cone — the arithmetic content is where the boundary sits and the anchor values.

Scope, in the same breath. Everything else in notes CDLXVII–CDLXXXIV remains sandbox: no other module, no marker moved, no gate closed or narrowed — in particular the round-163 Selberg-effectivity adjudication (note CDLXVIII, verdict HARVEST-NEGATIVE: no citable effective Selberg variance exists as of 2026-08) and the round-165 definitive 7-row census (note CDLXX) stay experiments-side, deliberately; the round-164 Bughunt-V audit's corrections of record were applied to the live surfaces in the CIX pass (no verdict flips, 0 MAJOR). Mid-promotion, the sixth adversarial bughunt (round 170, note CDLXXXV, 22/22, 0 MAJOR, no verdict flips, no arrow retypes) independently re-audited the whole endgame chain — the arrow table re-verified with own code, the Yoshida-1992 priority blockade validated against the source — and its live-surface wording corrections (the prime-power entry formulation, the re-run phrasing, the recoded cofinality legs) are adopted on the surfaces of this arc. All five promoted rows say nothing about zeta's Riemann Hypothesis in either direction — identities, recipe-derived laws and finite/classical closures with every open omega typed and standing; the quartic law stays measured, the jet-mass-floor stays open, the census cardinality stays 4. **No statement about the Riemann Hypothesis is made.**

12.53 The triple-cofinal arc: the jet-mass product floor, the TOPROOT statement, the H3-cofinal adjudication, the unconditional Gonek pricing and the mode-level Landau bridge promoted (rounds 171–175, notes CDLXXXVII–CDXCI, 2026-08-19/20; promoted 2026-08-20)

Five further discovery freeze rounds (notes CDLXXXVII–CDXCI, 2026-08-19/20, two run as concurrent lanes) are promoted to the load-bearing suite on 2026-08-20 as [`verification/v931_jetmass_floor_theorems.py`], [`verification/v932_toproot_theta_statement.py`], [`verification/v933_h3_cofinal_adjudication.py`], [`verification/v934_gonek_pricing_unconditional.py`] and [`verification/v935_thetainf_landau_bridge.py`] (suite 923 → 928, ledger 1074 → 1079), after the seventh adversarial bughunt (round 176, note CDXCII) covered the arc with 0 verdict flips and its corrections of record (F1–F8) were applied — and *adopted verbatim on every surface of this promotion.* The honesty fences travel first: **all five rows are certified finite theorems, unconditional classical pricings and honest wall records — they say nothing about the truth of the Riemann Hypothesis in either direction; they close no omega, narrow no gate and move no other marker; the omega census stays at cardinality 4; and the arc's terminal residue is stated in the Bughunt-VII-corrected form: the composed per-block hypothesis is the *triple* {H1 ∧ H2 ∧ H3}-cofinal (one rung per dyadic block, all three at the same h — *not* H3 alone; the limsup form holds only mod the measured defect $D = 0.004183$), its classical-evaluation face θ_∞ typed open-nonperturbative-variational, plus {census-forall-k == loop, flagged} + {L1, WPD}; the quartic law stays measured; every measured leg is typed verbatim.**

The jet-mass product-floor theorems (round 171, `jetmass_floor_probe.py`, frozen SPEC `57de8b2a83677a9c`, 36/36*, 2056 s run-of-record + deterministic re-run with timing-normalized diff empty, two post-run amendments disclosed (the CLEAN_FIT margin window; the control-refusal re-adjudication onto the BA3 bridge + size separator), all logs kept;* re-run green as typed at this promotion *with identical SPEC and gate count; promoted as [`verification/v931_jetmass_floor_theorems.py`],* 10/10*).* The proven layer: the census product identity with the Vieta trace at coefficient level (PF1 — Newton/Vieta resum the entire J-tail into root data), **the product floor** $F/A_0 \geq (1-c^*/z)^{(1+\kappa)/c^*}$ given H1 + H2 with *the exponent the trace* (PF2 — the round-140 far-field law is its $\kappa \to 0$ shadow, the measured 1.2-percent tightness explained), the source-pure half-plane exclusion (PF3 — H1 is one exact source evaluation per rung), and the assembled factorization [PF proven given H1 + H2] × [WF classical per census] × [RATE == TOPROOT] with the round-169 measured-lock ancestor *eliminated.* The certified curve sits under the truth at every one of ∼100k true tail zeros; the B2/B3 block certificates run through the product floor; the witness kills the certificate pair by

modus tollens — the toys are excluded *by theorem.*

The TOPROOT statement and the quartic certificate (round 172, `toproot_theta_probe.py`*, frozen SPEC* `cf27df22aa5dffbf`*,* 33/33*,* 2813 *s run-of-record + deterministic re-run, one smoke-stage fix disclosed, all logs kept;* re-run green as typed at this promotion*; promoted as* `[verification/v932_toproot_theta_statement.py]`*,* 10/10*).* The last λ-uniform edge becomes *one named inequality*: TOPROOT(p, C) with the exact SEQ-cofinal quantifier and three locked forms; the rate dictionary $a = p/2 - 1$ proven at $p = 3, 4, 5$; theorem TR-CAP proven *and* typed circular for TOPROOT (the cap is y_t-normalized; Fujiwara's sharpest term is the trace); the Vieta pinch killed ($1/A_0$ in every symmetric function); the A_0-triangle route flagged as the fourth loop; the admissibility ceiling recomputed ($p_{\max} = 10.2695$ — six powers of margin over the measured quartic); and the certificate **H3**: $y_t \le 0.155\, T_z^4$ per rung, source-pure, certified 26/26 with margin ≥ 1.99, *refutable* (the inflation witness breaks it at $\theta'' = 62.69$) and not norm-continuous (the witness price is exponentially small: any proof must consume the arithmetic sign structure).

The H3-cofinal adjudication (round 173, `h3_cofinal_probe.py`*, frozen SPEC* `876dafc977d3d8fc`*,* 36/36*, run-of-record + deterministic re-run, one smoke-stage fix disclosed, all logs kept;* re-run green as typed at this promotion*,* 1339.5 *s; promoted as* `[verification/v933_h3_cofinal_adjudication.py]`*,* 10/10*).* The three mechanisms priced: the saturation is *real* and equals exact-quartic convergence (the local exponent descends $4.30 \to 4.02$; θ_∞ stays open in the band $[0.0766, 0.0977]$ under the bar, $1/(4\pi)$ inside the band and recorded-not-claimed); block-average + pigeonhole is proven exact yet *subsidy-empty* (the quantifier gap is measured thin, $D = 0.004183$ — the limsup form is cited only mod D); and the witness boundary becomes a theorem boundary: the inflation witness breaks three already-certified constraints (ground state, two-sided lock, BA3 bridge $\times 3.6 \times 10^8$), the J_2-repairing escalation costs $\times 10^5$ and still breaks the same three, and H3 is Davis–Kahan norm-stable at the measured residuals — **scoped to the DK radius** (gated $h \le 13$, measured to $h = 20$, crossover $h = 16$; the Bughunt-VII F4 correction adopted: the lock leg excludes only large dials).

The unconditional Gonek pricing (round 174, `gonek_pricing_probe.py`*, frozen SPEC* `3050678b352eaa9a`*,* 20/20*,* 188 *s run-of-record + deterministic re-run, one smoke-stage fix disclosed, all logs kept;* re-run green as typed at this promotion*,* 244.2 *s; promoted as* `[verification/v934_gonek_pricing_unconditional.py]`*,* 8/8*).* The "Gonek constants" item leaves the residue: Landau 1912 and Gonek 1985/1993 identified *unconditional* (sums over all nontrivial zeros, no on-line hypothesis), the RH-conditional family Gonek 1984 machine-checked *not an ancestor* of either leg (the would-be hidden loop detected, flagged, never consumed — externally re-verified in Bughunt VII against the sources), the error envelope collapsed to a sympy-exact closed form with a finite census-depth-insensitive ceiling, the constants priced on the 20M-zero cache ($\hat{c} \le 0.086$ at signal/bound up to 2.7×10^4) and typed honestly *constant-empirical-per-census*; the controls refuse $\ge 100\times$ through the same instrument.

The mode-level Landau bridge and the conditioning wall (round 175, `thetainf_pin_probe.py`*, frozen SPEC* `3044558e5fa52e01`*,* 28/28*,* 577 *s run-of-record + deterministic re-run, one smoke-stage fix disclosed, all logs kept;* re-run green as typed at this promotion*,* 718.1 *s; promoted as* `[verification/v935_thetainf_landau_bridge.py]`*,* 9/9*).* The classical pin of θ_∞ does *not* land and the round proves exactly why: the program's **first linear per-mode classical identity** — the prime block of the world matrix is linear in two atom transforms per mode, and the mode-level Landau bridge evaluates the arithmetic fluctuation (≥ 99.2 percent of θ) from the verified zero census alone, residuals $1.4 \times 10^{-5} \to 5.4 \times 10^{-5}$ at 20M zeros with a clean $\sim 1/T$ tail law; the rate dictionary is a proven *bijection* (transports every θ_∞, pins none); the ceiling candidate list is enumerated and *empty*; and the variational conditioning wall is measured (the Bughunt-VII F7 typing adopted: a *measured-law* extrapolation, "gemessene" not "bewiesene" Verschwendung): $T_{\text{req}} = 3.76 \times 10^{14} \to 1.75 \times 10^{77}$ vs $T_{\text{PT}} = 3.0 \times 10^{12}$ — the $1/A_0$ circularity relocates into the variational conditioning; θ_∞ is re-typed **open-nonperturbative-variational** (deeper census improves the data $\sim 1/T$ but provably never the limit).

Scope, in the same breath. Everything else in notes CDLXXXV–CDXCIII remains sandbox:

no other module, no marker moved, no gate closed or narrowed. The seventh adversarial bughunt (round 176, note CDXCII, 21/21, 8 findings, 1 MAJOR, *no verdict flips*) covered rounds 170–175 before this promotion; its corrections of record — F1 (the residue is the *triple* {H1 ∧ H2 ∧ H3}-cofinal, not H3 alone), F2 (the limsup form only mod $D = 0.004183$), F3 (two definitional exact-layer legs), F4 (the manifold-exclusion verdict scoped to the DK radius), F7 (the T_{req} wall a measured-law extrapolation) — were applied to the frozen records as appended correction blocks (note CDXCIII; per the Bughunt-VIII F2 correction this application pass carries no round number of its own — canonical round 177 is the manifold-invariance lane) and are adopted verbatim on every surface of this arc. The later rounds — per the Bughunt-VIII F2 numbering correction the canonical rounds 177–182 (the manifold-invariance, L1/WPD, de Bruijn–Newman, CBJ confluent-jet-frame, CBJ sub-dof and alignment-law lanes, notes CDXCIV–D) — were covered by the eighth adversarial bughunt and promoted in the post-bughunt arc (Section 12.54). All five promoted rows say nothing about zeta's Riemann Hypothesis in either direction — certified finite theorems, unconditional classical pricings and honest wall records with every open omega typed and standing; the quartic law stays measured, θ_∞ stays open, the census cardinality stays 4. **No statement about the Riemann Hypothesis is made.**

12.54 The post-bughunt arc: the tau-free DK exclusion, the L1/WPD consolidation, the exact census semigroup, the CBJ adjudication, the sub-dof kill and the alignment law promoted (rounds 177–182, notes CDXCIV–D, 2026-08-20; promoted 2026-08-20)

Six further discovery freeze rounds (notes CDXCIV–D, 2026-08-20, several run as concurrent lanes; per the Bughunt-VIII F2 numbering correction the canonical rounds are 177 = manifold, 178 = L1/WPD, 179 = de Bruijn–Newman, 180 = CBJ frame, 181 = CBJ sub-dof, 182 = alignment law) are promoted to the load-bearing suite on 2026-08-20 as [verification/v936_manifold_invariance_exclusion.py], [verification/v937_l1wpd_closure_reduction.py], [verification/v938_dbn_heatflow_census.py], [verification/v939_cbj_frame_adjudication.py], [verification/v940_cbj_
and [verification/v941_alignment_law_factorization.py] (suite 928 → 934, ledger 1079 → 1085), after the eighth adversarial bughunt (round 183, note DI) covered the arc with 0 verdict flips and its corrections of record (F1–F7, note DII) were applied — and *adopted verbatim on every surface of this promotion.* The honesty fences travel first: **all six rows are certified finite theorems, exact instruments, honest kill records and frozen measured laws — they say nothing about the truth of the Riemann Hypothesis in either direction; they close no omega, narrow no gate and move no other marker; the omega census stays at cardinality 4; θ_∞ stays open; the λ_h limit stays open; and the arc's terminal residue is stated in the canonical note-DII form: the *triple* {H1 ∧ H2 ∧ H3}-cofinal (one rung per dyadic block, all three at the same h; the limsup form only mod $D = 0.0042$) + {census-forall-k == loop, flagged} + {H-pin = the one λ-uniform edge of the pair: L1 = TAIL proven + H-pin open; WPD($a < \gamma_1^2$) ⇐ H-pin; the WPD non-λ legs: extension instantiated, TAILWPD world-front} — with the Bughunt-VIII F1 clarification that the round-182 wall convergence is *class*-level (same-class-not-new-edge), not an object identity.**

The tau-free DK exclusion theorem and the transport ledger (round 177, `manifold_invariance_probe.py`, frozen SPEC `af86ec3b097ae8c4`, 39/39, 1354.1 s run-of-record + deterministic re-run, one smoke-stage fix and three amendments disclosed with kept fail records (the third the honest feasible-upper-value retype, monotone-safe, Bughunt-VIII re-verified); re-run green as typed at this promotion*; promoted as [verification/v936_manifold_invariance_exclusion.py],* 10/10*).* The Bughunt-VII F4 scope gap is removed within the instrument: the exclusion is now (i) a **tau-free directional theorem** — $\sin\theta \le (r_0/\tau)/(\mathrm{FG} - a)$ with τ cancelling in ratio form, proven for $h \le 13$ at the frozen 10^{-25} bar (measured to $h = 20$) plus the exact required-bar schedule $b^*(h)$ for every rung; (ii) a measured **full-space minimal-price ladder** ($1.35 \times 10^5 \to 6.73 \times 10^8$ times τ, strictly growing — 30–33 orders above the bar, *always cited paired with the proven schedule*: 23.8/21.7/15.7/4.4 orders at $h = 4/5/8/13$, none at $h \ge 16$, per Bughunt-VIII F7), with

the mechanism identified as **alignment-sharing** (the low eigenvectors share the source's $1/A_0$ cancellation — the r140/r143 alignment wall quantified for the first time from the defense side); and (iii) the adjudicated **non-universality of BA3** (the scale-gauge theorem plus the exact zmin passer whose own certificate price grows to 3.2×10^{68}). The forward-invariance/induction route is priced dead *as a reduction*: transport relabels, levels are cheaper than increments, the manifold protects the falsifiability boundary and not the H3 value — the infinite quantifier does not fall by induction.

The L1/WPD consolidation (round 178, `l1wpd_closure_probe.py`*, frozen SPEC* `0e306146bebbd9be`*,* 29/29*,* 228.9 *s run-of-record + deterministic re-run, no amendment;* re-run green as typed at this promotion*; promoted as* `[verification/v937_l1wpd_closure_reduction.py]`*,* 10/10*).* The sharpening is label-level but machine-gated: **WPD's λ-uniform content is the H-pin edge** — the AND-fire demonstration at the declared edges shows the H-pin grants fire the WPD battery and L1 while the jet-triple grants fire PF/HCOF and neither fires the other: the pair {L1, WPD} carries *one* open λ-uniform edge, not two. The three-part bridge-expressibility verdict: the pin's source data *are* bridge-expressible per census; the floor escapes linearity at exactly two machine-exhibited maps (the eigenvector map and the root map — the round-175 conditioning wall applies verbatim); and the classical SW pricing is vacuous at battery rungs — no closure. The v925 all-x sharpening is propagated onto the residue surface with own replication, and the r133-vs-r171 H1/H2/H3 name collision is flagged on the record.

The exact census semigroup (round 179, `dbn_heatflow_probe.py`*, frozen SPEC* `67eaf86c7bfa7d84`*,* 26/26*,* 672.4 *s run-of-record + deterministic re-run, one smoke-stage fix disclosed;* re-run green as typed at this promotion*; promoted as* `[verification/v938_dbn_heatflow_census.py]`*,* 11/11*).* The one big classical import never tried in 178 rounds — the de Bruijn–Newman heat flow, with Rodgers–Tao 2020 and Polymath15 2019 consumed as *cited ceilings* only — lands as an exact instrument: the flow transports onto the derived census polynomial as a **closed-form finite semigroup** e^{-tT} (no zeros, no ODE) with the exact trace law (measured deviation 0.0); the dictionary of "the flow" is proven **threefold-inequivalent** (every pinch claim is dictionary-dependent); the pinch is dead three ways (theta-flow-inert, wall-flow-incommensurate, and backward-persistence machine-flagged as the RH class); the **missing half of the generator is exactly the RH-conditional pair class** (the flagged loop — the Rodgers–Tao proof itself consumes pair correlation); and the round leaves a new instrument, the λ_h **criticality ladder** ($2.39 \to 0.216$, strictly decreasing toward the admissible window scale, the limit *open*) — honestly typed *not* an off-line-zero detector (the Epstein world measures larger backward slack), with the round-171 witness preserved-not-expelled.

Step-A exact and the CBJ adjudication (round 180, `cbj_frame_probe.py`*, frozen SPEC* `d7fbf2d956184674`*,* 41/41*,* 1042.5 *s run-of-record + deterministic re-run, no amendment;* re-run green as typed at this promotion*; promoted as* `[verification/v939_cbj_frame_adjudication.py]`*,* 10/10*).* The review's externally proposed primary route is fully instrumented and honestly adjudicated. **Step A is exact**: $\delta_h == |J_h|^2_{G_h}$ with remainder $R == 0$, proven generically — the single most promotable theorem of the arc — with the cluster normal form (the basis-change determinant *is* the spacing product) and the Montgomery–Vaughan pricing (killed exactly at the contract's linkage, carrying at 3.15π with the exact rational margin $1/21$). The *global* form of the review inequality dies at step C on three independent measured walls (the dof kill $n = 474$ vs rank 128; the seam coupling 0.994; the occupancy collapse), while the per-cluster floor carries k-uniformly at fixed r through the exact occupancy bound — the wall is the r-collapse, not the depth. The source-only selector ($\hat{h}_k = 7/13/31/\ldots$, Bertrand-cofinal, recomputed in the module) underperforms the block average yet clears the SF2 demand at every rung — disclosed as the honest adverse finding.

The sub-dof kill and the Carleson pricing (round 181, `cbj_subdof_probe.py`*, frozen SPEC* `2db82c76ce5f067c`*,* 37/37*,* 867.9 *s run-of-record + deterministic re-run, one spec-text amendment disclosed (the sympy-exact Fejér constant* 1/6*, Bughunt-VIII-verified);* re-run green as typed at this promotion*; promoted as* `[verification/v940_cbj_subdof_blockfloor.py]`*,* 9/9*).* The blockwise sub-dof-scoped floor statement — what the program would actually need after the global-form

death — is posed exactly and honestly **killed at the decisive measurement**: the sub-dof mass fraction falls at +0.999 dex per rung (recomputed) and saturates at zero; the scoped margin dies *at the selector rungs* (dead by factor ∼125 at $\hat{h}(B3) = 13$); the unscoped round-180 floor remains the measured carrier — killed is the proof route through the well-conditioned subspace. The 180-round Carleson debt is settled with exact citations: the unconditional embedding is **wrong-direction** for a floor (Ortega-Cerdà–Seip 2002), and every unconditional lower class misses for a named reason — the Nazarov–Turán constant, exponential in the term count, is the classical mirror of the measured occupancy law. New instrument: **fraction-world-separating** — the floor *value* is world-insensitive but the mass *location* separates worlds (only the arithmetic world rides the ill-conditioned tail).

The alignment law (round 182, `alignment_law_probe.py`*, frozen SPEC* `e4cdb9a932093196` *— verified invariant under the appended correction-of-record block —* 37/37*,* 775.6 *s run-of-record + deterministic re-run, five freeze retypings disclosed;* re-run green as typed at this promotion*; promoted as* `[verification/v941_alignment_law_factorization.py]`*,* 11/11*).* The sharpest unexplained regularity of the program is frozen as a **measured triple**: the half-power envelope (α inside the descriptive band $(0.38, 0.54)$ at all 14 rungs — never a prediction, per Bughunt-VIII F4), the quarter-depth wall lock ($\mu_m = 0.2372 \log_{10} g_{\min}$, R^2 0.9916), and the deep excess (5.64–7.58 dex — exactly the round-181 kill mass). The mechanism is factorized onto **one shared nonlinear link**: the Gram is source-free (proven three ways), the ray-to-law map is **projective-linear and A_0-free** (the round-178 obstruction does *not* bite — the key adjudication), and the only nonlinear step is the wall eigenmap, shared with δ itself: the law is exactly as bridge-expressible as δ, given d. Every deterministic deformation of the true ray breaks the law (alt-jets-break-alignment); the witness breaks it at exponentially small source cost. The terminal adjudication carries the Bughunt-VIII F1 *corrected* wording verbatim: **same-class-not-new-edge** — the floor form lives in the same variational-quotient *class* as θ_∞ and H-pin Omega-a (four distinct formal statements over the same source object, the wall argmin ray d, in A_0-quotient form), and *no* new residue edge opens, because the floor route is optional and machine-excluded from the delivered set — not because the objects are identical.

Scope, in the same breath. Everything else in notes CDXCIV–DII remains sandbox: no other module, no marker moved, no gate closed or narrowed. The eighth adversarial bughunt (round 183, note DI, 20/20, 7 findings, 1 MAJOR, *no verdict flips*) covered rounds 176–182 before this promotion; its corrections of record — F1 (same-class-not-new-edge, the class-not-object adjudication), F2 (the canonical round numbering), F3 (the G51 retype), F4 (the descriptive-band citation discipline), F6 (the refined residue form canonical), F7 (the paired refuter-margin citation rule) — were applied to the frozen records as an appended correction block (note DII) and are adopted verbatim on every surface of this arc. The two bughunt rounds themselves (176 and 183) are process rounds and stay experiments-side per precedent. All six promoted rows say nothing about zeta's Riemann Hypothesis in either direction — certified finite theorems, exact instruments, honest kill records and frozen measured laws with every open omega typed and standing; the census cardinality stays 4, θ_∞ stays open, the λ_h limit stays open. **No statement about the Riemann Hypothesis is made.**

12.55 The mechanism arc: the detector specificity, the ablation localization, the residue identity, the causal synthesis with its stratification, the Krein pencil, the Loewner–Pick dictionary and the sign-characteristic law promoted (rounds 184–191, notes DIV–DXIII, 2026-08-21; promoted 2026-08-21)

Eight further discovery freeze rounds (notes DIV–DXIII, 2026-08-21, several run as concurrent lanes; rounds 184–191, with round 190 a surgical follow-up that upgrades round 187 *by reference*) are promoted to the load-bearing suite on 2026-08-21 as [`verification/v942_pi_pattern_transfer_specificity.py`], [`verification/v943_mangoldt_ablation_localization.py`], [`verification/v944_ground_residue_observability.py`],

[verification/v945_zero_causal_stratification.py] (one *joint* module for rounds 187 + 190), [verification/v946_census_krein_pencil.py], [verification/v947_loewner_pick_dictionary.py] and [verification/v948_krein_sign_characteristic.py] (suite 934 → 941, ledger 1085 → 1093), after the ninth adversarial bughunt (round 193, note DXV, 44/44, 9 findings, 1 MAJOR, 0 verdict flips) covered the arc with the headline **X5: every claimed-exact object of the arc reproduced in fully independent code with zero failed recomputes**, and its corrections of record (K1–K5, note DXVI) were applied as appended CORRECTION-OF-RECORD blocks — and *adopted verbatim on every surface of this promotion.* The honesty fences travel first: **all eight rows are certified finite theorems, exact instruments, instrument/mechanism characterizations and honest partial-lift or refutation records — the mechanism-arc modules (v942, v943, v945) are typed "instrument characterization, not arithmetic novelty" prominently and make no claim about where the true zeros of zeta lie; they say nothing about the truth of the Riemann Hypothesis in either direction; they close no omega, narrow no gate and move no other marker; the omega census stays at cardinality 4; θ_∞ stays open; the λ_h limit stays open; and the arc's terminal residue is stated in the canonical note-DII/DXVI form: the *triple* {H1 ∧ H2 ∧ H3}-cofinal (one rung per dyadic block, all three at the same h; the limsup form only mod $D = 0.0042$) + {census-forall-k == loop, flagged} + {H-pin = the one λ-uniform edge of the pair: L1 = TAIL proven + H-pin open; WPD$(a < \gamma_1^2)$ ⇐ H-pin; the WPD non-λ legs: extension instantiated, TAILWPD world-front}.**

The detector-specificity theorem (round 184, `pi_pattern_scan_probe.py`, frozen SPEC `00fc85173fe07470` — verified invariant under the appended K3 correction block — 35/35*,* 191 *s run-of-record + deterministic re-run;* re-run green as typed at this promotion*,* 212.2 *s; promoted as [verification/v942_pi_pattern_transfer_specificity.py],* 9/9*).* The prime-front instruments are transferred unchanged in structure to pi-derived data and the expected honest null lands cleanly: **all five pi worlds classify top-heavy** ($-10.0.. - 17.9$, both holdout halves, all 100 non-Mangoldt instances, zero anomalies) while the C-MANGOLDT positive control **fires on every transferred detector** (spike $C = 131.04$ vs null max 12.60) and the structured controls prove the detectors *can* fire — **transfer validity proven**, the methodology the whole arc rides. The separator is **prime-specific**, not a universal-structure detector; the K3 smoke lineage is disclosed; P-BBP carries the in-spec PROVISIONAL flag. An instrument characterization — no pi-numerology claim, no new fact about pi.

The zero-coupled sharp localization (round 185, `mangoldt_ablation_ladder_probe.py`, frozen SPEC `504dcac5b2eac650`, 26/27 *— the single FAIL is the* disclosed *control-side Beurling density band (*5.74 *vs* $[0.2, 5.0]$*), Bughunt-IX-adjudicated an honest Poisson fluctuation, density-correct, the W4 headline unweakened;* re-run green as typed at this promotion*,* 167.2 *s, same single disclosed FAIL; promoted as [verification/v943_mangoldt_ablation_localization.py],* 8/8*).* The additive property ladder W0 random → W5 true Mangoldt switches **both arithmetic signatures on only at W5 and sharply** (S1 jump 12.07 dex; S2 jump 61×); **the Beurling headline answer is no** — full multiplicative semigroup structure without Riemann-zero coupling produces neither signature: the detectors are seeing the true zeros through the source side. Every subtractive ablation confirms necessity (A1 −powers kills S1 while S2 survives at 129.06: S2 sits on the exact prime positions); **even** ±0.01 **position jitter kills both**. The round-184 interpretation is sharpened: the separator detects *the explicit-formula zero coupling of the true comb*, not "prime arithmetic" generically.

The exact residue identity (round 186, `ground_residue_obs_probe.py`, frozen SPEC `48637c8898a1da5a` — verified invariant under the appended K1 correction block — 33/33*,* 782 *s run-of-record + deterministic re-run, one amendment disclosed;* re-run green as typed at this promotion*,* 792.7 *s; promoted as [verification/v944_ground_residue_observability.py],* 11/11*).* The H3/A_0-floor leg lands in **eigenvector-free coordinates at identity level**: $\mathrm{Res}_{z=\tau} R_h(z) = A_0^2/\|d\|^2$ gated at all 14 rungs to $\le 7.8 \times 10^{-45}$; the Feshbach scalar form with $w'(\tau)$ computable from $\{\alpha, m, B\}$ plus the one measured scalar τ (*source-side (eigenvector-free, given τ)* per the K4 glossary); **strict interlacing** ⇔ $A_0 \neq 0$, both directions exact; and the

honest headline **gap-is-residue-in-disguise** — the transversality gap is the residue in matrix coordinates, not an independent handle. The demand honestly *descends* (the compression ground overlap; $\|m\|$-pricing loses 11.2–90.7 orders — no floor claimed, the λ-uniform gap floor stays the named open input); the raw-gap floor leg rides τ and is relabeled-and-stopped. **The K1 (major) correction of record travels verbatim on every surface**: the off-diagonal of the raw wall is exactly the divided-difference form of *one* source function — displacement rank 2 fixes *only* the off-diagonal — while the full wall is $M_h \sim L_f + \mathrm{diag}(\Delta)$ with source-side $\Delta \neq 0$: *not* the canonical Loewner matrix, and $R_h(z)$ is the resolvent of $L_f + \mathrm{diag}(\Delta)$, not of the Loewner kernel alone.

The causal synthesis with its stratification (rounds 187 + 190, `zero_causal_synth_probe.py`*, frozen SPEC* `c20e87eec6d158b9`*,* 31/33 *with the two FAILs the* disclosed *adjudication gates (the f-cell monotonicity violation and the round-175-predicted S1 instrument death);* re-run green as typed at this promotion*,* 819.0 *s; and* `zb_wiggle_strat_probe.py`*, frozen SPEC* `8639b3a78503a0f9` *— verified invariant under the appended K2 correction block —* 23/23*;* re-run green as typed at this promotion*,* 64.5 *s; promoted as the* joint *module* `[verification/v945_zero_causal_stratification.py]`*,* 10/10 *— round 190 upgrades round 187 by reference, the r187 record itself stays PARTIAL, unedited).* The inverse experiment: comb worlds with **controlled zero spectra**, synthesized through the round-175 bridge run backwards — the engine is valid (Z0 $|dC| = 0.0006$ on $C = 131.04$), the off-line dose surface is **monotone in the dose everywhere** (first response $(0.10, 10^{-2})$, first death $(0.05, 10^{-4})$), and **exact-ordinates-required**: Poisson and GUE resamples at the true density both die. The one blemish is stratified and adjudicated as noise with a sharp mechanistic picture: the wiggle reads **1.07** in full precision (the K2 corrected figure — never 1.14), sits deep inside every matched-dose selection spread, the dose is monotone in expectation, the inversion is generic subset-overshoot, and the per-zero leverage ladder is measured (γ_2 carries the most, no first-zero dominance, strongly subadditive). The round-185 interpretation upgrades to **demonstrated mechanism on the S2 channel** — still no claim about where the true zeros lie.

The exact Krein pencil (round 188, `census_lift_probe.py`*, frozen SPEC* `8ada6b97d56aca46`*,* 72/72*,* 2066 *s run-of-record +* 2060 *s deterministic re-run, the log lineage disclosed in the spec;* re-run green as typed at this promotion*,* 2067.1 *s; promoted as* `[verification/v946_census_krein_pencil.py]`*,* 8/8*).* Spur 1 of the review plan closes as an honest **partial lift** with a real structural theorem: the census polynomial *is* the spectral determinant of a source-side self-adjoint pencil — but of a **Krein pencil** at every true rung: the residue-sign ladder $(1,5)/(7,3)/(6,14)/(27,14)/(45,34)/(43,73)$ is mixed everywhere, so realness (H2) is not delivered structurally — **it is equivalent to Krein strong definitizability** (the operator-form coordinate); the smooth world is one-signed while every atom-bearing world is mixed: **mixedness-is-arithmetic** (atoms-vs-no-atoms, the wrong orientation, explicitly not a sign source), and mixedness is **transform-invariant** in the predefined class. E1 is dyadic-exact at all six rungs, E2 integer-Bareiss-exact at $h \leq 13$; the bracket is certified via ENVJ *without computing a single root*; roots are never consumed and the construction is tau-free.

The exact Loewner–Pick dictionary (round 189, `loewner_pick_probe.py`*, frozen SPEC* `a547448468899af9`*,* 28/28*,* 1046 *s run-of-record +* 1058 *s deterministic re-run, three pre-freeze amendments disclosed;* re-run green as typed at this promotion*,* 1082.9 *s; promoted as* `[verification/v947_loewner_pick_dictionary.py]`*,* 9/9*).* The wall is *not* the canonical Loewner matrix of its potential but a **diagonally-shifted** one — $\mathrm{Raw}_h = L_f + \mathrm{diag}(\Delta)$ — with **the new exact cosine-quadrature diagonal law**: the prime-block diagonal shift is exactly $2a$ times the cosine quadrature of the same sieve atoms whose sine quadrature feeds the potential — **the wall carries both quadratures of the same atoms in different slots**. The pole leg is a genuine Pick function with an **exact Herglotz point mass** $2\sinh^2(a/2)\,\delta_{-1/4}$; the Pick-class reading is **refuted with margins** ($\lambda_{\min}(L_f) < 0$ at all 14 rungs, missing wall positivity by 9.4–87.9 orders, refinement deepening); the smooth cell is PSD at the nodes by the exact **commensurate-sampling mechanism** ($\sin(2a\,\omega_k) == 0$ — lattice geometry, not a prime fingerprint); the dream statement dies honestly at the prime leg (a positive rewrite would be the wall positivity itself — the barrier named, not crossed). The r45 LOEWNER-DEAD distinction is carried: a *third* reading is killed here,

the IIKS/RHP lane is untouched.

The sign-characteristic law (round 191, `krein_definitizer_probe.py`*, frozen SPEC* `332c1f48f48a6d82`*,* 57/57*,* 1950 *s run-of-record* + 1901 *s deterministic re-run, no post-freeze amendments;* re-run green as typed at this promotion*,* 1983.4 *s; promoted as [verification/v948_krein_sign_characteristic.py],* 10/10*).* The round-188 upgrade path is **closed by theorem, not by search**: the kernel vector at a census root is explicit, the sign characteristic is the crossing direction of the Weyl secular function, and **the round-188 residue ladder *is* the Krein sign-characteristic multiset** (two-route proven — in the module on a 32-cell exact Sturm battery with no floats). **The parity scaffold** is the new source-side (root-free) coordinate: root-count parity per pole gap from residue signs alone, verified at all 81 gaps. The crossing form is congruence-invariant (the W' terms die on the kernel), so the measured mixed ladder **kills the entire** $W(y) > 0$ **congruence class at once** — impossible-in-class, with the remaining doors named honestly. The strict definitizer is **root-dependent** (occupancy-minimality fails at $h = 8, 13$ by exactly one interior doubled gap); the r172 witness **breaks census realness while the raw ladder is blind** — a new detector; the Krein coordinate closes STRUCTURAL-ONLY (the reformulation adds no leverage).

Scope, in the same breath. Everything else in notes DIV–DXVI remains sandbox: no other module, no marker moved, no gate closed or narrowed. **Round 192 (the commensurability mechanism, note DXIV) stayed experiments-side at this date** — it landed after the Bughunt-IX scope froze and waited for bughunt coverage per precedent; it has since been covered by Bughunt X and promoted as [verification/v949_commensurability_mechanism.py] (Section 12.56). The ninth adversarial bughunt (round 193, note DXV, 44/44, 9 findings, 1 MAJOR, *no verdict flips*, zero failed recomputes) covered rounds 184–191 before this promotion; its corrections of record — K1 (the off-diagonal-only Loewner law, the major retype), K2 (the wiggle reads 1.07), K3 (the smoke lineage), K4 (the source-side glossary rule), K5 (the note-trail erratum) — were applied to the frozen records as appended correction blocks (note DXVI) and are adopted verbatim on every surface of this arc. The bughunt round itself (193) is a process round and stays experiments-side per precedent. All eight promoted rows say nothing about zeta's Riemann Hypothesis in either direction — certified finite theorems, exact instruments, instrument/mechanism characterizations and honest partial-lift, refutation and closure-by-theorem records with every open omega typed and standing; the census cardinality stays 4, θ_∞ stays open, the λ_h limit stays open. **No statement about the Riemann Hypothesis is made.**

12.56 The Bughunt-X arc: the commensurability mechanism, the zero-channel characterization, the cancellation functional, the Turán pricing, the $A_{v_0} \geq 0$ theorem with its Sturm certificates, and the Perron–Frobenius adjudication promoted (rounds 192, 194–198, notes DXIV–DXXIV, 2026-08-21/22; promoted 2026-08-22)

Six further discovery freeze rounds (rounds 192 and 194–198, notes DXIV, DXVII–DXXII, several run as concurrent lanes) are promoted to the load-bearing suite on 2026-08-22 as [verification/v949_commensurability_mechanism.py], [verification/v950_zero_channel_capacity.py], [verification/v951_cancellation_functional.py], [verification/v952_turan_extremal.py], [verification/v953_fewatom_
and [verification/v954_nodeless_pf.py] (suite 941 → 947, ledger 1093 → 1099), after the tenth adversarial bughunt (round 199, note DXXIII, `bughunt10_probe.py` 30/30, SPEC `5551aa7b967230f1`, 11 findings, 1 MAJOR, 0 verdict flips — and the X4 headline: **every exact claim of the arc reproduced in fully own code**, the ACF/pole/arch laws to $\leq 10^{-61}$, the Fejér cap re-proved by an independent single-node dual route, the classical citations web-verified verbatim) covered the arc, and its corrections of record (KA/KB/ KC/KE applied as appended CORRECTION-OF-RECORD blocks, KD note-only per precedent; note DXXIV) are *adopted verbatim on every surface of this promotion.* The honesty fences travel first: **all six rows are certified finite theorems, exact instruments, honest kill records and KA-scoped measured laws — the channel**

row (v950) is typed "instrument characterization, not arithmetic novelty" with *no storage claim*; no row says anything about the truth of the Riemann Hypothesis in either direction; no omega closes, no gate narrows, no other marker moves; the omega census stays at cardinality 4; and the arc's terminal residue is stated in the canonical note-DII/DXVI/DXXIV form: the *triple* {H1 ∧ H2 ∧ H3}-cofinal (one rung per dyadic block, all three at the same h; the limsup form only mod $D = 0.0042$) + {census-forall-k == loop, flagged} + {H-pin = the one λ-uniform edge of the pair: L1 = TAIL proven + H-pin open; WPD($a < \gamma_1^2$) ⇐ H-pin; the WPD non-λ legs: extension instantiated, TAILWPD world-front}.

The exact node mechanism (round 192, `commensurability_mechanism_probe.py`*, frozen SPEC* `dbc14014899fb286` *— verified invariant under the appended KE correction block —* 28/28*,* 276 *s run-of-record + deterministic re-run, three pre-freeze amendments disclosed;* re-run green as typed at this promotion*, 317.0 s; promoted as [verification/v949_commensurability_mechanism.py],* 11/11*).* The thin geometric hope "incommensurability feeds wall positivity" is **cleanly killed at the margin**: the defect ladder is O(1)-flat while τ falls 45 orders, the dose is **direction-blind** (dose = antidose within < 2 dex — the control round 185 lacked) and kills the margin at every $t \geq 1/8$ plus 0.01 jitter; no commensurate revival. What survives is exact: the smooth world's whole $2a$-periodic oscillation **collapses at the nodes to Möbius data** (closed forms recomputed in the module), the structural-zero census is the $e \mid 2fk$ law (exact integers), and $L_f(t = 1)$ is PSD at all four dose rungs **world-blind** — lattice geometry, not a prime fingerprint. **The first Baker-class pricing in the program's history** is honest and honestly useless: LMN 1995 misses the measured two-log forms by 7.1×10^3–3.1×10^4 dex and Matveev 2000 by 1.4×10^9–1.2×10^{10} dex (both bound formulas recomputed in-module from the published constants; the citations Bughunt-X-web-verified); even the elementary Liouville floor is orders below the measured equidistribution-scale minima. The sharpened next step travels with the record: the sought source functional *cannot* run through incommensurability lower bounds — it must consume the exact positions wholesale.

The zero-channel characterization (round 194, `zero_channel_capacity_probe.py`*, frozen SPEC* `fa49271201ba30fb`*,* 28/28*,* 170 *s run-of-record + deterministic re-run, the A0 design disclosures + A1 frozen before any decoding;* re-run green as typed at this promotion*, 209.6 s; promoted as [verification/v950_zero_channel_capacity.py],* 8/8*).* The v945 write→read arc characterized **as a channel**: the capacity surface reaches **error-free full ternary capacity** from $\varepsilon = 0.01$ in both placements; the resolution limit is LOW = 0.001 / MID = 0.003 — below the r187 resonance scale, *with the Bughunt-X F6 quantification adopted on every new surface*: the LOW call has $P = 0.0032$ under the 20000-trial null Monte Carlo, **marginal-but- nonnull** single-reading and decisive with averaging (the sqrt-n prediction held). Linearity breaks at $\varepsilon = 0.01$ (a linear codec below, interference-memory above); **parity- erasure recovery** 1.000 — added error-correcting redundancy survives end-to-end through the explicit-formula dictionary, *with the F7 corollary typing adopted*: the 1.000 is a corollary of the zero-symbol-error cell, not independent evidence. P3 refuted and disclosed (the off-line leverage law does not transfer). Typed **instrument characterization, no storage claim**; the round-107 code-reading verdict is cited, not upgraded.

The cancellation functional (round 195, `cancellation_functional_probe.py`*, frozen SPEC* `a50b85bb112513a1` *— untouched by the Bughunt-X corrections, the hash verified unchanged —* 24/24*,* 1036 *s run-of-record +* 1061 *s deterministic re-run, four pre-freeze amendments disclosed incl. A3, the substantive float64- direction-noise reversal;* re-run green as typed at this promotion*, 1102.8 s; promoted as [verification/v951_cancellation_functional.py],* 10/10*).* The arc's single most promotable round: **the wall is a closed truncated Weil-explicit-formula functional** of the PD test function g_x. Three exact laws, all recomputed in the module: **the ACF law** ($x^T W(u)\, x = -2 \int_0^{L-u} A_x A_x(\cdot + u)$ — the v947 "two quadrature slots" are one object; verified at every atom of $h = 4, 5$ on an own wall build), **the pole square** ($8 \sinh^2(a/2)(\int e^{-t/2} A_x)^2$, a manifest L^2 square), and **the arch law** ($c^* = (\gamma + \ln 2\pi)/2$ exactly, every mode of $h = 4, 5$). **The KD pin — the arc's external handoff object — recomputed in-run**: in the $L \to \infty$ limit the arch kernel *is* the classical Weil archimedean kernel, $\kappa + c^* \to -\frac{1}{2}[\mathrm{Re}\, \psi(1/4 + i\omega/2) - \ln \pi]$

(Bughunt-X verification 6×10^{-18}; the finite-L corrections $O(e^{-L/2})$ typed not-small): the wall's arch leg speaks the textbook explicit-formula language exactly in the window limit. **Per-atom manifest positivity is provably impossible** (Wiener–Khinchin); the pole alone carries positivity (budgets pinned); **the sign law**: every resolvable atom negative at v_0 at all 14 rungs, compensation *atoms-vs-pole*. Priced honestly as an **algebraic restatement** — the relabeling barrier named, not crossed; WEIL-ALLTESTS ↔ RH stays a flagged loop consumed by nothing.

The Turán pricing (round 196, `turan_extremal_probe.py`*, frozen SPEC* `a6edc3f911e8f069` *— verified invariant under the appended KB correction block —* 23/23*, 767 s run-of-record + 802 s deterministic re-run, amendments A1/A2 disclosed;* re-run green as typed at this promotion*, 795.6 s; promoted as* `[verification/v952_turan_extremal.py]`*,* 9/9*).* The classical PD-cone theory priced for the first time in 196 rounds, **two-sided**: every cone bound is *negative* at every rung while the truth is τ-deep positive — **cone geometry cannot see positivity**, and the cone-vs-truth gap *is* the τ ladder (slope -1.007, R^2 1.000: the relabeling barrier named at the cone level); yet **the classical Fejér cap describes the wall's drain capacity almost exactly** (capture 0.14–0.24 dex flat — the round's real news). The Fejér coefficient theorem is machine-proved in the module at $n = 1, 2$ (construction + single-node dual certificates, the Bughunt-X independent route); **the KB correction is adopted with its rational-lag identity recomputed**: the mode lattice attains the full-cone cap *exactly at rational lags only* ($M_{\mathrm{sub}}(L/2) == 1/2 == \cos(\pi/3)$ identically; near-attainment elsewhere — the "f64 precision" wording is retired); the arch leg does not classicalize (named); the new TURAN-CONE-POSITIVITY loop is flagged and consumed by nothing.

The $A_{v_0} \geq 0$ theorem (round 197, `fewatom_reduction_probe.py`*, frozen SPEC* `35fb341bb281b04b` *— verified invariant under the appended KA/KC correction block —* 25/25*, 790 s run-of-record + 811 s deterministic re-run, amendments A1/A2 disclosed;* re-run green as typed at this promotion*, 830.7 s; promoted as* `[verification/v953_fewatom_reduction.py]`*,* 10/10 *— with the Bughunt-X Sturm certificates recomputed in-module).* The collapsing direction's mode polynomial is **nonnegative on** $[0, L]$ **at all 14 rungs** — a nodeless positive bump peaked exactly at $L/2$ whose edge minimum *is* the jr_0 ladder (the KC rider: exact *un-normalized*; the r197/r198 pair consistent). **The KA scope rider travels verbatim**: the r197 gate class is GRID-MEASURED (at $h = 15/16/20$ the record minimum sits below the bar's resolution — print evidence, not gate certificate); **the headline is true and now stronger** — continuum nonnegativity is **certified by exact-rational Sturm chains** (Bughunt X at $h = 4, 5, 13, 20$; *recomputed in the module at* $h = 4, 5$: exact dyadic Fractions, Chebyshev conversion, integer primitive-PRS chain, zero roots in $(-1, 1]$, $P(\pm 1) > 0$ — no floats). **The one-line mechanism**: $A \geq 0$ forces every ACF sample nonnegative, hence every atom negative — the v951 sign law reduced. The horizon is bounded ($Q_{99} \in \{2, 3\}$ incl. the holdout) *with the mandatory scale-relativity rider*: at the demand scale the horizon consumes every resolvable atom (NOT-A-COMPLEXITY-DROP-IN-H; any lossy tail bound is certification-dead). **Object-A is named** (the Bughunt-X X6 targeting result): "prove $A_{v_0(h)} \geq 0$ for all h" is the *one genuinely new tractable target* of the arc — $v_0(h)$-quantified, yields the sign law only, **no lever on wall positivity**.

The Perron–Frobenius adjudication (round 198, `nodeless_pf_probe.py`*, frozen SPEC* `7499c39a026d0d0f`*,* 27/27*, run-of-record + deterministic re-run, amendment A1 disclosed;* re-run green as typed at this promotion*, 817.1 s; promoted as* `[verification/v954_nodeless_pf.py]`*,* 10/10*).* The r197 candidate route for Object-A is **fully adjudicated — dead in the mode basis**: the wall's off-diagonals are mixed at every rung (recomputed exactly at $h = 4, 5$: $(15, 6)/(38, 17)$), v_0 is not parity-aligned, and the trivial route is killed both ways ($v = e_1$: nonnegative coefficients yet $A(L/2) = -1$ exactly). The obstruction is **localized**: the cone tests show *the pole's rank-one positive kernel is the sole obstruction* — removing it repairs cone invariance at every rung (recomputed at $h = 4$: wall operator 11..14 of 20 violations, no-pole operator zero): *the operator piece that fails the nonnegative-function cone is exactly the piece that carries the positivity* — the v951 pole-vs-atoms compensation as cone-geometry tension. The surviving structure: the **attractive-hopping Z-structure** persists in band-limited position coordinates on prime + arch

(pole kernel negfrac 0.000 exactly); SMOOTH keeps nonnegativity with *no* Z-structure (the shape needs none). Object-A stays open, **genuinely function-space**; the X6 discipline travels: "pole-vs-hopping balance" is the *class name* of Object-W (wall positivity itself), not a new attack object.

Scope, in the same breath. Everything else in notes DXIV–DXXIV remains sandbox: no other module, no marker moved, no gate closed or narrowed. The tenth adversarial bughunt (round 199, note DXXIII, 30/30, 11 findings, 1 MAJOR, *no verdict flips*) covered rounds 192 and 194–198 before this promotion; its corrections of record — KA (the GRID-MEASURED retype with the verbatim scope rider, the major), KB (rational-lags-only attainment), KC (the un-normalized identification), KE (the vacuous symbolic leg) — were applied to the frozen records as appended correction blocks (note DXXIV) and are adopted verbatim on every surface of this arc; KD (the classical-Weil-kernel pin, a *positive* finding) lives note-only per precedent and is recomputed in [verification/v951_cancellation_functional.py]. The bughunt round itself (199) is a process round and stays experiments-side per precedent — *its Sturm-certificate deliverable is promoted inside v953.* All six promoted rows say nothing about zeta's Riemann Hypothesis in either direction — certified finite theorems, exact instruments, honest kill records, an instrument characterization with no storage claim, and KA-scoped measured laws with every open omega typed and standing; the census cardinality stays 4, θ_∞ stays open, the λ_h limit stays open. **No statement about the Riemann Hypothesis is made.**

12.57 The integrable-dictionary arc: the IIKS tau identification executed, the degree-1 Lax adjudication, the signed-Toda dictionary, the half-filling boundary, the free moment window law, the four-path dictionary, and the L-gauge midpoint connection promoted (rounds 224–226 and 228–231, notes DL–DLII and DLIV–DLVII, 2026-08-23/24; promoted 2026-08-24)

Seven further discovery freeze rounds (rounds 224–226 and 228–231; round 227, the Fermi-edge classification, stays experiments-side as a classification round by design, its builders consumed as a read-only library) are promoted to the load-bearing suite on 2026-08-24 as [verification/v955_tau_iiks_toda_dictionary.py] and [verification/v956_signedmoment_halffilling_duality.py] (suite 947 → 949, ledger 1099 → 1104). The frozen probes are embedded byte-exact and executed verbatim, all seven at 100% first pass (18/18+15/15+17/17 and 14/14+17/17+17/17+13/13, zero fails, deterministic records, sealed specs); the intermediate rounds 199–223 stay experiments-side (adjudication and classification rounds). The honesty fences travel first: **both rows are certified finite theorem sets and exact dictionaries — every wall statement is typed wall-equivalent and consumed nowhere; no row says anything about the truth of the Riemann Hypothesis in either direction; the mincut stays base 4 / refined 5; the claim splitting of notes DLI/DLVII is carried to the ledger** — PRIME.PORT.TAU.FINITE.IIKS.01 [E] and PRIME.PORT.TAU.NOPOLE.COFINAL.01 [O] split out of the v883-registered umbrella PRIME.PORT.TAU.01 (which stays [O] for the fully symbolic arbitrary-h statement), and PRIME.FREEMOMENT.JFRACTION.01 [E] + PRIME.JACOBI.DUAL.REVERSAL.01 [E] split from PRIME.FREEMOMENT.POSITIVEPREFIX.01 [O] — the strong finite mathematics no longer hides under the open umbrella, and the open sign question stands alone, honestly.

The tau identification (round 224, `tau_symbolic_probe.py`, 18/18 *first pass, SPEC `fb04cd7ce9fb306b`; promoted in [verification/v955_tau_iiks_toda_dictionary.py]).* The v883 question — is the wall determinant a tau function or a rank-two displacement structure mistaken for one? — is answered TAU-IIKS-EXACT at the finite-identity level on all five heavy rungs (h = 151..591, ports 13..54): Sylvester holds as a *full characteristic identity* $\det(I - sQ_{\text{state}}) = \det(I - sE_{\text{node}})$ on the whole s-grid; the s-family of the wall carries the *s-dressed* port, $\det(I - sE) = \det(I - sR)\det(I - s\,D_P(s))$ with $D_P(s) = P + sX(I - sR)^{-1}X^T$ exact, while the fixed-kernel family is a *different* family (alias gap 0.06–0.20 at $s = 0.5$, named and blocked); the contract's declared death spot dissolves — the diagonal is the confluent limit of

the *same* generator formula (classical CD confluence, $\leq 3.2 \times 10^{-13}$); the dressed resolvent is integrable again (IIKS dressing, $\leq 7.9 \times 10^{-13}$) with the dressed port generators **source-explicit in closed form** (the G31 surprise, $\leq 1.5 \times 10^{-13}$); the Fredholm variational identity holds in *two* independent times ($\leq 1.4 \times 10^{-9}$); and the relative tau transfer holds exactly over -212.84 / -195.50 log-unit determinant collapses ($\leq 3.4 \times 10^{-12}$). The controls are *inverted* per contract: Epstein and scramble *satisfy* every identity and differ in the *value* ($\operatorname{sign}\tau(1) = -1$) — the IIKS structure is operator geometry, the arithmetic sits in the value.

The Lax degree adjudication (round 225, `lax_conditioned_probe.py`*,* 15/15*, SPEC* `93fd9759db7e7b3c`*).* The word "2" in `PRIME.PORT.LAX2.01` was a hypothesis, and the split verdict retires it: **the true minimal degree in the closing direction is one, and the closing direction is** h**, not** s (LAX1-H-EXACT) — the h-step is exactly rank one ($\leq 2.9 \times 10^{-16}$), the dressed relative kernel has displacement rank exactly two with explicit generators, and the degree-1 source-chain connection *predicts* the next generator pair (3.4×10^{-16}) and the next tau step via the Christoffel scalar (3.2×10^{-11}) on the sealed blind rungs — the next RHP and its sign never consumed; zero curvature holds identically, world-blind; mpmath wards at dps 80/120. The honest negatives are typed, not hidden: the s-time has *no* fixed degree (basis-invariant ε_d plateau 0.34–0.60, CONDITIONING-ONLY; the old 0.244 degree-2 proximity was a regressive shadow), and across rungs there is *no* common carrier.

The signed-Toda dictionary (round 226, `hirota_sign_probe.py`*,* 17/17*, SPEC* `d78e236bf633de7b`*).* The round's theorem is a new coordinate system for the wall: $I - Q_n$ is exactly the Gram of the **signed defect measure** $\tilde{\mu} = \mu - \nu$ in the μ-orthonormal basis, hence $\tau_{w,n} = D_n(\mu - \nu)/D_n(\mu)$ (a Hankel-determinant *quotient*), the Christoffel scalar is $r_n = h_n(\mu - \nu)/h_n(\mu)$, and the Hirota coefficient comes source-pure from a scaled *signed* Stieltjes recursion — no determinant, no tau, no next RHP in the coefficient path (worst 1.7×10^{-11} to 4.0×10^{-8} at full depths 184..591, dps-60/80 warded). The sign adjudication is the round's value: $\hat{\gamma}_n > 0$ for all $n \iff$ **quasi-definiteness of the signed defect measure** $\iff$ the wall (WALL-EQUIVALENT, with the explicit SIGNED-TODA $X - Y$ structure $h_n(\tilde{\mu}) = \|\pi_n\|_\mu^2 - \|\pi_n\|_\nu^2$); MAIN carries *all* 184 $r_n > 0$ (min $+0.3666$ — the wall's full degree chain measured positive for the first time), the controls flip at $n = 25/21/27$; HIROTA-CONE-GO was *not* reached (the expected base verdict, now a theorem instead of a suspicion); "RH wall inside a window" now reads as a classical moment-problem statement about the von Mangoldt double zone.

The half-filling boundary (round 228, `holedual_probe.py`*,* 14/14*, SPEC* `a823d4be3f0b1c06`*; promoted in* `[verification/v956_signedmoment_halffilling_duality.py]`*).* The particle-hole illusion is removed before any RHP campaign (NOT-A-HOLE-EDGE), and the measurement behind it is the lane's sharpest structure find: the window depth obeys the exact law $N_w = \lceil \#\mathrm{supp}(\tilde{\mu})/2 \rceil$ on all five windows — **the window cap is the half-filling of the union support** — and extending the ladder past the cap the wall dies *immediately* (first r-flip at $N_w + 0/2/2/3/1$, $O(1)$ offsets with no N-growth, two independent paths plus an mp ward): $\mu - \nu$ is quasi-definite *exactly* up to half-filling — **the wall is maximal**. The complement identity and the h-duality $h_{N-m}(\tilde{\mu})\, h_{m-1}(\tilde{\mu}^{\#}) = 1$ are exact at dps 80 (a permanent dictionary item); the r-quotient duality is structurally unavailable for the wall pair (R-DUAL-OBSTRUCTED).

The free moment window law (round 229, `pontryagin_maxpos_probe.py`*,* 17/17*, SPEC* `b062906cb458da2a`*).* An honest double twist: the reviewer's signature explanation is **refuted** ($n_{\mathrm{flip}} = 184/153/170/367/592 \neq p = 263/211/237/503/816$; the offset is not $(p - q - 1)/2$; the wall dies far below the Pontryagin ceiling, on MAIN too), and the question "why half-filling" gets a sharper answer: an S-atom signed measure has exactly S free moment parameters, the largest Hankel window inside them is $n = (S+1)/2 = N_w$, and **MAIN survives the entire free moment window** plus $O(1)$ forced-tail degrees while the controls die deep inside it ($n = 25/21/27 \approx 0.11$–$0.15\, N_w$) — the free-window survival *is* the arithmetic. The inertia theorem is exact in all parts; the maximal Lagrange contractor is exact on the toy (the equivalence $H_p > 0 \iff \|C\| < 1$ verified in both truth values) and astronomically non-contractive at full degree ($\sigma_{\max}(C) = e^{110.5}$) — the wall never operates at the Lagrange endpoint; at window degree the wall family *is* the same evaluation

contractor with $\lambda_{\max}(Q_{N_w}) = 0.999832..0.999999 < 1$ (SAME-CONTRACTOR-EXACT, family form — the `v881` testing law closes the circle).

The four-path dictionary (round 230, `jfraction_probe.py`, 17/17, SPEC `124cda6f00caeeca`). The finite reparametrization program completes: **moment prefix = Jacobi chain = Euclidean chain = value chain** — one object, four exact source-pure paths ($m_k = h_0(J^k)_{00}$ exact in rationals; the tail anatomy exact; the r228 offsets *are* forced-coupling survival counts 0/2/2/3/1; the polynomial remainder algorithm reproduces the chain with no Hankel, no Cholesky, no tau anywhere, and Epstein's *first wrong Euclidean pivot* appears exactly at its known flip $n = 25$). The reversal theorem is the dictionary bonus: the dual signed measure carries the *mirrored* Jacobi chain ($\alpha^{\#}_m = \alpha_{S-1-m}$, $\beta^{\#}_m = \beta_{S-m}$), yet it is *not* a builder transformation of the original (NO-SOURCE-REVERSAL — half-filling is not the center of a self-dual chain); $\beta > 0$ stays the wall in *every* coordinate.

The midpoint connection theorem (round 231, `rhp_midpoint_probe.py`, 13/13, SPEC `90d2c5a8926d820a`). Derived by residue calculus before the run, frozen, then gated: **the dual FIK problem is an explicit L-gauge transform of the original**, $Y^{\#}_{N-1+l}(z) = J\,Y_{N-1-l+1}(z)\,W(z)$ with $W = \mathrm{antidiag}(1/L, L)$ — the original chain arrives at half-filling from the left, the dual chain arrives mirrored from the right, and the gauge consumes *the node polynomial alone* (no h, no tau, no wall data; exact in rationals and on the real w9 comb at 9.1×10^{-94}, dps 100). The structure consequence: the gauge is *h-free*, so **the wall's orientation sits exclusively in the h-chain the gauge does not see** — the decisive Stokes multiplier of the two-sided problem is $c_w\beta_n$ with $c_w > 0$ (SIGNED-STOKES-WALL-EQUIVALENT). The honest negatives are typed (NODE-LOG-POLE-REMOVER-ONLY; NO-SOURCE-CRITICAL-FILLING — t_c stays open; the quenched midpoint model supported at median spread 0.44), and **the micro falsifier is frozen**: any future parametrix must blindly predict the forced tail 0/2/2/3/1 *and* the control flips 25/21/27 from the same connection — a model that only paints the positive side is a fit, not a mechanism. Half-filling is the *location*; its positive reachability stays the result to be proven. **No statement about the Riemann Hypothesis is made.**

12.58 The bordered-dictionary arc: the bordered-RHP dictionary, the Schlesinger rank-1 insertion, the centering normal form, the three exact error formulas, the contour R_1, and the exact augmented tau quotient promoted — with the campaign contract registered at the post-round-255 state (rounds 244–255, notes DLXXIII–DLXXXV, 2026-08-24; promoted 2026-08-24)

The RHP campaign of rounds 232–255 turns the compact end-object of the free-moment arc — *bordered Hankel PSD = wall + budget* — into an exact Riemann–Hilbert dictionary, and its six durable certified theorem rounds are promoted to the load-bearing suite on 2026-08-24 as [`verification/v958_bordered_tau_readout_dictionary.py`] (suite 950 → 951; probes embedded byte-exact and executed verbatim, all six at 100% first pass, 23/23+29/29+24/24+20/20+22/22+26/26, zero fails, SPEC SHAs pinned; plus a module-own exact-rational PSD base theorem). The honesty fences travel first: **the row is a certified finite dictionary and adjudication set — every wall statement remains typed wall-equivalent and is consumed nowhere; the measurement rounds 250, 254 and 255 stay experiments-side (typed measurements, not identities) and enter only as the frozen state of the open campaign contract; no row says anything about the truth of the Riemann Hypothesis in either direction; the mincut stays base 4 / refined 5.** The theorem set is registered as `PRIME.PORT.RHP.BORDERED.READOUT.01` [E]; the campaign stands alone as `PRIME.PORT.RHP.FULLSOURCE.BASEFIBER.01` [O].

The PSD base theorem (module-own, exact rationals). Before any probe runs, the module gates the trivial-but-load-bearing structural anchor: for *any* positive measure the Gram/Hankel matrix is positive definite and the border readout $T = t^T G_N t = \int (\sum_i t_i x^i)^2\, d\mu \geq 0$ for *every* real border t (exact Sylvester minor chain; exact Schur complement — the bordered matrix $[[G_N, u], [u^T, B]]$ is

PSD iff $B \geq u^T G_N^{-1} u$, precisely the wall-plus-budget shape of the end-object); the frozen signed witness fires the must-fail (a $\mu - \nu$ surrogate with $m_0 > 0$ breaks the minor chain and the border $t = (-2, -\frac{1}{2}, 2, 0, 0)$ reads negative, exactly). Any bordered-positivity route that consumes only positive-basis Gram structure proves the *wrong* statement — the wall content is exactly the quasi-definiteness of the signed defect measure (the `v954` trivial-route lesson, re-anchored on the bordered object).

The bordered dictionary (round 244, ***`bordered_hankel_probe.py`****, 23/23, SPEC* ***`c21e15b5852126b9`****).* The Uvarov border column $C_n^\sigma(z) = \int \hat{\pi}_n \, d\sigma/(z - x)$ carries F_n as its z^{-1} coefficient *without* orthogonality cancellation (6.7×10^{-13}; the $\tilde{\mu}$-column coefficient is 0 — the contrast *is* the orthogonality); the strongest piece is the CD-kernel form of the budget, $S_{n-1} = \iint K_n \, d\sigma \, d\sigma$ with $K_n(x, y) = (Y_n^{-1}(y) Y_n(x))_{21}/(y - x)$: **the budget is an RHP functional of the terminal Y_n data alone** (3.7×10^{-15} on $42 + 3$ worlds) — any later asymptotics needs only Y_n; the Uvarov tau step $\tau_{n+1}^b/\tau_n^b = h_n(1 - \rho_n/(B - S_{n-1}))$ is rational-exact (mp 7.1×10^{-58}), the bordered determinant identity is affine in B (gated at two budgets, hence for all B), and the bordered LAX1 flow $F_{n+1} = T_n - \hat{\alpha}_n F_n - \hat{\gamma}_n F_{n-1}$ with source $T_n = \int x \, \hat{\pi}_n \, d\sigma$ and autonomous corner $D_{n+1} = D_n - \rho_n$ runs mp-exact through all 184 w9 degrees (7.0×10^{-159}). Measured and typed: all three sealed canonical corner candidates *fail* the PSD census (0/42 each — CORNER-IMPORTED-ONLY: S_{N-1} exceeds every source-pure positive geometry on every window), and the budget is **head-heavy, not edge-heavy** ($\arg\max \rho$ at degree 0 on 42/42, Gini 0.91 — IRREGULAR-BULK: the later Szegő/steepest-descent analysis must control a highly non-uniform bulk, not a terminal boundary layer).

The rank adjudication (round 245, ***`bordered_finite_rank_probe.py`****, 29/29, SPEC* ***`886c65618145b838`****).* The last open FINITE question is decided in the most favorable case: **the border is a Schlesinger rank-1 insertion** — no irreducible 3×3, no growing rank. The RHP matrix size is 3×3 but unipotent/triangular (the third column is the F-sourced Uvarov column, determined by linear quadrature after the 2×2 is solved; $\det Y_3 = -h_{n-2} F_{n-1}$ newly derived, unique gauge; augmented normalization $\det Y_3^{\text{aug}} \equiv 1$ to 5.9×10^{-55}); the IIKS generator rank is *exactly* 2 (4.8×10^{-15} f64 / 5.9×10^{-160} mp razor — the nonlinear kernel stays the old one); the reduction to a source-pure 2×2 RHP plus one linear column is exact (the T-source appears as the displacement entry of the unipotent extension — the inhomogeneous three-term recursion for F becomes homogeneous three-dimensional); the budget fiber is one-dimensional (a scalar cocycle), and the formulation carries the *full* pivot chain — the DETERMINANT-ONLY kill does not fire. **The finite phase is closed**: the complete finite architecture of the end-object is a canonical 2×2 RHP $\ltimes$ a one-dimensional budget cocycle (the skew-product reading machine-verified — the border creates no second nonlinear geometry, it reads the existing solution out), and the irreducible-3×3 steepest-descent campaign is dropped without replacement.

The centering normal form (round 248, ***`border_centering_probe.py`****, 24/24, SPEC* ***`767d16cff674f312`****).* With $c = F_0/h_0$ the geometric rank-1 head is **algebraically completely eliminable**: PSD of the bordered matrix $\iff$ PSD of the centered form with $B^\circ = B - \rho_0$ (exact in rationals, world-blind, mp 2.6×10^{-62}); the centered border functional $\sigma^\circ = \sigma - (F_0/h_0)\tilde{\mu}$ annihilates constructively, the triangle converse $F_1 .. F_m = 0 \iff \sigma^\circ \perp P_m$ stands, and $Q_m = t^T H^{-1} t$ is exactly the squared dual norm; the **tail-kernel compression** $T = \iint [K_N - K_8] \, d\sigma^\circ d\sigma^\circ$ is exact at dps 160 *through the full depth* $N = 184$ (rel 3.3×10^{-160}) — the extensive tail is *one* terminal CD readout. The three-zone normal form is frozen: GEOMETRIC-RANK1-HEAD (ρ_0 world-blind) + LOWMODE-ARITHMETIC-SILENCE (the silence *is* the arithmetic: scramble breaks immediately at $k = 1$, median +2.21 decades, while Epstein holds +0.000) + EXTENSIVE-PAIRING-SENSITIVE-TAIL (tail excess fraction 0.836; no universal and no quenched macroprofile exists — the full discrete source must be carried, but thanks to the compression as one integrated readout). The honest limit typing: the quiet zone is ASYMPTOTIC-MOMENTS but marginal (DEV median 4.99×10^{-4}, slope -0.18 against bar -0.10; the builder source check is *negative* — no construction identity, the silence is real arithmetic).

The three exact error formulas (round 251, ***`targetreadout_error_probe.py`****, 20/20, SPEC*

b76487877383f645; seven pre-freeze amendments disclosed). The complete evaluation infrastructure, exact: $\delta h_n = (R_1)_{12}$ with $R = YM^{-1}$ (own mp pass at dps 700 against ∼600 digits of far-Cauchy cancellation); the central rest identity $T^Y - T^M = \iint([K_N^Y - K_N^M] - [K_8^Y - K_8^M])\, d\sigma^\circ d\sigma^\circ$ (route consistency 5.2×10^{-18}); the error-kernel sandwich $\mathrm{row}_2(M^{-1}(y))[R^{-1}(y)R(x) - I]\,\mathrm{col}_1(M(x))/(y-x)$ at 1.1×10^{-81}. The four direction decisions are measured and typed: amplification *extensive* ($\mathfrak{A}_{\sup} \sim N^{1.01}$ — "$\varepsilon \to 0$ suffices" is dead), annihilation *neutral* (the centering buys 0.00 decades in the error — the 3.5 decades of round 248 live in the value), the fiber sits *fully* in the error (the outer model carries ∼2% of T_{true}; the MAIN–SCRAMBLE difference sits completely in ΔT), and the full round-250 model is *not* σ°-pairable (the arcsine-cell Szegő layer has atom poles with exponent 380 decades — an obstruction, not an error; no such value was ever formed).

The contour R_1 and the gauge adjudication (round 252, `base_gauge_constant_probe.py`, 22/22, SPEC `26276cb2758152e9`). The proof-ready instrument: R_1 as the contour integral $\frac{1}{2\pi i}\oint(R(z) - I)\,dz$ instead of evaluation at astronomical z, verified against the round-251 readouts at 3.2×10^{-14}. And the reviewer gauge thesis is **refuted on all three prongs** (859 QP equilibria over the full degree ladder): the quotient test kills the premise (the "degree-constant offset" was a bounded 4-window measurement — on the full ladder $\Delta_{w,n}$ drifts by $-1.1.. - 2.1$ decades end-to-end), the h_0 normalizer leaves up to 2.32 decades, the diagonal gauge is pinned $c = 1$ by the normalization at infinity; and the hardest gate: **the outer model is orientation-blind** (positive γ^{out} on *all* worlds — no model flip on Epstein/smooth where the true chains flip early). Round 251's BASE-BLOCKED-BY-OFFSET is corrected on record to BASE-BLOCKED-BY-DRIFTING-LAYER: the missing layer is degree-dependent *and* orientation-relevant — the base needs the same class of work as the fiber.

The exact augmented tau quotient and the pole classification (round 253, `schlesinger_pairing_probe.py`, 26/26, SPEC `e5f0498c98112da4`). The theorem of the round: $D = B^\circ - Q_7 - T = \tau^{\mathrm{aug}}/\tau$ **holds machine-exact at full depth** $N = 184/168$ on three independent routes (dual-norm solve, slogdet ratio, chain) at 10^{-11} with mp cross-check 10^{-16} — the fiber target is a pure tau quotient of the augmented problem, not an integral afterburner. The pole removal is closed: the rank-1 border poles are bookkeeping (residue closure $\mathrm{Res}\, C_N = w_j\hat{\pi}_N(x_j)$, triangularly removable) while **the D-layer is essentially singular** (order measurement $p = 0.997$: $\exp(c/(z-s))$ type — no finite meromorphic divisor lifts it). The measured refutations are typed, not hidden: both sealed main-term candidates miss the ×2 window (the Gram *diagonal* carries only 19% of the world gap and has the wrong sign on scramble), the scramble separation before readout is 0.47 decades (gate 1.0), and the relative problem $Y_{\mathrm{MAIN}}Y_{\mathrm{SCR}}^{-1}$ is structureless — **the pairing lives in the full quadratic form, not in its diagonal**, and it is extensive over all atoms.

The campaign contract at the post-round-255 state (`PRIME.PORT.RHP.FULLSOURCE.BASEFIBER.01` [O]; measurement rounds 250/254/255 experiments-side: `centered_basefiber_probe.py` 21/21 SPEC `f9bf299d`, `offdiag_gram_probe.py` 19/19 SPEC `1083f7a0`, `nodebands_base_probe.py` 17/17 SPEC `45875904`). The opening measurement fell *badly and precisely*, as designed (OUTER-MODEL-FAILS): the best global outer model from exact comb data carries the readout *rates* — the base plateau $\hat{\gamma}^{\mathrm{model}} = ((b-a)/4)^2 = 0.2500$ exact within 0.040 on 4/4, h-rate slopes ≤ 0.007 dec/degree — but fails pointwise and misses the fiber entirely (wrong sign on w9; the naked electrostatic diagonal model beats the fully dressed one 2.2–3.8× on all four windows: the arcsine-cell Szegő dressing is the wrong pointwise object). The operator-spectral compression test (OFFDIAG-EXTENSIVE) finds no compact eigenspace on the exact tau coordinate (participation dimension $D_{\mathrm{part}} \approx 0.195\,N$ over 42 rungs) and returns the arc's conceptual surprise, **the compactness inversion**: arithmetic worlds are spectrally *spread*, destroyed worlds spectrally *compact* — scramble collapses to 2 eigendirections, Epstein to 6, MAIN needs 28–30; compactness is the signature of destruction, and the atom diagonal carries 78.6% of the world gap — every one-index compression leaves the complement in true pair structure (the Epstein fiber anomaly is closed as artefact kernel coupling: the round-252 orientation problem and the fiber are the *same* layer, seen through the kernel). On the base front the node-band route is dead (transitions dense at fraction 0.55–0.63, the drift anti-correlated with them), but the round-255 find is the arc's first

low-dimensional result (ORIENTATION-LOWDIM(1)): the Heine saddle point is the first outer-layer object that holds the h-rate 4/4 *and* owns a sign channel, and a single parity-flipping one-node swap within 0.2 decades flips the model sign at the true flip degrees — **the base failure is a selection error of $O(0.2\,\mathrm{dec})$, one bit**, not an extensive information gap. The synthesis: the base flip is a threshold event of an extensively accumulated quantity; the fiber *is* that accumulation, read through the kernel. The admissible target form is frozen — an *integrated* statement over the compressed CD readout (no uniform Y_n parametrix), the full von Mangoldt comb in the main problem, base $H_N > 0$ with Epstein/smooth as breakers, fiber $D_N(B) \geq 0$ with scramble as falsifier. Round 256 was still in flight at this promotion cut and is not consumed. **No statement about the Riemann Hypothesis is made.**

12.59 The coupled-tau terminal arc: the positive-prefix firewall and the causal census, the coupled two-term tau recursion with its bilinear corollary, the blind coefficient-field pass of the frozen micro-falsifier, the exact budget telescope with its one-inequality terminal reduction, and the three parametrix no-gos promoted — with the campaign re-localized onto its two named terminal edges (rounds 256–259, notes DLXXXVII–DXC, 2026-08-24; promoted 2026-08-25)

The four rounds after the bordered-dictionary cut execute the reviewer's causal program and give the wall problem its most compact form to date; their durable certified content is promoted to the load-bearing suite on 2026-08-25 as [verification/v959_coupledtau_terminal_dictionary.py] (suite 951 → 952; probes embedded byte-exact and executed verbatim, 21/21 + 26/26 + 27/27 + 19/19, zero fails, SPEC SHAs pinned; plus a module-own exact section deriving the round's algebra in sympy and exact rationals). The honesty fences travel first: **the row is a certified finite dictionary and adjudication set — every wall statement remains typed wall-equivalent and is consumed nowhere; the measured census, covariance and ensemble findings are typed numerical (structural facts, not laws); the three parametrix negatives are load-bearing honest no-gos; rounds 260 and 261 were in flight at that promotion cut and are not consumed in v959 (the durable content of rounds 260–278 is promoted separately in the wave-4 arc, sec:surface-orientation-arc); no row says anything about the truth of the Riemann Hypothesis in either direction; the mincut stays base 4 / refined 5.** The theorem set is registered as PRIME.PORT.RHP.COUPLEDTAU.TERMINAL.01 [E]; the campaign contract PRIME.PORT.RHP.FULLSOURCE.BASEFIBER.01 [O] is carried forward with its two named terminal edges registered as successor contracts, PRIME.PORT.COUPLEDTAU.TERMINAL_CROSSRATIO.01 [O] and PRIME.PORT.FULLSOURCE.PREFIX_RESUMMATION.01 [O] (both progressed in wave 4, sec:surface-orientation-arc).

The positive-prefix firewall and the causal census (round 256, `baseborder_factorial_probe.py`, 21/21, SPEC `c7aa9f46889414c4`). The reviewer-demanded causal separation runs *before* any further compression search. The firewall type table: MAIN carries the positive prefix through its *entire* window, $p_{\max} = 184 = N$ with zero negative modes — the only positive-prefix world — while the controls type INDEFINITE-CONTINUATION at $p_{\max} = 21/25/27$ (scramble/Epstein/smooth, with 37/55/4 negative modes); every prefix kernel is numerically PSD (worst $-\lambda_{\min}$ rel 5.7×10^{-16}) — the reviewer's PSD core statement holds machine-level, and the module re-proves it as exact algebra (H_p is positive definite iff the whole h-prefix is positive, $\det H_p = \prod_{i<p} h_i$ exactly, with the explicit Schur-complement border reading exactly $h_{p_{\max}} < 0$ past the prefix). The raw 2×2 census reproduces the r254 priors exactly (worst rel 4.9×10^{-5}) — but the causal reading of those numbers *was contaminated*: base and interaction effects at full degree are bookkeeping over an indefinite kernel. On the causally clean surface (the shared positive prefix $n^* = 21$, both kernels PSD) the double inversion appears: the base rotation is *null* ($D_b^* = -1.5 \times 10^{-6}$ — the chain does nothing on the clean prefix) and the border carries the prefix gap ($D_{bo}^* = -0.1759$); the sealed load-bearing rule honestly does not fire (the raw border effect has the wrong sign), residual verdict

EFFECTS-MIXED-COUPLEDTAU. **Two contamination downgrades go on record**: the round-254 headline "scramble spectrally compact ($K_{80} = 2$)" is base contamination (the exact h-sign mode split $+3292.6 - 3297.1 = -4.494$ — the net census entry is carried by the $h < 0$ mode space at 734× excess, and the two carrying eigendirections sit at $f_{\text{neg}} = 0.93$ in it), and the r253/r254 Epstein fiber anomaly lives entirely in the indefinite continuation degrees $k \geq 25$ (on the shared prefix the Epstein border is fiber-*identical* to MAIN, 1.3×10^{-16}) — neither finding may be cited as a border/fiber discriminator any more. The consequence, per the sealed reviewer pre-declaration for the mixed case: **the separate treatment of base and fiber is artificial** — the right target is the indivisible full-source object $D = \tau^{\text{aug}}/\tau$.

The coupled-tau recursion (round 257, `coupledtau_probe.py`*,* 26/26 *in the first full pass, SPEC* `73d8247f6de36a2b`*).* The theorem of the round: the pair closes under the exact two-term recursion $\tau_{n+1} = a_n\tau_n$, $\tau^{\text{aug}}_{n+1} = a_n\tau^{\text{aug}}_n + b_n\tau_n$ with $a_n = c_n^2 h_n$ and $b_n = -(c_nF_n)^2$ (c_n the exact Chebyshev leading coefficient, F_n the border Cauchy readout; seeds closed), equivalently the Riccati drain $D_{n+1} = D_n - \rho_n$ with $\rho_n = F_n^2/h_n$ (signed-log propagation vs direct bordered slogdets at 1.2×10^{-9} on MAIN; anchor $D_N = 5/7 - \rho_{N-1}$ rel 4.8×10^{-11}; mp wards dps 220 *through* the scramble flip). The module derives and gates the canonical integrable corollary, **the bilinear tau form** $\tau^{\text{aug}}_n\tau_{n+1} - \tau^{\text{aug}}_{n+1}\tau_n = (c_nF_n)^2\tau_n^2$ — a manifest square: the drain increment is a *positive tau cross-ratio*. The coefficient typology is machine-gated: a_n reads the h-chain (its sign *is* the world pivot sign) while b_n is a negative square, source-sign-pure in every world — **the base alone carries the sign; the border contributes only nonnegative magnitude** — and the cocycle law breaks exactly at the control base flips 21/25/27 (the r256 firewall reproduced gradewise inside the coupled dynamics). The one-line form of the whole problem follows: on MAIN (all $h_n > 0$) the drain is monotone through the entire free window — fiber positivity *is* the budget inequality $\sum F_n^2/h_n < B$, and wall positivity supplies the increment signs: base and fiber are coupled in one scalar law. And the frozen micro-falsifier of `PRIME.FREEMOMENT.POSITIVEPREFIX.01` is **passed blind for the first time, at the coefficient-field level**: the AST-separated predictor (consuming node positions and signed weights only) hits the control flips $21/25/27 = 3/3$ *and* the forced tail 0/2/2/3/1 on all five windows (flips 184/153/170/367/592, mp wards dps 40) — honestly typed: mechanism coordinate yes, analytic model no; the parametrix half stays open. The no-pole measurement approaches cofinally (slope -2.87 dec/dec, Spearman -0.81, $\lambda_{\max} < 1$ on 10/10), but *both* augmented border variants cross the wall — any no-pole path must go through the pair dynamics itself, not through a bordered spectral bound.

The budget anatomy (round 258, `budget_anatomy_probe.py`*,* 27/27*, SPEC* `28026ae6b0750098`*).* The collapse structure is confirmed with a sobering precision: $B = S_{N-2} + 5/7$ is exact *by construction* (the r243 a_4 definition) — the nontrivial content is the Schur telescope $D_N = B - S_{N-1}$ (12 windows including $N = 878$, mp wards dps 220 at 2.4×10^{-16}; f64 loses deep with slope $+4.16$ dec/dec, disclosed) — and the 5/7 is *neither* Q_7 nor level-8 structure: it is the inverted r241 floor 7/5 of the *measured* minimum $h_{N-1}/F_{N-1}^2 = 1.4278$ (margin 0.028, not saturated), a rounded rational *under* the measurement, typed FLOOR-IMPORTED — the source-pure derivation of the floor is now *the* named terminal task. The consequence gate is exact: margin $> 0 \iff q_N = \rho_{N-1}/(5/7) < 1$ — **the entire fiber positivity is one terminal inequality**. The census: $q_N < 1$ on all 42 rungs $N = 142..878$ (min/med/max 0.0015/0.4188/0.9805, minimum margin 0.0139) with a *flat* trend — the wall margin does not shrink with N (the census is upgraded to a *certified* surface in wave 4, sec:surface-orientation-arc). The sharpest single finding: Spearman$(2\log F_{N-1}, \log h_{N-1}) = +1.000$ — border-readout square and terminal norm are rank-perfectly covariant over the ladder, the residual correlations tied: **the driver is neither F nor h but the quotient itself** (a measured structural fact, not a law) — any terminal proof must bind the *ratio*. Consistently, all three sealed mass majorants (sup, Cauchy–Schwarz, triangle) break with 0/42 coverage, median ×490 excess, break locus $0.97N$ exactly in the near-terminal zone where h collapses with a +8.2-decade terminal rebound — **the mass-bound class is dead**, as compression, constants and band locality before it. The corner ray $B(t) = S_{N-2} + t \cdot (5/7)$ makes the terminal pivot affine (collinearity 3.4×10^{-13}, slope exactly 5/7, machine root exactly q_N on 12/12) — the budget question *is* a t^* coordinate

in the r238 normal form, yet Spearman$(q_N, t^*_{\mathrm{r238}}) = +0.027$: one normal form, two statistically unconnected data instances. The r256 side-finding resolves honestly: the prefix border dominance is sign-universal (10/10 rungs) but the "81%" was a w9 coincidence — the stable object is the sign.

The parametrix pass (round 259, `parametrix_pass_probe.py`*,* 19/19*, SPEC* `e79da4f43d63213a`*) — three honest negatives.* Can a closed analytic model, without minors or slogdets, hit the control flips and the forced tail blind? **No — three times, with mechanisms.** (i) *Level crossing refuted*: the parity-resolved saddle energy branches cross 5–16 degrees *too early* (Epstein 9 vs 25, smooth 19 vs 27, scramble 16 vs 21), and on MAIN w9 they cross *45 times without a single true flip* — the $|\gamma| > 1$ jump sits at the true flips and belongs to the *resummed chain*, not to the branch crossing. (ii) *The one-swap no-go*: the critical swap at every control flip is a short-range countersign edge pair (median distance 2) — but MAIN carries the *same* structure at the *same* degrees with fully overlapping gap distributions: the one-bit question is answered negative, and **round 255's orientation-lowdim(1) is revised on record** — the 0.2-decade swap exists, but the missing information is not one-dimensional. (iii) All three sealed candidates fail blind (0/3 control flips, 0/5 forced tail). The named obstacle, measured: **the parametrix gap is a resummation gap, not a selection gap** — the true sign is carried by coherent cancellation over the configuration ensemble; no dominance-configuration model at any k-swap resolution sees it, and the r257 minor scan performs exactly this resummation exactly — the analytic counterpart is the missing object.

The campaign at the post-round-259 state (the two named terminal edges). After rounds 258 and 259 both cores of the campaign contract carry precisely named obstacles of the same construction: **the last fiber edge is a positive tau cross-ratio** — prove the terminal ratio $h_{N-1}/F^2_{N-1} > 7/5$ (equivalently derive the true floor source-pure) on the cofinal ladder, one ratio inequality with a flat measured margin whose driver is the perfect F-h covariance (`PRIME.PORT.COUPLEDTAU.TERMINAL_CROSSRATIO.01` [O]) — and **the last base edge is a globally oriented prefix resummation** — find the closed form of the coherent ensemble resummation, the determinant as a signed partition sum with no saddle dominance and no k-swap truncation (`PRIME.PORT.FULLSOURCE.PREFIX_RESUMMATION.01` [O]). In both cases the object is a coherent totality rather than a dominant component — the most consistent explanation of why two years of single-component attacks (compression, constants, saddles, majorants, bands) all measurably failed. Rounds 260–278 have since executed on these contracts; their durable certified content is promoted in the wave-4 arc (sec:surface-orientation-arc). Mincut base 4 / refined 5 unchanged. **No statement about the Riemann Hypothesis is made.**

12.60 The surface closure and the orientation arc: the 42-rung census $q_N < 1$ certified as a finite fact, the universal pair theorem with its single open hypothesis, the Wronskian/Casoratian dictionary, the blind Maslov census GO with the honest atom-Sturm refutation, and the metric-firewall measurements (rounds 260–278, notes DXCII–DCXI, 2026-08-25; promoted 2026-08-25)

Wave 4 promotes the durable certified content of the nineteen rounds after the coupled-tau cut in two modules, [`verification/v960_terminal_surface_closure.py`] (rounds 260/262/263/268–271 as the theorem set, rounds 272/273/275 as typed measurements; 31/31 + 26/26 + 26/26 + 28/28 + 29/29 + 29/29 + 28/28 + 28/28 + 24/24 + 24/24 first-pass gates, zero fails) and [`verification/v961_midpoint_orientation_dictionary.py`] (rounds 274/277 as the theorem set, rounds 276/278 as typed measurements; 32/32 + 25/25 + 29/29 + 31/31), suite 952 → 954. Both modules embed their probes byte-exact and execute them in the sealed `-smoke` stage (SPEC SHAs pinned, byte-equality wards live); the full records stay sealed experiments-side and are re-verified by the RH-workspace suite `rh/verification/run_rh.py`, which since this wave is a gate of `bash build.sh audit`. The honesty fences travel first: **the surface closure is a census-plus-certificate result, not a cofinal theorem — the window quantifier stays; every wall statement remains typed wall-equivalent; the measured trends, censuses**

and firewall curves are typed numerical; the honest negatives are load-bearing; round 279 (`oriented_theorem_probe.py`) is in flight at this cut and is not consumed; no row says anything about the truth of the Riemann Hypothesis in either direction; the mincut stays base 4 / refined 5.

The terminal-surface closure (`PRIME.PORT.COUPLEDTAU.SURFACE_CLOSURE.01` [E]). The milestone of the wave: **the 42-rung surface census $q_N < 1$ is certified on all 42 rungs as a finite fact** — 41 rungs mechanism-certified target-blind and kz15 exact-finite. The pieces: (i) the round-263 **two-branch theorem** — with U_w the sealed round-260 terminal-triangle form ($|r_{N-1}| \le |t_{N-2}| + |a'||r_{N-2}| + |b'||r_{N-3}|$, prefix data only, target-blind AST-audited), $M_w = \sqrt{5/7}$ and $g_w = M_w - U_w$: the cheap branch $g_w \ge 0$ closes exactly 35/42 rungs *including both mains without any cancellation* (w9 $g = +0.442$, w13 $+0.212$), and the exception set is exactly $\{kz15, kz20, kz22, kz36, kz38, kz39, kz52\}$ (all seven $Z_w < 0$, all drive-dominated); (ii) the sealed **phase bounds** — the round-268 edge truncation delivers the first source-pure certification of an exception rung ever (kz22, margin $+0.04$) with the remaining gaps measured sub-paircorr, and the round-269 **block alternation** (adjacent sign-run pairs as exact pair sums, then the triangle on the pairs) gains $+0.26$ to $+0.41$ decades on all 42 rungs and certifies kz20/22/36/38/39/52 with *no* detector firing; (iii) the round-270 **kz15 interval certificate** — the whole bordered chain in outward-rounded interval arithmetic (dps 640, 202 steps, end width 1.5×10^{-92}, all 810 source atoms exact): $\sup|Z_{kz15}| < \inf\sqrt{5/7}$ *strictly*, margin $+0.02680$, with the dps-halving ward destroying the certificate as demanded — honestly typed an exact *finite* enclosure certificate, no mechanism; and (iv) the round-271 **universal pair theorem**: hypotheses H1–H4 are structural and proved (the finite pair algebra also in Lean, `rh/lean/RH/PairBound.lean`, no `sorry`), while H5 — the margin $|Z_{\mathrm{local}}| + \varepsilon < \sqrt{5/7}$ — is the *single* window-dependent hypothesis: the formal entry gate of the cofinal step. The honest negatives travel with the row: the eventual-triangle no-go (the deepest rung kz52 is itself an exception), the three identity routes dead with break loci (target restatement + support geometry; squares on the drain side; paircorr miniature), the retracted w13 Gram provenance, and the round-275 **KYP memory no-go from structure** (the exact-rational double obstruction: fiber forcing $p_{11} = 0 \Rightarrow p_{12} = 0$ plus the descent window $\sum b^2 \le 0$, on 46/46 worlds; the unique admissible Riccati memory is target-inverse, sp $= +1.00$, and falsely certifies scramble).

The cofinal state (rounds 272/273, typed measurements). The round-271 scaling alarm $\mathrm{sp}(N, \varepsilon) = +0.67$ is dissected exactly: pair count $\alpha = +1.01$, amplitude $\beta = -0.81$, bound-side cancellation exactly flat ($\gamma = -0.01$) — while the *truth* side runs opposite ($\gamma_{\mathrm{true}} = +0.453$; the truth margin *rises* with N, $+0.264 \to +0.414$): **the bound loses cancellation, not the world** (BOUND-COARSENESS; H5 is contradicted by no measured truth quantity). The quantified flip condition: a non-adjacent/global mechanism (address c3) must supply $N^{-\delta'}$ with $\delta' > 0.21$ of the measured available 0.45. The round-273 Euler surgery ladder adds the double finding: no surgery collapses the truth-side cancellation (the $N^{-0.45}$ decay is generic root-scale, not an arithmetic fingerprint) while *every* surgery kills the free-prefix positivity at degrees 33–101 — **the wall, not the cancellation rate, is what is arithmetically special about MAIN**.

The midpoint-orientation dictionary and the Maslov census (`PRIME.PORT.RHP.MIDPOINT.ORIENTATION_DICTIONARY.01` [E]). The round-274 Wronskian dictionary closes exactly: the base Casoratian is the pivot chain itself ($\hat\pi_{n+1}q_n - \hat\pi_n q_{n+1} = h_n$ with $c' = 1$; right solution triple-constructed with exact coincidence, the node polynomial *is* the right Dirichlet boundary condition), the h-free midpoint form *is* the node polynomial (the r231 sign-blindness as an identity — it provably carries no orientation), and the augmented telescope $D_{n+1} = B - W_n^{\mathrm{aug}}/W_n^{\mathrm{base}}$ is exact against the independent round-263 route; the honest c-typology: the entire orientation sits in $c_n = 1/W_n^{\mathrm{base}}$ — the dictionary relocates the question, it does not solve it. The round-277 **Maslov census** then delivers the blind GO: rule R2 — the interlacing/reality structure of the *Jacobi* zeros — passes the sealed examiner protocol on all 42 rungs with the mains safe at full depth and the controls firing *exactly* at flip+1 (26/22/28); R2 is provably safe on every positive prefix, not h-equivalent, and its break never heals: a one-way detector, the cofinally monotone

object the proof plan needs. The honest refutation travels with it: **the raw atom-Sturm census is not the right winding quantity** (on MAIN $c_n \equiv n$ breaks at 56/48 at throughout-positive h: the classical separation theorem fails genuinely for the signed comb; STURM-CHAIN-VERIFIED is not awarded). The named consequence is the *contraposition* form — an independent index obstruction forbidding the R2 break before half-filling on MAIN — registered as the successor contract `PRIME.PORT.RHP.MIDPOINT.ORIENTED_THEOREM.01` [O] (round 279 in flight at that cut; consumed in wave 5 together with rounds 280/281, sec:pinning-theory-arc). The metric-firewall measurements (rounds 276/278) complete the row: the wall is a graded *continuum* ($D \sim \theta^b$, no jump), support exactness is the most wall-critical property (the arithmetic firewall is more precisely a *metric* firewall), the exact position gradients (Hellmann–Feynman class, identity fine type) make the u-profile predictive (the top-3 atoms are the small primes 2, 3, 5) — but the stability law is PERTURBATIVE-ONLY, and the MAIN positivity itself stays the open center; the global form is registered as `PRIME.PORT.WALL.METRIC_FIREWALL.01` [O].

The campaign after wave 4. The two ex-edges converge on one formal target — the master theorem `augmented_prefix_positive` (`rh/lean/RH/Window.lean`): $A_{w,n} \succ 0$ for every admissible MAIN window and prefix degree, with `MainWindow` an opaque predicate that per the wave-4 measurements must encode *metric* exactness (the sub-gap-exact placement of the von Mangoldt comb), not combinatorial Euler bookkeeping. The open fronts, precisely: (i) the cofinal front = hypothesis H5 / lemma L2 on `PRIME.PORT.COUPLEDTAU.TERMINAL_CROSSRATIO.01` [O]; (ii) the oriented midpoint theorem on `PRIME.PORT.RHP.MIDPOINT.ORIENTED_THEOREM.01` [O] (executed as rounds 279–281 and consumed in wave 5, sec:pinning-theory-arc); (iii) the global metric firewall on `PRIME.PORT.WALL.METRIC_FIREWALL.01` [O]. Mincut base 4 / refined 5 unchanged. **No statement about the Riemann Hypothesis is made.**

12.61 The half-filling pinning theory: four theorems, four refutations, and the north star in reinstform (rounds 279–281, notes DCXII/DCXIV/DCXV, 2026-08-25; promoted 2026-08-25)

Wave 5 executes the reviewer demand to freeze the pinning rounds *as a small mathematical theory, not just as experiments*: [`verification/v962_halffilling_pinning_theory.py`] (rounds 279/280/281; first-pass records $32/32 + 29/29 + 28/28$, zero fails; the three probes embedded byte-exact in their sealed `-smoke` stage with SPEC SHAs pinned, plus a module-own exact section in pure Fractions that re-derives the theory from scratch), suite $954 \to 955$. **The four theorems, as named gates.** (T1) *The moment counting theorem* (r281, Identity): the pivot h_n consumes the moments $m_0..m_{2n}$; an S-atom signed measure has exactly S free moments $m_0..m_{S-1}$, every later moment being forced by the monic node-polynomial recurrence — hence the free pivots are *exactly* $h_0..h_{N_w-1}$ and h_{N_w} is the first forced one: **half-filling is the end of the free moment space** (arithmetic-exact for all $S = 2..2000$; the freedom demonstration is exact in rationals: $dm = e_{S-1}$ moves h_{N_w-1} alone). The reviewer question "why half-filling" is thereby *answered by counting* — no secret. (T2) *The crossing budget theorem* (r279, Identity): $\#\{n < S : h_n < 0\} = S_-$ exactly (Jacobi's minor-sign rule plus the Sylvester congruence $G_S = VWV^T \sim W$), *world-blindly*: the total Maslov crossing budget over the full algebraic continuation is the negative-atom count (w9 104, w13 98, controls 141/94/6, kz15 121, kz52 551). (T3) *The two-sided parity theorem* (r279, Identity): $\operatorname{sign} Q_n(x_j) = e_j(-1)^{S-1-j}\operatorname{sign}(h_n)$ at every node and degree, so the union occupancy of every open atom gap is odd in every weight-agreement gap (never empty) and even in every disagreement gap — h-blind, at every degree; the census bilanz $c_n + c^{\#}_{S-1-n} = (S-1) - |D| + 2\,\mathrm{scD}(n)$ holds exactly at every degree and by exhaustion over all 87376 sign-vector pairs $k \le 8$. (T4) *The main window reduction*: given T1 and T2, **the entire open statement of the wall is** $\min C \ge N_w$**, exactly equivalent to** $\forall n < N_w : h_n > 0$ — no size question, no cancellation question, no orientation question remains: one placement question, whose reinstform is the reformulated north star, *why is the signed prime moment form quasi-definite up to the maximally free order?* **The four refutations, as named negative gates** (the negative information is itself load-bearing): NO-UNIVERSAL-O1-PINNING (r281, exact: the one-negative toy has offset $N_w - 2$, unbounded in S — any $O(1)$ upper side must consume the comb structure; the

only provable ceiling today is the T2 pigeonhole $\min C \le S - S_-$, and $C \le 5$ is a 42-rung *measurement*); NO-EXTREMALITY (r280, measured plus exact witness: three structured mp-confirmed directions push the w9 crossing $184 \to 185$ *past* half-filling, and a 4-atom one-negative measure has $\min C = N_w + 1$ exactly — N_w was the free-window bound, never an absolute maximum); NO-GENERIC-MASLOV-OBSTRUCTION (r279, exact: three rational counterexamples — the two-sided machinery is world-blind classical structure and provably carries no arithmetic); NO-SIMPLE-OFFSET-LAW (r281, measured: six sealed source-pure candidates reach max $|\mathrm{sp}| = 0.273 \ll 0.75$, all world-blind). Ledger: `PRIME.WALL.HALFFILLING_PINNING_THEORY.01` [E]; rounds 279–281 consumed on `ORIENTED_THEOREM.01` [O] (theorem share promoted, the center precisified); the successor contracts `PRIME.PORT.REPRESENTATION.CONTEST.01` [O] and `PRIME.PORT.RHP.FULLSOURCE.QUASIDEFINITENESS.01` [O] are in flight at this cut and *not* consumed (both executed and consumed in wave 6, sec:lstar-arc). In Lean the hole is now fog-free: `free_window_positivity` (`rh/lean/RH/Window.lean`) is the one central `sorry` in base coordinates (via T4 equivalent to the base half of the master theorem), T1 and T4 are *proved*, T2 is stated, and the exact one-negative instance is a permanent counterexample guard. Mincut base 4 / refined 5 unchanged. **No statement about the Riemann Hypothesis is made.**

12.62 The L* reduction: the canonical form of the open center, the four-language elimination, and the honest margin decay (rounds 282–285 and the standalone problem, notes DCXVI/DCXVII/DCXIX–DCXXI, 2026-08-25; promoted 2026-08-25)

Wave 6 freezes the L* arc: [`verification/v963_lstar_reduction_dictionary.py`] (rounds 282/283/284/285; first-pass records $30/30 + 26/26 + 30/30 + 33/33$, zero fails; the four probes embedded byte-exact in their sealed `-smoke` stage with SPEC SHAs pinned, plus a module-own exact section that re-derives the reduction from scratch in pure Fractions), suite $955 \to 956$. **The canonical reduction (r283, exact).** In the μ-orthonormal frame the full Hankel form of the signed defect measure $\tilde{\mu} = \mu - \nu$ is congruent to $D_\mu - G$ with the leading minors *preserved* exactly ($\mathrm{minor}_k(D_\mu - G) = D_k(\tilde{\mu})$ at every $k \le S_+$); quasi-definiteness through the free window is exactly the strict contraction of the ν-dressed μ-Christoffel-Darboux kernel ($h_0..h_{m-1} > 0 \iff \lambda_{\max}(E_m) < 1$, both truth values gated), and $\lambda_{\max}$ crosses 1 *exactly* at $\min C + 1$; the pigeonhole ceiling $\min C \le S_+$ is re-proved in the frame, and capacity-as-counting is refuted by the exact rank-one pair (same rank, same support, opposite window fate — the deciding values are *metric*, 1.25×10^{-13} vs 1.25×10^{2}). Hence the **entire** free-window question is ONE two-measure subordination scalar, **the missing lemma L***: for every real polynomial p with $\deg p < N_w$,

$$\int p^2 \, d\nu \;<\; \int p^2 \, d\mu, \qquad \text{equivalently} \qquad \lambda_{\max}(E_{N_w}) < 1$$

— registered as the standalone contract `PRIME.LSTAR.SUBORDINATION.01` [O], with the fully self-contained problem statement (no project vocabulary) in `rh/problem/lstar_problem.tex` for external mathematicians. **The four-language elimination (r282, exact negative gates).** The reviewer dream "positivity by representation instead of estimate" is closed in the four classical languages, each with the exact reason: the SOS/$|m|^2$ monodromy structure exists IFF the negative register is empty (the fake SOS misses h_2 by exactly $N_2 = 48360721965/70120631072$; the surviving clause is $h > 0$ verbatim); the Kasteleyn orientation class is exactly $\{\pm \mathrm{sign}\, w\}$ by full 2^S exhaustion and is value-preserving IFF $S_- = 0$ (defect = 2×negative mass, exact); the canonical-system Hamiltonian positivity is $h > 0$ bookkeeping; and the dual pair is sign-synchronized *by theorem* ($h_n h^{\#}_{S-1-n} = 1$ exactly) — verdict CONTEST-ALL-DEAD, with the common deep reason: *every classical positivity language forces positivity exactly for the positive measure class, and MAIN is signed.* **The decomposition bookkeeping (r285, exact).** $\lambda_{\max} = \mathrm{maxdiag} \times (1 + \mathrm{assist})$ exactly, with the budget equivalence sign-exact at every degree — L* splits into (D) the per-atom diagonal condition $v_k K_{N_w}(y_k) < 1$ and (C) the coherence condition. **The measured anatomy (r284/r285, typed).** MAIN's wall crossing is a *near-single-atom Christoffel event* ($n_{\mathrm{DIAG}} = 187$ vs

crossing 185) while all four controls die collectively far below their single-atom degree; the binding family is the shallow-u ARCH edge below the first prime on 42/42 rungs, and its kernel growth is *sub-classical* ($p = 0.38$ vs bulk law 1; classical coverage 0/42: any (D) proof must be a discrete bound); the shielding geometry is world-blind and the Nyquist/resolution formula is refuted at the discriminant; MAIN's wall assist is the extreme LOW-OUTLIER of both conservation-gated replicate ensembles (pct 0.00), and the round delivers **the first two positively MAIN-separating detectors of the program** (assist-at-crossing 0.0195 vs dead 1.69..2.99; random-sign $z = -3.15$ destructive vs dead $+4.95.. + 12.44$ all constructive) — the first positively localized arithmetic signals of the L* coordinate. **The honest margin finding (DCXX; open).** All 42 ladder margins $1 - \lambda_{\max}(E_{N_w})$ are strictly positive, but the family minimum is 1.4175×10^{-7} (anchor $z = 233$, $S = 1717$) while the flagship $z = 16$ carries the *largest* margin 1.68×10^{-4}: the spectral L* margin falls ∼3 orders of magnitude over the tested family (driven by the maxdiag → 1 trend of the per-atom budget; the r258 FLAT finding concerned the terminal ratio q_N, a different object) — the unbounded-family version of L* was, at this cut, genuinely uncertain in both directions; the r286 margin-scaling round was in flight at this promotion cut and is consumed in wave 7 (sec:coherence-arc: resolved *harmless* — 15 new anchors, no counterexample, the $O(1)$ offset survives). Ledger: `PRIME.LSTAR.REDUCTION_DICTIONARY.01` [E] and `PRIME.LSTAR.SUBORDINATION.01` [O]; `FULLSOURCE.QUASIDEFINITENESS.01` consumed (MECHANISM-PARTIAL(A2), L* established); `REPRESENTATION.CONTEST.01` consumed (CONTEST-ALL-DEAD). In Lean the canonical form is stated (`lstar_subordination`, `rh/lean/RH/Window.lean`, documented `sorry`) and the direction L* ⇒ free-window positivity is *proved* (`lstar_implies_free_window`). Mincut base 4 / refined 5 unchanged. **No statement about the Riemann Hypothesis is made.**

12.63 The L* coherence census: the margin warning resolved, the van der Corput theorem for L2, the carrier map, and the metric-only adjudication of the diophantine question (rounds 286–289, notes DCXXIV/DCXXII/DCXXV/DCXXVI, 2026-08-25; promoted 2026-08-25)

Wave 7 freezes the coherence arc: [`verification/v964_lstar_coherence_census.py`] (rounds 286/287/288/289; records $29/29 + 25/25 + 31/31 + 42/42$, zero fails after the disclosed calibration amendments; the four probes embedded byte-exact in their sealed `-smoke` stage with SPEC SHAs pinned, plus a module-own exact section that proves the van der Corput and Abel/Erdős–Turán inequalities from scratch in pure Fractions), suite 956 → 957. **The margin warning resolved (r286).** The DCXX finding is cleanly defused: the ladder is extended by 15 new anchors ($N_w = 942..1218$, 40% beyond the old family cap) and *every* margin is mp-sign-safe positive (minimum $+1.806 \times 10^{-8}$, each anchor mp dps-30/45 staggered) — no counterexample; the $O(1)$ census offset survives beyond the cap (off distribution $\{0{:}1, 1{:}10, 2{:}2, 3{:}1, 4{:}1\}$, max +4); the margin law is a *flattening* power law ($\alpha \approx 3.05$ vs S, halves 3.67/2.07, curvature −1.60) driven by the cancellation sharpening $c_w \to 1$, and the HARMLESS reading is quantified: the margin falls *slower* than the local eigenvalue loading speed — the decay is the kinematic signature of the fixed $O(1)$ census offset at growing eigenvalue density, not an approach to a crossing; q_N is reconciled (the r258 FLAT verdict reproduced: two different objects); the frozen 42-rung power fit *predicts* all 15 extension margins inside the sealed 1-decade band (EXTRAP-CALIBRATED 15/15). **The van der Corput theorem (r287, the 30 % lane).** The generic L2 half bows to a *named classical theorem*: the exact, constant-free, arithmetic-free van der Corput inequality $|\sum_j P_j|^2 \le \frac{m+H-1}{H} \sum_{|h|<H} (1 - \frac{|h|}{H}) A(h)$ at the frozen window $H = \lceil \sqrt{m} \rceil$ delivers $\delta' = +0.309 > 0.21$ world-blind (admissible: L2 is the generic half) and certifies 6/7 exceptions +38/42 rungs; the F1 discrepancy/Erdős–Turán frame dies loud ($\delta' = -0.21$, 0/7, detector fire +1.44 dec); every surviving frame misses only the kz15 razor (0.07..0.12 dec), which is exact-finite certified per r270; the autocorrelation surprise relocates the c3 address honestly — on 39/44 worlds the block sequence *reinforces* at root scale: the cancellation is already absorbed into the P_j magnitudes, the classical frames win through the $\sqrt{m}$ count economy, not lag structure. **The carrier map (r288).** The destructive coherence has a named carrier class: the neg fraction 0.503

is globally coin-like but sharply band-structured — fold distance 1–2 is 87 % positive (sampled in phase) while distance 3–4 is 80 % negative (antiphase); the wall destructivity is carried by the *antiphase next-nearest ARCH–ARCH pairs* ($X_v = -0.0517$, $z_v = -3.149$ exactly reproduced, $C_{\text{off}} = -0.1046$) with total control reversal (all four controls constructive at their own crossing) — and it sits in *finest alignments* far below the median phase resolution: the chain zeros co-move with the atoms (turn rate 0.24 per unit dose = the static prediction 0.23) yet z_v flips already at dose 0.005. Three honest negatives are named: SAMPLING-BLIND (the sinc/CD predictor is only partial, $0.728/0.878 < 0.90$), SOURCE-SEPARATOR-NOT-FOUND (plain phase dispersion is world-blind and ensemble-generic), DIFFERENT-OBJECTS (the r287 magnitude absorption and the kernel destructivity are two coordinates). **The metric-only adjudication (r289; the central refutation).** The *rational twin* — every tent center $\log n$ replaced by a rational $p/q \times \Delta$ (denominators ≤ 56801, position cost $\leq 2.1 \times 10^{-9}$), destroying *every* exact log-linear-form relation — keeps the full r288 signature *identical* ($\min C = 184$, crossing 185, $z_v = -3.149$, mp-confirmed at the 1.7×10^{-4} margin): the effect of the diophantine structure is null; the shuffle twins die purely metrically ($\theta_{\text{eff}} = 0.12$ is ∼40–120x beyond the measured coherence threshold $10^{-3}..3 \times 10^{-3}$ of the local gap); the only sub-gap entry point is exactly identified (the tent-split fractions, completeness identity 4.2×10^{-14}); and the Baker need is documented, never applied: ∼7900x too weak in the exponent *and* unnecessary per the verdict. *The deep clarification:* the firewall reads the fraction *profile* metrically, not its arithmetic — every profile of 10^{-3}-gap precision carries (the rational twin proves it); the why-MAIN search continues in profile space, not in number theory. The precise open front: *which functional of the fraction profile forces the destructive coherence?* (The r290 profile-functional round was in flight at this cut; rounds 290–295 are consumed in wave 8, sec:curvature-arc.) Ledger: `PRIME.LSTAR.COHERENCE_CENSUS.01` [E] (honest negatives explicit: SAMPLING-BLIND, SOURCE-SEPARATOR-NOT-FOUND, DIOPHANTINE-EXCLUDED(METRIC-ONLY)); `PRIME.PORT.L2.VDC_LEMMA.01` [O] — the vdC form as a provable chain statement (the remaining input: the chain origin of the P-variance scaling); `PRIME.LSTAR.SUBORDINATION.01` [O] carried at the wave-7 state; `PRIME.PORT.COUPLEDTAU.TERMINAL_CROSSRATIO.01` [O] records the vdC candidate for the H5/L2 front. In Lean no new statement is needed — the hole stays `lstar_subordination` (`rh/lean/RH/Window.lean`). Mincut base 4 / refined 5 unchanged. **No statement about the Riemann Hypothesis is made.**

12.64 The L* curvature arc: the geometry of the working set measured, the closed-functional search sealed honestly negative (rounds 290–295, notes DCXXVII/DCXXIX–DCXXXIII, 2026-08-25/26; promoted 2026-08-26)

Wave 8 freezes the curvature arc: [`verification/v965_lstar_curvature_arc.py`] (rounds 290–295; full records $32/32 + 30/30 + 36/36 + 43/43 + 42/42 + 44/44$, zero fails after the disclosed calibration amendments; the six probes embedded byte-exact in their sealed `-smoke` stage with SPEC SHAs pinned, plus a module-own exact section that proves the r292 polarization identity, the r291 budget telescope, the r293 $\alpha = 1$ simple-zero doubling law and the sealed r294/r295 decision bars from scratch in pure Fractions with mutant must-fails), suite 957 → 958. **The basin geometry (r290).** The working set around MAIN is a *soft-shouldered, strongly anisotropic tube* in the exact θ_{eq} LAG coordinate: kill fraction 0.38/0.62/1.00 at distances $5 \times 10^{-4}/10^{-3}/2 \times 10^{-3}$ (NEAR radius ∼5e-4..2e-3 gap-equivalent); the world-directed axes kill 5–50x *earlier* than random (onsets $4 \times 10^{-5}..2.3 \times 10^{-4}$ vs the r289 jitter threshold $10^{-3}..3 \times 10^{-3}$); the SMOOTH (arithmetic) axis is a privileged killer (onset 1.36×10^{-4}) yet *orthogonal* to the first-order wall gradient ($\cos = -3 \times 10^{-5}$) — its lethality is collective, not gradient-linear; and the r280 ridge is *real*: $\min C = 185$, one degree above MAIN, at extension factors 1..8, first death at factor 16. **The budget mechanism (r291).** The lift is a *first-order budget phenomenon*: one threshold in $(1.280, 1.291]$ separates lift from no-lift over all 18 matched-dose cases (PRIME/HEAD/XIPOS lift, POWER/MID/TAIL/XINEG do not; $k_{\min} = 9$, top atoms $2, 3, 5, 13, 11, 4, 29, 7, 89$ — small-prime head-heavy, not one-atom), with exactly one overdrive retraction (TOP6 at factor 8, margin 9.09); there is *no fixed point* (the iteration saturates on the one-degree plateau $\min C = 185$, never 186, while the axis decoheres),

and the lift is MAIN-specific (EPSTEIN is first-order flat, $\theta_{\text{up}} = 5.03 \times 10^{10}$, and never lifts); the SMOOTH valley is collective-quadratic ($d^2 = -23.3$ vs -0.98 random, ratio 23.7). **The curvature spectroscopy (r292).** The wall Hesse form at MAIN is negative on all 29 sealed directions and *rank-1 dominated* in the L2 spectral metric (92.5 % of $\sum|\lambda|$, $\lambda_{\text{top}} = -0.418$), and the top axis is a pure on-support DENS combination — *not* the SMOOTH axis ($|\cos| = 0.07$) and *not* the ridge (the lift sits in the flattest sector, L2-rank 28/29); the m^* threshold and the retraction are near-flip nonlinearities invisible to jets up to third order (q_R median -0.911: a second-order *assist*; $B_2 = 27.6/B_3 = 150.9$ over-predict); EPSTEIN is second-order structureless too (5.4×10^{-15}). **The metric reconciliation and the flip anatomy (r293).** Three metrics sealed *before* any evaluation (distortion factors 11/59/124, largest sector DENS); against the baseline of its own metric the split-sealed curvature energy F10 wins for the first time in four rounds (home margin $+0.024 = 0.884 - 0.860$; the mixed r292 margin $-0.023 = 0.884 - 0.907$ re-derived from the same pins: MIX-IS-CAUSE) and carries partial information (+0.423 test / +0.826 train-side); all 8 selected flips are *simple zeros* of h_{184} ($\alpha = 1.000 \pm 0.003$ both sides, 225 gated wall evaluations), and the TOP6 retraction *is* a second simple h_{184} zero at $f_{\text{ret}} = 7.107$ — but MSTAR-NO-LAW: the per-direction thresholds scatter 1.1295..1.3931 (the r291 bracket was a fixed-dose artifact), with no fixed safety distance in any unit (spreads 3.0/3.0/3.8/10.2 vs bar 1.25). **The honest rejections (r294/r295) — the frozen content.** The r294 stability test: the |sp| win replicates on all five fresh seed-disjoint corpora (margins $+0.003.. + 0.115$) but the partial channel does not (median $0.299 < 0.3$, 2/5) — sealed verdict F10-FRAGILE, promotion *rejected*; robustness typed TRAIN-ROBUST ($\sigma = 0.0101$), RANK2-CARRIES (top DENS eigenaxis alone +0.855, DENS share 0.989 — the mechanism hint), STEP-STABLE; window transport a knife edge (w11 +0.002 / w7 −0.009) with DENS mechanism constancy (0.795/0.989/0.834). The r295 hardening test: on twenty fresh corpora (192 forbidden seeds enumerated, zero overlap; H_{tr} hash-gated `3447ed198a56`) even the partial-free form wins only 14/20 (margin median +0.028, min −0.079): the sealed three-clause HARDENED bar fails on two clauses — F10-SP-MAJORITY, a *documented regularity* (19/25 combined census), not a theorem; the r294 5/5 was itself top-of-distribution, exactly as the r293 partial was (R293-LUCK: the composition-matched subsample gains +0.067 but misses the level clause by 0.004); L2-VIA-CONSERVATION is near-tautological (η_0 share 2.9×10^{-30}) — why L2 stays open in substance. *The frozen consequence:* after **five** sealed class families (local scalars best 0.263; non-local/low-rank best 0.471; curvature two-form in the mixed metric $0.884 < 0.907$; the metric-reconciled two-form, fragile; the hardened |sp| form, majority-only) *no closed predictive profile functional of the working set exists over the honest bar* — F10 stays unpromoted, and the sealed rejections are themselves the load-bearing content of the wave. Ledger: `PRIME.LSTAR.CURVATURE_ARC.01` [E] (honest negatives verbatim: ALL-FUNCTIONALS-BLIND (5 class families), F10-SP-MAJORITY(14/20, not a theorem), MSTAR-NO-LAW, R293-LUCK); `PRIME.LSTAR.CLOSED_FUNCTIONAL.01` [O] — the closed-functional question with the sealed kill catalog and the named open fronts (loss-corpus forensics K05/K07/K19, the rank-2 DENS core, why L2); `PRIME.LSTAR.SUBORDINATION.01` [O] carried at the wave-8 state (the characterization question functionally negative-sealed, the geometry measured; no round in flight at this cut). In Lean no new statement is needed — the wave carries no new formal target form; the hole stays `lstar_subordination` (`rh/lean/RH/Window.lean`). Mincut base 4 / refined 5 unchanged. **No statement about the Riemann Hypothesis is made.**

12.65 The L2 reduction chain: the DENS fork closed, the generic-half target reduced to one inequality (rounds 296–300, notes DCXXXV–DCXXXIX, 2026-08-26; promoted 2026-08-26)

Wave 9 freezes the DENS fork and the L2 reduction chain: [`verification/v966_l2_reduction_chain.py`] (rounds 296–300; full records $39/39 + 27/27 + 28/28 + 32/32 + 31/31$, zero fails after the disclosed calibration amendments; the five probes embedded byte-exact in their sealed `-smoke` stage with SPEC SHAs pinned, plus a module-own exact section that proves the σ^*-composition arithmetic, the Fejér-block decomposition identity, the participation identity with the machine-checked DIAG↔NEFF equivalence, the kernel envelope with exact equality witnesses, and the five sealed

verdict bars as exact decision logic from scratch in pure Fractions with tipping mutants), suite 958 → 959. **The DENS fork (r296) — the lane closed honestly.** The reviewer's hard identity fork on the r292 curvature axis lands DENS-WORLD-BLIND under sealed rules: the coupling number $\cos(e_{\text{top}}, \nabla\lambda_{\max}) = +0.394$ sits *below* the sealed 0.40 bar (noise max 0.331; the miss is 0.006, honest), the λ_3 control couples *harder* (-0.574: what coupling exists is a near-crossing eigenvalue-*band* property, not top-specific), every arithmetic candidate of the hash-sealed library stays ≤ 0.38, and the only above-chance structure (the 4-dim moment-gradient subspace, 0.825) holds on both dead controls (0.571/0.603) — construction-adjacent low-moment smoothness, so the sealed world-downgrade fires (disclosed tension: the absolute collapse bar is conservative for subspace members; rule unmoved). Per the pre-adjudicated fork the resources moved to the L2 front. **The target inequality (r297).** The exact slope decomposition makes $\delta'(F2) > 0.21$ *equivalent* to $\sigma := \text{slope}(S_F/M_W^2) \leq \sigma^* = 2(\text{sl}_{c2} - 0.21) - \text{sl}_{\text{pref}} = -0.516$; measured $\sigma = -0.714$ — the truth clears the target with 0.198 sigma margin (0.099 in δ'): *what is missing is not room, it is provenance* (σ^* composes back to $\delta' = 0.21$ exactly, the pad-dropped mutant to 0 — gated module-own in Fractions); all three classical provenance routes are VALID 47/47 but slope-fail ($-0.112/-0.097/-0.097$) ⇒ CHAIN-PROVENANCE-PARTIAL(B3, slope miss 0.307 exact), with the surprise that the chain is *exactly orthogonal* w.r.t. the WINDOW measure (devs 0.000 world-blind: the Fractions-proved Parseval sum rule attaches verbatim). **The exact transfer (r298).** With $\Delta := \text{border} \ominus \text{window}$ and the frozen positional Fejér block kernel, $S_F = B(\beta, \beta)$ splits per-rung *exactly* (8.8×10^{-16} of scale, 47 worlds) as $S_F = B(\omega, \omega) + B(\Delta, \omega + \beta)$ — sign-preserving (the $|\Delta|$ mutant breaks by ∼1.7 of scale, loud); adjudicated TRANSFER-DOMINANT: the window main term is *empty* (med -3.94 dec, slope -1.386), the cross term negligible (-1.4×10^{-4}), so $S_F \approx B(P\Delta, P\Delta)$ — *the vdC input is the Fejér energy of the difference measure itself* (share 0.99, $T > 0$ on 42/42; the two r297 gap halves are one structure, sp = +0.69, broad-based). **The decay split (r299).** The exact spectral representation is LOWPASS (band shares 0.93/0.04/0.02); the structural find: full-support overlap on 42/42 rungs — $\Delta_{\text{fresh}} \equiv 0$, Δ is a *pure c-value difference measure* with no pointwise convergence (cconv 0.86, +0.045); the ET/Abel composition fails loud (composed $+1.948 = 1.504 + 0.444$ exact — the r297 magnitude wall recurs at the frequency level), but the pair split fires: the diagonal already falls fast enough ($\text{sl}_D = -0.571 \leq \sigma^*$, margin 0.055) with falling ratio B/D (-0.168) ⇒ DECAY-SPLIT with the frozen rest pair DIAG-TARGET + ratio flatness; the O-sign class separates MAIN (O-POS) from both dead controls (O-NEG) — the first world-separating sign class of the lane, disclosed. **The diagonal anatomy and the refined freeze (r300).** In exact participation coordinates $\text{sl}_D = 2\,\text{sl}_{L1} - \text{sl}_{\text{neff}} = 2(+0.196) - (+0.963) = -0.571$ (dev 6.7×10^{-16}) — *the entire diagonal decay is participation growth*; both magnitude factorizations fail exactly as r297 predicted (chain-norm -0.384 *and* the $|dw|$ sum-rule attach breaks; max $\times L_1$ -0.346, miss 0.170 — the fill decay -0.225 is invisible to the class); the ratio half is settled structurally (RATIO-BOUNDED-STRUCTURAL: the exact kernel envelope $F_H \leq \min(H, 1/(H \sin^2))$ majorizes $B/D \leq R_{\text{env}}$ med 1.61, itself falling -0.122; the lobe-width heuristic refuted, sp = -0.47); the FILL class separates MAIN from both dead controls — the second world-separating class. *The chain state:*

$$[\text{NEFF-TARGET}] \Rightarrow \text{sl}_D \leq \sigma^* \Rightarrow \sigma \leq \sigma^* \Rightarrow \text{r297 target} \Rightarrow \text{v964-S0 vdC} \Rightarrow \delta' > 0.21 \text{ (generic half)},$$

with exactly **one inequality open**: NEFF-TARGET, $\text{slope}(n_{\text{eff}}) \geq +0.908$ (measured +0.963, margin 0.055; the D_{rank} bridge sp = -0.81 is real but correlational; *r301 in flight at the wave-9 cut — consumed in wave 10, Section 12.66*). Ledger: `PRIME.L2.REDUCTION_CHAIN.01` [E] (the exact decompositions and the target reduction, with the honest negatives verbatim: the magnitude-route catalog r297/r299/r300, the ET failure +1.948, no c-value convergence); `PRIME.L2.NEFF_TARGET.01` [O] — the one remaining inequality; `PRIME.LSTAR.CLOSED_FUNCTIONAL.01` [O] updated (the DENS fork closed, front (2) answered negatively at the identity level); `PRIME.PORT.COUPLEDTAU.TERMINAL_CROSSRATIO.01` [O] carries the reduced L2 state. In Lean no new statement — NEFF-TARGET is a measured ladder-slope aggregate, not an exact finite identity, and a universal ladder form would be refutable (the r273 guard pattern); the hole stays `lstar_subordination`

(`rh/lean/RH/Window.lean`). Mincut base 4 / refined 5 unchanged. **No statement about the Riemann Hypothesis is made.**

12.66 The cascade closure and the documented lane stop (rounds 301–304, notes DCXL–DCXLIV, 2026-08-26; promoted 2026-08-26)

Wave 10 is the honest end of the reduction cascade — *the rejections and retypings are the content*: [`verification/v967_l2_cascade_closure.py`] (rounds 301–304; full records 32/32+30/30+26/26+37/37, zero fails after the disclosed calibration protocol; the four probes embedded byte-exact in their sealed `-smoke` stage with SPEC SHAs pinned, plus a module-own exact section that proves the margin-invariance algebra, the coherence identity, the χ-lag decomposition, the zero-sum tautology with the truncated net covariance, the period-4 comb certificate, the sign-pattern certificate and the four sealed verdict bars as exact decision logic from scratch in pure Fractions with tipping mutants), suite 959 → 960. **The rest targets executed (r301/r302).** r301 proved $n_{\text{eff}} = n_{\text{act}}/(1+CV^2)$ *exactly* (ward $\le 6.8\times10^{-16}$ on 47 worlds) with the *perfect* count link $n_{\text{act}} = m$ on 42/42 — the effective-carrier count *is* the constructive level-2 block count (+1.002); the whole Rényi order family grows at one exponent (+0.963/ + 0.926/ + 0.894/ + 0.738: echt anti-concentration), the margin is jackknife-stable (0/42 below NEED), and the rest relocated onto UNIF-TARGET (slope$(1+CV^2) \le +0.094$, measured +0.039) — NEFF-DERIVED missed only the sealed CV-flat clause. r302 fired the *first* DERIVED verdict of the lane (UNIF-DERIVED(B1+A2), all hypotheses typed MEASURED): the normalized block profile is PROFILE-STATIONARY (KS 0.043 ≪ 0.125 over a 3× depth range) with an exact $1/N$ transient onto $m_2^{\infty} = 1.973$ (held-out dev 0.002; the r299 pointwise-cconv negative was the wrong convergence notion), the coherence identity $1+CV^2 = n_{\text{act}}\,\chi/(\text{surv}^2\, n_{\text{eff}}^{\text{atom}})$ is exact ($\le 6.0\times10^{-16}$) with $\chi = 0.63$ *destructive* and falling ($\rho_1 = -0.22$ its atom-level mechanism), and the rest relocated onto ATOM-TARGET (+0.888 needed, +0.942 measured). **The regress audit (r303) — the retyping.** The three 0.055 margins are *one algebraic number*: $m_D = m_{\text{NEFF}} = m_{\text{UNIF}} = m_{\text{ATOM}} = +0.0547$ (invariance devs $\le 9\times10^{-16}$; Fractions re-proof $3/50\times4$ exact); the core is $S = \sigma^* - \text{sl}_D = +0.0547$, the r299 DIAG margin; the σ level is *not* the core ($\sigma^* - \sigma = +0.1976 = S+$ ratio surplus +0.1429 exact) and the $\frac{1}{2}$-conversion conjecture is *refuted* ($0.0988 \ne 0.0547$) ⇒ REGRESS-CONFIRMED: **the cascade r297→r302 is retyped as an exact reduction dictionary (coordinate finding), not six proof steps**; the hard reviewer rule binds from r303 on — *a round counts as progress only if it adds new information, not if it re-expresses the same slack in a new coordinate.* The first *causal* coordinate: the mechanism ladder on 1008 sealed marginal-preserving builds is monotone in ρ_1 (χ 0.630 → 0.764 → 1.029 → 1.342; end margin +0.055 → +0.057 → +0.032 → −0.044: *the sign flip kills the target inequality*) — but ρ_1 matching misses the destructive χ level by 0.134 ⇒ MIXING-INSUFFICIENT; bycatch at theorem grade: $n_{\text{eff}}^{\text{atom}} = (L_1^a)^2/Q$ is a pure *marginal* functional (invariance 10^{-15} on all 1008 builds); the ρ-sign census is honest at 41/42 (Fractions-exact negative certificates kz18/ kz23 — no raw per-rung theorem). **The short-range law (r304) — the reviewer stop case.** The global lag profile $\rho_{1..16}$ of the *dc* profile is a *stable, world-specific period-4 comb* (med −0.222/ − 0.140/ + 0.089/ + 0.130, then strong lags at every $k = 4m$ and $4m{+}2$ up to 16; halves-stable 3/3), *not* a decaying tail — no $k_0 \le 8$ exists: LONGRANGE-STRUCTURE + LAW-LONGRANGE fire on the letter of the sealed rule; the reviewer condition *splits*: the net covariance $NC(16) = 0.712 < 1$ holds (net-negative) while summability-with-small-tail fails (SUM(16) = 1.563; the full sum is the gated zero-sum tautology — only the truncated NC carries content); the structural counterpoint is exact ($\chi = 1 + 2\sum_k T_k/Q$, ward 4.7×10^{-16}): the χ-relevant within-block structure *is* short-range (the r303 miss 0.134 sits at $k \le 3$, lag-2-dominated) — the comb is invisible to χ; lag-8 matching reproduces the χ *level* (0.652 vs 0.630, $|d| = 0.022$) but breaks the *slopes* (0.028/0.027 > 0.02) — **the mechanism is two-scale** (level from the short lags, slopes from the rung-wise ρ_1 trend); the graduated ladder is monotone in χ, *not* in the margin (the r303 kill came from per-rung flipped targets). *The consequence (the sealed stop case):* **the global-profile**

mixing route of the L2 lane is documented closed — the honest stop state is

$$\text{L2 generic} \iff \text{anti-concentration of the explicit } dc \text{ block field} \quad (\text{core slack } S = +0.0547 \text{ measured}),$$

with long-range (period-4 comb) structure blocking the global-profile route; return later with new tools. What stands honestly: the exact identity dictionary (count, coherence, χ-lag), the two-scale split, $NC < 1$, the $-/-/+/+$ sign pattern (Fractions-exact on kz18/kz23), the marginal-functional invariance of $n_{\text{eff}}^{\text{atom}}$ (10^{-15} on all $2100 + 1008$ builds). Ledger: `PRIME.L2.CASCADE_CLOSURE.01` [E] (the retyping and the stop are the frozen content); `PRIME.L2.NEFF_TARGET.01` [O] *retyped to the documented stop state* (the rest is no longer NEFF/UNIF/ATOM — all one slack — but the stop state above, hard rule verbatim); `PRIME.L2.REDUCTION_CHAIN.01` [E] carries the retyping note (the exact identities stand, the reading is refined). In Lean no new statement — the stop carries no new exact target form (measured ladder aggregates, the r273 guard pattern); the hole stays `lstar_subordination` (`rh/lean/RH/Window.lean`). Mincut base 4 / refined 5 unchanged. **No statement about the Riemann Hypothesis is made.**

12.67 The architecture adjudication (rounds 305–316 incl. 310b, notes DCXLV–DCLVIII, 2026-08-26; promoted 2026-08-26)

Wave 11 is the *architecture day*: after the wave-10 lane stop the reviewer set a three-lane contest (A: block-Green / constructive PSD-G rule; B: paired-cone / rank-one stream; C: fixed head) in parallel with the sharpest new fiber attack (the pointwise Rényi-3 law) and a Lean reconstruction of the formal layer — [`verification/v968_architecture_adjudication.py`] (rounds 305–316; full records 27/27 + 28/28 + 30/30 + 33/33 + 34/34 + 32/32 + 32/32 + 29/29 + 29/29 + 35/35, zero fails after the disclosed calibration protocols; the TEN probes embedded byte-exact in their sealed `-smoke` stage with SPEC SHAs pinned, plus a module-own exact section that re-proves the Hill/Lagrange chain with the Rényi bridge, the opening-flux lemma with the flux telescope, the collision counting identity, an r311-class exact rational Farkas mini-certificate, the ten sealed verdict bars and the four-level composition gate from scratch in pure Fractions with tipping mutants), suite $960 \to 961$. The day is frozen *in the reviewer's binding four-level structure.* **Level 1 — real formal theorems** (Lean, `rh/lean/`, already built and green; no Lean change ships with the promotion): `lstar_terminal_implies_master` (r305: L* + terminal $q_N < 1 \Rightarrow$ full master positivity via Schur bordering + principal submatrices; `augmented_prefix_positive` and `free_window_positivity` now *corollaries* — no third arithmetic axiom); the four closed Inertia theorems (incl. a from-scratch Sylvester criterion); the real PrimeWindow construction (r310: nodes $= \log p^k$, weights $= \Lambda(p^k)$ *derived*); source exactness by `rfl` through derivation, mass conservation of any folding map (aggregation is a quotient, not a projection) and the *strong* stabilization form onto the Weil prime side (r310b); the r309 rank-one update identities; the Hill monotonicity + Rényi-3 $\Rightarrow N_2$ bridge (two Lagrange witnesses, Fractions dev 0); the sorry census fell $9 \to 5$ — the two *true* formal holes stand alone (`lstar_subordination`, `terminal_positive_main`). **Level 2 — certified finite statements**: the Rényi-3 inequality $\sum_j q_j^3 \le 1.069\,(\log m)^2/m^2$ on all 57 rungs (r306, growing reserve, trend -0.322; $A = 1$ fails exactly at kz53/kz67 — polylog-squared is the honest form; the fiber target $n_{\text{eff}} \ge m/(1.034 \log m)$, above every demanded power $m^{0.888}$); the fixed-head census 77/77 (r307: the full flat arch edge restores macroscopic reserve, min 2.19×10^{-3}); the block-Green identity $Q = \sum \Delta^{\mathsf{T}} G \Delta + \frac{5}{7} t^2$ on the finite objects (r308, toy t^2 pin 4/7, f64 4.0×10^{-14}) *plus* strictness of the cone with exact rational Farkas certificates (r311: MINI#0 + SM1#0 fully exact in-span duals, 0/16 in-span PD samples decompose — membership strictly stronger than positivity); paired-cone soundness on 20714 steps (r309, 0 violations, 56 % certified); fold multiplicity *exactly* 2 and 93 % far-triple cancellation (r313); the signed cubic identity with *vanishing* boundary term (r314: $\sum_j |x_j|^3 = \Delta F + C_{\text{collision}} + C_{\text{boundary}}$ decided in exact Fractions *on the real w9 data*, every deviation the rational 0; the opening-flux lemma $F_1 \equiv 0$ makes $C_{\text{boundary}} \equiv 0$; f64 4.5×10^{-17} on all 57 rungs); the 21 small-m certificates (r316, $C_{\text{small}} = 1.0694$). **Level 3 — negative architecture decisions** (verbatim): FIXED-HEAD-DEAD (r307 HEAD-GROWS: no fixed $r \le 8$,

best reserve 6.5×10^{-6} four decades under the sealed bar — the one-atom anatomy a *coordinate*, not a *dimension*); PAIRED-CONE-NO-INDUCTION + PAIRED-CONE-WORLD-BLIND (r309 B1–B4: B2 partial at 56 %, B3 not reached, B4 missed — *B does not overtake A*); the r308 discriminator demarcated as the budget sign / a chordal restatement (r311: the Δ-sparsity graph chordal on every size, the trivial dual $Y_t = e_t e_t^{\mathsf{T}}$ closes the r308 stalls two-sidedly); COEFFICIENT-SIGN-WALL — **Lane A closed as a cone language** with the mechanism *named* (r312: block-psd membership without rank-one-SDD membership, obstructed at the budget/border sign; Farkas mass on the border/budget row, t-share 0.583/0.725; the entire obstruction in 2.4 % SDD-forbidden negative cross-mixings); FLOQUET-EXPANDING (r313: med sr 1.0600, 0/57 contracting — no transfer-operator contraction form remains open at chain level); TRIPLE-TYPE-MAJORANTS-WRONG (r313); PHI3-ALL-BLIND (r315: the entire break on kz55+kz67, FCIX 0.955/0.915 vs 0.629 — exactly where the flux cancellation dies); TWO-REGIME-DEAD (r316, with the anatomy: the obstruction family cuts *across* the FCIX stratum, kz53 at bulk-normal FCIX 0.654; the r306 first-5 protocol was *load-bearing*; kz55/kz67 near-one-block, TOP1 0.558/0.785 vs 0.18). **Level 4 — open mechanisms** (typed): the constructive PSD-G rule (closed as a lane, open as a question), the signed cubic flux theorem, the quantitative source functional, the `crossing_budget` mathlib gap, the definitional source bridge, L*, the terminal. *The claim split (reviewer verbatim, binding — not one* TRANSFER *title):* `PRIME.SOURCE.RANKONE.UPDATE.IDENTITY.01` [E] carries *only* the exact update identities (determinant dictionary, Sherman–Morrison chain, $\pm(\eta \pm e_t)$ border split, local Schur reserve, signed source updates); `PRIME.SOURCE.TENSOR.MECHANISM.01` [O] carries the open question whether a source-pure cofinal control arises from them — registered *after* r313/r316 with the documented negatives. `PRIME.L2.RENYI3.PROVENANCE.01` [O]: the pointwise GO stands as a certified finite statement, the provenance is open (trilogy state: identity exact, functional blind, two-regime dead; the R317 fork verbatim: a near-critical family coordinate *or* a certified exception family — reviewer adjudication pending). The universality-class framing: the *base* is fine-metric (METRIC-ONLY, twin invariance everywhere), the *fiber* is recursive (EPSTEIN holds, SCRAMBLE breaks) — shared algebra, not necessarily a shared mechanism; the reviewer maturity raise $4 \to 5$ (formal architecture 9.5) is documented as a *reviewer assessment*, not a machine-checked fact. Ledger: `PRIME.LSTAR.ARCHITECTURE_ADJUDICATION.01` [E] (the four-level structure verbatim); the claim-split pair above; `PRIME.LSTAR.CLOSED_FUNCTIONAL.01` [O] carries the Lane-A closure note and `PRIME.LSTAR.SUBORDINATION.01` [O] the wave-11 state (the two formal holes, the formal source interface, the adjudicated architecture). Mincut base 4 / refined 5 unchanged; no round in flight at this cut — R317 waits on the reviewer fork. **No statement about the Riemann Hypothesis is made.**

12.68 The red-team morning and the two forks (rounds 317–322, notes DCLXI–DCLXVI, 2026-08-27; promoted 2026-08-27)

Wave 12 is the *red-team / fork morning*: the reviewer's R317 fork executed both ways on the fiber, the new indefinite idea class adjudicated on the base, and — the center of the wave — an independent red-team reconstruction of the extraction chain from the two lemmata to Weil/RH, with its complete Lean repair — [`verification/v969_forks_and_redteam.py`] (rounds 317–322; full records $38/38 + 25/25 + 39/39 + 25/25$, zero fails; the FOUR probes embedded byte-exact in their sealed `-smoke` stage with SPEC SHAs pinned — `exception_families 04fbe5c0`, `indefinite_fork f2d98683`, `continuous_coordinate e68883ad`, `antiphase_sign_law 761b51d4` — plus a module-own exact section S0 that re-proves the two-sided concentration bracket $q_{\max}\Phi_{H1} \le \rho_2 \le \Phi_{H1}$ from scratch, re-establishes the U1–U3 red-team witnesses in exact arithmetic (the $B = -1$ window against the terminal type; the mesh-level-0 total node collision reduced to three rational facts about e on the certified bracket $685/252 \le e \le 685/252 + 1/35280$, kill $0 < 0$ exact; the adversary $(|F|+1)^2 < 5/7$ false for every F), gates the r320 separation/canonical-split certificates (witness mesh level 4 via $2^7 < 3^5 < 2^8$, reduction $1/8 < 2/5$ exact) and seals six verdict bars with tipping mutants under a wave-12 composition gate; no Lean call — the r319/r320 Lean rounds are consumed as *reports*, their artifacts in `rh/lean/` re-verified by `lake build` and the RH suite),

suite 961 → 962. **The red team (r319) and the repair (r320).** The audit reconstructed the chain independently and found three kernel-checked *type inconsistencies* three levels above the boxes (the r273 disease one level up): U1 — the r310b source bridge bound neither u nor B, so a $B = -1$ window represented the empty spec against `terminal_positive_main`; U2 — at mesh level 0 the tolerance log(anchor) admits the total node collision, and $p = X - 1$ forces the strict subordination to demand $0 < 0$; U3 — the pre-r320 `pair_margin_main` quantified $(Z_{\mathrm{loc}}, \mathit{runs})$ freely and forces its source predicate *empty*; plus the mesh-vs-anchor *cofinality seam* (`cofinal_prime_windows` proves the anchor direction at fixed mesh, not the H_{cof} refinement direction — **the false cofinality direction is documented, not solved**). The honest chain verdict is binding for every downstream surface: "two lemmata ⇒ RH" does *not* survive the literal reading — literally the two lemmata give *only window-local master positivity* for the opaque, currently witness-less `MainWindow`. R320 repaired the interface completely (bridge retype with u/B fidelity + separation discipline + `budget_pos`; the additional arch/border finding sealed by the new opaque `SourceExact`; `pair_margin_main` retyped onto the canonical extraction with `signRuns_sum` and `canonical_split` *proved*; three permanent sorry-free guards; the sorry-free witness at anchor 2, atoms $\{2, 3, 4\}$, mesh level 4; census 5 → 5 with two typed retypes). *The three-level proof graph (reviewer adjudication, binding; the two holes are named* ***window-local*** *on every surface from wave 12 on):* Level A — local window mathematics, *proved*: $L^*(w) + T(w) \Rightarrow \mathrm{MasterPositive}(w)$ ("for every canonical finite window, L* and terminal positivity imply augmented prefix positivity — this is a window-local theorem"); Level B — the universal window family, *the* two arithmetic window-local holes: $\forall w : \; L^*(w) \wedge T(w)$; Level C — extraction to Weil/RH, *open*: source-exact realization + admissible transport + convergence/stabilization in the correct directed order + density/continuity for the Weil criterion. Two binding caveats: the sorry census does *not* measure the Level-C distance (the missing extraction architecture appears in no `sorry`, because the correct theorem statements do not yet exist), and `SourceExact` — a good firewall — must not stand as a free assumption in a final proof: the named open Lean target is `CanonicalPrimeWindow` + exactly one construction theorem `sourceExact_buildPrimeWindow` (load-bearing, not formalization polish). **The fiber fork (r317/r321).** Fork (b) sealed WHAC-A-MOLE(kz53, kz83) by contract (38/38: the blind gap rule recovers the real spike family $B = \{kz55, kz67\}$ with finite certificates $C_B = 1.0536$, but class A stays empty and kz53/kz83 stay uncovered — the census find: the F_A top-3 are *exactly* kz53/kz83/kz67 = 2.47/2.39/2.38, one source-pure rank-local QMAX coordinate ranks the complete near-critical family on top, but as a *continuum*: the exception-family form is the wrong statement shape); fork (a) in the r317-sharpened form fired SLIDING-BOUND-GO(G_{SQ}) (r321, 39/39): $\sum_j q_j^3 \le 1.3056\, F_A(w)^2\, (\log m)^2/m^2$ for $m \ge 73$ with 0/39 test violations and *all four* named r316/r317 violators inside at reserves 7.0–9.6 — registered as `PRIME.L2.RENYI3.SLIDING_BOUND.01` [O] (honest: $C_{\mathrm{impl}} = 7.97$ is 7.5× looser than the r306 first-5 constant — form, not sharpness; the provenance question is now the two-part F_A/qmax split). **The base fork (r318/r322).** r318 adjudicated the Pontryagin/Krein idea class P2-MAIN-SPECIFIC (25/25): the P1 index route closes *as language* (the index statement is a total restatement of L*, the global inequality vacuous, the invariants world-blind — banked, no lever), classical sign regularity is dead on MAIN; r322 then closed the antiphase fingerprint as ALGORITHM-ARTIFACT (25/25): the $(D3, D4)$ sign law and the r312 97.6/2.4 anatomy are properties of the least-norm-proximal *Dykstra basin*, not of the psd solution set (the sealed random-start variants carry modal pair $(0, 2)$; the law is not identity-forced — exact Fractions analysis, positive forced value); the named residual is the *selection geometry. The typed reviewer assessment (supersedes the wave-11* 4 → 5 *scalar; an assessment, not a machine-checked fact):* finite dictionaries 9.5/10, formal window-local architecture 9/10, terminal mechanism 6/10, L* mechanism 2/10, source/extraction architecture 2–3/10, complete RH proof path ∼ 3/10, audit quality 9/10 — "stronger as a window-local research program; as a complete RH proof chain currently less far than previously assumed — the removal of a shortcut that did not exist." Ledger: `PRIME.REDTEAM.EXTRACTION_AUDIT.01` [E] (the audit + repair + the honest chain reading + the reviewer adjudication verbatim); `PRIME.L2.RENYI3.SLIDING_BOUND.01` [O] (the theorem candidate); `PRIME.L2.RENYI3.PROVENANCE.01` [O] carries

the executed fork, `PRIME.LSTAR.SUBORDINATION.01` [O] the wave-12 state — **the L* lane is internally paused** (allocation 0 %, reviewer decision; the selection geometry is not pursued further; reopening only under the six documented conditions; the external memo candidate: the two-weights sampling formulation $\|J_N\| < 1$). The terminal is sharpened to the QMAX $\times M_2$ shape route (R324 in flight, not consumed); the extraction-repair fork R325 is in flight (not consumed). Mincut base 4 / refined 5 unchanged. **No statement about the Riemann Hypothesis is made.**

12.69 The extraction repair and the terminal composition (rounds 323–327, notes DCLXVII–DCLXXII, 2026-08-27; promoted 2026-08-27)

Wave 13 executes the two lanes the wave-12 reallocation opened (55 terminal / 35 extraction / 10 formalization / 0 internal L*): [`verification/v970_extraction_and_composition.py`] (rounds 323–327; full records 36/36 + 36/36 + 18/18 + 34/34, zero fails; the FOUR probes embedded byte-exact in their sealed `-smoke` stage with SPEC SHAs pinned — `fa_provenance 9a6696f8` (r324-pre, banked), `qmax_m2_origin dc36cacb`, `extraction_order 31277f91`, `group_mass_cap 11e4fd40`; r324 imports the embedded r324-pre and r327 the embedded r324-pre *and* r324 — plus a module-own exact section S0 that re-proves the r324 interpolation $M_3 \le q_{\max} M_2$ with slack exactly 1/36 and double equality at the flat profile, the F_A identity $F_A B \log m = m\, q_{\max}$ as exact constructional algebra, the r327 group ledger (partition, cap chain, two-ancestor bound; both mutants break by exactly 1), the r325 onset formula $\alpha^* = (n_g + 1)D_0/2$ as an *exact toy stabilization*, and the m_0^* log-space solver verbatim (landing $10^{59.6}$; the disclosed pre-freeze linear-space bug rebuilt and caught) plus the r328B chain-audit gate S0-T5d, under six sealed verdict bars and a wave-13 composition gate; no Lean call — the r326 Lean round is consumed as a *report*, the r323 abort as a *note*), suite 962 → 963. **The clean abort (r323).** The planned L* indefinite-signature fork was aborted by the binding reviewer adjudication (L* paused at allocation 0 %) at the end of its design phase, *before any write access*: no probe, no measurement, nothing counts as a round. **The terminal composition (r324, with the pre-work banked as r324-pre; r327 the anatomy).** The round ran under a binding mid-round course correction, disclosed in full: the original F_A-boundedness contract was executed and sealed *first* (36/36 — FA-BOUNDED-DISTRIBUTIONAL: the F_A distribution matches the iid stationary max-statistic null, envelope $C_F = 3.357$; the honest tension disclosed: the per-rank p values at the named spikes are 0.005/0.000/0.000), then superseded — the cleaner decomposition is QMAX $\times M_2$. The binding contract fired PILEUP-GROWS-SUBCRITICAL ($e_{\text{tot}} = +0.172 < \text{CRIT} = 0.224$, both halves subcritical, the second falling — decided; a first-class outcome by the reviewer contract): S0 exact — $M_3 \le q_{\max} M_2$ Fractions-proved and F_A **de-black-boxed** by the identity $F_A B \log m = m\, q_{\max}$ (2.7×10^{-16} live: F_A is the rank-local normalization of $m\, q_{\max}$, not an independent coordinate); the pointwise M_2 export refuted (7/39, exactly the banked spike set) while the distributional r302 export holds; the mechanism half and half — the $O(\log m)$ *scale count* certifies pointwise ($C_{\text{NSC}} = 2.0258$, 0/39) but the per-scale mass is refuted (11/39): the near-critical windows are *one near-maximal heavy source scale* at the argmax block, not multiscale convergence; the composition typed MEASURED — $\sum_j q_j^3 \le 8.941\,(\log m)\, m^{+0.172}/m^2 \Rightarrow N_2 \ge m^{0.888}$ for all $m \ge m_0^* = 10^{59.6} \Rightarrow$ the full chain to `terminal_positive_main`; m_0^* is astronomical and **the gap** $(274, 10^{59.6})$ **is the disclosed extrapolation hypothesis** — no cofinal claim. An independent chain audit (r328B, same day, read-only) found the one moderate transcription error of the wave: the bar CRIT = 0.224 used the *atom-level* need 0.888, while $N_2 = n_{\text{eff}}$ is block-level (need 908/1002 ≈ 0.9062 in m units) — the chain-honest bar is 0.188, **the verdict survives** ($+0.172 < 0.188$, exact v970 gate S0-T5d), the chain-honest m_0^* is 10^{238} (the record $10^{59.6}$ belonged to the atom-level need; both astronomical), and the going-forward bar is 0.188; the sealed record is not edited — the correction is carried as a chain annotation. r327 then located the heavy scale structurally (CAP-PARTIAL with anatomy **SOURCE**): on *all 65 rungs* the heaviest group of the argmax block is *exactly one β/ω fold pair* (median alignment 1.000 — the ancestors reinforce); at kz53 the single pair carries 88.8 % of the argmax block at gap 0.076 — *the spike is literally one bulk/window coincidence*; the Λ-pair cap is refuted as a mid-ladder polylog (19/39, named 0/4 — the exact algebra holds, the constant

does not close), **the group count certifies** ($C_{NG} = 2.6351$, 0/39, named 4/4 — the third certified $O(\log m)$ count of the lane); the route halves straddle, so HEAVY-GROUP-UNBOUNDED does *not* fire — the r324 MEASURED chain stays the end state, and the named direction is the *coincidence geometry*: WHERE can the single heavy coincidence sit (source geometry, not group count or multiplicity). Registered as `PRIME.L2.RENYI3.MEASURED_COMPOSITION.01` [O]. **The extraction repair (r325) and the Level-C architecture (r326).** r325 adjudicated the three reviewer variants for the R319 cofinality seam: ELEMENTWISE-STABILIZATION-GO primary — the repair is a *quantifier statement*, not a mesh theory: on the native v749 class *all three channels* (comb/arch/pole) of the canonical tower windows stabilize exactly at the predicted finite anchor onset $\alpha^* = (n_g + 1)D_0/2$ and are constant under dyadic mesh refinement (comb 3.3×10^{-18}, arch 1.5×10^{-15}, pole 2.0×10^{-17}) — no mesh-cofinal ladder, no transport, H_{cof} **is not needed and is replaced as the target route**; variant B is the construction substrate of A (v749 T1.4b at rel dev 0.0), variant C the quantified density step (one-signed per-channel defect inside the rigorous interpolation envelope, no wall margin anywhere). r326 implemented the architecture in `rh/lean/RH/Elementwise.lean`: `sourceExact_buildPrimeWindow` *proved* (the wave-12 reviewer target — `SourceExact` eliminated as a free assumption from the extraction route), the comb stabilization elementwise *proved* (onset predefined from the element's support alone), and the extraction without the ladder `weil_nonneg_of_windowlocal` *proved* (one finite instantiation per element: Euclid anchor + native mesh); sorry census 5 → 8 with three *named typed Level-C statements* (arch/pole transcription classical, source completion definitional) — the wave-12 census reservation *partially discharged*. **The honest formulation of the new connection (binding, verbatim):** *two window-local lemmata (over the canonical family) + the elementwise architecture + cited classics (density, v912 continuity, the Weil criterion) ⇒ RH; open links: the arch/pole transcription (classical), the compression bridge, the source completion (definitional).* Registered as `PRIME.EXTRACTION.ELEMENTWISE.01` [E]; the window-local positivity premise is proved *nowhere*, and the two true window-local Lean holes are unchanged. L* stays paused. Mincut base 4 / refined 5 unchanged. **No statement about the Riemann Hypothesis is made.**

12.70 The wave-14 consolidation: terminal martingale, cover and growth, the closed margin chain, the Borodin duality and the second arithmetic (rounds 334–357, notes DCLXXXI–DCCIX, 2026-08-27; promoted 2026-08-27)

Wave 14 — reviewer-authorized after the double substance condition (r355 K2 NU-free + r357 second arithmetic) — freezes the E-ripe finite content of rounds 334–357 as *five* modules; unlike the embedding waves 8–13, every module re-derives its exact layer *from scratch* (sympy symbolic + pure Fractions, no probe imports) and re-runs the sealed round verdicts as exact decision logic on the frozen record aggregates with tipping mutants (the eighteen discovery probes stay sealed in `experiments/tfpt-discovery/`, pinned in `rh/INVENTORY.json`, re-verified by `run_rh.py`). **(1) The terminal density-martingale layer** ([`verification/v978_terminal_density_martingale.py`], rounds 339 + 341; `PRIME.TERMINAL.DENSITY_MARTINGALE.01` [E]): the density per remaining descendant $X(v) = d(v)/d(\mathrm{root})$ is a nonnegative martingale from *mass conservation alone*; the moment dictionary $E[X_\infty^2] = mM_2$, $E[X_\infty^3] = m^2M_3$, $\max X_\infty = mq_{\max}$ makes the terminal target a martingale moment (the banked r306 $C_2 = 1.0694$ certifies the $A = 2$ polylog form on the 57-rung set); the tilted tower $E[X_\infty^3] = E_Q[\prod \Gamma]$ with steps $\tilde{p}_c = p_c R_c^3/\Gamma$ (the untilted reading breaks by an exact Fraction), the exact hand-off $E_{3h} \le (mq_{\max})^2 \mathrm{msh}$ and the algebraic pair ceiling $\Gamma \le 4$ (via $4R - R^3 = R(2-R)(2+R) \ge 0$) are theorem-grade; the r339/r341 letters stay honest-negative on the budget side (the target grows 8× slower than the worst-case budget; $R^* = 3/2$ is a tuning surface) and the r324 MEASURED composition stays the end state. **(2) The three-arm cover, the growth law and K2** ([`verification/v979_cover_growth_k2.py`], rounds 344/346/351/353/355; `PRIME.L2.COVER_GROWTH_K2.01` [E], *Frame-A-typed census*): the FAB identity $F_A B = mq_{\max}/\log m$, the data-free K1 formula $R_{\mathrm{ALG}} = 4^{1/3} \to 8/5$ (interior, certified by exact cube midpoint comparisons) and the r355 mesh identity $h - \nu u \in (0, 3/2]$ are exact;

the first certifying three-arm cover (0/51 + named 4/4 + midband 6/6, $m_0^* = 10^{22.6}$), the P02 predictor ($F_A \geq 1.5$, 51/51) and the growth ceiling $C_{\mathrm{FAB}} = 14.93$ (fresh EXT5 tranche 0/6; polylog $m_0^* = 10^{18.9}$) are census at frozen constants — *all bound by the r353 Frame-A restriction*: the second construction family breaks the FAB ceiling (+21 %) and kills the reserve floor at $m \sim 760$ (the sliding coverage is *finite*); the one cross-family survivor is K2 ($F_A B\, g_{\mathrm{rel}} \leq 11.87$), certified NU-free over three window aspects in r355 — the gap geometry is the ν-free coordinate of the lane, its source chain exactly closed but with vacuous census caps (K2-SOURCE-CHAIN-OPEN). **(3) The closed L* margin-law chain** ([verification/v980_lstar_margin_chain.py], rounds 345/347/348/350/352/354; PRIME.LSTAR.MARGIN_CHAIN.01 [E]): the one-line identity $m_2'(p' + q' - m_2') = p'q' - c'^2$ (Vieta), the two-level dressed-scalar theorem (121/250, 79/250, 72/250 by hand; cross-tie $r_2' = 4375/9559$), the pinning theorem (orthonormal top-2 geometry ⇒ the dressed characteristic polynomial factors as $(t-m)(t-mg_{21})$, so m_2' = margin *identically*), the rate-equality theorem in exact ratio form and the ρ_K identities are theorem-grade; the censuses GAP-RATIO-PRIMARY-CERTIFIED (the pre-sealed curvature-honest protocol; the r343 soft decay re-adjudicated as a protocol artifact — the wave's method lesson), ALPHA-CLOSED ($\alpha = a_c + \delta$, residual 0.033), RHOR-REDUCED (one object at 500×), PHI-DICTIONARY-GO (computability, no closed form) and DELTA0-UNRESOLVED (pool exhausted at $10^{3.90}$) freeze the chain typed — weights dictionary, deficits candidate law, pinning theorem, $\rho_r = \varphi$-difference computable — and the lane goes to the specialist memo. **(4) The Borodin particle-hole duality** ([verification/v981_lstar_borodin_duality.py], round 356; PRIME.LSTAR.DUAL_HOLE.01 [E] — the wave's strongest single theorem): the complementation $\Pi_N^u + G\,\Pi_{N-1}^{u^\vee} G^{-1} = I$ holds bit-exact at half filling $S = 2N - 1$ with the reciprocal dual weight $u^\vee = 1/(uP'^2)$ and a *rational* conjugator; the exact spectral map gives $L^* \Leftrightarrow R > \frac{1}{2} I$ with margin $= 2 - 1/\lambda_{\min}(R)$; the r228 half-filling law *is* Borodin's rank condition (integer lemma); on the cosine lattice the dual weight is exactly $u^\vee \propto c_j(1 - x_j)/|f|$ — the original weight carries $+\log|f|$, the dual $-\log|f|$: *the r354 anti-correlation of the two φ blocks is duality algebra by design.* The honest main verdict is DUALITY-REPARAM-ONLY (no dual carrier, no compression): a reparametrization of the open condition, not progress on its truth — the lane is final at the memo, with RESERVE-LOCALIZED (the local dual 2×2 block predicts the log reserve without readback) and the AC-class exclusion ($a_{\rho K} = 1.4222$ vs the Lubinsky-type constant prediction) banked census-side. **(5) The matched Dirichlet frame and the second arithmetic** ([verification/v982_dirichlet_matched_frame.py], round 357; PRIME.WORLD.DIRICHLET_MATCHED_FRAME.01 [E], census on 126 windows): the χ-archimedean side is *derived* from $\Lambda(s,\chi) = (q/\pi)^{(s+a)/2}\Gamma((s+a)/2)L(s,\chi)$ — $F_A^\chi(\xi) = -\log(\pi/q) + \mathrm{Re}\,\psi((1+2a)/4 + i\xi/2)$, trivial reduction bitwise to zeta, conductor shift $\log 3$ exact, parity kernel $e^{-w/2} \to e^{-3w/2}$ at 30 digits — and with it SECOND-ARITHMETIC-LIVES: the r330 death was a *frame artifact*, the wall holds on 42/42 windows for χ mod 3 and mod 4 (living-world evidence $n = 1 \to n = 3$ arithmetics), the verbatim r330 battery retypes all four splits; mechanisms transfer (two-atom pair, determinant condition, dictionary) while the zeta-graded signatures do not (exact pinning, saturated reserve, φ suppression 439× vs 3×, counting constants): *the L* margin legislation is a statement about how close zeta's world sits to the wall*; K2 holds 0/126 (the third independent test of the v979 law); the matched scramble breaks — the wall needs the arithmetic; everything census-side is a finite census, kein Satz. **Adjudication.** The Block-B negatives are consolidated as prose and row updates only (the r334 capacity cap, the r336 parity and r340 transport world-blindness with the wall-is-global-rescue find, the r349 class law inside the v979 third arm, the binding r353 FLOOR-KILLS); PRIME.LSTAR.SUBORDINATION.01 [O] and PRIME.L2.RENYI3.SLIDING_BOUND.01 [O] carry wave-14 notes with markers *unchanged*; the L* lane is formally frozen at `rh/problem/lstar_problem.tex` with the one-object question halved. Mincut base 4 / refined 5 unchanged. **No statement about the Riemann Hypothesis or the Generalized Riemann Hypothesis is made.**

12.71 Kernel-Loewner positivity at $L = 0.3$ (rounds 494–495, promoted 2026-09-01)

Rounds r494 + r495 freeze as one module [verification/v1017_kernel_loewner_positivity.py] (PRIME.RDAGGER.KERNEL_LOEWNER.01, Numerical/certified float64 — not interval arithmetic, not [E]; suite

1009 → 1010, ledger 1173 → 1174). Unlike the embedding waves, the module re-derives the bound from scratch (no probe imports). *Scope is a single finite support:* $\mathrm{supp}(h) \subset [-0.3, 0.3]$, so $2L = 0.6 < \log 2$ and the classical prime term is empty. The x-space Weil form

$$Q_W(h) = \int_0^{2L} k(x)(g(0) - g(x))\,dx - c_L\, g(0) + \Pi(h),$$

with $k(x) = e^{x/2}/\sinh x$ and $\Pi(h) = 2(\langle h, \cosh(x/2)\rangle^2 - \langle h, \sinh(x/2)\rangle^2)$, agrees with the defining digamma multiplier at the calibration gate ($\sigma_A(0) = -5.3721834192256654$, Dirichlet Rayleigh 1.180×10^{-2}, odd Legendre sector 0.2226, c_L in hull). After zero-extension to $L^2(\mathbb{R})$, $I - \mathrm{Re}\, R_x$ is PSD by the Loewner identity $\langle h, (I - \mathrm{Re}\, R_x)h\rangle = \frac{1}{2}\|h - \tau_x h\|^2_{L^2(\mathbb{R})}$; a quintic C^2 cutoff books the omitted diagonal as κ_w. A 401-dimensional Legendre section of a degree-48 Chebyshev surrogate is enclosed by Bernstein Hilbert–Schmidt bounds; the exact tail is charged three times. The enclosed floor is 2.122×10^{-3}, so

$$Q_W(h) \;\geq\; 2.1\times10^{-3}\,\|h\|_2^2.$$

Independently (r495): the translation identity holds 6/6 exactly over $\mathbb{Q}$ on two step functions (boundary-strip mass strictly positive), and doubling c_L produces a negative certificate budget. *Honest boundary:* r496 is a method no-go for the same compact-tail completion at $L = 0.8$ — this row does *not* claim $\lambda_*(L) \geq 0$ for general L, is not cofinal, and is not evidence for or against the Riemann Hypothesis.

12.72 The T-B census at 69/70

The P4 main strike ([`verification/v712_rank3_transverse_deck.py`], 5/5) finishes what the density round started (sec:tb-chain): the v692 margin identity is *scalarized along the transverse direction* — the penalty pays only the transverse zero mass, not the full matrix norm — which improves the ten open windows by ×4.23 median and closes nine of them at the cited height 3×10^{12}. **The census now stands at** 69/70 **complete family windows closed unconditionally-modulo-citations**; the single remainder is $h = 5690$ with certificate height $T^* = 1.6 \times 10^{14}$ (down from 8.5×10^{14}). The deck-triplication lever, by contrast, is honestly dead for T-B (triangle inequality: channel sups can only lose) — the deck geometry helps the arch reading ([`verification/v705_chain_deck_sector.py`]), not the margin. The zeros $\leq 2 \times 10^4$ enter the *margin* only (RvM-checked); the penalty uses no zero data. T-B itself — full W3 positivity — remains the conjecture; no RH statement is made.

Part VIII
Status, contracts, and open problems

The closing part. Section 13 separates what is certified from what is measured and what is hypothesis input, states the asymptotic mountain that stands between all of it and any infinite statement, and names the extraction candidates — the community-checkable pieces. The open research contracts of the prime line (`PRIME.Z1.OPERATOR.01` — sharpened 2026-08-05 with the measured qf handover constraints after the closure of `PRIME.GRAM.DIAGONAL.01` (Section 12.11), and executed the same day as the `v780` trilogy (Section 12.12): the compactness half is measured carried, and the selection half — once the one shared open object with `PRIME.KMS.INDUCTIVE_STATE.02`, merged target Z1-COMPACTNESS — was finite-level solved in round 23 by Mosco+Friedrichs (Section 12.16), leaving the sector floor as the single analytic remainder (`PRIME.FLOOR.RATIO.01`, narrowed in round 24 by the certified floor skeleton of Section 12.17 and depth-hardened in round 25 (Section 12.18): the kill gates survived at full sieve depth, the α-bound fell at citation grade, the envelope is certified-explicit at $c_{\mathrm{cert}} = 4.335$, and the alias-phase proof blocker is named — and closed on its analytic-envelope flank in round 26 (Section 12.19): the blocker resolves into a typed pair-correlation circularity boundary, the route is stop-listed, and the demand moves to the

bridge contract `PRIME.FLOOR.PAIRCORR.01` [O] — still open; round 27 (Section 12.20) then decides both corner-era routes at that wall: the corner route's structural half closes weight-generically (the identity is comb-blind, the Hjelmslev CP tower strictly projective and structure-only — the wall relocates into the identification step), the commutant SOS route closes definitively by exact certificates, and both are stop-listed with the bridge contract unchanged; round 28 (Section 12.21) completes the closure quantifier (doors, tower levels, expectations, position carriers — the identity is an extremal state condition), quantifies the zero side (the demand saturates the GUE boundary structurally, the bootstrap loop is short), and REOPENS the route with the relational input: the identification carrier exists (`PRIME.RELATION.MULT.01` [O]), the identified corner's excess is positive on all 67 rungs, and the certified skeleton (`PRIME.RELATION.SKELETON.01` [O]) types the wall as the infinite quantifier over strictly-positive certified enclosures — the sharpened demand: a uniform lower bound on the excess margin; round 29 (Section 12.22) closes the arc: the margin becomes the doubly-derived typed law $\tau = e_1\, h^{-3/2}\, \tau_{\mathrm{pnt}}$ (the envelope constant reproduced exactly across both coordinate systems), the tower gives scale not recursion, the excess is a growing cancellation (the wall self-similar at cell level), and the finite-table limit is kernel-checked in Lean (`PRIME.MARGIN.LAW.01`, `FORM.PRIME.EXCESS.SKELETON.01`) — the quantifier now sits on the one scalar series $e_1 \geq 4.335$; and round 30 (Section 12.23) decides the three candidate proof architectures — the relational Schur–Gram (structural: the forced Cauchy–Schwarz price, geometry-independent by the unitary spectral mother), the sign-register wedge (exact lift, uniformity failure), the completed-cell cone transport (broken at the $n = 2$ cell, the arithmetic in the violations) — and completes the continuum extraction: the implication chain *cofinal finite positivity* (cofinal in the mesh-refinement order) $\Rightarrow$ *Weil positivity* $\Rightarrow$ *the target* is theorem-grade modulo named citations (`PRIME.EXTRACTION.CHAIN.01` [O]), so the wall is exactly hypothesis (H) and nothing else; and round 31 (Section 12.24) draws the invariance atlas — the nuclear-norm tax theorem closes the positive-dilation class in any basis, the index route is endpoint-determined, the corner cluster route is position-geometric, the cone violations resolve into the graded kernel law $D(X) \sim 0.75\, X^{-0.55}$ (`PRIME.CELLCONE.GRADEDKERNEL.01` [O]), the criteria align (BD $= 2\lambda_1$; the NB span is the spectral-mother geometry; the Li transfer fails by cone geometry), the sieve input class reaches the band in scope but not in grade — the wall is invariant across compressions, geometries, criteria and input classes, and its minimal hypothesis is kernel-checked: (H_{cof}), with `UniformMarginBound` formally demoted to an over-strong sufficient lemma (`FORM.PRIME.COFINAL.WEIL.01` [F]); and round 32 (Section 12.25) unifies the graveyard under the kernel-checked *grade law* — grade-1 targets never live inside grade-2 Gram squares, the affine escape is priced by an exact PSD tax, and the measured exchange rate is 10^4–10^5 (`FORM.PRIME.GRADE.NO_GO.01` [F]) — closes the finite-head phase-rescue family by theorem (`PRIME.PHASE.LEVER.01`), closes the Lévy/conditional-positivity elevator class on the deployed sides (both densities signed; the arch digamma band ends at $\tau^* = 6.27$), and adds the two typed positive cores: the tax-evading pair-correlation instrument (`PRIME.RELATION.CONNECTED_COVARIANCE.01`) and the conditional kernel-checked frame *three hypotheses* $\Rightarrow$ (H_{cof}) (`FORM.PRIME.COFINAL_CURRENT.01` [F], its second hypothesis measured empty at the deployed frame); and round 33 (Section 12.26) gives the wall its operator form — the exact Krein normal form with the Douglas pencil identity, the rank-one defect ($\lambda_1 =$ the τ-margin exactly), the kernel-checked defect theorem with `SourceContractor` the named hypothesis (`FORM.PRIME.KREIN.DEFECT.01` [F]), the closed-form target-free contractor formula $C = W_- F G_+^{-1} F^{\mathsf{H}} W_+$ with certified defect transfer, the *Perron closure* ($\rho(|C|) > 1$ everywhere: no absolute bound can ever certify — the certificate must be phase-aware), the quadrature reach with its intrinsic depth price, and the register canonical up to proven gauge — the sharpest demand registered as `PRIME.KREIN.CONTRACTOR.01` [O]); and round 34 (Section 12.27) names the kernel and preregisters the next test — the contractor's reproducing kernel is *exactly* the Christoffel–Darboux kernel of an explicit source measure (rank-2 displacement collapse with a 10^{13} gap, firewalled chain-only reconstruction; the honest boundary typed: the collapse is window geometry, the arithmetic is the Jacobi chain and the Christoffel weights), the defect splits exactly as $I - C^*C = T_1 + T_2$ (indefinite Jacobi cross-geometry plus PSD Christoffel damping) with the

exact global minimizer equal to the soft direction and the wall as two explicit source numbers ($s = \kappa\tau$), the classical pole transformations and finite-rank compensations are closed with typed residues (`PRIME.CD.DAMPING.COMPENSATION.01`), and the Prüfer/Cotlar route is committed as the frozen preregistration `PRIME.PRUEFER.COMPENSATION.01` — committed byte-exact, not run; and round 35 (Section 12.28) executes that contract — the program's first fully preregistered uniformity test returns COTLAR-GROWING by its own frozen rules on the data as measured (`PRIME.PRUEFER.COMPENSATION.01`, executed); and round 36 (Section 12.29) resolves and *reverses* it — the deep-holdout explosion was a silent comb-truncation artifact of the deployed atom table (DEEP-ALPHA-ARTIFACT: complete-comb floors strictly positive, no negative canonical window), the identically frozen v1 rule returns BOUNDED on complete combs and the route *reopens* with h-stable envelope constants (`PRIME.PRUEFER.COMPENSATION.02`; the remaining gap typed: $B_C \sim 4$ vs 1, local defects 7–10), the Carleson chain is made machine-exact with embedding constant $1 - \tau_h$ and the surviving analytic target is registered in embedding coordinates (`PRIME.CARLESON.PRIME.01` [O]), the polygon and holonomy compressions are closed with typed reasons, the residual quadrature certifies the floor constructively at full degree on all 44 floor-positive rungs, and the π-resonance is measured *thick* — the wall is phase-distributed (`PRIME.TRUNCATION.AUDIT.01`); and round 37 (Section 12.30) closes the partition chapter entirely — the pointwise bound is *derived* with h-uniform source constants ($C_{\text{int}} \in [1.23, 2.33]$, the route's first analytic uniformity statement), but the cell-envelope decay is cancellation-carried, no cell accounting reaches below 3.57, and honest tight wave-packet frames leave $\leq 18\%$ diagonal mass with a growing Schur constant — the cancellation is *atomically global* and the certificate demand is retyped non-decompositional (`PRIME.PARTITION.CLOSURE.01`); and rounds 38/39 (Section 12.31) give that non-decompositional demand its coordinates — the wall is one operator seen four ways (the rank-2 Jacobi displacement, the gauge-exact Carleson Gram with the classical testing diagonal, the exact port Schur reduction with $(1 - \lambda_{\max}(D_P))/\tau = 1.00$ on all rungs — the port *is* the wall — and the preserved IIKS integrable class), its ∼39% port mass is the classical PNT edge law $1 - e^{-1/2}$ under $\sqrt{n/X}$-uniformization with the Mellin–Cauchy port symbol $1/(1+2i\tau)$, the criticality budget closes ($+1.405 - 1.674 + 0.284 = +0.015$: the wall is a prime error-term statement at the port frequencies), the hardness is decided (`PRIME.ERRORTERM.SCALE.01`: the reduction chain is an exact reformulation at RH strength — no unconditional slack), and the port/scalar/cocycle/sum-rule/Riemann–Hilbert coordinates now exist as named contracts (`PRIME.PORT.LIMIT.01`, `PRIME.PORT.SCALAR.01`, `PRIME.PORT.COCYCLE.01`, `PRIME.CASE.SUMRULE.01`, `PRIME.PORT.RIEMANNHILBERT.01`, all [O]); and rounds 40/41 (Section 12.32) open the flow corridor — the wall's sign is an exact finite determinant chain $\operatorname{sign}\tau_h = \operatorname{sign}\sigma_h$ with $\tau_h := \det(I - D_P)$ (`PRIME.PORT.TAU.01` [O]: the symbolic IIKS tau identification is the demand — since executed at the finite-identity level in round 224, `PRIME.PORT.TAU.FINITE.IIKS.01` [E] via `[verification/v955_tau_iiks_toda_dictionary.py]`, with the sign question refined to `PRIME.PORT.TAU.NOPOLE.COFINAL.01` [O]), the parametrix question is decided (`PRIME.PORT.LATTICE.PARAMETRIX.01`: the Euler-product comb is globally rigid — every smoothing-based parametrix is dead, the small parameters live in the flow direction), the first rigorous positivity statement of the program lands (`PRIME.PORT.CERTIFIED.HEAD.01`: $\sigma_h > 0$ proven on three head rungs by exact integer Sylvester certificates, modulo the declared conservative evaluation error model), the asymptotic one-sidedness is registered as `PRIME.PORT.LEADING.SIGN.01` [O] with `PRIME.PORT.TAIL.01` [O] carrying the head-plus-tail composition (head delivered on three rungs; the rollout and the ball-arithmetic closure named — both since delivered, `v887`/`v897`), and the induction anatomy is measured (`PRIME.PORT.ISOFLOW.01`, `PRIME.PORT.LADDER.TRANSFER.01`, `PRIME.PORT.PRIMELADDER.01` — positivity is a property of the complete comb — and `PRIME.PORT.LAX2.01`: the conditioned flow coordinate is the named open object, since adjudicated LAX1-H-EXACT in round 225, `v955`); and rounds 42–49 (Section 12.33) complete and decide — the certificate rollout finishes (`PRIME.PORT.CERTIFIED.LADDER.01`: $\sigma_h > 0$ proven on all 42 reachable rungs, 12 exact-rational plus 30 validated-precision, modulo the declared evaluation error model — since re-proven interval-rigorously in rounds 54/55, `PRIME.PORT.BALLLADDER.01`: 15 exact-rational plus 27 validated-precision with the informal error model retired), the four route

decisions land (PRIME.PORT.DIAGSEP.01, PRIME.CASE.ZONESPLIT.01, PRIME.CASE.PNTGAMMA.01, PRIME.CASE.SIGNEDHOMOTOPY.01 — the wall dies through off-diagonal coherence, PNT is insufficient, and the sum-rule route PRIME.CASE.SUMRULE.01 is retyped conditional: its input is pair-correlation class), the surviving positivity architecture is measured (PRIME.PORT.HIROTA.01 [O]: flag positivity — all leading principal minors positive, Sylvester, not classical total positivity — on 37/37 rungs with the wall margin as the pole distance of the s-flow; PRIME.PORT.TAU.MOEBIUS.01: inheritance is multi-factor), and the Möbius/carrier-invariance route is closed with mechanisms (PRIME.PORT.MOEBIUS.KILLS.01; the deep-core coherence is the surviving remnant); and rounds 50–56 (Sections 12.34 and 12.35) assemble and rigorize — the bounded-margin skeleton stands with one open statement (PRIME.PORT.ETASOURCE.01, PRIME.PORT.RELFLAG.01), the certificate base becomes interval-rigorous (PRIME.PORT.BALLLADDER.01: the informal error model retired), the soft pivot is an exact deflated-Christoffel evaluation with its positivity premise honestly typed as a coordinate change (PRIME.PORT.CHRISTOFFEL.RATIO.01), the conditional pair contract becomes a positive-weight norm square on the finite band with the boundary term carried exactly (PRIME.CASE.KERNEL.SOS.01, PRIME.CASE.EDGEDEFECT.01 — the hypothesis itself stays conditional), and the core update anatomy is exact and two-dimensional with the region program's boundary conditions registered (PRIME.PORT.NORMALIZED.CORE.01, PRIME.PORT.GRAPH.REGION.01); and rounds 57–59 (Section 12.36) draw the mechanism map — the wall splits exactly into two source-only scalars over a measured $O(1)$ co-block floor with the first substantive τ-screen pass and a dead classical certification route (PRIME.PORT.TANGENT.SCHUR.01, PRIME.PORT.BFLOOR.01), the pivot rewrite and the moment-matrix identity relocate — never remove — the difficulty (PRIME.CASE.EDGE.CHRISTOFFEL.01, PRIME.PORT.GRAM.COMPLETION.01, PRIME.PORT.RADAU.WEIGHT.01), and the 12-bit health word is typed a pure diagnostic with zero RH content (PRIME.ARITH.HEALTHCODE12.01)) — PRIME.DETECTOR.WINDOW.01 (registered in round 25: the comb-native window detector, validated out-of-sample) — and PRIME.Z1.MOONSHOT.01) stay open, exactly as stated in place. The final fence at the end of the section governs the entire document: no claim of progress on the Riemann Hypothesis is made anywhere.

13 Status and outlook

13.1 What is certified, what is measured, what is hypothesis input

The three-way split, stated once

Certified (exact identities or closed-form bounds, machine-checked): the forty-nine ledger modules of Section 4; and, in the sandbox arc, the bare bound b_0 in closed form, the parity superselection $JQJ = Q$, $JB = -BR$, the pole splitting $ab^{\mathsf{T}} + ba^{\mathsf{T}} = ss^{\mathsf{T}} - tt^{\mathsf{T}}$, the channel inertia, the avoidance law $Q_{\text{full}} \succeq 0 \Rightarrow \|B^{\mathsf{T}} v_i\|^2 \leq w_i \Lambda_-$, the growth inequality, the accumulation bounds, the Sherman–Morrison/Woodbury reduction of (R) to $r \leq 1$, the three T108 identities (ε as a $Q|_{\text{odd}}$ energy, the rank-one overlap $hh^{\mathsf{T}}/\varepsilon$, and ε as the square of the last Cholesky pivot), the two T109 certificates (the graded matrix cap $\omega_{\text{cert}} = 0.2561$–$0.7569$, no hypothesis input, and the residual certificate for the boundary value, sharpness 1.0000–2.4937, 16/16 in budget), the three T110 pieces of the propagation circle (the exact splitting of the growth step, embedding error $\leq 2.6 \times 10^{-14}$, with the new atom's restriction to the old window the exact zero matrix; the graded Loewner minorant step, valid exactly under the induction hypothesis, holding 15/15 with retention 1.0000; and the base-case Cholesky certificate at $n = 2$, $\lambda_{\min} \geq 6.93 \times 10^{-2}$ at three resolutions), and their T111 deep-ladder extension (117 handover certificates at retention 1.000000, largest single loss 5.96×10^{-8}; atom entry the exact zero matrix on 117/117; and the arithmetic ladder-wall statement at frozen cell width, gap $> 2D$ needed against the twin-prime gap 0.003831 at $521 \to 523$), the T114 margin-free step certificates (Albert/pseudo-inverse Schur, 27/27 ladder steps to $n = 1331$ including all seven torn zones, with the exact 4×4 Schur complement at 42–67% of the block scale — no margin division anywhere), and the T115 compression certificates (the exact identity $X_{\text{mixed}} = Q_{\text{old,mixed}}$; Albert on 66/66 (zone, q) combinations; the one-sided second-order compression error; the deep margin-free step at $n = 155\,921$ and the ten-step chains — certificates for a coarser Galerkin space, declared as such), and the T116 boundary-recursion identities (the partial-minimisation identity $t^{\mathsf{T}} T^{-1} t = r^{\mathsf{T}} \Sigma^{-1} r + \sigma$ on six zones; the Haynsworth double-Albert

equivalence 6/6 with sharp negative controls; the exact Riccati chaining, bit-exact on real windows; and the exact carriage of the global rank-one pole in the bordered state — the deep march itself certifies the band model's cost geometry, declared as such), and the T117 epsilon-theorem package (the four Galerkin identities — the cell functional of $2\sinh(x/2)$, the exact nesting $T_c = P^{\mathsf{T}} T_f P$, the two-level Pythagoras, the dual-norm residual form, all to 10^{-10} or better; the two-level lower-bound chain, every link an identity or a completed Cholesky, positive on all 19 nested pairs; and the exact jump formulas — the Levinson bordering drop r_0^2/s_0 and the rank-$(k_0{+}1)$ Woodbury corner update, both to 10^{-10}), and the T118 symbol package (the closed-form two-grid symbols — σ_S the weighted harmonic and σ_z the arithmetic mean of the aliasing pair; the saturation identity $\varepsilon_c - \varepsilon_f = y^{\mathsf{T}} S y$ to 4.9×10^{-11}; the CBS identity $\lambda_{\min}(L_z^{-1} S L_z^{-\mathsf{T}}) = 1 - \gamma^2$ to 2.1×10^{-15}; the isometry ordering onto $T_h(\sigma_z)$; $\inf \sigma_z > 0$ with explicit second-order margins on 7/14 windows; and the pole-mass identity in the oscillation space, $\le 2.1 \times 10^{-5}$ via the DTFT factor $|\sin(\theta/2)|$), and the T119 arithmetic package (the closed-form comb symbol — one cosine per prime-power atom, linearly interpolated over the two nearest lags — with the parity rule for σ_z; the atom-count bound $|\sigma_z^{\mathrm{comb}}| \le \Xi(\alpha)$, uniform in D; the archimedean growth identity $\inf \sigma_z^{\mathrm{arch}} = \log(1/D) + B$ with universal $B = -1.0474$; the resulting D_0 theorem; the large-sieve section certificate on one window (floor $+0.396$ where $\inf \sigma_z = -0.349$); the Levinson corner identity and innovation mass budget; the identity $\kappa_{\mathrm{end}} = 1/(1+R)$ to 1.1×10^{-16}; and the coarse-graining fence $\lambda_{\min}(T_h(\sigma * K)) \ge \lambda_{\min}(T_h(\sigma))$ by unitary averaging), and the T120 Harnack package (the exact increment system $Kv = s - u_0 G\mathbf{1}$ to 2.4×10^{-12}; the exact symbol identity $\sigma_c \sigma_z - \tau^2 = f_1 f_2$ to 3.9×10^{-16}; the pairing certificate (C1) — a proved implication whose single hypothesis is one sign of the corner increments, giving $|R - 1| \le 0.04745$ and $\kappa_{\mathrm{end}} \ge 0.480827$ for $n_C \ge 8$; the frame identity $\alpha_o = u_k/2 + O(D)$ over all 732 handovers; and the universal-B evaluation of the D_0 criterion on all 1492 zones, $\alpha^* = 0.693$), and the T121 wide-window package (the shifted-Cholesky section floors on the dense frame windows; the Christoffel/Fejér per-dip budget, positive on 8/16 frame windows up to $\alpha = 3.446$ at sharpness 0.20–0.76; and the exact deficit identity $y^{\mathsf{T}} S y / \mathrm{SUPPLY} = r_{\mathrm{ray}} r_{\mathrm{cbs}}/(1-\gamma^2)$ to 2.8×10^{-14}), and the T122 net-balance package (the closed form of the compressed Hankel term, $(Z^{\mathsf{T}} H Z)_{jk} = b_{j+k}$, to 7.0×10^{-16} on 36/36 rows; the two-grid isometry identity $A_z = W^{\mathsf{T}} \mathrm{Toeplitz}_M(c) W$ with $W^{\mathsf{T}} W = I$ exact; the certified band bound (5R) and its uniform y-free variant, non-vacuous on 36/36 rows to $\alpha = 6.31$; the structural Rayleigh bound, positive and sound on 36/36 rows with residual slack 1.00–1.03; and the exact residual decomposition of the fitted balance, closing to 2.9×10^{-16}), and the T123 structural-CBS package (the two-isometry completion $V = BP$ with $V^{\mathsf{T}} V = I$ and $V^{\mathsf{T}} W = 0$ exact, the three-compression identities to 3.4×10^{-15}; the closed forms of the coupling — half a first difference of the lag sequence to 1.8×10^{-15}, the aliasing-difference symbol to 5.7×10^{-16}; the uniform oscillation positivity (5R*), $\lambda_{\min}(E_z) = +0.303\ldots + 5.017 > 0$ on every row at slack 1.0007–1.007; the exact dyadic depth-layer split of $y^{\mathsf{T}} G y$ to 1.1×10^{-15}; and the two-term residual split of the certified balance, closing to 2.8×10^{-17}), and the T125 assembly package (the integer window nesting $M_n(k) = M_o(k+1)$ on all 29 consecutive pairs of the composing run with the incoming atom block bit-exactly zero and the composition residual 2.9×10^{-14}; the four assembly identities on 88/88 rungs of 52/52 zones; (8R) certified on 88/88 rungs at $\mathrm{cb}/\delta = 0.9071$–$0.9971$; the ε lower bound on 52/52 zones at 3.2×10^{1}–8.7×10^{10} over the declared floating-point bound; the base-case equivalence $\operatorname{sign} \lambda_{\min}(Q^{(L)}) = \operatorname{sign} \varepsilon$ on 52/52; and the margin-free handover on 62/62 steps with the exact Schur complement 3.0×10^{10}–4.6×10^{13} above its floor while the norm surrogate is negative throughout — 430 completed Cholesky certificates in total), and the T126 seam package (the exact DP decision of the partition question on the real gap table — $\rho_{\mathrm{crit}} = 2.0258$, no partition at 1.83, refinement-only infeasible at any ratio ≤ 64; the Haynsworth reproduction of $\lambda_{\min}(S)$ to 2.8×10^{-14} with the transport bracket containing the fine value on 30/30 real pairs; the twelve certified free-resolution seams, eleven in one re-gridding and the one at $n = 31$ through a three-stage ladder; the end-to-end seam at $\rho = 2.251$; and the exact fractional-Dirichlet identity of the window form).

Measured (holds on the resolved windows; no uniform constant): ρ_-; the ratio $r = 0.0053$–0.1814 and $\hat{r} < 1$ at all 92 resolution points; $\omega = 0.2366$–0.7531; the strict odd-channel margin $\lambda_{\min}(Q|_{\mathrm{odd}}) = 1.4 \times 10^{-5}$–$5.1 \times 10^{-2}$; the margin map $m_k \ge$ need109 on 16/16 zones (ratio 2.09–179.6), extended by T111 to 199 zones with the crossing *measured* between $n = 461$ and $n = 463$ ($n^* \approx 462$, an upper bound; the T110 extrapolation $n \approx 170$ was a fit artefact); the reserve trend $f_{\mathrm{crit}} \sim n^{-0.39}$ (fit); the requirement wall $n = 727$; the closure map 44/64; the handover margins; the two race slopes; the T114 ratio closure $r = 2 \times 10^{-4}$–0.103 on 27/27 steps and the D-powers of ε and κ (fits); the T115 transport split (the bracket's lower end positive on 7/11 regrid pairs, threshold $\rho^* = 1.83$ against a median ratio 2.001) and the fall laws $\lambda_{\min}(S) \sim \rho^{-1.69}$, $\varepsilon \sim \rho^{-1.71}$ (fits); the T116 law $\varepsilon \approx 8.34\, D^{1.790} \alpha^{-6.04}$ (jackknife ≤ 0.041), the factor-120 jumps at prime-power entries, the comb truncation budget growing like $n^{2.17}$,

and the criticality $\tilde{t}^{\mathsf{T}}T^{-1}\tilde{t} \to 0.999975745$ (fits/measurements); the T117 rates $\theta'(\text{sum}) = 1.74$ against $\theta = 1.76$ (loss 0.04 powers of D) with bound/$\varepsilon \in [0.111, 0.185]$, the growth of $\lambda_{\min}(S)$ under refinement, the entry direction (all 23 prime-power entries *raise* ε — correcting the T116 factor-120 reading, a sweep artefact), and the saturation ratio in $[0.675, 0.744]$; the T118 measurements: the corner-exponent drift $+0.238$ per halving (3–7.4σ), the concentration growth $\sim h^{0.99}$, the CBS floor $1 - \gamma^2 \geq 0.181$, the computed saturation band $\beta \in [0.252, 0.336]$ on 20 triples, the log-versus-power growth decision on the 128× FFT lever (fits with jackknife bands), the reduction sharpness 32–76% on 14/14 windows, and the zero crossings of $\inf \sigma_z$ on 3/5 zones; the T119 measurements: the mass distribution of $\|y\|^2$ (corner 0.18–0.42, middle 0.19–0.38, inner half 0.36–0.48), the exponents $\varepsilon \sim D^{1.761}$, $\|y\|^2 \sim D^{1.961}$, $\lambda_{\min}(S)\|y\|^2 \sim D^{1.757}$ (fits), the flat band $\kappa_{\text{end}} = 0.4906$–$0.5093$ with drift ≤ 0.0019 per level, the ratio $R = 0.9634$–1.0349, and the wide-zone crossing depths $M \sim 10^{13}$–10^{187} (extrapolations of the certified growth law); the T120 measurements: the corner sign structure (the T^{-1} corner block positive share 1.0000 against a TP2 minor share of only 0.51–0.88 and a K^{-1} positive share of 0.44–0.51 — the two refutations), the sign purity 1.0000 on all 3915 rows with $R \in [0.9098, 1.1011]$, the per-cell ratio $|a_j/w_j|$ up to 4.8×10^3 (the per-cell inequality is false), the (C1) decay $n_C^{-0.58\pm0.05}$ (fit), the comb-supremum incoherence 0.038–0.603 of Ξ (a grid supremum), the large-sieve floor usable on 2/6 zones, and the γ drift $1 - \gamma^2 = 0.079$–0.283 with fitted slope -0.297 ± 0.013 in α; the T121 measurements: section positivity on all 16 frame windows to $\alpha = 6.277$ (the ten large rows Lanczos measurements, upper bounds by construction), the net balance $\text{SUPPLY}/\varepsilon_c \sim D^{-0.177\pm0.096}\alpha^{-1.565\pm0.135}$ with per-resolution refits $-1.88 \ldots -1.62$ (fits), the link-(5) ratio 0.357–0.916 on 42/42 rows (the Hankel refutation), the Rayleigh and CBS slacks 1.29–13.19 and 3.42–16.67, the one-sign run 24/24 at $n_C = h/16$ against 21/24 at $h/8$ (run-share slope -0.0258 ± 0.0042, a fit), the Fisher–Hartwig drift $-0.008 \ldots + 2.934$ against the allowed -0.120 (inconclusive, declared as the result), the no-cancellation ratio 4.6×10^{-8}–7.1×10^{-5}, and the pairing quotient $\delta = 0.0126$–0.1331 over 360 rows ($\kappa_{\text{end}} \geq 0.4688$, $\delta \sim n_C^{+0.29}$); the T122 measurements: the Hankel/Toeplitz norm ratio 0.123–0.781 (the Weyl floor positive on only 12/36 rows), the band-bound recovery 0.429–0.995, the high-band mass of Wy (99.3–100.0% against 0.1–1.7% for y itself), the actual CBS coupling $\kappa_y = 0.0067$–0.5311 against $\gamma^2 = 0.7173$–0.9647, the certified balance $S_2/\varepsilon_c \sim D^{-0.003\pm0.075}\alpha^{-0.729\pm0.090}$ with the measured-CBS balance $\alpha^{-0.113\pm0.063}$ against the ceiling $\alpha^{-0.116\pm0.062}$ (fits), and the band margin 0.024–$0.363 \sim D^{-0.042}\alpha^{-0.525}$ (a fit); the T123 measurements: the band mass of Wy (99.64–100.00% at $d = |\theta - \pi| < \pi/2$ against 8.9×10^{-6}–1.5×10^{-3} in the $\theta = 0$ cell) and of the worst-case y^* (81.22–100.00% middle-band), the coupling range $\kappa_y = 0.0067$–0.4622 against $\gamma^2 = 0.7248$–0.9467 (a factor 2–109), the coarse conditioning $\text{cond}(A_c) = 2.1 \times 10^4$–$3.1 \times 10^7$ with $\lambda_{\min}(A_c) = 1.7 \times 10^{-5}$–$2.9 \times 10^{-4}$ (envelope-margin ratio 8.4×10^2–4.0×10^5, rising like $\alpha^{+4.202}$, a fit), the two-factor overshoot 1.20–3.51 with the certified margin positive on 5/24 rows, the certified balance $S_6/\varepsilon_c \sim D^{-0.069\pm0.099}\alpha^{-0.544\pm0.113}$ against the ceiling $\alpha^{-0.044\pm0.076}$ (fits; gap $\alpha^{+0.500}$), and the lid share $\varepsilon_f/\varepsilon_c = 0.177$–$0.546 \sim \alpha^{+0.030}$ (a fit); the T125 measurements: the retention against the measured ε_0 (0.0013–0.9694) and against the L-level ceiling (0.9140–0.9929), the pairing band $C_1 = 0.01063$–0.20927 over the ladder (worst case a single shallowest zone; $C_1 \leq 0.06721$ on the remaining 51), $R = 0.8243$–1.0377 and $\kappa_{\text{end}} = 0.4907$–$0.5481$, the head-room minimum 31.6 at $n = 243$, the rung-level balance $\text{cb}/\delta \sim D^{-0.010}\alpha^{+0.001}$ and the run-level $\sum \text{cb}/\sum \delta \sim \alpha^{-0.012}$ (fits), and the (S4) cancellation ratio $|r_{\text{corr}}| = 0.3888$–$14.8018$ with the norm-only variant returning exactly 0; the T126 measurements: the transport reach (positive to $\rho = 2.251$, first failure at 2.061, an overlap band rather than a clean split), the fixed-ratio zone sweep (12/22 zones at 2.0256), the free-resolution overhead $D_{\text{free}}/\text{cap} = 0.021$–$1.000$ (median 0.201) with re-gridding at 764/9 700 boundaries, the seam retention (median 0.578), the pencil alignment 0.866–0.987 (median 0.975), the Galerkin consistency orders $D^{1.798\pm0.057}$ and $D^{1.802\pm0.065}$ with Richardson limit 2.1256×10^{-4} (fits with jackknife bands), and the diverging continuity constants of the refuted naive continuum route; every scaling exponent quoted in this document.

Hypothesis input / classical (used, named, not proved here): Kneser, Witt/Eichler, Shimura, Waldspurger/Baruch–Mao, Cohen, Weil–Guinand, Bombieri, Yoshida, Connes–Consani, Douglas, Slepian–Landau–Pollak, Gronwall/Robin (unconditional form only), Grenander–Szegő, Sherman–Morrison/Woodbury, Christoffel–Darboux, Szegő–Levinson, the Weyl bound, Combes–Thomas/Jaffard, Prager–Synge, first-order Kato perturbation, Albert (the pseudo-inverse Schur criterion), Haynsworth (the partial-minimisation form of the Schur complement), Céa/Strang's first lemma, Bramble–Hilbert, Brandt/Yserentant two-scale spaces, Szegő–Kolmogorov, Aubin–Nitsche duality, Payne–Weinberger, Levinson/Durbin, Kac–Murdock–Szegő/Widom/Fisher–Hartwig, Fejér–Riesz, Hartman/Peller (Hankel smoothing), Gershgorin, Parseval, the local Fourier / two-grid symbol (Brandt, Hackbusch, Fiorentino–Serra), Yserentant's strengthened Cauchy–Schwarz constant, Axelsson/Axelsson–Vassilevski (the band-split strengthened-CBS mechanism), the Bank–Smith/Dörfler–Nochetto saturation assump-

tion (with Bornemann–Erdmann–Kornhuber as the honest caveat that saturation can fail), the Montgomery–Vaughan/Selberg large sieve (upper direction only), Chebyshev/Mertens (the atom count), Trench/Böttcher–Silbermann (explicit Toeplitz inverses), Delsarte–Genin (the split Levinson recursion), Wilkinson/Higham backward error.

13.2 The asymptotic mountain

Three separate reasons why the cascade of Figure 8 is not a route to a theorem as it stands, and why the document does not present it as one. Since T110 the mountain has a *precise* formulation — the three gaps of Section 7: no reserve, no scalar step law (structurally excluded by the boundary layer), and no uniformity in k. Since T111 it also has a *measured* one — the three walls: the margin wall at $n^* \approx 462$ (measured, an upper bound; the earlier $n \approx 170$ was a fit artefact), the arithmetic ladder wall at the twin-prime pair $521 \to 523$, and the requirement wall at $n = 727$. The mechanism itself never failed on the deep ladder — all 117 handovers certify at retention 1.000000, and the reserve even opens with depth ($f_{\mathrm{crit}} \sim n^{-0.39}$, fit) — so the actionable content of the measurement is the reinterpretation: the operating variable is the window depth, not the zone index, and a proof along this route must beat the exponent gap $-0.974 = -0.681 - 0.293$ and couple the depth δ_0 to the local prime gap. T112 (`adaptive_scaling_probe.py`) then performed that coupling: the ladder wall and the depth dependence of the requirement wall fall *structurally* in the gap-coupled frame, but the margin wall is frame-invariant at exponent -0.974, with the prime-gap dependence disclosed (Bertrand–Chebyshev suffices upward; Zhang/Maynard force θ-uniformity downward). T113 (`limit_operator_probe.py`) then settled what that invariant number *is*: the wall carries the same exponent in every admissible currency — it is substance — but it measures the *discretisation*, not the spectrum: at a fixed window both leading eigenvalues carry the same cell-width power under refinement (the continuum window form has no gap), the positive floor is a cancellation of relative size 10^{-7}–10^{-4}, and the T109 requirement chain divides by an artifact margin. T114 (`margin_free_step_probe.py`) then discharged the leading part of that balance: the margin-free step certificate exists (Albert 1969, the exact Schur complement), runs eleven steps past the old wall to $n = 1331$, certifies all seven torn zones, and shows the wall was an $O(1)$ numerator divided by an artifact floor — chains now stop only at the $h \leq 1500$ window cost, never at a step, and the (R) demand itself is grid-bound and deleted ([P5]). T115 (`schur_transport_probe.py`) then split the two-object core: the transport of the exact Schur complement across a real regrid certifies only mild refinement ($\rho \lesssim 1.8$) — and for a principled reason, since on nested ladders, where the transport error is exactly zero, $\lambda_{\min}(S)$ itself falls like $\rho^{-1.7}$ — while the two-scale compression breaks the compute cap: a certified margin-free step at $n = 155\,921$ (117× deeper), ten-step chains, the stopper always cost and never a step, with the contiguous reach moving from $n \leq 125$ to $n \leq 5437$ (the $n \log n$ barrier divided, not broken). T116 (`boundary_formulation_probe.py`) then ran the boundary reformulation itself: the induction step *is* a boundary process — the global pole rides exactly in a $12 \times 12 + 12 + 1$ state and one unbroken Riccati march ran 169 236 prepends to 1.35 million cells (903× the old cap) at flat cost, declared as a cost-geometry demonstration and not a Weil certificate — but the state cannot be bounded, because the prime comb itself refuses compression (every incoming cell reflects off every prime power in the window; the truncation budget grows like $n^{2.17}$). T117 (`epsilon_theorem_probe.py`) then made that inequality theorem-shaped: the Galerkin reading is four exact identities, the lower-bound direction is a certified two-level chain that loses no power of D ($\theta' = 1.74$ against $\theta = 1.76$), the jumps have exact closed forms — and, correcting T116, the prime-power entries *raise* ε, so the factor-120 falls were a sweep artefact and the jump side-condition disappears. Since T117 the mountain is three named analytic lemmas about one symbol, each with a classical address — the Kac–Murdock–Szegő/Widom corner asymptotics of T^{-1}, a one-level lower bound for $\lambda_{\min}(S)$ at the Nyquist frequency, and the Bank–Smith saturation constant, two of the three constants rather than rates — plus the comb as the honest support obstruction. T118 (`symbol_lemmas_probe.py`) then measured the three against their addresses, and two of the three stand: saturation is an *identity* here; the Schur floor is rescued by the

strengthened Cauchy–Schwarz shift onto the oscillation Gram, certified on 7/14 windows with the remainder shown to be under-resolved rather than obstructed on the 15 680-point FFT lever; and the corner lemma corrects itself — a log singularity is the boundary of all powers, so the chain is repaired at its last step and the rate now lives on the oscillation mass $\|y\|^2$. Since T118 the mountain is *one* genuinely new analytic statement — (H2), a $D^{1.75}$ lower bound for the oscillation mass — plus the standard CBS uniformity (H1), two arithmetic reductions for the comb dips, and the **[SUPPORT]** obstruction. T119 (`oscillation_mass_probe.py`) then closed the arithmetic half as a theorem — $\inf \sigma_z > 0$ for all $D < D_0(\alpha) = \exp(-(\Xi(\alpha)+B))$, with Ξ the prime-power atom count and $B = -1.0474$ universal — proved the energy route to (H2) empty (the hypothesis is genuinely new content), and reduced the oscillation-mass half to one discrete Harnack inequality through the exact identity $\kappa_{\text{end}} = 1/(1+R)$. T120 (`harnack_probe.py`) then *explained* that inequality — the two increment families are one sequence offset by a single fine cell, so given one sign of the corner increments the summed bound $|R-1| \le 0.047$ is a proved two-line implication, while the per-cell version is provably false and the two obvious structural routes to the sign (total positivity, the discrete maximum principle) are refuted by measurement — and at the same time sharpened the wide-window defects: the frame consumes $\alpha = u_k/2$ by construction, so the unconditional D_0 route covers not one handover of the real ladder ($\alpha^* = 0.693$, below the first admissible zone), and $1-\gamma^2$ decays with α. Since T120 the mountain is four named defects — one sign of the corner increments (a corner-localised Fisher–Hartwig estimate), a uniform bound on the pairing ratio (a discrete gradient estimate), D_0 for the whole ladder, and the α-dependence of γ — plus the **[SUPPORT]** obstruction; the chain contains no zeta input anywhere. T121 (`wide_window_probe.py`) then ran the σ_z route against the real ladder and restructured two of the four: the route survives as a *section* statement ($\lambda_{\min}(T_h(\sigma_z)) > 0$ on 16/16 frame windows to $\alpha = 6.277$, half of them where the symbol infimum is negative; the certified Christoffel budget reaches $\alpha = 3.446$) and dies as a symbol statement, and the net α-balance of the whole chain is one poly-log ($\alpha^{-1.57}$, uniform in D), split exactly into the Rayleigh and CBS steps with nothing in γ — while one link recorded as classical was refuted (the odd sector's Hankel term). Since T121 the mountain is the same four defects changed in kind — the corner sign (now a head-versus-tail inequality at $n_C = h/16$), the pairing bound, the section lemma with the Hankel term, and one poly-log of α-slack with a structural address — plus the **[SUPPORT]** obstruction. T122 (`net_balance_repair_probe.py`) then repaired the two worst-case steps that poly-log named: the Hankel term is the reflection half of an exact isometry ($A_z = W^{\mathsf{T}}\text{Toeplitz}_M(c)W$, $W^{\mathsf{T}}W = I$), so link (5) is retired by an identity — and the per-dip Nikolskii budget with it — while the certified band floor reaches $\alpha = 6.31$ where the Christoffel budget tore at 3.45; the structural Rayleigh bound is sharp (residual slack 1.00–1.03, flat in α), the certified deficit halves to $\alpha^{-0.73}$ (exactly uniform in D), and with the measured CBS coupling the balance closes to within the chain's own ceiling ($\alpha^{-0.113}$ against $\alpha^{-0.116}$). Since T122 the mountain is the two unchanged sign/pairing defects plus two scalar estimates of the same band-profile kind — the band margin (F3) and a structural CBS bound (F4) — plus the **[SUPPORT]** obstruction. T123 (`structural_cbs_probe.py`) then ran the same band machinery at both scalars, and they *split*: the margin closes almost for free (the reduction to the band profile of Wy costs 0.7%, and the oscillation block gains uniform certified positivity, (5R*)), but the coupling resists structurally — every certified route to κ_y needs the coarse form from below, whose near-null direction is smooth and lives on a cancellation no pointwise symbol minorant can see — and the remaining $\alpha^{+0.500}$ gap to the ceiling is identified with the ε_f lid itself, the same object 4–8 recursion levels deep. Since T123 the mountain is the unchanged Harnack pair (F1)/(F2), one band-profile lemma (F3), and coarse-level positivity (F4) — no longer an estimate to sharpen but the recursion base case — plus the **[SUPPORT]** obstruction. T124 (`multilevel_telescope_probe.py`) then built that recursion, and the telescope carries: the ladder is one window form on nested spaces ($P^{\mathsf{T}}A^{(\ell+1)}P = A^{(\ell)}$ exact — the two-level system is literally one rung), the rung contribution is a residual in the inverse norm — a maximum, not a minimum — so it is certified from *above*, where the envelope works ((8R), valid on 400/400 rungs at a drift seven times weaker than T123's), and (F4) collapses from a quantitative estimate to a sign the coarse-to-fine induction already carries,

with base case pure semidefiniteness — $\lambda_{\min}(A_c)$, $\mathrm{cond}(A_c)$ and γ^2 leave the chain entirely. T125 (`grand_assembly_probe.py`, the series finale) then mounted all of it on one ladder and changed the *shape* of the mountain rather than its height: on 52 of 52 zones — 30 of them a literally composing run of consecutive atoms on one common resolution — the load-bearing spine [A][B][C][E] is at CHOLESKY-CERT, so the Harnack pair (F1)/(F2) *leaves the spine* and survives only in the second, independent route (iv). Since T125 the mountain is therefore *one* declared accounting convention ((C-F5)) on the spine, the Harnack pair as an alternative route rather than a bottleneck, and **(F6) uniformity in the zone index** — no uniform-in-k handover Schur complement, no uniform lower bound on cb_ℓ — plus the **[SUPPORT]** obstruction. That uniformity gap, not any missing estimate, is the honest distance from the chain to any infinite statement (Section 9), and it is where the series closes. T126 (`uniformity_seams_probe.py`, the first part of the second phase, Section 10) then opened the attack on exactly that gap and finished the seam architecture: the segment-partition question dissolves into a monotone free-resolution construction (re-gridding only at record-small gaps, all twelve affordable real seams certified, a full seam end to end at the ratio the arithmetic demands), the continuum step closes through an exact fractional-Dirichlet identity onto Lipschitz test functions in the reached window, and the audit splits (F6) into three named lemmas of which exactly *two* are genuinely new — the U3 direction lemma and the U5 zone-uniform seam floor. T127 (`two_inequalities_probe.py`) then dissected both, and both changed: the U5 separating variable is pure interval geometry (the border protrusion $s_{l,r}$, not the atom and not the gap), U5 as posed is *refuted* and replaced by a band covering 99.26% of the real seams with the 8 exceptions enumerated (Mersenne/Fermat pairs and one twin), and U3 collapses to a coarse floor through the harmonic-mean identity, with exactly one direction type to exclude. T128 (`teml_probe.py`) then worked the four-point list cheapest first, and three of the four stand at their preregistered bars: (L) is derived and closed (the three affordable seams certified, the five open ones each lacking exactly one Cholesky beyond the cap), (T) becomes an exact identity plus majorants valid on every transport, (E) certifies through the Q-inner-product chain on all nine in-band seams, and (M)'s boundary-layer exclusion is now a proof with an 11.6× floor margin — while the fourth point, the protrusion concentration κ, missed its own bar by 3.6%, systematically in the ratio. T129 (`kappa_deep_seams_probe.py`) then tested the κ law that miss preregistered, and broke it productively: the fitted law falls once on 331 fresh transports with the bar frozen, but the structure underneath it is solved with identities — the flat border vector sits at $\kappa = 1$ exactly, a linear density profile at $\kappa = 2$ exactly, everything above is curvature, and the curvature chain is a per-transport theorem with an explicit constant holding on all 436 transports — while the two affordable deep seams ($n = 127, 256$) carry complete certificates on the graded two-scale space, honestly downgraded for a measured 8% false-positive rate of the compression, declared before the results. T130 (`curvature_bridge_probe.py`) then attacked the two named pieces and exactly one of the two stands: the graded-to-uniform *bridge* stands as an identity — the matrix-form Céa/Strang defect reproduces the directly computed uniform floor on 84 pairs with zero overshoot, explains the 8% false positives completely (they sat in a bracket the bridge makes unnecessary), and carries both deep seams to positive fine floors at up to 3.8× the factorisation cap — while the curvature bound honestly broke its frozen shape band on 13 of 545 disjoint test transports and is reduced to a zone-uniform bound on *one* exponent. T131 (`self_supply_probe.py`) then built the self-supply loop and left it one number short of closed: the $\varepsilon \to \lambda_{\min}$ secular sandwich is a *theorem* — sharp to a factor ≈ 1.3, with the sign half an equivalence — that replaces the Lanczos estimate on all 84 bridge pairs with zero brackets lost (and exposes that the old Ritz value overestimated the floor by up to 7.9×), sign constancy became a theorem via Perron–Frobenius on S^{-1}, the one-hump property honestly broke at depth, and M25 is reduced to positivity of the pole-free section with nine decades of slack. Since T131 the mountain is the map V6: two irreducibles — the word "for all" (M19) and the RH address itself (M21) — plus **M25 reduced-to-pole-free**, M22 collapsed onto one measured exponent, the vacuous M17 assembly with its named whitening fix, the conditional comb bound (M18), the per-side law (M24), the (C-F5) convention, the all-α step that no computation can close, and the **[SUPPORT]** obstruction. T134 (`pole_free_floor_probe.py`,

PARTIAL) then closed the *existence* half of that pole-free floor — every Cholesky pivot positive on 79/79 windows, per-window certificates throughout — while every cheap lower-bound route fails by *sign*, not size: the surviving opening is an M-matrix question, and the whitening honestly corrected T131's diagnosis (the comb dominates the pole in the norm; what is missing is a localisation statement). T136 (`m_matrix_pair_probe.py`, ONE-CARRIES) then took that question and closed one of its three items outright — Varga's regular splitting makes $\rho(J) = \tau/(1+\tau)$ an *identity* and Collatz–Wielandt bounds it a priori to 1.00–1.03 of the measured gap on 900/900 blocks — while the exact bookkeeping $\lambda_{\min} \sim D^{-0.56} \cdot D^{2.72} \cdot D^{0.12}$ shows the whole degradation sitting in the margin and M17 closes *negatively* (the bad subspace is delocalised). T137 (`long_lag_probe.py`, BOTH-RESIST) made the perturbation explicit (comb-generated anti-diagonal stripes at the prime-power atoms, each a perfect matching, amplitude certified) and *killed a whole family by certificate*: $\rho(|E|) \geq 1.32$ from below on 35/35 windows, so no absolute-value bound can ever supply a floor — the residue is one named object, a sign-preserving bound. T138 (`sign_compensation_probe.py`, PAIR-EXACT) kept the signs and found the mechanism — the coupling sign follows the *interval geometry* of the two edges, and the m-paired Neumann certificate removes the arithmetic wall on all 77 dead blocks (pool 563 → 875/900) — while T139 (`green_decay_probe.py`, DENSE-RESISTS) refuted the classical decay lemma *at its hypothesis*, arithmetically, derived the sign law from one exact telescoping identity and killed the never-truncating layer series from below: the measured core shrinks to *one signed inequality at stripe distance* $b \leq 16$. T140 (`signed_band_probe.py`, FINITE-CORE) then gave that inequality an exact finite core per zone — $\rho(W) = \lambda_{\max}(K^{1/2} H K^{1/2})$, a closed-geometry covering kernel times a mass-plus-Dirichlet form, with the checkerboard split replacing the $O(nb)$ Weyl steps by three D-independent ones and *all* the D-dependence located in the geometry; what remains is a zone-uniform discrete Hardy inequality. T141 (`discrete_hardy_probe.py`, HARDY-RESISTS) attacked exactly that inequality: four exact identities put it in classical two-weight shape, but the certified constant is *not* zone-uniform ($D^{-0.366 \pm 0.036}$ against a bar of 0.25) while the object it bounds *is* ($D^{-0.229 \pm 0.007}$) — the growth sits in the diagonal profile; the additive shape is dead as a *shape* by its own exact Weyl floor and the joint shape fails at the normalisation alone ($\Omega = 20.7$–2724), so the residue is one closed conductance profile. The identity and structure blocks underneath the map are no longer sandbox: they are promoted as v542, v543, v544 and v545 (Sections 4.8–4.11). And the *reverse-flow* parts have landed — T132 (`bd_seam_probe.py`) made the Beurling–Deny triad an operator discriminator for the seam DtN, SPECTRUM-ONLY with the gap in the killing measure, T133 (`cert_floor_probe.py`) audited the suite's own PSD rows and produced the exact positive-mixture Gram certificate that hardened v379 in this same round, and T135 (`comb_compressibility_probe.py`, BOUNDED-STATE) showed the seam DtN admits a bounded faithful state ($m_{\text{cert}} = 12$, from the pre-declared set) where the Weil window provably has none — with the driver named as weight summability and `QEC.SEAM.01` explicitly *not* advanced. The reverse flow resumed at the far end of the phase: T177 (`cp_invariant_probe.py`, DEGENERATE) turned the gauge-degree toolkit on the four CP ledger rows of the physics side — Tier-1 invariance of the $\pi/3$ structure empty by algebra, two inequivalent Jarlskog-class invariants carrying $\pi/3$ with neither coupled to the frame, four ledger readings narrowed with no marker moves, and the exact cores promoted as v561.

1. **Sixteen zones are sixteen zones.** Everything below the top line of the cascade is established on the resolved zones $n = 2 \ldots 29$ (T111's deep ladder extends the measurements to $n = 521$, T114's margin-free ladder to $n = 1331$, T115's compressed run to $n = 155\,921$, T116's boundary march to 1.35×10^6 cells with atom reach $n = 173$ — finitely many zones either way). Not one constant in the chain is certified uniformly in k. The T101 requirement list (A)–(D) is unchanged: the hardness would sit in (A), a sharp, non-perturbative arithmetic lower bound at a Weil-positivity edge.

2. **Finite resolution is a wall, and a measured one.** Numerics as a witness is exhausted beyond $\alpha \approx 0.55$ ($O(M^{-1.8})$, T96); the finite-block route was judged the structurally wrong instrument, not an under-resourced one (T92); $\lambda_{\min}$ collapses geometrically by a factor 11.4 per

mode, so double precision ends at $N \approx 10$ while $N = 24$ would need 10^{-27} entries.

3. **The induction base is not free either.** The recursion terminates arithmetically (240/240 in ≤ 4 steps to the classical zone, T99; 960/960 with longest chain 13 steps, T101), but the zone peaks need percent-level bounds, not 0.1% (T99: extrapolated peak margins $+2.7 \times 10^{-2}$, $+1.4 \times 10^{-2}$, $+1.7 \times 10^{-2}$, i.e. 2.2–5.5% of $\mu_k/2$, with zone 5 saturating exactly).

13.3 Extraction candidates: the community-checkable path

The useful question at this point is not "how close is the program to RH" — it is not close, and no such claim is made — but "which pieces of it are unconditional theorems that someone else can check". Three candidates are named in the diary and are repeated here as the outlook; the third one is the finale's own recommendation and now leads:

- **The assembled chain, as a conditional lemma paper in numerical linear algebra** (the T125 recommendation). Stated without one word of this project's vocabulary: *a certified multilevel lower bound for the Galerkin prediction error of a log-singular Toeplitz form with a prime-power comb*, with the (8R) direction reversal as the theorem and the margin-free bordering induction as its corollary — the comb is then merely the example that makes the symbol log-singular, and no arithmetic conjecture is mentioned anywhere. The reassessment scored it against the matching-lemma package on the five criteria of the July extraction analysis and found it *stronger* on three (prior-art independence — the ingredients are classical but the assembly is a corollary of none of them; self-containedness — decisive, since the statement needs no project vocabulary at all; referee surface — three Schur-complement facts, Parseval, Galerkin orthogonality and one floating-point convention) and *comparable* on the other two (machine-checkability: 430 Cholesky certificates and 440 identities against v541's 33/33 with a 10^6 enumeration; honest boundary). Two conditions come with the recommendation: carry (C-F5) in the *abstract* rather than in a remark, together with the measured failure of the solution-free rung as a separate proposition — that negative result is publishable content and it is what keeps the gap from looking overlooked — and do *not* extract (F1)/(F2) with it, since they are window certificates outside the spine and their classical address belongs in a separate note whose whole content is a discrete Harnack inequality for the increments of the inverse.
- **Zone extension beyond** $\log 2$**.** Unconditional Weil positivity is classically known exactly up to support width $\log 2$ (Yoshida 1992; Bombieri 2000). T91's target inequality — for $a \in (\log 2, a^*]$ and $\|f\| = 1$, $(P_{\text{pole}} + A_{\text{arch}})(f) \geq \sqrt{2} \log 2 \cdot h_f(\log 2)$ — is typed as *provable-shaped* and would, if proved, be a standalone classical result: a zone extension past Bombieri's boundary. Its typing is explicit and unflattering: RH $\Rightarrow$ (T), but (T) $\not\Rightarrow$ RH. T92 showed the finite-block route to it is the wrong instrument; that narrows, rather than closes, the target.
- **The matching lemma.** Already proved exact-integer on $[4, 10^6]$ with an explicit margin inside v541, and reduced by T77 to a restricted σ_{-1} divisor bound in the Gronwall regime — with the note that Robin's *unconditional* 1983 bound suffices at the measured headroom, so the RH-equivalent sharpening is not needed. Closing the tail (the correlated-cancellation lemma on credit-rich non-coherent atoms) would be a self-contained classical statement.

Final fence

The prime line of TFPT has produced seven machine-checked modules about lattice-native Hecke structure, a finite relative-trace identity, and an exactly closing transport ledger. It has *not* produced evidence for the Riemann Hypothesis, and the one place where RH appears — I5 — appears as an equivalence typing that names the residual difficulty rather than reducing it. The corresponding open gates in the ledger (`ZETA.CENSUS.TO.GL1`, `ZETA.HP.CARRIER`) are untouched by everything in Sections 5–7, and no marker moves anywhere on account of this document.

Appendices

A Probe index, diary parts T11–T199

Every row is one exploratory probe in `experiments/tfpt-discovery/`; the file column omits the common suffix `_probe.py`. "Checks" is the probe's own assertion count, all green. Verdict strings are the diary's preregistered verdict enums. Rows marked "→ vN" were later consolidated into the corresponding load-bearing module; the promotion, not the probe, is the load-bearing artefact. The index covers the 189 parts T11–T199 that the current diary file records (190 runs — T104 was a double run with two independent arms — and 4752 green checks); the ten earlier probes T1–T10 predate that file and are not indexed here. Series totals as recorded by the diary at T125, the final part of the series: 125 **probes,** 3139/3139 **sandbox checks**, plus v535–v564, v569–v570, v573 and v576–v596 promoted; with T126–T199 (T176, the endgame measurement, landed after the reverse-flow part T177 and is numbered by contract; T178–T199 are the post-closure backflow parts), the first seventy-four landed parts of the second phase, the running totals are 199 **probes and** 4944/4944 **checks**.

Part	Probe	Contract / finding	Verdict	Checks
T11	`e8_glue_chi4_signed_theta`	the χ_4-signed glue theta welds parts 6 and 7	NO PROMOTION	19/19
T12	`e8_glue_lseries_selfdual`	L-series of the signed glue count; Apéry/$\zeta(3)$ bridge; Poisson self-dual width 2π	NO PROMOTION	16/16
T13	`e8_glue_character_atlas`	glue-character atlas of E_8	NO PROMOTION	11/11
T14	`seam_selfdual_width`	SEAM.SELFDUAL.WIDTH; unimodularity gate	DEFLATED	13/13
T15	`zeta_arith_completion`	ZETA.ARITH.COMPLETION; shell Hamiltonian	PARTIAL	12/12
T16	`zeta_local_euler`	ZETA.LOCAL.EULER; one weight-4 Hecke polynomial	PARTIAL	16/16
T17	`zeta_funceq_duality`	ZETA.FUNCEQ.DUALITY; Poisson involution	PARTIAL	9/9
T18	`zeta_weil_recovery_trace`	ZETA.TRACE.COMPLETENESS; Guinand–Weil balance	BENCHMARK-STANDING	17/17
T19	`prime_thinning_seam_truncation`	PRIMON.SEAM.TRUNCATION; thinning vs. the finite tower	MIXED	19/19
T20	`seam_archimedean_kernel`	archimedean kernel from the seam density of states	KILLED-AS-NAIVE	25/25
T21	`prime_prediction_mechanics`	four prediction channels quantified	MAPPED	19/19
T22	`seam_halfcut_dilation_arch`	non-DOS successor of the T20 kill	PARTIAL (K1, K2 pass; K3 fails)	25/25
T23	`seam_boost_arch_dictionary`	arch dictionary via the seam boost	DEAD	23/23
T24	`seam_truncated_trace_rvm`	truncated trace / Riemann–von Mangoldt route	DEAD	25/25
T25	`seam_tate_phase`	scattering phase as the last observable	DEAD (arch route closed)	29/29
T26	`shell_multiplicity_one_quotient`	shell quotient towards multiplicity one	PARTIAL	18/18
T27	`e8_pneighbor_hecke`	Kneser p-neighbours carry the Eisenstein half	PARTIAL → v535	27/27
T28	`e8_kneser_transport_holonomy`	cusp coefficient from mod-3 lattice geometry	TRANSPORT-VISIBLE	22/22
T29	`primon_transport_category`	primon monoid relations	RELATIONS-VERIFIED	16/16
T30	`e8_transport_cusp_uniformity`	uniformity of the T28 reading at $p = 5, 7$	BROKEN (reading); transport holds	32/32
T31	`e8_neighbor_operator_decomposition`	cusp extraction is a rule $\nu_p = a\,\mathrm{Id} + b\,T_p$	REPAIRED-BY-DICTIONARY → v535	49/49
T32	`census_newform_recovery`	census redundancy is 2-adic oldform structure	REDUNDANCY-IS-OLDFORM → v535	24/24

Part	Probe	Contract / finding	Verdict	Checks
T33	`e8_nu_rule_many_primes`	hardening of the ν-rule to $p \leq 100$; Eichler finding	HARDENED → v536	26/26
T34	`compiler_functor_theta`	project ZETA.COMPILER.FUNCTOR founded	FOUNDED-WITH-ANCHOR	13/13
T35	`rankin_selberg_functor`	Rankin–Selberg translates the abelian shadow	RS-TRANSLATES-EISENSTEIN	13/13
T36	`e8_lambda_charsum_evaluator`	cusp remainder isolated geometrically	TWO-SIDED → v536	29/29
T37	`e8_salie_signed_cusp`	signed cusp channel, scalable	SCALABLE-AND-SIGNED	37/37
T38	`cuspidal_bridge`	Shimura preimage of f_8 inside the compiler monoid	BRIDGE-FOUND → v537	13/13
T39	`zeta_center_atlas`	the abelian drop	ABELIAN-DROP-CLOSED	13/13
T40	`weil_recovery_compiler_sector`	Weil-recovery sector terrain	TERRAIN-MAPPED	24/24
T41	`functor_wiring_halfintegral`	the half-integral bridge is Hecke-equivariant	FUNCTOR-WIRED → v537	13/13
T42	`e8_local_densities`	local densities closed	CLOSED → v536	25/25
T43	`kohnen_plus_waldspurger`	Kohnen plus space	PARTIAL	12/12
T44	`kohnen_operator_projection`	central-value route via Kohnen plus	OUTSIDE-KOHNEN-SCOPE → v537	17/17
T45	`baruch_mao_metaplectic`	central-value family as $b(d)^2$; R constant	CENTRAL-VALUES-WIRED → v537	13/13
T46	`stage4_prolate_prototype`	stage-4 prolate prototype	IMPL-OPEN	16/16
T47	`stage4_prolate_uv_dirac`	pointwise UV/Dirac prototype	UV-NO-SIGNAL	17/17
T48	`lambda_scale_uniqueness`	does the compiler fix the Λ scale?	SCALE-TORSOR (claim killed)	17/17
T49	`rankin_positivity_miniature`	finite Hecke/Jacobi/Rankin–Selberg identities survive; mean-to-pointwise exponent drop corrected	RANKIN-EXPONENT-DROP-FALSIFIED → v1021	30/30
T50	`half_integral_selfconvolution`	the central-value family aggregates exactly	FAMILY-AGGREGATED	17/17
T51	`waldspurger_family_kernel`	first non-collapsing infinite carrier candidate	INFINITE-CARRIER-CANDIDATE	22/22
T52	`hecke_orbit_transfer`	path-space vacuum resolvent + classical local Satake determinants; no path-space Fredholm determinant	TRANSFER-REALIZED (narrowed)	16/16
T53	`relative_trace_compiler`	v535/v536/v537 are three projections of one RTF identity	ONE-FORMULA → v538	16/16
T54	`waldspurger_canonical_measure`	the "canonical discriminant measure" claim	MARGINAL-WEIGHT (killed)	26/26
T55	`rtf_period_pairing`	canonical distributional limit of the finite RTF	DISTRIBUTIONAL-RTF-TYPED → v539	36/36
T56	`rtf_pairing_sharpening`	all T55 identifications to sub-percent	SHARPENED	27/27
T57	`infinite_rtf_packing`	both infinity directions in one double series $Z(s,w)$	PACKED	24/24
T58	`zeta_rtf_xi_residue`	GL(1) shadows of $Z(s,w)$	GL1-SHADOW-CLASSICAL (route A closed)	24/24
T59	`weil_gns_identification`	residual factor in closed form	TRANSFORM-REQUIRED	19/19
T60	`plancherel_descent`	Sato–Tate moment contraction	PARTIAL	21/21
T61	`st_isotype_gl1_core`	the trivial Sato–Tate isotype is a GL(1) core	GL1-CORE-EXACT → v539	24/24
T62	`core_isolation`	d-twist mixing vanishes fibrewise	CORE-ISOLATED → v539	24/24
T63	`weil_core_completion`	Plancherel correction irreducibly non-automorphic	EXTRA-TERMS → v539	33/33
T64	`rtf_stabilization`	$\det_2$ absorbs obstruction 2; the minus stays	HALF-STABLE	34/34

Part	Probe	Contract / finding	Verdict	Checks
T65	`pi_digits_prime_distribution`	prime distribution in the digits of π	PI-NULL	16/16
T66	`pi_prime_four_level`	the π/prime question on four levels	FOUR-LEVEL-NULL	23/23
T67	`amplitude_dirac_sqrt`	Dirac square root of the amplitude plane	DIRAC-SQRT-EXACT → v540	27/27
T68	`geometric_polarization`	geometric polarisation $b = N_+ - N_-$	RESHUFFLE-ONLY → v540	34/34
T69	`mixed_channel_full_weight`	even-k deletion is a determinant invariant	DELETION-UNIVERSAL → v540	33/33
T70	`linear_measure_weil`	positive linear Θ carrier; pure plus combination	LINEAR-PLUS-COMBINATION → v540	29/29
T71	`transport_terrain`	FE of the linear family exactly anchored	TERRAIN-MAPPED → v540	40/40
T72	`cone_enlargement`	the cone library saturates at 5/24	COMPLEMENTARY-PAIR → v540	34/34
T73	`continuous_cone`	per-direction hybrid cones in the continuum	HYBRID-GAINS	28/28
T74	`door_a_spectral_polarization`	the spectral side is provably sign-blind	VACUUM-STRUCTURE	32/32
T75	`door_b_lambda_transport`	the gap functional λ^* has its own structure	LAMBDA-STRUCTURED	28/28
T76	`hybrid_universality`	the recipe certifies 91/91 adversarial Weil directions	RECIPE-UNIVERSAL-ON-BATTERY	22/22
T77	`matching_lemma_structure`	the matching lemma reduces to a restricted divisor bound	LEMMA-CLASSICAL-SHAPED	21/21
T78	`lemma_window_proof`	matching lemma exact-integer on $[4, 10^6]$	WINDOW-PROVED → v541	25/25
T79	`transport_ledger`	the transport ledger closes; I5 named and typed	LEDGER-CLOSES → v541	22/22
T80	`tail_correlation_lemma`	character-exact signed envelope; tail to $N^* \approx 10^{23}$	RESERVE-PARTIAL → v541	29/29
T81	`recipe_coherent_avoidance`	reachability theorem: coherent targets need coherent m	AVOIDANCE-FAILS → v541	31/31
T82	`heat_arch`	Δ_{arch} is the internal Γ difference	ARCH-INTERNAL → v541	22/22
T83	`fe_symmetrization`	symmetrisation absorbed; the wall is FE-transversal	INVARIANT-NULL	27/27
T84	`gaussian_lift`	Grössencharacter phases give $L(1, \lambda_k)$ convergence	LIFT-WORKS-UNANCHORED	29/29
T85	`lambda_equivariant_design`	the λ design closes the coherent class	LEMMA-CLOSES-LAMBDA → v541	24/24
T86	`correlated_cancellation`	the Q-pairing involution closes the non-coherent tail	LEMMA-FULLY-CLOSED	29/29
T87	`i5_connes_dictionary`	I5 one-family form vs. the semi-local Connes structure	DICTIONARY-EXACT-CORE	22/22
T88	`i5_tightness`	eight near-vertical tight curves; Γ-side spacing	TIGHT-SET-PARAMETRIZED	27/27
T89	`sonin_crossing`	window compression is Bombieri's classical object	CROSSING-MAPPED	19/19
T90	`residual_dissection`	the residual vectors are explicit Gaussian modes	CORE-DISSECTED	17/17
T91	`thin_band_analytic`	the band is the one-atom zone; atom rescue	BAND-PARTIAL (3/4 closed)	19/19
T92	`zone_extension_proof`	the finite-block route is the wrong instrument	T-SKELETON	36/36
T93	`band_reconciliation`	third independent implementation of the T89/T91 band map	MIXED (map recalibrated)	41/41
T94	`prime_blind_demo`	753/753 primes of an unseen window from lattice counting	BLIND-100	16/16
T95	`continuum_extension`	C1 chain proved; band top behaves as a positivity edge	T-CONTINUUM-NUMERIC	28/28
T96	`relay_test`	the T95 edge was a map artefact; the relay is real	EDGE-ARTIFACT + RELAY-CONFIRMED	21/21
T97	`relay_induction`	the induction step has proof shape; one cross-block lemma	ALIGNMENT-ONLY	105/105
T98	`cross_lemma`	the $\sqrt{\lambda_0}$ law is Douglas range inclusion (circular)	LAW-CONFIRMED-MECHANISM-OPEN	44/44

Part	Probe	Contract / finding	Verdict	Checks
T99	`dk_recursion`	exact mirror-parity selection rule; recursive inequality	DECAY-LAW-FOUND	23/23
T100	`bulk_remainder`	closure 11/24 → 24/24 samples; the drift was a grid artefact	REMAINDER-CLOSES-ZONES	27/27
T101	`k_asymptotics`	16 zones resolved; primitives flat, instrument loses the race	CROWDING-TRENDS	31/31
T102	`arithmetic_bound`	the onset is manufactured by the Schur dressing; C/g was a proxy	MECHANISM-IDENTIFIED	42/42
T103	`instrument`	slope halves; closure map 7/64 → 44/64	INSTRUMENT-IMPROVED	29/29
T104	`schur_profile_bound`	arm B: naive margin route dead, threshold split closes 16/16	CHAIN-PARTIAL	21/21
T104	`schur_profile_chain`	arm A: independent twin of the same contract	CHAIN-PARTIAL	47/47
T105	`bare_wing_bound`	bare bound certified in closed form; avoidance is a theorem	ONE-OF-TWO	28/28
T106	`friedrichs_angle`	parity split of the Weil pole; hardness in the odd channel	DENSITY-MAPPED	32/32
T107	`odd_channel_closure`	Sherman–Morrison/Woodbury reduction to $r = \kappa/\varepsilon$	SCALAR-TRACTABLE	30/30
T108	`ratio_certificate`	ε is an exact $Q\|_{\mathrm{odd}}$ energy; (R) on two scalars; first non-vacuous margin chain	EPSILON-IDENTITY	44/44
T109	`boundary_decay`	cancellation, not decay; both scalars certified (graded cap + residual certificate); one strict-margin input left	BOUNDARY-CERTIFIED	29/29
T110	`margin_propagation`	the induction circle closes on the measured zones (certified base case + graded Loewner minorant; atom entry free); three gaps named	MARGIN-PROPAGATES	28/28
T111	`deep_zone_stress`	the crossing measured, not extrapolated ($n^* \approx 462$, upper bound); three walls (margin / twin-prime ladder / requirement); mechanism never fails, 117/117	CROSSING-CONFIRMED	23/23
T112	`adaptive_scaling`	gap-coupled frame: ladder wall and ω depth-dependence fall structurally; margin wall frame-invariant (-0.974); trade to [P1]/[P2], prime-gap inputs disclosed	SCALING-PARTIAL	20/20
T113	`limit_operator`	the wall is currency-invariant (-1.168 in all five currencies) but measures the discretisation: no continuum gap, floor a 10^{-7}–10^{-4} cancellation, need109 an artifact margin; new balance [P1]–[P4]	SUBSTANCE-CONFIRMED	27/27
T114	`margin_free_step`	the margin-free step: Albert/pseudo-inverse Schur certifies 27/27 steps (11 past the old wall, to $n = 1331$) and all 7 torn zones; the wall was an $O(1)$ numerator over an artifact floor; (R) grid-bound, deleted ([P5]); new two-object core	WALL-DISSOLVES	22/22
T115	`schur_transport`	transport certifies only mild refinement (split at $\rho^* \approx 1.8$; on nested ladders $\lambda_{\min}(S)$ itself falls like $\rho^{-1.7}$), but the two-scale compression breaks the cap: margin-free step at $n = 155\,921$, ten-step chains, stopper always cost; remaining list three points, one inequality (Szegő–Levinson)	TRANSPORT-BLOCKED	26/26
T116	`boundary_formulation`	the induction as a boundary process: the (Σ, r, σ) state carries the global pole exactly and marches 169 236 prepends to 1.35×10^6 cells at flat cost (cost-geometry demo, not a Weil certificate); the prime comb refuses compression (budget $\sim n^{2.17}$); $\varepsilon \approx 8.34\, D^{1.790} \alpha^{-6.04}$ with factor-120 jumps — a Galerkin error under Aubin–Nitsche duality	RICCATI-PARTIAL	33/33

Part	Probe	Contract / finding	Verdict	Checks
T117	`epsilon_theorem`	the one inequality theorem-shaped: four exact Galerkin identities ($\varepsilon > 0 \iff \tilde{t} \notin T(V_c)$); a certified two-level lower bound on 19/19 pairs at $\theta' = 1.74$ against $\theta = 1.76$ (no power of D lost); exact jump formulas (Levinson bordering, rank-(k_0+1) Woodbury) — entries *raise* ε, the factor-120 falls were a sweep artefact; three named analytic lemmas remain, two of them constants	THEOREM-SHAPED	23/23
T118	`symbol_lemmas`	the three lemmas against their classical addresses: saturation is an *identity* here ($\varepsilon_c - \varepsilon_f = y^{\mathsf{T}} S y$, $\beta \in [0.252, 0.336]$); the S-symbol is a weighted *harmonic* aliasing mean, vacuous on the comb — rescued by the CBS shift onto the oscillation Gram, whose *arithmetic* mean suppresses the comb quadratically ($\inf \sigma_z > 0$ on 7/14 windows; on the 15 680-point FFT lever the floor rises logarithmically and crosses zero on 3/5 zones — under-resolved, not obstructed); the corner exponent drifts (log is the boundary of all powers) and the chain is repaired on $\|y\|^2$; one analytic statement remains — (H2), $\|y\|^2 \geq c\, D^{1.75}$	TWO-OF-THREE	36/36
T119	`oscillation_mass`	the arithmetic half closes as a theorem: the comb symbol in closed form (one cosine per prime-power atom; parity rule under σ_z), the atom-count bound $\lvert\sigma_z^{\mathrm{comb}}\rvert \leq \Xi(\alpha)$ uniform in D, the archimedean growth *identity* $\inf \sigma_z^{\mathrm{arch}} = \log(1/D) + B$ with universal $B = -1.0474$ — hence $\inf \sigma_z > 0$ for all $D < D_0(\alpha) = e^{-(\Xi(\alpha)+B)}$, deep-verified ($M$ up to 65 536, matrix-free); the large sieve certifies one window's section *without* pointwise positivity ($+0.396$ where $\inf \sigma_z = -0.349$) and dies structurally for $\alpha \geq 2.5$ (a moment problem); the energy route to (H2) is proven empty (genuinely new content); and $\kappa_{\mathrm{end}} = 1/(1+R)$ *exactly* (1.1×10^{-16}) — one discrete Harnack inequality remains	ARITHMETIC-DONE	27/27
T120	`harnack`	the Harnack core is proof-shaped: a_j and w_j are the odd/even half of *one* increment sequence, offset by a single fine cell, so one sign of the corner increments gives $\lvert R - 1\rvert \leq \sum \lvert v_{2j+1} - v_{2j}\rvert / \sum \lvert v_{2j}\rvert \leq 0.04745$ *unconditionally* (C1; the per-cell inequality is provably *false*, $\lvert a_j / w_j\rvert$ up to 4.8×10^3); $R \in [0.9098, 1.1011]$ on 3915 rows, $\kappa_{\mathrm{end}} \geq 0.480827$ for $n_C \geq 8$; two structural routes refuted (T^{-1} corner positive but not TP2; the discrete maximum principle false — u oscillates in the bulk); the α-audit makes (D2) total ($\alpha_o = u_k/2$ by construction, D_0 holds on 3/1492 zones, $\alpha^* = 0.693$ below the first admissible handover); γ gains an exact symbol identity but $1 - \gamma^2$ falls with α (slope -0.297 ± 0.013); theorem V4: 18 links, defect list $3 \to 4$	HARNACK-EXPLAINED	21/21

Part	Probe	Contract / finding	Verdict	Checks
T121	`wide_window`	the σ_z route survives the real ladder as a *section* statement and dies as a symbol statement: $\lambda_{\min}(T_h(\sigma_z)) > 0$ on 16/16 frame windows ($n = 7 \ldots 283\,303$, α up to 6.277, h up to 107 790; 8 with $\inf \sigma_z < 0$, the section at 0.64–$3.23 \times \lvert\inf \sigma_z\rvert$ positive), the Christoffel/Fejér per-dip budget certified on 8/16 up to $\alpha = 3.446$ (each dip 0.156–0.720 of one Fejér width; the dip count grows like $e^{2.41\alpha}$, a fit); the net balance SUPPLY/$\varepsilon_c \sim D^{-0.18}\alpha^{-1.57}$ — one poly-log, uniform in D, split exactly (2.8×10^{-14}) into the Rayleigh ($\alpha^{-1.02}$) and CBS ($\alpha^{-0.51}$) steps, nothing in γ; link (5) refuted on 42/42 (ratio 0.357–0.916 — the odd sector's Hankel term); one-sign 24/24 at $n_C = h/16$ (only 21/24 at $h/8$; certificates restated, free since $\delta \sim n_C^{+0.29}$); the FH comparison inconclusive at reachable h (drift $-0.008 \ldots + 2.934$ against an allowed -0.120); $\delta = 0.0126$–0.1331 over 360 rows, $\kappa_{\text{end}} \geq 0.4688$; theorem V5: 19 links, the defect count stays 4 but every entry changes kind	WIDE-RESTRUCTURED	21/21
T122	`net_balance_repair`	the Hankel term is the reflection half of an isometry, not an error term: $A_z = W^{\mathsf{T}}\text{Toeplitz}_M(c)W$ with $W = BZ$, $W^{\mathsf{T}}W = I$ exact (the compressed Hankel is half a second difference of the lag sequence, closed form to 7.0×10^{-16}; norm ratio 0.123–0.781, $O(1)$ — Z suppresses both halves equally; the Weyl floor positive on only 12/36); the certified band bound $\lambda_{\min}(A_z) \geq \text{thr} - \lambda_{\max}(G)$ non-vacuous on 36/36 rows to $\alpha = 6.31$ (the T121 Christoffel budget tore at 3.45; recovery 0.429–0.995; caveat: the optimum sits at $\text{supp}(g) \approx 1$); the structural Rayleigh bound is *sharp* ($q_{\text{str}} = 1.00$–1.03, drift $\alpha^{-0.002}$), removing the whole $r_{\text{ray}} \sim \alpha^{+0.83}$ slack; the certified balance halves the deficit, $S_2/\varepsilon_c \sim D^{-0.003}\alpha^{-0.729}$, exactly D-uniform, and with the measured CBS coupling ($\kappa_y = 0.0067$–0.5311 against γ^2) it reads $\alpha^{-0.113}$ against the chain's own ceiling $\alpha^{-0.116}$ (residual decomposition closes to 2.9×10^{-16}); theorem V6: the count stays 4, link (5) retired by an identity, (F3) the band margin (0.024–$0.363 \sim D^{-0.042}\alpha^{-0.525}$) and (F4) a structural CBS estimate absorb (E3*)/(E4*)	NET-IMPROVED	20/20

Part	Probe	Contract / finding	Verdict	Checks
T123	`structural_cbs`	the coupling resists structurally: the coarse block is the other half of the T122 construction ($V = BP$ an isometry, $V^{\mathsf{T}}V = I$, $V^{\mathsf{T}}W = 0$ exact — A_c, B_x, A_z three compressions of one window Toeplitz form, identities to 3.4×10^{-15}); the coupling is half a *first* difference of the lag sequence, in symbol form $-\frac{i}{2}\sin(\varphi/2)[\sigma(\varphi/2) - \sigma(\varphi/2+\pi)]$ — the aliasing difference, vanishing where the coarse space lives, which explains the factor 2–109 between $\kappa_y = 0.0067$–0.4622 and $\gamma^2 = 0.7248$–0.9467; both certified routes to κ_y need the coarse form from below (band-split CS is correct 24/24 and non-vacuous 0/24, costing $\mathrm{cond}(A_c)$ up to 3.1×10^7; the Schur route yields (5R*) $\lambda_{\min}(E_z) = +0.303 \ldots + 5.017 > 0$ on every row at slack 1.0007–1.007 but needs $E_c \succ 0$, true on 0/24) — the near-null direction of the window form is smooth, and its eigenvalue is a cancellation no pointwise symbol minorant sees (envelope margin 8.4×10^2–$4.0 \times 10^5 \times \lambda_{\min}(A_c)$, rising $\alpha^{+4.202}$); the band margin (F3) reduces to the band profile of Wy at a cost of 0.7% (dyadic split exact to 1.1×10^{-15}, overshoot 1.20–3.51); the certified balance $S_6/\varepsilon_c \sim D^{-0.069}\alpha^{-0.544}$ against the ceiling $\alpha^{-0.044}$ — gap $\alpha^{+0.500}$, all of it the coupling (two-term residual split closes to 2.8×10^{-17}) — and the gap is identified as the ε_f lid itself, 4–8 recursion levels deep; theorem V7: the count stays 4, restructured — (F1)/(F2) the Harnack pair, (F3) one band-profile lemma, (F4) coarse positivity = the recursion base case	CBS-RESISTS	20/20
T124	`multilevel_telescope`	the telescope carries: the ladder is one window form on nested spaces ($P^{\mathsf{T}}A^{(\ell+1)}P = A^{(\ell)}$ exact to 2.4×10^{-14} — the T122/T123 two-level system is one rung), every rung is the saturation identity and the Galerkin energy of the level correction ($\delta_\ell = \|u_{\ell+1} - Pu_\ell\|_A^2$ to 3.7×10^{-8}; telescope $\sum \delta_\ell = \varepsilon_0 - \varepsilon_L$ to 9.2×10^{-10}; top rung 0.27–0.88, geometric decay, $L = 5$ ceiling 0.9911–0.9987); the rung is a *maximum* (dual residual form $\delta_\ell = \hat{s}^{\mathsf{T}}S^{-1}\hat{s}$; $r_\ell = W\hat{s}$ by Galerkin), so it wants S from *above*: $S \preceq A_z$ needs only $A_c \succeq 0$ (Haynsworth), $A_z \preceq U = Z^{\mathsf{T}}T_M(\mathrm{up})Z$ is Parseval against the certified majorant — (8R) $\delta_\ell \geq \hat{s}^{\mathsf{T}}U^{-1}\hat{s}$ valid on 400/400 rungs (80 chains, 20 zones), $\mathrm{cb}/\delta = 0.4739$–$0.9938$ at drift $\alpha^{-0.080}$, 7× weaker than T123's $\alpha^{-0.544}$ ($\lambda_{\min}(E_c) < 0$ on 399/400 is irrelevant — (8R) never forms E_c; the Levinson bordering route falls: refinement is not a bordering, tail/δ = 13–863); the near-null direction stays smooth and self-similar (prolonged Rayleigh quotient exactly $\lambda_{\min}(A^{(\ell)})$, overlap 0.944–1.000), and (8R) needs only the *sign* of A_c — the previous rung's conclusion: the non-circular induction runs coarse → fine, base case $\varepsilon_L \geq 0$; balance $\sum \mathrm{cb}/\varepsilon_0 = 0.6404$–$0.9729$ against the ceiling, two-level $\mathrm{cb}/\varepsilon_c \sim D^{-0.090}\alpha^{-0.100}$ against T123's $\alpha^{-0.544}$ — $+0.444$ of the $\alpha^{+0.500}$ gap; theorem V8: (F1)/(F2) the Harnack pair, (F3) closed, (F4) *collapsed* to a sign, (F5) new, a bookkeeping convention — the solution-free rung provably fails	TELESCOPE-CARRIES	28/28

Part	Probe	Contract / finding	Verdict	Checks
T125	`grand_assembly`	the series finale — the chain composes: all five stages ([A] base Cholesky, [B] telescope with (8R) rungs, [C] ε lower bound, [D] κ_{end}/parity, [E] margin-free handover) complete on 52/52 ladder zones plus 10 handover-only zones to $n = 1331$, and ladder 1 is a *literally composing* run — 30 consecutive atoms $n = 2 \dots 79$ on ONE common resolution $D = 3.472446 \times 10^{-3}$, $M_n(k) = M_o(k+1)$ as an integer identity on 29/29 pairs, the incoming atom block bit-exactly zero, so $X = Q_{\text{old}}$ and the next zone's leading block is that X to 2.9×10^{-14} (T114's [P2] seam constructively *circumvented*: 0/84 gap ratios dyadic, so one frame is chosen for the whole run, at the price of its length); the load-bearing finding: the weakest stage of the whole mounting is [D] at WINDOW-CERT, but the SPINE [A][B][C][E] is CHOLESKY-CERT on 52/52 — *the Harnack pair (F1)/(F2) no longer carries*, it lives only in the independent second route (iv); (8R) on 88/88 rungs at cb/$\delta = 0.9071$–0.9971 (primal form above δ on 88/88 — an upper bound, never a supply), base-case equivalence $\operatorname{sign}\lambda_{\min}(Q^{(L)}) = \operatorname{sign}\varepsilon$ on 52/52 at head-room 4.4×10^{5}–5.6×10^{10}, ε lower bound on 52/52 at 0.9140–0.9929 of the L-level ceiling and 3.2×10^{1}–8.7×10^{10} over the declared fp bound, Albert Schur 3.0×10^{10}–4.6×10^{13} above its floor on 62/62 steps while the norm surrogate is negative throughout; 430 completed Choleskys, 440 identities; theorem V-final printed with line-by-line attribution (25 lines: 10 identity, 9 Cholesky, 3 window — all in (iv) —, 1 hypothesis = (C-F5)) and a five-point "what is not claimed"; series balance: kill list 24 routes over 18 parts (4 of them one cause), cascade 7 stations, certification degree 90.3% chain / **96.2**% spine identity-or-Cholesky, RH distance in six checkable points; new defect (F6) uniformity; extraction reassessment: stronger on 3/5 criteria than the matching-lemma candidate	ASSEMBLY-GREEN	34/34

Part	Probe	Contract / finding	Verdict	Checks
T126	`uniformity_seams`	the first part of phase 2 (“the full proof”) — the seam architecture is finished: the partition question is decided *exactly* by DP on the real gap table (9 700 zones, $n \le 10^5$) and refuted at T115’s $\rho^* = 1.83$ (dies at $n = 127$, the twin-after-a-large-gap pattern; true requirement $\rho_{\text{crit}} = 2.0258$, essentially the single quotient $g(125)/g(127)$; refinement-only infeasible at any ratio ≤ 64; coarsening fails at the destination grid, and ladders make it worse); the decisive new measurement: at fixed ratio 2.0256 the certificate survives on only 12/22 zones — the seam dies at the *zone*, not the ratio, so no threshold closes the programme; what carries is the *free-resolution route*: the running minimum $D_k = \min(\text{cap}_k, D_{k-1})$ is monotone by construction, re-gridding at only 764/9 700 boundaries (7.88%, record-small gaps), all 12 affordable real seams certified (11 in one re-gridding, $n = 31$ via a three-stage ladder, median retention 0.578), a full seam end to end at $\rho = 2.251 \ge 2.0256$; the uniformity audit (35 rungs, 14 zones) gives every spine constant a status — U1 self-certifying, U2 one Bernstein bound, U3 measured-flat but *directional* (alignment 0.975), U4 a compute wall, U5 *falls* (the load-bearing gap), U6 resolved; the continuum step closes via the exact fractional-Dirichlet identity (86.1% negative off-diagonals; consistency $D^{1.798}/D^{1.802}$, Richardson limit $2.1256 \times 10^{-4} > 0$); the full-proof map has 16 points, 5 done/classical, 9 conditional, exactly 2 genuinely new — the (U3) direction lemma and the (U5) zone-uniform seam floor at ratio ≤ 2.026	SEAMS-CERTIFIED	31/31

Part	Probe	Contract / finding	Verdict	Checks
T127	`two_inequalities`	both remaining inequalities are proof-shaped, and neither is the statement T126 named: the fixed-ratio sweep reproduces (12/22, T126's list — the *certificate* dies, not the positivity: $\lambda_{\min}(S_f) > 0$ on all 22); of 29 candidate separating variables 5 separate perfectly (22/22), all the same *geometric* quantity — the border protrusion $s_{l,r}$, i.e. what the fine border block protrudes past the coarse block's inner edge (corr $+0.997$ with τ_{dn}; every arithmetic candidate fails, best 0.727); U5 as posed *refuted*, replaced by **U5-band**: zone-uniform for every $\rho \le 1.49531$, at least one zone dies for $\rho \ge 1.49766$ (bisected to 2.3×10^{-3}, binding $n = 19$), covering 99.26% of real seams — the remaining 8 are an enumerated list of next-atom-distance ≤ 2 pairs (Mersenne $3, 7, 31, 127, 8191$; Fermat-like $4, 16, 256, 65536, 131071$; twin $101, 103$; Mihăilescu 2004 cited, not used); the band buys $\tau_{\mathrm{dn}} \approx 1$, not small η ($\eta = \eta_{\mathrm{cons}} + \eta_{\mathrm{proj}}$ exact; max $|\eta|/\lambda = 1.146$ at the tightest ratio; floor $c \ge 0.0090$ over 307 in-band transports); the sharpest finding: the near-identity census (43 deep zones, $\rho = 1.00140$) fails *once* ($n = 479$, $|\eta|/\lambda = 1.472$) — the band is a *proxy* for the border geometry, repaired by a three-stage ladder (retention 0.5027), and the risk indicator $\tau_{\mathrm{dn}}/\tau_{\mathrm{flat}}$ (Spearman $+0.749$, the failure its argmax) sees it before any factorisation; U3 defuses via the harmonic-mean identity $\mathrm{cb}/\delta = 1/\sum w_i/\mu_i$ ($\hat{s}$-mass on $\{\mu \ge \frac{1}{2}\} \ge 0.9665$ over 42 rungs, so a *crude* floor $\mu_{\min} \ge 1.62 \times 10^{-2}$ gives $\mathrm{cb}/\delta \ge \frac{1}{4}$); "$\hat{s}$ avoids the bad direction" refuted — flatness is generic, exactly one direction type fails (a boundary layer at the window edge, concentration 3.1× vs 0.03× for $\hat{s}$); map V2: 16 points, 10 carried, 2 conditional-classical, the 2 genuinely new reformulated as (T)/(E)/(M) plus the enumeration (L) — three inequalities of the same craft as the proven links, residual risk in the word *uniform*	BOTH-SHAPED	28/28

Part	Probe	Contract / finding	Verdict	Checks
T128	`teml`	the four-point list of map V2, worked cheapest first, and three of the four stand at their preregistered bars: **(L)** is *derived* and closed — the out-of-band class $\rho > 1.49531$ has exactly 8 seams up to $n = 300\,000$, all at next-atom distance 1 with identical border geometry ($\mathrm{gc}_c = 2$, $\mathrm{gc}_f = 4$); $n = 7, 16$ certified in one step (retention 0.179/0.156), $n = 31$ through a two-rung ladder (0.091), and 127/256/8191/65536/131071 BUDGET-OPEN at $h_f = 2\,476 \ldots 6.2 \times 10^6$ (1.7×–4119× over the cap; each lacks exactly *one* Cholesky, nothing conceptual; Mihăilescu 2004 cited as the address, not used); **(T)** from predictor to bound — the exact identity $\tau_{\mathrm{dn}} = \|y\|^2 - m_{\mathrm{prot}} + m_{\mathrm{fill}} - V_{\mathrm{bord}}$ (9.0×10^{-13} against an independent full projection, 126 transports), $\tau_{\mathrm{dn}} \geq 1 - m_{\mathrm{prot}} - V_{\mathrm{bord}}$ with explicit majorants valid 126/126 (exact bound positive 126/126, fully explicit 125/126; slack median 0.275, the oscillation majorant the loose part at 7×) — *but* the protrusion concentration $\kappa = m_{\mathrm{prot}}\,\mathrm{gc}_f/(t_l + t_r) = 0.086$–$2.071$ misses the preregistered bar $\kappa \leq 2$ on 1/120 by 3.6%, *systematically* in ρ (group maxima 1.889/1.965/2.071 at $\rho = 1.001/1.250/1.495$); the bar is not moved, the next hypothesis is $\kappa \leq 2 + C(\rho - 1)$; and κ splits by side (1.61–2.07 right against 0.09–0.25 left) — (T) is an *eigenvector* statement; **(E)+(T)** jointly: the ℓ_2 route certifies 0/12 (median 55× overshoot), the Q-inner-product chain $\sqrt{\lambda_f} \geq \sqrt{\lambda_c \tau^* - |\eta_{\mathrm{cons}}|} - \sqrt{e_d}$ ($e_d/\lambda_c = 0.041$–0.154) certifies 9/9 in-band seams at floor ≥ 0.0850, 11/12 overall; the one tear ($n = 31$, already on the (L) list) is η-limited ($|\eta_{\mathrm{cons}}|/\lambda_c = 0.392 > \tau^* = 0.363$ — no sharper (T) would have helped, nor a smaller ratio), repaired by the two-rung ladder, every rung closing the chain by itself; **(M)** earned: the boundary-layer exclusion is a *proof* from the closed form (adjacent-component ratio in $[e^{-D/2}, 1]$, outer-k-cell share $\leq kD(1 + \cosh\alpha)/\sinh\alpha$, separation $\geq 84\times$), $\mu_{\min}(S, A_z) = 1 - \gamma^2$ exact (5.9×10^{-15}), $U \preceq (1 + \varepsilon_{\mathrm{env}})A_z$ with $\varepsilon_{\mathrm{env}} \leq 0.038$ Cholesky-certified — constructed floor 0.188 against the 1.62×10^{-2} demanded, an 11.6× margin on all 31 rungs; the comb residual stays measured but is measurably no layer (half its mass needs 12–39% of the half-window); map V3: 21 points — 10 done, 3 certified on real objects, 3 conditional-classical, 5 genuinely open (M9 the κ bar, M17 the pencil mass, M18 the comb, M19 "for all", M21 the RH address)	THREE-OF-FOUR	27/27

Part	Probe	Contract / finding	Verdict	Checks
T129	`kappa_deep_seams`	the most productive break of the phase: the fitted law falls, the theorem underneath stands — $C^* = 0.3506$ frozen from T128's three calibration groups (a fit, labelled a fit, not refitted; calibration itself 0/105, not the test); on 331 *new* transports (14 ratios 1.05–4.00, none in the calibration set; $h_f = 29$–1450) the law $\kappa \le 2 + C^*(\rho - 1)$ is violated **once** ($n = 7$ at $\rho = 3.00$, slack -0.0334; the bare unmoved bar 2 would fall 78 times), the per-side law on κ_r 1/293, the side asymmetry reproduces ($\kappa_r = 1.70$–2.91 vs $\kappa_l = 0.05$–0.21); the bar is not moved; **the structural ground is solved with identities**: $\sum(1 - g_i) = t_l + t_r$ (7.1×10^{-13}) makes the flat border vector $\kappa = 1$ *exactly* (T128's bar 2 was twice the flat value), the profile identity $p_{N-1} = 2 - p_0 + 2E$ (6.7×10^{-16}) makes a linear density profile $\kappa = 2$ *exactly* (everything above is curvature; $2E > p_0 \iff p_{N-1} > 2$, 114/436 with no exception), and the chain $\kappa \le 2 - p_0 + \frac{N-2}{N} \sum |\Delta p - \overline{\Delta p}| + (p_{\max} - p_{N-1})$ is a *theorem* with an explicit constant (Hölder/telescope/Abel), valid on all 436 (slack median 0.344; last term vanishes 392/436); depth block: 14 graded transports at $h_f = 1656$–$17\,669$ (12× the cap, matrix-free, $m/h = 0.064$–0.512), 0 violations — the residual risk is a ratio effect, not a resolution effect; **the deep seams**: machinery verified (two-scale assembly $= J^{\mathsf{T}} Q J$ to 2.7×10^{-16}, $X = Q_{\text{old}}$ to 2.4×10^{-17}), $n = 127$, 256 compressed to $m/h = 0.507$, 0.255 carry *complete* graded seam certificates (X PD, Schur floors above Wilkinson, bracket positive, retentions 0.307/0.417), the floor survives 6 further coarsening rungs — honestly downgraded to GRADED-CERTIFIED: Rayleigh–Ritz points the wrong way (1.53–5.18×, no exception) and a measured 8% false-positive rate (4/48 pairs graded-positive where uniform is not), declared before the seams; $(L) =$ [3 uniform, 2 graded, 3 budget-open]; **M18**: the Cauchy–Schwarz row bound is vacuous (0/40, reported as the failure it is), the S-procedure operator bound works (40/40 below 1, 0.170–0.774, 1.3–182× above the truth, median 7.8; CERTIFIED-CONDITIONAL on $\theta_1 \sim D^{1.219}$, a fit; saturates at $(kD)^{0.263}$ instead of decaying); map V4: 9 entries — M9 reduced, L2 graded, M18 bounded, M17 untouched, **M19 "for all" and M21 the RH address irreducible**, M22/M23/M24 new and named; promotion: *ready in principle* as a narrow module on the five identity/theorem pieces	KAPPA-WILD	28/28

Part	Probe	Contract / finding	Verdict	Checks
T130	`curvature_bridge`	the two named pieces attacked head on, and exactly one stands — the bridge, not the bound: **the Céa/Strang defect identity** $S_{\text{graded}} - S_{\text{uniform}} = R^{\mathsf{T}} X^{-1} R$ verified to 1.3×10^{-13}, matrix-free (FFT matvec + BPX-preconditioned CG, one graded Cholesky under the cap); the *scalar* form is vacuous (loses 3–28×), the *matrix* form reproduces the directly computed uniform Schur floor on **84 pairs (21 zones) to** 2.5×10^{-11} **relative with zero overshoot** — T129's 8% false positives explained completely (16/84 at bracket level, **0/84** at floor level: they sat in the transport bracket, which the bridge makes unnecessary); **both deep seams carry**: $n = 127 \to \lambda_f = 0.2255$, $n = 256 \to \lambda_f = 0.2614$ ($h_f = 2476/5694$, $1.7 \times$ /3.8× the cap), BRIDGED-CERTIFIED-CONDITIONAL on *one* measured number per grid (the Lanczos floor for $\lambda_{\min}(X_{\text{fine}})$; the Grenander–Szegő route for it *dead*, symbol indefinite, floor $-76 \ldots -27$; conclusion unmoved at $100 \times /10^4\times$ smaller floor); **the curvature front**: the TV rewrite $\sum |\Delta p - \overline{\Delta p}| = \mathrm{TV}(P) \le (2n_{\text{run}} - 2) \cdot \text{sag}$ is *exact* (one hump on 83/765, sign purity 765/765), the power profile wins 635/635 ($a = 0.703$–0.999, median 0.856), the T108 Levinson corner identity independently reproduced (24 zones to $M = 1236$, 4.2×10^{-9}) but the pole solve only a proxy (cosine 0.992–0.996, curvature ratio 1.5–5.3) — and the frozen shape band ($a \in [0.734, 0.897]$, $S^* = 1.1926$) **breaks on 13/545** disjoint test transports (bar not moved; per-transport exponent 520/545) ⇒ M22 reduced to a zone-uniform bound on *one* exponent; theorem battery: 0 violations of the exact chain on 765, the fully explicit chain closes 635/635 but 0 under the bare bar 2 ($\kappa > 2$ measured on 130/765); map V5 (12 items): I1–I4 + **I5 new** certified, I6 bridge + I7 seams certified-conditional, M23 conditionally closed, **M25 new** (a certified positive floor for $\lambda_{\min}(X_{\text{fine}})$), promotion check list (1)–(6); M19/M21 untouched	ONE-OF-TWO	30/30

Part	Probe	Contract / finding	Verdict	Checks
T131	`self_supply`	the self-supply loop is built, one number short of closed: **the $\varepsilon \to \lambda_{\min}$ secular sandwich is a theorem** — $\varepsilon/(\|u\|^2 + \varepsilon/\mu_1) \le \lambda_{\min}(Q) \le \varepsilon/\|u\|^2$ with the sign half an *equivalence* (Albert 1969) — *sharp* (relative width 1.2663–1.3894, true value at 1.06–1.31× the lower bound, 0 violations on 10 dense windows), ε itself certified at the deep grid (Céa/Strang + CG, retention 1.0000); **correction to T130**: the Lanczos Ritz value overestimates $\lambda_{\min}(X)$ by up to 7.9× (an upper bound by construction, never a floor); the chain floor replaces it on **84 bridge pairs with 0 brackets lost** (retention 0.766–1.059 against the certified Cholesky floor, median 0.875), both deep seams CHAIN-POSITIVE ($n = 127$: $\varepsilon \ge 2.63 \times 10^{-7} \to$ floor 1.08×10^{-6}; $n = 256$: $3.83 \times 10^{-8} \to 2.52 \times 10^{-7}$), μ_1 tolerating **nine decades** of underestimate ⇒ **M25 reduced-to-pole-free**; the mirror rung dead ($T(\ell)$ indefinite 4/4 — the Szegő indefiniteness; the telescope runs coarse-from-fine only); **M22**: sign constancy a **theorem** via Perron–Frobenius on S^{-1} (entrywise positive 575/575; Metzler fails 11/575), the one hump **broken at depth** (530/575 with more than two monotone runs; the certified $(2n_{\text{run}} - 2) \cdot \text{sag}$ majorant replaces it on 575/575), exponent and S^* stay measurements ($a = 0.727$–0.999, 55/492 outside the old band; the deeper sweep lifts the required S^* to 1.8472, the frozen 1.1926 exceeded 16 times); **M17** assembled and *vacuous* (majorant 27/27, non-vacuous 0/27; dominant loss the U-metric mismatch 2.18; named fix: M18 whitened); map V6 (13 items), promotion list now **9 items** (new: the sandwich, the Perron sign theorem, the run-count chain); shortest remaining list four points	SUPPLY-PARTIAL	25/25

Part	Probe	Contract / finding	Verdict	Checks
T132	`bd_seam`	the first *reverse-flow* part: the phase's toolkit turned on the theory side, and the Beurling–Deny triad becomes an **operator discriminator** where the spectrum is blind — the BD identity exact to 2.4×10^{-16} at every N, the triad reconstructing the *operator* (1.4×10^{-14} at $d = 256$), diffusion part identically 0 (finite atomic space; the continuum DtN is the pure-jump $\frac{1}{2}$-Laplacian, Caffarelli–Silvestre 2007, fitted decay 1.980), and the **killing measure = the μ_4 mark-sum profile pointwise** (1.3×10^{-14}): the marks enter the operator *only* through the killing term; the **Markov remainder is exactly zero** with the closed form $J_{rs} = 1/(d \sin^2(\pi(r-s)/d))$ for odd $r - s$ and 0 for even — which explains T126's 86.1% count fraction; discriminator verdict SPECTRUM-ONLY: spectra agree to 7.5×10^{-13} but the triad gap sits in the **killing measure at** 0.1746 **at every** N (spread 1.9×10^{-14}) while the jump gap shrinks ($7.7 \times 10^{-3} \to 3.5 \times 10^{-4}$), so "same spectrum + μ_4-equivariance" is provably too weak; controls break the triad (0.520–0.982 against mark-local 7.5×10^{-16}) and the grading too; covariance limit named: random orthogonal (90.4) and generic μ_4-equivariant conjugation (86.5) destroy the triad, the mark-preserving lattice symmetries (D_4: clock rotation, sheet reflection) leave it **exactly** invariant (1.6×10^{-14}) ⇒ `QGEO.MARKS.01` and `QGEO.KERNEL.01` are **coupled**; ceiling stated: this is the *model* DtN, `QGEO.KERNEL.01` **not** closed	SPECTRUM-ONLY	21/21
T133	`cert_floor`	the audit that changed a load-bearing module: shifted-Cholesky certificates (Wilkinson 1968 / Higham 2002 / Rump 2006 bars) applied to the suite's own RP/PSD rows, machinery itself validated on **840 controls** (0 false certifications, 0 false negative-direction fires, sharpness median 1.000) — the 10×10 Hankel v379 builds is **as a matrix of doubles certifiably NOT PSD** (certified negative direction -7.75×10^{-18} at 60 digits; also at $n = 14, 20$) and the module's $\lambda_{\min} > -10^{-10}$ passes it; the cause is priced, not guessed (cond $\approx 1.8 \times 10^{18}$ over the certifiability ceiling 1.0×10^{13}; entry bar 7.6×10^{-12} dominates every fp floor; 16/17 window variants MEASUREMENT-ONLY), the blind band exhibited (a certified negative direction passes the test), and the *mathematical* matrix is fine (certified floor 2.7×10^{-25} at 40 digits, while rounding to doubles moves it 3.5×10^{-17} — eight orders more; precision ladder 25/40/70/70 digits at $n = 6 \ldots 20$); **the exact repair needs no precision**: $M = V^{\mathsf{T}} \mathrm{diag}(1/1600) V$ is a positive-mixture Gram, so $M \succeq 0$ in exact arithmetic with no bar — the route v171 already takes; other rows clean (v171 exactly rank-3 Vandermonde Gram, $\lambda_{\min} = 0$ exactly; v527/v519 CERT-UP, $N = 8, 16$ even in binary64); recommendation table explicit: **no marker moves**, method and wording only — executed in the same round as the v379 hardening and the `SEAM.S3.RP.01` rewording	MIXED	23/23

Part	Probe	Contract / finding	Verdict	Checks
T134	`pole_free_floor`	the three items of T131's rest list, and nothing else — **the floor is never an existence problem**: every Cholesky pivot of the *pole-free* form positive on 79/79 windows (T119's negative pivot belonged to the form *with* the pole), per-window Cholesky certificates R4 79/79 / R2b 68/68 — but **every cheap route fails by sign, not size** (Dirichlet+Gershgorin, global, Jacobi dominance, Ostrowski all 0/68; path comparison infeasible, congestion up to 4.7×10^5): the nine decades of slack are worthless to them; anatomy: the good half $\mathrm{diag}(w) + L_N$ definite 68/68 ($\lambda_{\min} = 0.13$–3.06, 8–85,000× above target), the *entire* loss the positive-off-diagonal Laplacian L_P at the long lags ($\lambda_{\max}(L_P)/\mu_1$ up to 4.7×10^5, $k_{\mathrm{eff}} = 24$–397 — not low-rank, T131's rank-one trick does not generalise); R5 measured: compression and Schur both *above* $\lambda_{\min}$ on 56/56 — no coarse recursion can supply a floor; degradation cert $\sim D^{2.26}$ (fit); **the inverse-positivity gets its class**: the lumped Stieltjes comparison $S_B = S + L_\Delta$ is Stieltjes and definite by construction, $\rho(\text{Jacobi}) = 0.709$–$0.990 < 1$ on 900/900 $\Rightarrow S_B^{-1}$ a convergent nonnegative series (discrete Green function; naive split only 659/900; killing reading refuted, row sums negative 900/900; Neumann majorant 558/558, coverage 26.9%; $S^{-1} > 0$ certified a posteriori 900/900 at residual $\leq 2.6 \times 10^{-11}$; RP-on-floor-eigenspace weakening refuted 535/900); **whitening corrects T131**: the 2.18 metric mismatch was not the dominant loss — the comb dominates the pole by 4.7–81×; first finite bound 2.81–5.04 (sharp 2.52–4.26) vs bar 0.50, $\sigma_b = 0.77$–1.00 exhausts the budget alone $\Rightarrow$ the rest is a localisation statement; promotion list 9 → 15, rest list five points	PARTIAL	21/21

Part	Probe	Contract / finding	Verdict	Checks
T135	`comb_compressibility`	the third *reverse-flow* trace: the T116 Riccati machinery ported *verbatim* to the seam DtN Λ_Σ (bordered identity 1.5×10^{-16}, banded m-march = dense state to 8.2×10^{-16}; pole candidate the rank-8 Calderón polarisation on the 16 residue classes, documented as a *model*) — census not a kill: 100.00% of the off-band mass on the mod-4 stripes (complement machine zero), stripe count exactly $\lfloor W/4 \rfloor = \Theta(W)$ (exponent 1.0000 ± 0.0000) against the true prime comb's $\pi(W)$-like 0.726 ± 0.018; **the core number** $m_{\mathrm{cert}} = 12$: error/margin 3.3×10^{-16} exact ($h = 64$–1500) and 2.1×10^{-17} certified ($h = 1000$–$100{,}000$, *falling* in h) against T116's 1.3×10^{2}–1.1×10^{6} *above* the margin; $m = 4 \to 48$ divides the error by 4.7×10^{8} (T116: no decay); $12 \in \{8, 12, 16, 24, 32\}$ pre-declared ⇒ MATCHED, not derived (bar-fragile: $8 \le m_{\mathrm{cert}} \le 16$ defensible); controls: mod-3 also closes ($m = 8$) ⇒ the driver is **weight summability**, not comb periodicity; the *true* prime comb *fails* under identical machinery (error/margin 0.16–0.27 rising, 12× state buys 1.69×) — non-vacuity established; sober: `QEC.SEAM.01` *not* advanced, m_{cert} partly circular (16 channels by hand), model DtN not the raw seam; new: finite-dimensionality visible in a *recursion*, not only a spectrum, with the Weil window as the proved contrast	BOUNDED-STATE	13/13
T136	`m_matrix_pair`	the M-matrix question T134 left open, and **one of three goals closes outright**: the classical row-sum criterion is *vacuous* at the border (900/900 negative row sums — the same sign failure), but **Varga's regular splitting makes** $\rho(J) = \tau/(1+\tau)$ **an identity** for the lumped Stieltjes comparison and the Collatz–Wielandt bound at the M-matrix anchor vector reproduces the measured $\rho = 0.709$–0.990 with a gap shortfall of only 1.00–1.03, *flat* in D and in the zone on 900/900 (a priori $\rho \le 0.7160$–0.9903, bar 10) ⇒ rest item *(3)* **closed**, two solves instead of a spectrum; the pair itself resists with an *exact* anatomy: anchor half certifiable losing ≤ 1.51, but margin $1 - \rho(E)$ only 2.5×10^{-6}–2.5×10^{-2} wide against cheap $\rho(E)$ bounds at 1.82–3.11, the shifted ladder empty *by* monotonicity of M-matrix inverses (Berman–Plemmons), and the three-way split $\lambda_{\min}(A) \sim D^{2.27} = D^{-0.56} \times D^{2.72} \times D^{0.12}$ (anchor × gap × slack; fitted exponents) putting the *whole* degradation in the margin against an anchor that improves with depth; **M17 closes negatively**: the bad pencil subspace is *delocalised* (participation 0.54–0.98, 91% of cells for 90% of the mass, flatness 0.98), its dimension tracking the atom count ($n_{\mathrm{bad}}/n_{\mathrm{at}} = 1.64$) not the grid, so $\sigma_b(k) \ge 0.646$ and no cut brings M18 under 0.50 (best 2.76–7.00) — what M18 needs is a localisation *statement*; promotion list 15 → 22 (→ v543)	ONE-CARRIES	30/30

Part	Probe	Contract / finding	Verdict	Checks
T137	`long_lag`	the long lags named, and **a whole bounding family killed by certificate**: the archimedean half carries *no* positive off-diagonal (0 of 40 windows), so L_Δ is entirely **comb-generated** — supported on the anti-diagonal stripes $r+s = M-1-\mathrm{round}(u_j/D)$ at the prime-power atoms (1.00–1.00 of the positive entries within one grid cell of an atom, 4–192 stripes on 3–99 atoms, 1.33–1.97 per atom), each stripe a *perfect matching* (orthogonal edges, 2×2 blocks), with the certifiable amplitude law $\Delta \le c_{\mathrm{comb}}(\mathrm{anti})$ (the single-atom $\mu_j/2$ only *typical*, worst ratio 2.268); **the $\lvert E\rvert$-envelope family is dead by a lower bound**: Collatz–Wielandt from below gives $\rho(\lvert E\rvert) \ge 1.3218$–$2.2963$, above 1 on 35/35, so no Gershgorin bound, no row-sum norm and no rescaling can *ever* produce a floor; the surviving edge/Gram family (second differences of the Green function) reaches 1.1530–2.3502 against a true ρ of 0.9752–1.0000 and clears the target on 0 of 35 (overshoot 24–7.1×10^5 gaps), the coarsening ladder crossing 1 only when the entire Gram is one block ⇒ *the inter-stripe coupling is the object*, decay algebraic $\mathrm{dist}^{-0.63}$ (fit; Demko–Moss–Smith *not* applicable, A_B dense); **rest item (2) delivered**: the Neumann series bounds the Green function from below with *no* error term (recovering 0.011–0.406 of the minimum entry) and the anchor-weighted ceiling from above (overshoot 2.49–12.24), yet the $S^{-1} > 0$ certificate still stops at 563/900 (716/900 k-exact, 66/900 structural) because $\rho(\lvert F\rvert) \ge 1$ on 77 blocks — *the same wall*: the absolute value destroys the cancellation, and the residue is a sign-preserving bound; promotion list 22 → 30 (→ v544)	BOTH-RESIST	22/22

Part	Probe	Contract / finding	Verdict	Checks
T138	`sign_compensation`	the cancellation **kept instead of majorised**, and the mechanism found: the sign of an inter-stripe coupling follows the **interval geometry** of its two edges — nested positive on 0.76–0.94 of the area, crossing 0.59–0.63, disjoint only 0.06–0.36 (mostly negative) — *not* the lag distance (flip rate near 0.5), with global cancellation $\lvert\Sigma\rvert/\Sigma\lvert\cdot\rvert = 0.29$–$0.70$ (the one-number reason for every absolute-value overshoot); honest negatives: Karlin TP_2 *fails* (0.54–0.65 of the minors), Green rank-one dominated but *not* semiseparable; the best **certified** signed bound (band-exact Cholesky + block row-sum tail) reaches 0.9758–1.1020 against target 0.998556–0.999979 — 1/16 windows, overshoot 0.02–565 gaps against T137's 27–8×10^5, three orders better and no floor; the clamp *localises*: the band crosses the target at $b = 2$–8 with the certified tail still 0.46–0.70 (never small together), the far tail is **net negative** (truncation raises ρ on 59/142 rungs), Leibniz pairing buys nothing, two-body exact but thin ($\leq 0.6\%$; three-body up to $2.5\times$ — a *many-body* effect); **the m-paired Neumann certificate** $((I-F)^{-1} = (\sum_{j<m} F^j)(I-F^m)^{-1}$, only the outer factor majorised, condition $\rho(\lvert F^m\rvert) < 1$) removes the arithmetic wall on *all* 77 dead blocks, certifies 52, pool $563 \to 875/900$ ($\max \lvert F^m\rvert/\lvert F\rvert^m$ median 0.295); M17 whitened: $\mathrm{St} = I - W_S$, $W_S \succeq 0$, direct Kantorovich harmonic-mean route with no bad subspace, margin relaxed $0.5 \to 0.8284$ (854/900, HARMONIC-PARTIAL); promotion list $30 \to 37$; the margin question returns one level down ($\rho(W_S)$)	PAIR-EXACT	26/26

Part	Probe	Contract / finding	Verdict	Checks
T139	`green_decay`	the decay lemma **refuted at its hypothesis, arithmetically**: the certified kernel envelope decays $d^{-0.15}$–$d^{-0.04}$ where Jaffard 1990 needs $p > 1$ — the archimedean lags sit *on* the borderline ($\sim 1/d$), the prime-power atoms lay $O(1)$ spikes across the window (no envelope sees the comb's thinness); constant chain constructive: Demko–Moss–Smith *verified where applicable* (75/75 banded rungs), operator gate 0.97–1.83 (1/75), algebra gate 3.4–1.6×10^3 (0/75); exponent *not* inherited (kernel $d^{-1.5\ldots-1.1}$ vs inverse $d^{-0.52\ldots-0.37}$); the replacement is an **identity**: $b_e^{\mathsf{T}} G b_{e'} = \sum_{\text{box}} H$ exactly (4×10^{-15}), $H < 0$ on 0.71–0.87 off-diagonal, $H > 0$ on 100% of the diagonal $\Rightarrow$ **T138's whole sign law derived** from one kernel plus box geometry (Gantmacher–Krein one-pair quantitatively dead); the never-truncating layer series **killed from below**: 3.388–6.171 vs target 0.999182–0.999972 (0/15, overshoot 681–46900× = the effective layer count), zero trace forbids dropping any layer, Gram $\preceq$ Band$_b$ *never* (85/85), Weyl floor $\max_b$ Rayleigh(Band$_b$) = 1.0303–1.1116 > target on 15/15 — no decay lemma can rescue it; clean-up: m-ladder to $m = 48$ certifies 619/620 border blocks (the one open: need 2.15, tightness at index distance 15), $\rho(W_S) = 0.667$–0.939 inherits the structure verbatim, three-body cumulant negative on 0.93–1.00 of 4200 triples (certified *benign*); promotion list 37 $\to$ 47; **the core shrinks to one signed inequality at stripe distance** $b \leq 16$	DENSE-RESISTS	30/30

Part	Probe	Contract / finding	Verdict	Checks
T140	`signed_band`	the one signed inequality acquires an **exact finite core per zone**: the H-telescope lifted from entries to the *form*, Gram $= CHC^{\mathsf{T}}$ exactly (3×10^{-15}, $C = \mathrm{diag}(\sqrt{\Delta})M$ with the edge-interval incidence M), so $\mathrm{rank(Gram)} \le h - 1$ (44–202 vs 128–1499 edges) and $\rho(W) = \lambda_{\max}(K^{1/2}HK^{1/2})$ exactly (6×10^{-14}), $K = M^{\mathsf{T}}\Delta M$ the covering kernel in closed geometric form $K_{rs} = W([r \wedge s, r \vee s])$ (nonnegative, monotone by inclusion; Green-like, *not* one-pair); energy reordering $H = \mathrm{diag}(s) + L_N$ exact (5×10^{-16}) — the sign law as mass + long-range Dirichlet form (weights positive on 0.71–0.87); the certified chain **tears at the drop of the positive H part** (6.4–239× alone; the Hardy step adds 6.4–9.0×, input $\max_k Q_k = 18.5$–339; 0/16 on all four routes); **the checkerboard split solves R2**: $\mathrm{Band}_b = D + A_{\mathrm{even}} + A_{\mathrm{odd}}$ entrywise exact ⇒ three D-independent Weyl steps instead of $O(nb)$; blocks zone-uniform ($D^{0.13}$, $D^{0.02}$) vs $\lambda_{\max}(K) \sim D^{-2.99}$ — **all D-dependence in the geometry**; band from below > target on 16/16 (rank inflation 1.2–10.1× as mechanism), index-grid far part Loewner-*positive* on 256/256 (no truncation licence in either grid); Floquet not applicable (spread 0.15–0.69), Bendixson certified at 1.07–1.08 × $\rho(W)$; border pool 420, m-ladder to 24 certifies 402, the 18 open far-carried (0.635–0.912); core inherited one level down ($\rho(W_S)$ to 4×10^{-15}, = 0.541–0.925); map V12 (18 rows), 17 new promotion items (stock 47 → 64); **the rest is one zone-uniform discrete Hardy inequality**	FINITE-CORE	31/31

Part	Probe	Contract / finding	Verdict	Checks
T141	`discrete_hardy`	the Hardy ingredient **resists, and the resistance gets an address**: four exact identities put R1″ in classical two-weight shape — the K^+ Rayleigh form $\rho(W) = \sup\{u^\mathsf{T}Hu/u^\mathsf{T}K^+u\}$ (3×10^{-11}), the signed crossing kernel $L_N = D^\mathsf{T}BD$, $B_{kl} = \sum_{r \le k\wedge l,\, k\vee l < s} N_{rs}$ (7×10^{-16}), $Q = B_+\mathbf{1}$ (3×10^{-15}; T140's Cauchy–Schwarz step *is* the Schur/Gershgorin diagonal step) and $DKD^\mathsf{T} = L_\Delta$ on the interior nodes (2×10^{-13}; the Hardy denominator is the original Laplacian, its diagonal the *endpoint edge mass*); **the uniformity separates**: the certified constant $A_M = 2.20$–31.23 runs as $D^{-0.366\pm0.036}$ (spread 14.21× over a D range of 11.21×) — *not* zone-uniform at bar 0.25 — while the object it bounds, the exact Dirichlet share, is uniform in the strict sense ($D^{-0.229\pm0.007}$, band 0.235): the growth sits in the diagonal profile, and the closed Muckenhoupt product ($D^{-2.687}$) and the Gershgorin variant ($D^{-0.993}$) overshoot the signed constant by 25–15903× and 1.2–14.3×; **the additive shape is dead as a shape** — its own exact two-term Weyl floor is already 1.694–3.855× the target (0/60; additive Hardy 2.775–32.152×, the Hardy step costing only 1.62–16.86× over the exact share, so the shortfall is the *split*); **the joint shape fails at the normalisation alone** — $\Lambda(H, Y_{\mathrm{tri}}) = 0.500$–$0.942$ is fine, $\Omega = 20.71$–2723.99 (ideal 1, $D^{-1.867}$) is not, with an entrywise off-tridiagonal defect of K^+ of only 0.027–0.285: Gantmacher–Krein returns, the *profile* is the whole remaining freedom; R3′ shaped as a first-moment statement (cancellation gain of the signed kernel 31.95–15561.41× against the T140 drop cost); border pool 256, ladder to $m = 24$ certifies 245, 11 open and far-carried (0.835–0.902); map V13, 13 statements ripe (stock 62 → 75); **the rest list collapses to R1b: one closed conductance profile with $Y \preceq K^+$ and $\Omega \approx 1$** (→ v545)	HARDY-RESISTS	22/22

Part	Probe	Contract / finding	Verdict	Checks
T142	`conductance_profile`	the optimal Hardy weight **found exactly, not guessed**: the capacity decomposition $K^{-1} = D^{\mathsf{T}} J^{-1} D + xx^{\mathsf{T}}/\mathrm{cap}$ ($J = DKD^{\mathsf{T}} = L_{\Delta}$ the endpoint Laplacian, $x = K^{-1}\mathbf{1}$ the equilibrium charge, $\mathrm{cap} = \mathbf{1}^{\mathsf{T}} K^{-1} \mathbf{1}$) verified to 5.5×10^{-12} on 26/26 windows; the Dirichlet half is an *orthogonal projection*, so $\Omega = 1$ **exactly** (T141 guessed 20.7–2724, $D^{-1.87}$) — the weight is geometry (Miclo 1999, Maz'ya capacity), and the exact capacity Rayleigh form follows with no inequality taken; the variational class optimum is $\Lambda\Omega = 1.3160$–1.4235 at $D^{-0.029 \pm 0.014}$ — **the failure is a constant factor, not a power** — with the stable shape $c_k \sim 1/J_{kk}$ (correlation 0.988–0.995) and essentially no mass; the certified chain reads $\Lambda\Omega = 2.2671$–2.4536 vs target ~ 1 (0/26, flat at $D^{0.006 \pm 0.004}$; gain over guessing 31.9–137×); **the rank ladder closes the comparison path**: the diagonal minorant + r dominant residual modes stalls at 1.39–1.41 for all $r \leq 128$ and crosses 1 only at $r^*/m = 0.995$–0.998 (the tautology $Y = K^{-1}$; anchored to ρ at 1.7×10^{-11}), and since $\rho(W) = 1 - \Theta(D^3)$ with equality only at $Y \propto K^{-1}$, **no comparison argument can deliver D-uniformity**; R3′ corrected — only 0.03% of the first moment of $(-H)_+$ inside distance 8 (support radius 204–873), the long-range Loewner step certified, its constant $\sim D^{-1.73}$ not uniform ⇒ R3″; border pool 72, 68 certified, 4 open (required far shrink 0.656–1.052); map V14, 9 statements ripe (stock 75 → 84); **rest list: R1c the sharp capacity–Rayleigh route, R1d the LP/SDP dual, R3″, R4**	PROFILE-RESISTS	24/24

Part	Probe	Contract / finding	Verdict	Checks
T143	`sharp_capacity`	the sharp route **carries**: the exact capacity–Rayleigh form $1-\rho(W)=\inf_v[d^{\mathsf{T}}(J^{-1}-B)d+(x^{\mathsf{T}}v)^2/\mathrm{cap}-\sum_k s_k v_k^2]/[d^{\mathsf{T}}J^{-1}d+(x^{\mathsf{T}}v)^2/\mathrm{cap}]$, $d=Dv$, is an *identity* on 26/26 windows (assembly $10^{-11}/5\times10^{-12}$, eigenvalue 3×10^{-6} relative under the declared 1/gap conditioning), with a structurally new bookkeeping — the minimiser *orthogonal* to the equilibrium charge (the capacity rank one carries nothing), the mass share *negative* (-0.0698 to -0.0029), the gap a cancellation between Green share (exactly 1) and crossing share (1.0029–1.0698), the naive split certified dead at 6.19–$66.63\times\rho$; **Maz'ya's criterion lands inside its window**: $\Phi_{\sup}\lambda=0.5438$–$0.6457$ on 26/26 (window $[1/4,1]$) for a *non-Markovian* form (72–84% positive off-diagonals), the bound $\lambda\geq 1/(4\Phi_{\sup})$ captures 0.387–0.460 of the true gap with a **zone-uniform loss factor** $D^{-0.048\pm0.010}$ (bar 0.25); the supremum lives on closed families (0.9606–1.0000 of the best, local search $\leq 1.041\times$; in node coordinates *intervals* dominate by 8.3–129.5 — Muckenhoupt's one-dimensional structure) and is *certified by enumerating all* 255–16383 *subsets* on 6 border blocks ($\Phi_{\sup}\lambda=0.6087$–$0.9016$, inside the window); Miclo's constructive chain loses 46–2561 ($D^{-2.13}$) — the conclusion holds, the classical proof mechanism does not; R1d dead as a direction sample (the LP dual saturates at the trivial floor, $\rho(W)$ to a factor 1.000000; needs the full SDP constraint), border pool 36, 32 certified, 4 open, the capacity argmax entirely the far tail; map V15, 10 statements ripe (stock 84 → 94); **rest list: S1 one named capacity inequality** $\mathrm{cap}_E(A)\geq|A|\lambda_0/c_0$ **with absolute** c_0**, S2 the Muckenhoupt/interval route to it, S3 a closed capacity lower bound, R1d′, R4**	SHARP-CARRIES	24/24

Part	Probe	Contract / finding	Verdict	Checks
T144	`capacity_inequality`	the interval route **carries, and the chain has exactly one unproven input**: with the *Jacobi whitening* as the single comparison licence ($\kappa_{\mathrm{up}} = 1.1195$–$1.3162$ certified per window; Gershgorin would allow ≤ 1.983; a denominator comparison, untouched by T142's obstruction), two interval-reduction statements separate — the *pointwise hull comparison is false* (up to 425, drifting $D^{-1.137}$, scattered level sets as offenders $\Rightarrow$ S2 must be a testing condition) while the *best-interval comparison holds*, $c_{\mathrm{glob}} = 1.0000$–$1.6876$ zone-uniform at $D^{-0.068\pm0.024}$, measured on 2.24 million sets incl. *all* 16383 subsets of 136 enumerated blocks (capacity monotonicity holds on all of them, though the form is not Dirichlet); **the two-weight sup is exact, not sampled**: 11 390 676 intervals exhausted via a Cholesky prefix-sum identity (verified to 10^{-11}), giving the core number $B_{\mathrm{res}}\hat{\lambda} = 0.6694$–$0.7813$ inside the Maz'ya window $[1/4, 1]$ on the whole surface, flat at $D^{0.013\pm0.005}$ and coordinate-robust (T143: 0.5438–0.6457), with the free half $\Phi_{\mathrm{sup}}\hat{\lambda} \leq 1$ passing on all 2.24M sets; **the chain** $\lambda \geq 1/(c_0\kappa_{\mathrm{up}}c_{\mathrm{glob}}B_{\mathrm{res}})$ certifies 0.1002–0.2653 of the exact gap per window, the residual D-dependence in the Jacobi transfer ($D^{-0.178}$), not the capacity calculus; dead: the naive Muckenhoupt trial-energy transcription (89–6995, $D^{-1.885}$); honest counter-side exact: $\Phi_{\mathrm{int}}\hat{\lambda} = 0.0928$–$0.3578$ drifts $D^{0.654\pm0.016}$ — intervals saturate S1 only from above; **the Markov perturbation route is certified dead** ($\lambda_{\min}(E - P) < 0$ everywhere, $t^* \sim D^{3.96}$; 3–12% positive off-diagonal entries carrying 36.7–64.5% of the mass); **S3 stronger than asked**: the Cauchy–Schwarz floor makes Φ_{sup} a max-density-subgraph value (Charikar 2000; Goldberg 1984), $\Psi_{\mathrm{all}}\hat{\lambda} = 0.7399$–$0.8515$ at $D^{0.015\pm0.005}$ over *all* 2^m sets — the flattest number of the probe, argmax on intervals (B_{res} only 1.078–1.129 below); R4: 22 blocks, 19 certified, 3 open, $c_{\mathrm{glob}} = 1.0000$ exact on 5 enumerated; map V16, 9 statements ripe (stock 93); **rest list: S1′ — S1 under a Green mean-density bound, a Muckenhoupt-type instead of a Markov-type hypothesis — S2′ as testing condition, R4, R5; the only unproven input is the absolute constant** c_0	INTERVAL-CARRIES	31/31

Part	Probe	Contract / finding	Verdict	Checks
T145	`mazya_proof`	the proof attempt for S1′, **one step missing, and it has a name**: the classical Maz'ya proof transcribes step by step onto the non-Markovian form — M1 level arithmetic (theorem, factor 1.649–2.670 against the ceiling 3), M2 the Green floor (theorem, Cauchy–Schwarz), M3 truncation admissibility (theorem, ×4, slack ≤ 1.036 on 168 levels) — and **the Markov property sits in exactly one line**, M4 truncation subadditivity, which is *void* here: 3.2–14.9% positive off-diagonals carrying 18.3–58.7% of the mass, the conductance share of the minimiser's energy negative on 61/64 windows, while the same code on the Stieltjes surrogate $E - P$ reproduces the classical step; **but M4 splits** — the mass half is a theorem ($\sum_k f_k^2 \leq \psi^2$ pointwise; ratio 0.2639–0.2995) and dominates, so $\sigma_{\mathrm{tot}} = 0.2145$–$0.4425 < 1$ everywhere; two routes make c_0 **explicit**: energy $12\sigma_{\mathrm{tot}} = 2.5734$–$5.3099$, sign-free $G_{\mathrm{dy}} = 2.1559$–6.3937, better route $c_0 = 2.2480$–4.2265 at $D^{0.028\pm0.017}$ (zone-uniform) against Maz'ya's Dirichlet value 4, and **S1′ is certified per window**: cert_lam_max$(R) \leq c_0\Psi$ on 64/64 (sign price $\lvert R\rvert$ vs R^+: 1.12–1.41, witness built in); the **no-go** $R = aa^{\mathsf{T}} + \varepsilon I$, $a_i = i^{-1/2}$ (entrywise nonnegative, proved density ceiling $4 + \varepsilon$, $\lambda_{\max} \sim \log m$) shows $\Psi_{\mathrm{all}}\hat{\lambda}$ decaying $0.702 \to 0.488$ and G_{dy} growing $7.6 \to 36.8$ — a level-profile hypothesis is *necessary*; S2′ retired as a requirement ($c_{\mathrm{test}} = 1.0782$–$1.1327$ against Charikar's cited 2), R5 two-sided but width $D^{0.543}$ (downgraded; used side flat $D^{0.065}$), R4 all 24 blocks closed; map V17, 11 statements ripe (stock 104); **rest: L1 the level lemma — an a-priori bound for σ_{tot} or G_{dy}, either alone closes S1′ — L2 the profile route to it, R4 bookkeeping**; identity spine of T142–T145 promoted as v546	ONE-STEP-MISSING	33/33

Part	Probe	Contract / finding	Verdict	Checks
T146	`level_lemma`	the proof attempt for L1, **and the level lemma stands**: closed on the measurement surface as a chain of theorems and certified window inequalities, no step reading the minimiser — four hand-written closed profiles (sine, ramp, pole vector, equilibrium potential) lose by **5–9 orders of magnitude** (every guess sees $O(1)$, the gap is $O(D^3)$); what survives is the **Green column** itself ($\lambda_{\mathrm{up}}/\hat{\lambda} = 1.0497$–$1.3764$), and **Davis–Kahan is instrumented and discarded** (informative on 0/64: the spectrum bottom is a near-degenerate block, $\lambda_2/\hat{\lambda} = 1.2542$–$3.4471$, separation in the denominator) because $\psi = \lambda R\psi$ is an *identity* — no transfer step exists in the argument; the lever: $\|\psi\|_\infty \le \lambda_{\mathrm{up}} \max_j \|Re_j\|$ within 1.055–1.426 of the truth, the trivial ceiling would give $\Gamma \le \sqrt{m} = 5.1$–$34.2$, measured $\Gamma = 1.8356$–2.2797 (columns 2.7–16.0 below trivial — that factor *is* the delocalisation, read off the form); the localisation contradiction stays qualitative (leak tolerance $\le 2.38 \times 10^{-3}$), Perron only on the lumped partner; **the cake base is free** (the chain uses only $\|\psi\| \le \psi_t$, Maz'ya's dyadic 8 falls to $2b^2 \approx 2$, gain 3.76, price zero via the $O(m)$ union identity) and $c_0^{\mathrm{ap}} = 2b^2\Gamma \min(1, \Gamma_1) + \varepsilon =$ **3.9042–4.8488** ($b = 1.031$, 64/64) against T145's measured 2.156–6.394 — at the size of Maz'ya's Dirichlet value 4 — chain loss 0.0422–0.1586; stress tests discriminate (no-go Γ 3.676 → 13.800, $m^{0.420}$, unbounded; theorem half survives, slack 1.087–1.242; Dirichlet control flat 1.7183–1.7315); border pool 24/24 closed, a-priori chain positive on 24 ($c_0^{\mathrm{ap}} = 2.855$–$3.176$); map V18, stock 108; **rest: all-D asymptotics of the Green columns ($\max_j \|Re_j\|\lambda_{\min}$), σ_{tot} stays measured (not a hole), the actual 3 T134 blocks**; a-priori core promoted as v547	LEVEL-CARRIES	30/30

Part	Probe	Contract / finding	Verdict	Checks
T147	`green_asymptotic`	the asymptotics itself, **and the last open object becomes an exact identity**: $\Gamma = \sqrt{Q^\star}\, S_w$ (verified to 2.3×10^{-16} on all 85 windows $m = 26$–1491 and both stress forms) — D-uniformity *equivalent* to the boundedness of two named scalars, $S_w = \lambda_{\mathrm{up}} \|R\|_F$ purely spectral (effective bottom multiplicity $n_{\mathrm{eff}} = 1.251$–2.831, certified ≤ 4.6438 at a certified cut by $\mathrm{LDL}^{\mathsf{T}}$ inertia) and $Q^\star = m \max_j (R^2)_{jj} / \operatorname{tr}(R^2)$ purely geometric, scale-free (certified ≤ 2.8634 against the localised extreme $m = 1491$); **the classical decay route is computationally dead** (Demko–Moss–Smith $q = 0.994815$–0.999999, envelope $q^{m/2} = 0.9346$–0.9995 above the 0.05 bar everywhere; Jaffard exponent of E itself 0.06–0.82) and the columns are *delocalised* (half-mass radius 0.048–0.105 of the window) — delocalisation *is* the bound: 0.9864–0.9996 of $\|Re_j\|^2$ from the bottom block alone, $Q_B/|B| = 1.375$–1.839 against the exact floor (naive $Q_B \sim m$ wrong by 5–258), T146's unexplained factor 2.7–16.0 identified as $\sqrt{m/Q_B}$ up to $\lambda_{\mathrm{up}}/\lambda_{\mathrm{lo}}$; honest negative: the crude free-tail split grows $m^{0.538 \pm 0.020}$, reported, not used; **the mechanism is named**: Toeplitz-minus-Hankel section, bottom eigenvectors Fourier modes by Szegő/Widom (Kac–Murdock–Szegő 1953, Widom 1958, Basor–Ehrhardt 2009), sharp prediction $Q_B \leq 2|B|$ **holds**, chain $Q_B \leq \sum_B m \|\psi_k\|_\infty^2 \leq 2 \sum_B \|a_k\|_1^2$ pure arithmetic (sparsity ≤ 8.773 per mode, 0.999+ of the energy in the lowest $4|B|$ modes); four D-strata with certified maxima, six trend exponents flat at bar 0.25, all fits; congruence structure discarded (scatter 0.465); σ_{tot} does not fall but is located (block leakage 0.9627 vs required 2.7×10^{-7}); **the 3 open R4 border blocks close** (8/8 rebuilt border windows, $c_0^{\mathrm{ap}} = 2.9135$–$3.0383$, same mechanism); stress separates: no-go diverges and hits $Q_B = m/H_m$ to 1.0000, Dirichlet control flat (1.4142–1.4376 vs sine value 2); honest limit: translation covariance broken by the whitening (Toeplitz defect 0.21–0.54), the spectral factor needs its own a-priori input; map V19, stock 114; **rest: a Szegő theorem for the reweighted section (lifts the chain to all D), an a-priori bound for S_w, the σ_{tot} classification**	ASYMPTOTIC-SHAPED	39/39

Part	Probe	Contract / finding	Verdict	Checks
T148	`szego_bottom`	the lifting statement dissected, **and the second factor closes on the surface**: the passage from the pure to the reweighted section classified in seven typed steps — the order-2 model ladder and the pure section's Fourier sparsity THEOREMS (KMS 1953, Widom 1958, Basor–Ehrhardt 2009; model $\nu = 4.236$–4.257 flat), the ℓ^1 ceiling $\|a\|_1 \le 2\sqrt{\nu} + 1$ CERTIFIED ($\nu = (m+1)^{3/2}\|L_0\psi\|_1/(2\sqrt{2})$, every m-power cancelled; excess -2.6×10^{-6}), the chain $Q^\star \le 2\langle\|a\|_1^2\rangle_w \le 2\langle\mathrm{ceil}^2\rangle_w$ CERTIFIED 48/48, the commutator/Davis–Kahan route DEAD (5.4×10^2–5.3×10^5, 2.7–5.7 orders vacuous), the live route the LIOUVILLE transform $m\|\psi\|_\infty^2 \le 2\kappa_\Lambda \mathrm{ceil}(\hat{\varphi})^2$ with $\kappa_\Lambda = 1.733$–3.933 certified (gain 50.6), and $\nu \le F(\mathrm{form})$ THE OPEN INPUT (a-priori ceiling 81.4–469.0, grows $m^{0.615}$; ν_L needs $\lesssim 34$, measures 282, $x^{0.166}$); **the hypothesis isolated**: weight-class experiment at identical $\kappa_\Lambda = 4$ — two BV classes flat ($x^{0.000}/x^{-0.007}$), the rough class $x^{1.994}$ — $\mathrm{TV}(\log\Lambda)$ is the named scalar, measured 3.63–11.93 on the real surface, $x^{0.444}$ NOT flat; S_w **closes**: LDL^T layer cake $\Rightarrow S_w \le 1.9587$ certified (true 1.282–1.683), $x^{0.007 \pm 0.007}$ flat, per D-stratum; **honest negative**: the Toeplitz symbol has $f(0) < 0$ on 48/48 ($-87.3 \ldots -22.5$) — positive definiteness is a finite-section effect of minus-Hankel + lumping, the KMS order-2 hypothesis survives only as a measurement ($\alpha = 1.643$–1.989 in $[1.5, 2.5]$); no-go violates exactly the smoothness hypothesis (ν diverges, $n_{\mathrm{eff}} = 1.0000$ innocent), Dirichlet exact ($\nu \to \pi$, $m\|\psi\|_\infty^2 \to 2$); **the big number**: the certified chain delivers 8.36–15.86% of the true gap (median 11.74%, no window loses more than 12×), a-priori-shaped factors still a lower bound at factor 10.5; balance 6 certified / 2 measured / 2 fits, fits in no inequality; σ_{tot} RETIRED AS A ROUTE; map V20, stock 114; core promoted as v548; **rest: $\nu_L \le F$(form) m-free (= bounded $\mathrm{TV}(\log\Lambda)$ or weaker, target $\lesssim 34$ vs 282); cosmetic: an order-2 theorem from the minus-Hankel structure**	ONE-INPUT-MISSING	31/31

Part	Probe	Contract / finding	Verdict	Checks
T149	`weight_smoothing`	the one open input attacked through the gauge freedom, **and the hypothesis is refuted rather than weakened**: the whitening diagonal is a FREE GAUGE (identity plus one Rayleigh step, 9 preregistered candidates, every one a valid lower bound, family maximum valid); the preregistered winner `const` (geometric mean of the Jacobi diagonal) delivers the three ordered numbers — sandwich $\sigma \leq 5.5789$ CERTIFIED ($x^{0.154}$, bar 8.0), TV($\log \tilde{\Lambda}$) **exactly** 0.0000 (T148's 11.93 at $x^{0.444}$ ELIMINATED, not reduced), $\nu_{\tilde{L}} = 222.70$ in target on 21/44 windows only ($\rho = 1.347 \to 0.485$) — at price $\tilde{\kappa}_{\mathrm{up}} \leq 2.3146$ vs 1.2647; **the two-regime decomposition**: flutter-killers cheap (sandwich 1.129–1.572) but move $\nu_{\tilde{L}}$ by $\leq 0.9\%$, only macro-removers improve it (factor 1.27) — TV($\log \Lambda$) was the MISLOCATED hypothesis, the roughness sits in the FORM; chain: $\Gamma = \sqrt{Q^\star} S_w$ gauge-invariant at 4.4×10^{-16} (89 assemblies), family maximum 9.52–18.45% of the true gap certified (median 12.68%), a priori 0.800–1.566% (up to 1.284× better), Jacobi still wins certified 44/44; $Q^\star$ ceiling $697.9 \to 149.1$, exponent $0.551 \to 0.330$, NOT flat (driver $\nu_{\tilde{L}} = 265.2$, $x^{0.772}$, above the bottom block); stress: no-go already smooth, ν diverges $x^{1.420}$ under EVERY gauge; Dirichlet exact ($\nu \to \pi$, 5.9×10^{-14}) and CORRECTS the question to the ladder $\nu_k \leq Ck^2$ with m-free C ($C = 18.66$–44.61, $x^{0.272}$); TV anatomy: 85%+ flutter, amplitude 0.064–0.182 FLAT, edges innocent; 7 promotion candidates, unpromoted; **rest: m-free C in $\nu_k \leq Ck^2$ (load-bearing); an a-priori flutter-amplitude bound (closes it outright); cosmetic order-2 theorem**	PARTIAL-SMOOTHING	30/30

Part	Probe	Contract / finding	Verdict	Checks
T150	`mode_ladder`	the ladder attacked through the flutter amplitude, **and the mechanism has a name — parity**: the form is EXACTLY the compression of the full symmetric Toeplitz section onto its antisymmetric parity sector ($U_-^{\mathsf{T}} T_M U_- = T - H$, cross block $\leq 4.0 \times 10^{-16}$ on real windows; Basor–Ehrhardt 2009 the address) and **the entire negative inertia of the sign-changing symbol sits in the EVEN sector** (odd 0 / even 1 on 72/72, $\mathrm{LDL}^{\mathsf{T}}$) — $f(0) < 0$ EXPLAINED, not worked around; the diagonal is an EXPLICIT zone function ($\Lambda_r = c_0 - c_{M-1-2r} + \sum \max(c_{\lvert r-s\rvert} - c_{M-1-r-s}, 0)$, residual 0.0 on 72/72) with the exact split $c = c^{\mathrm{arch}} + c^{\mathrm{atom}}$, $c^{\mathrm{atom}} \leq 0$; the flutter amplitude a CERTIFIED form functional, fit-free ($\mathrm{amp}_{\mathrm{loc}} = 0.0606$–$0.1993$, $x^{-0.165 \pm 0.015}$, largest log step 0.1927; chain $\leq \mathrm{half} \cdot d_1^{\lambda} / \min \Lambda = 1.345$–$3.954$; atom budget $4B\sqrt{N}$ CLOSED); honest negative: the archimedean diagonal is NOT the macro profile (level shift 0.734–2.647, $x^{0.244}$); **the perturbation step precisely dead, twice**: multiplicative (Loewner) transfers with $\exp(\mathrm{amp}) \leq 14.11$ — the gauge step CLOSES — additive (Weyl / Bauer–Fike / Davis–Kahan) misses by 2.4–5.8 orders ($\lVert A^{\mathrm{atom}} \rVert = 4.3$–$12.3\%$ of the section) AND the archimedean-only section is itself INDEFINITE (2–7 negatives) — **the atoms are co-responsible for the positivity**; third valid ceiling from the parity sines ($\nu_k = \pi k^2$ on the control to 0.0002, gain 1.000–1.130); BUT $C = 14.24$–$43.39 = 13.81\pi$, $x^{0.258 \pm 0.009}$ — the one gate falls; max at $k = 1$ (67/72), the gap $\pi \to 13.81\pi$ exactly the ARITHMETIC LEAKAGE of the bottom mode (ℓ^1 1.41–1.79, $m\lVert\psi\rVert_\infty^2 = 2.45$–$3.41$ vs 2); chain 0.0680–0.1845 certified (T149's best reproduced exactly), ladder-shaped $Q^\star$ ceiling valid by monotonicity (price $\leq 2.555\times$ for ONE constant); 5/10 factors flat; stress LOCALISED: the no-go violates the ladder alone ($x^{1.840}$, multiplier smooth), controls $C = \pi$ (0.0004/0.0002); 9 candidates (stock $121 + 9 - 6 = 124$); core of T149+T150 promoted as v549; **rest: $\nu_k \leq Ck^2$ with m-free C for the ODD parity sector at a sign-changing symbol — THE one inequality; a closed bound on $\min \Lambda^{\mathrm{arch}}$; Basor–Ehrhardt at $f(0) < 0$**	ONE-TERM-MISSING	36/36

Part	Probe	Contract / finding	Verdict	Checks
T151	`odd_ladder`	the one inequality attacked head-on, **and it closes by REROUTING OFF THE LADDER**: the positivity of the odd sector is a GRID fact — the parity sines live on $\theta_k = 2\pi k/N$ with $\mu_k^P = 2 - 2\cos\theta_k$ exact, and the symbol's negative window ($\theta_c = 7.06 \times 10^{-4}$–$1.32 \times 10^{-2}$) lies UNDER the odd spacing θ_1 (ratio 0.328–0.407, $x^{-0.057}$) — stepped over on 72/72; BUT the symbol language fails ($f''(0) = 3.8 \times 10^5$–6.8×10^8, f moves 0.47–0.85 across ONE mode spacing, Fejér averaging instead of sampling, deviation 1.39–1.83 at $k = 1$, margin vacuous 72/72) — **T150's rest 3 answered NEGATIVELY: the local model is a MATRIX statement** — $\lambda_k(A) \le S\mu_k^P$ for $k \le 8$ via LDL^T (Sylvester), $S = 1.1019$–2.3870, $f(0) < 0$ ABSORBED (Rayleigh floor $L_P \ge \mu_1^P I$); leakage = LOW-k ROTATION ($\sin\angle(\psi, t_1) = 0.816$–$0.894$, shares 0.355/0.687/0.005 at $k = 1/2$–$4/{>}64$), no arithmetic sideband; **the Sobolev step IS the multiplicative mode comparison**: virtual node $\Rightarrow \|\psi\|_\infty^2 \le 2\|\psi\|_2\|\nabla\psi\|_2$, certified pencil floor $\kappa = 0.2273$–0.4502 $\Rightarrow b_k \le 2m\sqrt{K_{\mathrm{bot}}\mu_k^P/\kappa} \le C_S k$ — **LINEAR,** $C_S = 11.5137$–19.5731 **(median** 13.82**), trend** $x^{0.020\pm0.007}$ **NON-GROWING** vs 13.81π at $x^{0.258}$; factor 7.5–34.9 better at the bottom; rotation route priced and discarded ($4\sqrt{m}$, overshoots 4.2×10^3–1.0×10^6); global pencil useless ($x^{2.695}$) — ALL carried by $R = K_{\mathrm{bot}}/\kappa = 3.3634$–$9.7108$ ($x^{0.037\pm0.015}$), the ONE fit; $\min\Lambda^{\mathrm{arch}} = c_0^{\mathrm{arch}} - c_{M-1}^{\mathrm{arch}} = 2.5928$–$4.8981$ CLOSED AND ATTAINED (rest 2 done); $Q^\star_{\mathrm{sob}} = 12.39$–$23.94$ beats the ℓ^1 ceiling 2.69–10.13×, end to end $\to 2.01 \times 10^{-2}$–3.52×10^{-2} (gain up to 3.18×), dominant loss now $\Psi = 4.7 \times 10^3$–6.1×10^6; no-go breaks IN THE PENCIL only ($x^{1.986}$), controls exact (pencil $(1,1)$ to 3.7×10^{-12}, Sobolev price EXACTLY π); 10 candidates (stock $124 + 10 - 5 = 129$); core promoted as v550; **rest: the m-freeness of $R = K_{\mathrm{bot}}/\kappa$ — ONE scalar; a closed κ floor from the lag vector; Ψ**	ODD-CARRIES	27/27

Part	Probe	Contract / finding	Verdict	Checks
T152	`pencil_ratio`	the one scalar attacked head-on, **and both hoped-for arch gifts are REFUTED while the floor closes structurally**: A^{arch} is NOT positive in the odd sector on any window ($\lambda_{\min} = -2.8132$–-1.8396, certified by completed Cholesky; in pencil normalisation $\kappa_{\mathrm{ar}} = -5.28 \times 10^5$–$-4.00 \times 10^3 = O(-m^2)$, the lowest parity sine having L_P-energy $O(m^{-2})$ against an $O(1)$-negative form) and the atom part is negative too (-1408–-25.6): **odd-sector positivity is a CANCELLATION between geometry and arithmetic, not a property of either half** (at t_1: arch -5.26×10^5–-3.98×10^3 plus atom 8.19×10^3–2.88×10^6 — the atoms OVERSHOOT); additive Weyl misses κ by 1.5×10^4–1.8×10^6, the ℓ^1 budget by a further 2.3×10^5–9.9×10^7; BUT the arch diagonal is negative on only $k \leq 4$–7 modes (m-independent), above them quotient ≥ 0.989 ⇒ **the Schur two-block criterion (Schur 1917, both Cholesky floors subtracted) with a FIXED 16-mode low block certifies** $\kappa \geq t = 0.2250$–0.2500 **on every window,** $m^{0.006 \pm 0.036}$**, quartile medians identical — the floor is no longer a fit**, consuming ONE unproven inequality $B_{HH} \succeq tI$ (entrywise/Gershgorin provably insufficient: -8.99×10^5–-58 even after the cut; median row coupling 3.9–19.6 vs diagonal 0.36–0.89); the arch part does not carry the CEILING either ($K_{\mathrm{bot}}^{\mathrm{arch}} = 24.6$–$1326$, $m^{1.519}$, one to two orders above the true $K_{\mathrm{bot}} = 1.102$–2.118 — the atoms CREATE the alignment); new number $R \leq K_{\mathrm{bot}}/t = 4.408$–$8.471$**,** $m^{0.099 \pm 0.167}$**, certified on BOTH ends**, quartile walk 1.269 over a 4.54× m-range; two structural m-free routes lose 4–6 orders (Courant–Fischer: atom overshoot $O(m^2)$; two-sided bulk: KMS ratio a theorem, 285.95–$288.97 \leq 289$, but $T_{HH} = 26.8$–3116 grows $m^{2.044}$); no-go breaks exactly at the unclosed step ($m^{1.994}$), control $(1, 1, 1)$ to 3.5×10^{-11}; $\Psi \leq 8.70 \times 10^3$–$6.24 \times 10^5$ IS $1/\lambda_{\min}(E)$ to within 1.12–1.19, grows $m^{2.18}$ because $\mu_1^P = O(m^{-2})$, and 89–92% of $\mathrm{trace}(E^{-1})$ sits on the eight lowest modes — the ladder-sum rebuild is worth 3.46–5.29× end to end; honest dual reading printed (band estimator alone: RATIO-RESISTS); T151's $x^{0.037}$ was fitted against the zone variable, refit in m: 0.099; 10 map entries, NO promotion (the candidates consume the unproven block positivity); **rest:** $B_{HH} \succeq tI$ **as a block statement; an** m**-free** K_{bot} **ceiling; the** Ψ **rebuild, subset sup → ladder sum**	ONE-TERM-MISSING	37/37

Part	Probe	Contract / finding	Verdict	Checks
T153	`psi_ladder`	the mapped rebuild carried out, **and refuted by exactly its own factor — Ψ is PINNED**: for the subset maximising the first mode every term of $\sum_k \lambda_k^{-1}\langle v_k, \mathbf{1}_S\rangle^2$ is nonnegative, so $a_1/\lambda_1 \le \Psi \le 1/\lambda_1$ with $a_1 = 0.7695$–0.8306 — exactly the $8/\pi^2 = 0.8106$ of the first parity sine — Ψ determined to within $1/a_1 = 1.204$–1.300: **T152's** 3.46–5.29× **reserve never existed**, and the head/tail replacement lies 2.31–6.12× BELOW the hard lower bound (the maximising subset is aligned with v_1 — the head IS the value of Ψ); three rigorous rebuilt forms, all upper bounds on the same Ψ: mode-resolved ladder sum 1.195–1.761×, arithmetic-free operator form $(\text{const}/t)\Psi_P$ and theorem-grade collapse $\Psi \le \text{const}/(t\mu_1^P)$ both 3.73–9.70×; net gain 0.50–1.11× (median 0.62), end to end 1.01×10^{-2}–3.92×10^{-2} (the retired $c_0^{\text{ap}} = 4.157$–4.870 and the collapse cost cancel); collapse cost named: $\lambda_1(A)/(t\mu_1^P) = 3.25$–$8.40$ — the pencil minimum not at the bottom mode; Ψ **CLOSED as a term** (form (c) retires Charikar's licence and every per-window diagonalisation, needing nothing beyond t); **SIGN REVERSAL on the bulk**: the arch part is POSITIVE on the bulk parity block ($\lambda_{\min}(B_{HH}^{\text{arch}}) = 1.0362$–$1.4143 > t = 0.25$, peeling ≤ 8 modes, m-free) while the ATOMS are negative there (-484.6–-14.95); four block candidates dead (entrywise Gershgorin -1929–-21.35; block Gershgorin -333.4–-7.68, long-range coupling; mode-index T+H residual 0.70–0.95; recursive Schur scale-invariant, $t = 0.25$ at levels 0/1/2), the live Kato-type route positive per window ($(1 + \lambda_{\min}(C) = 8.3 \times 10^{-4}$–$2.9 \times 10^{-2}$) but shrinking $x^{-1.778}$; t certified 0.2400–0.2500 on 18 windows, flat; ceiling: no fixed family flat (sines $x^{2.47}$, Hann $x^{2.46}$, Fejér degenerate; the overshoot is the HANKEL REFLECTION half, +9.45 vs Toeplitz −1.99), but **two inverse-iteration steps give FLAT $K^F = 1.432$–2.369 on a fixed-size-8 certificate** vs true $K_{\text{bot}} = 1.102$–1.896; 9 map entries (v580–v588), NO promotion; **rest: TWO terms — the block inequality (sharp form: ONE number, the Kato distance against the m-free arch reserve 1.04–1.41) and the Green/alignment estimate of $(A^{-1}L_P)^2 t_k$, $k \le 8$ — both losing to the SAME missing object**	REBUILD-RESISTS	33/33

Part	Probe	Contract / finding	Verdict	Checks
T154	`green_align`	the Green/alignment estimate attacked head-on, **and the CEILING CLOSES, exactly and at fixed size** — with a direction lesson: the ceiling needs NO residual argument (Ritz values are UPPER bounds of the same-index eigenvalues, Courant–Fischer 1920/Cauchy 1829; Temple 1928/Kato 1949 is a FLOOR device) — the certificate $\mathrm{span}\{t_1..t_8\} + A^{-1}L_P\,\mathrm{span}\{t_1..t_8\}$ (16 columns, FIXED size, 8 Choleskys $\le 8\times 8$) carries $K^F = 1.1019$–1.9964 flat ($x^{0.094\pm0.058}$) **agreeing with the inertia-certified K_{bot} to** 5.17×10^{-7} **on** 12/12 — the 8 size-m $\mathrm{LDL}^{\mathsf{T}}$ counts RETIRED from the ceiling step (sines alone: 11.2–2.1×10^{4}, $x^{2.47}$); FLOOR at fixed size REFUTED $(\|R\|^2/(\beta\lambda_1) = 6.3\times10^{3}$–$5.6\times10^{9}$, $x^{4.50}$; scan closes 1/12 at $k_c/m = 0.64$; scalar Temple vacuous 12/12); **the obstruction NAMED: seven bottom directions agree to** 0.15–1.35° **(median), ONE sits at** 82.93–89.79° **— that single misalignment IS the collapse price**, recovered IN FULL per window by one Cholesky (4.408–7.985); the missing fixed-size ingredient reduces to ONE arithmetic-free number (the L_P floor on the Ritz complement, flat 5.93–$8.25\,\mu_1^P$, worth 91–100%); Kato anatomy: the loss is NOT the floor ($\lambda_{\min}(B_{HH}) = 0.266$–$0.415 > t$) but misalignment (1.97–121 at 24.9–89.8°; minimiser 96.5–100% on the 8 modes above the cut), Hankel culprit refuted TWICE (both halves hurt at the minimiser; without the reflection $\lambda_1 = -7.49\ldots-2.64$), the only working peel needs 0.40–0.69 of the window; end to end, two numbers never conflated: m-free-in-shape UNCHANGED 1.01×10^{-2}–3.92×10^{-2}, **per-window certified** 4.45×10^{-2}–3.13×10^{-1} **— inside the target band**; no-go: floor survives ($t = 0.05$), ceiling explodes $x^{1.99}$, certificate WITH it ($x^{2.29}$, overshoot 1.12–2.19) — attainment is a property of the REAL section; parity control 3.5×10^{-6}, direction violated 0/26; **rest: TWO uniformity terms — R1′ the m-free block floor (direction-aware) and R2′ the m-free bottom-mode floor on the Ritz complement (arithmetic-free)**; certificate core promoted as `v551`	ALIGN-RESISTS	29/29

Part	Probe	Contract / finding	Verdict	Checks
T155	`bottom_floor`	the two uniformity floors attacked head-on, **and the complement floor becomes an EXACT fixed-size certificate**: $\min_{v\perp W} v^{\mathsf{T}} L_P v \geq \mu^P_{K+1} - \lambda_{\max}(M^{1/2}(I - GG^{\mathsf{T}})M^{1/2})$, $M = \mathrm{diag}(\mu^P_{K+1} - \mu^P_k)$, $G = T_K Q_W$ — an identity plus one Rayleigh bound, a THEOREM for every m and every subspace, direction checked (the certified $\lambda_{\max}$ is SUBTRACTED and the certificate never exceeds the exact value) — reproducing the m-sized number to 0.999999783–0.999999958 on 16/16 windows, $K = 12$ sufficient everywhere (ladder $8 \ldots 64$ preregistered); controls hit from both sides ($\mathrm{span}\{t_1..t_8\} \to \mu^P_9$ at 1.05×10^{-12}; the WORTHLESS configuration $\mathrm{span}\{t_2..t_9\} \to \mu^P_1$ at 8.89×10^{-11}, reported rather than flattered); **the defect is localised at $k = 1$, the bottom mode itself**: free scale 80.74–81.00 against measured 5.82–7.86 (factor 10.31–13.92), largest uncovered fraction 0.539–0.611 ($x^{-0.063\pm0.016}$), and the 82–90° direction is DEMYSTIFIED — it lives on modes 9–12 (L_P Rayleigh 81.2–103.4 μ^P_1, $\leq 1.9 \times 10^{-2}$ of its mass on modes 1–8): exactly why $K = 12$; **the collapse price** 3.250–8.471 **is recovered** 78.8–100.0% **AT FIXED SIZE** (in full on 12/16, $\geq 90\%$ on 14/16; 32 Green columns + an 8×8 Ritz block + one 12×12 certificate, no size-m factorisation anywhere); two repairs refuted with mechanism (the pencil-ceiling chain dies on $\kappa = \lambda_{\max}(B) = 3.1 \times 10^3$–$2.7 \times 10^6$, $x^{2.82}$, attained where L_P is small; wider $r = 12/16/24$ raises the floor ×1.19–3.71 but the residual ×12.4–8514, Temple 0/32); block side: $\lambda_{\min}(B_{HH}) = 0.2430$–$0.4249$ certified positive, T154's anatomy belongs to a DIFFERENT vector (the B_{HH} minimiser puts only 1.5×10^{-5}–0.69 on the eight-mode window), the direction-aware 2×2 split dies on the coupling ($\|B_{wd}\| = 5.81$–389.41, ×16.8–908 too large, $x^{1.94}$), the local atom norm exceeds the local arch reserve by 1.50–5.12, the deeper cut does not terminate; **the arch reserve IS the symbol infimum** (1.000022–1.000521 on 14/14; band 1.0004–1.4049, factor 4.0–5.6 over t — theorem candidate, Szegő 1915/Widom 1958) while **the FULL symbol infimum is $-714.2 \ldots -7.6$, negative on every window against a positive section floor: NO symbol argument can produce R1′** — the mechanism must be finite-section Fejér cancellation (min diagonal 0.3840–0.7444); end to end 3.28×10^{-2}–2.83×10^{-1} CERTIFIED AT FIXED SIZE, the declared 4×10^{-2} bar missed by 1.22 and reported (like-for-like: the share 78.8–100%); no-go breaks on TWO axes (ceiling $x^{2.010}$; the new instrument reads defect 0.999–1.000 at exactly 90.00°, recovery 0/8); parity control 4.85×10^{-14}, Dirichlet hits its own $2/\sqrt{N}$ prediction to 4.1×10^{-7}; **rest: R2″ an m-free upper bound on ONE 12×8 object; R1″ the m-free atom part on the bulk, with every symbol argument excluded**	FLOORS-RESIST	27/27

Part	Probe	Contract / finding	Verdict	Checks
T156	`twelve_by_eight`	the two rest objects attacked head-on, **and both become SINGLE SCALARS**: on $\mathrm{span}\{t_1, A^{-1}L_P t_1\}$ the coupling is the identity $t_1^{\mathsf{T}} A y_1 = t_1^{\mathsf{T}} L_P t_1 = \mu_1^P$ (no A in it), so the t_1-loss of the bottom Ritz mixing is an EXACT closed function $F(P, r)$ of the Kantorovich product $P = (t_1^{\mathsf{T}} A t_1)(t_1^{\mathsf{T}} A^{-1} t_1) = 5.6 \times 10^2$–$1.0 \times 10^6$ and the inverse moment ratio $r = \|A^{-1}t_1\|^2 / (t_1^{\mathsf{T}} A^{-1} t_1)^2 = 2.7158$–$3.4089$, FLAT ($x^{-0.047 \pm 0.020}$; reduction verified to 2.2×10^{-16}, against the true 2×2 Ritz pair to 4.4×10^{-16}); the box route is VACUOUS despite the new m-free bound $P - 1 \geq L^2 s^2 (r - 1)$ ($\sup F = 1.0000$ where ≤ 0.632 is needed, attained at $P \to 1$); the best route is the two-line theorem $r \leq 1/(Ls) \leq 1/p_1$, TIGHT to 1.03–1.16, so **R2″ is ONE m-free lower bound on** $p_1 = \cos^2 \angle(t_1, e_1(A)) = 0.2010$–$0.3282$ (55.05–$63.36°$, flat $x^{0.054 \pm 0.029}$; loss at the ceiling 0.6718–0.7990 against truth 0.5392–0.6316); interlacing refuted in BOTH forms (rank hole at $K = 12$: eigenvalue 1 exactly; empty at $K = 8$: floor 1.02–$2.58\,\mu_1^P$ against 5.78–7.83); the model dominates the truth 16/16 (ratio 0.833–0.917) — MEASURED, and the stress kills it; block side: **J2a in T155's ≥ 1 form REFUTED** — $\inf f^{\mathrm{arch}} / (4 \sin^2(\theta/2)) = 0.8226$–$1.3973$, BELOW 1 on 3/12 (T155's 1.00002–1.00052 was the upper surface); survives as the weaker $\inf \geq t$ (factor 3.29–5.59, non-decreasing, grid-stable 1.4×10^{-5}, minimum at $\theta/\pi = 0.990$–1.000); Fejér damping real but ONE order (5–31 where the split needs 3–1981 growing); additive split DIVERGENT ($\|B_{HH}^{\mathrm{atom}}\| = 2.2$–$2673.4$, $h^{2.31}$, closes 0/12); **the mechanism is ALIGNMENT**: on the minimiser arch 1.4685–91.7130 + atom $-91.2694\ldots - 1.1151$ $= \lambda_{\min}(B_{HH}) = 0.2661$–$0.4436$, the minimiser seeing 6.2×10^{-3}–5.0×10^{-1} of the atom operator ($x^{-1.044}$) at 52.7–$90.0°$ from the atom-extremal vector; end to end UNCHANGED 3.28×10^{-2}–2.83×10^{-1}, recovery median 99.8% ($\geq 75\%$ on 12/16, like-for-like); balance 9 THEOREM (3 new), 6 CERTIFIED, 3 MEASURED (zero-MEASURED target missed); no-go breaks on THREE axes (K^F $x^{2.047}$; p_1 COLLAPSES to 7.4×10^{-15}–6.6×10^{-10}, $x^{-4.818}$; r-ceiling tightness 12.5–24.6 vs 1.03–1.16) and kills the domination step (16/16 real, 0/8 no-go); controls: parity 4.8×10^{-14}, μ_9^P at 2.3×10^{-13}, μ_1^P at 2.0×10^{-11} in the worthless configuration, Dirichlet $2/\sqrt{N}$ to 7.4×10^{-5}; **rest: R2″ one m-free lower bound on the angle p_1; R1″ one m-free alignment statement (the arch half ready: a one-variable grid infimum)**	TERMS-RESIST	37/37

Part	Probe	Contract / finding	Verdict	Checks
T157	`angle_floor`	the two angles attacked head-on, **and neither falls, but both change shape**: the best p_1 route is the RESOLVENT, $p_1 \geq \hat{g}_1^2(1-\text{tail}) = 0.1968$–$0.3228$ (97.9% of the measured 0.2010–0.3282), splitting into a NEW THEOREM times ONE measured number — **the sine-block confinement** $\|\gamma_H\|^2 \leq \lambda_1/(t\mu_{17}^P) \leq (S/t)/\rho_{17} = 0.0165$–$0.0293$ (from the certified floor t and ladder S alone: $e_1(A)$ is 97.1–98.4% inside the first 16 parity sines; with κ instead of S the bound is VACUOUS 43.5–38632) times $\hat{g}_1^2 = 0.2010$–0.3282 (flat, ONE entry of the bottom vector of a 16×16 Schur null problem); three routes refuted with numbers: the classical Rayleigh angle bound EMPTY already measured (16/16 negative, $\lambda_2/\lambda_1 = 1.73$–$5.87$ nearly degenerate), its block version empty at every J although $p_{\text{blk}}(2) = 0.9369$–$0.9993$ (t_1 lives in the bottom PAIR — the T156 question answered), the angle-free Cauchy–Schwarz route loses exactly the Kantorovich $P = 5.6 \times 10^2$–1.0×10^6; **the structural gem:** $1/s = (S_L)_{11} = 2.3359$–$6.2049$ **FLAT** — R2″ is a bound on ONE diagonal entry of the 16×16 Schur complement the chain already forms, and the inversion-free CS ceiling is off by 5.4×10^2–5.2×10^5 (the cancellation is nearly complete); **the Lipschitz deckel is EXECUTED**: adaptive bisection with the local constant certifies $\inf f^{\text{arch}}/(4\sin^2(\theta/2)) \geq t$ on 12/12 (0.2500–0.5353, 255–3139 evaluations, cost $x^{0.850}$) — the arch half is CERT-UNIF for the first time; mechanism localised: atom extremal $\theta/\pi = 0.0158$–0.3762 against arch 0.9901–0.9995 — OPPOSITE ends, so a proof must use the θ-GROWTH of the arch ratio, not its infimum at π; the domination $B_{HH}^{\text{arch}} - tI \succeq (-B_{HH}^{\text{atom}})_+$ holds 12/12 (quotient 1.0003–1.0907, margin 7.3×10^{-4} at $h = 970$, trend shrinking — per window, NOT uniform); substitution costs 1.00–1.62; balance 9 THEOREM, 2 CERT-UNIF, 2 CERT-WINDOW, 3 MEASURED (target 1 missed — but all three now fixed-size in the statement); no-go breaks on FIVE axes (p_1 $x^{-4.818}$; S $x^{2.289}$; confinement to vacuum; resolvent identically ZERO; $(S_L)_{11}$ $x^{2.268}$), floor survives; parity 4.8×10^{-14}, Dirichlet $2/\sqrt{N}$ to 7.4×10^{-5}; **rest: an m-free UPPER bound on $(S_L)_{11}$ (cancellation-seeing); an m-free non-shrinking R1″ domination (pointer: θ-growth); the T156 model-domination debt**; the four instrument candidates of T155/T157 promoted as v552	ANGLES-RESIST	32/32

Part	Probe	Contract / finding	Verdict	Checks
T158	`schur_entry`	the cancellation-seeing bound attacked head-on, **and it EXISTS, as a THEOREM**: the Thomson dual form $s = (B^{-1})_{11} = \max_x(2x_1 - x^{\mathsf{T}}Bx)$ (Maz'ya 1985) makes every trial vector a LOWER bound on s, hence an UPPER bound on $1/s$ — exactly the variational structure Cauchy–Schwarz lacks (CS evaluates a MAXIMUM at one direction and missed by 3.13×10^3–5.18×10^5; reproduced, direction intact 28/28); the Cholesky ladder $g_K = \sum_{j \le K} y_j^2$, $y = L_K^{-1} e_1$: terms STRICTLY POSITIVE, partial sums monotone 28/28, $1/g_1 = \hat{a}$ = T157's route; at $K = 16$ **certified** $1/g_{16} = 2.9670$–7.9664 **against true** $1/s = 2.3359$–6.3868**, TIGHT to** 1.1323–1.2738**, flat (**$x^{0.059 \pm 0.017}$**)**; the Green route is the IDENTITY — span$\{t_1, A^{-1}L_P t_1\}$ attains s to 10^{-9} because $L_P t_1 = \mu_1^P t_1$ (KMS 1953): **the entry is TWO-dimensional** once one Green column is granted, the sine truncation pays exactly 1.13–1.27; floor $L\,g_{16} = 0.2143$–0.2724 lands in the band of T157's measured p_1 WITHOUT an angle; **the T157 growth pointer REFUTED**: the growth is real and sub-quadratic ($q = 1.259$–1.438) and the binding vector does sit in the lowest dyadic band (0.6818–0.9988 of its mass — localisation confirmed), BUT the atom negative mass grows FASTER ($\theta^{-1.546\ldots-1.744}$ vs arch $\theta^{-1.203\ldots-1.458}$), band-local domination fails in EVERY band 21/21 (lowest-band cover 0.4161–0.4507), the off-band coupling exceeds the margin by 660–7.71×10^5 — what carries the inequality is the structure a dyadic argument discards; margin 3.57×10^{-5}–2.86×10^{-3}, SHRINKING ($x^{-1.649 \pm 0.643}$); the T156 debt RELABELLED MEASURED → CERT-UNIF ($F(P, 1/(L\,g_{16})) = 0.7276$–$0.7857$ vs $d_1 = 0.5392$–0.6241, margin 1.1969–1.3568 on 28/28, both sides fixed-size); end to end survives the substitution (0.2872–1.0 of $\lambda_1(A)$, cost 1.000–1.724); balance 6 **THEOREM** / 3 **CERT-UNIF** / 4 **CERT-WINDOW** / 1 **MEASURED** (was 5/2/3/3 — the measured count falls three → one, p_1 RETIRED); no-go breaks FIVE-FOLD (p_1 $x^{-4.818}$; $1/g_{16}$ $x^{2.267}$ tracks the collapse; $L\,g_{16}$ $x^{-0.275}$; tightness stays 1.001 — the size breaks, not the sharpness); parity 4.8×10^{-14}, ladder exactly constant in K on $A = L_P$ (4.0×10^{-15}), Dirichlet $2/\sqrt{N}$ to 7.4×10^{-5}; **rest: an m-free upper bound on ONE quadratic form** $x^{\mathsf{T}} B_{LL} x = 2.9670$–$7.9664$ **(halves cancel to depth h^2 — the explicit formula's own cancellation); the SIGN STRUCTURE of the off-band arch entries**; five candidates (P1–P5) bundled with T159, theorem cores promoted	ENTRY-RESISTS → v553	36/36

Part	Probe	Contract / finding	Verdict	Checks
T159	`exact_form`	the cancellation executed ALGEBRAICALLY — **seven machine-checked identities** (to 10^{-12} against the absolute scale $\sum\lvert c_d w_d\rvert$; relative to the cancelled total only 10^{-12}–1.2×10^{-8}: the cancellation eats 4.0–8.1 decimal digits, reported as a measurement): (P1) $x^{\mathsf T}B_{LL}x = y^{\mathsf T}Ay$, pencil gone; (P2) the EXACT LAG SUM $=\sum_d c_d w_d$, $w_d = T_d - H_d$, arithmetic and geometry separated; (P3) w_d in CLOSED form (256-term Dirichlet kernel, 1.4×10^{-15}); (P4) **the GAUGE IDENTITY $\sum_d w_d = 0$ EXACTLY** — Toeplitz-minus-Hankel annihilates constant lag vectors: the form is BLIND to the lag mass, exactly where the h^2 halves live; (P5) $w_0 = \sum x_k^2/\mu_k$ and $2w_0 - w_1 = \lVert x\rVert^2$; (P6) p-fold Abel exact to $p = 5$; (P7) the TmH signature $A_{r,s} - A_{r+1,s+1} = f(r+s)$; **$m$-free NOT reached, four routes closed with exponents**: the h^2 does NOT telescope (both halves grow $h^{+2.104}$ = the common WEIGHT); T158's fixed sixteen-vector REFUTED ($h^{+3.510}$, $\mathrm{cond}(B_{LL}) \sim h^{+2.887}$, needed accuracy $h^{-3/2}$ vs optimiser drift 2.5×10^{-2}–1.5×10^{-1}); one sine $h^{+3.066}$; ansatz family $h^{+3.001}$; Abel-$\ell^1\times$sup best rung $h^{+3.604}$ (misses by 5.6×10^6–2.2×10^{10}, $W_1^1 = -w_0$ makes $\sup\lvert W^1\rvert \geq h^2$ unavoidable); two m-free-shaped pieces NEW: $\lVert\Delta c^{\mathrm{arch}}\rVert_1 = 3.65$–$5.38$ flat (BV ≤ 6) and the T150 budget at the atom cutoff $X = e^{2\alpha}$; **(S1) EXACT: B_{HH}^{arch} is a symmetric Z-MATRIX** (100% off-band nonpositive, raw, 18/18), checkerboard refuted (0.489–0.494 = coin flip); Collatz–Wielandt BITES on the arch half: **closed sign-based floor** $\min_k(B_{HH}^{\mathrm{arch}}\mathbf{1})_k = 0.819$–$1.165$**, flat, above** $t = 0.25$ — but the atom block has NO sign law (0.31–0.49) and the full-block M-matrix criterion is vacuous ($-9.32\ldots-0.61$ vs exact 0.074–0.197); end to end $t = 0.0705$–0.1583 unmoved (M1/M2 structural, not quantitative); no-go breaks 5/5; balance 6/3/4/1 → 13 **THEOREM** / 6 **CERT-UNIF** / 4 **CERT-WINDOW** / 1 **MEASURED** (seven new theorems, three new CERT-UNIF); NUMERICAL HORIZON declared: $\mathrm{cond}(B_{LL}) > 10^{12}$ from $h = 1292$, one window dropped, the same obstruction as its own measurement; **rest: ONE fully explicit pairing inequality** $\sum(\Delta c)_d W_d^1$ needing CORRELATION instead of sizes ($\ell^1\times$sup off by $h^{3.7}$, the cancellation exactly h^2); a third device for the atom off-band block; theorem cores promoted	FORM-RESISTS → `v553`	41/41

Part	Probe	Contract / finding	Verdict	Checks
T160	`pairing`	the one pairing $\sum(\Delta c)_d W_d^1$ attacked through its CORRELATION structure, **and the watershed of the phase arrives — the SAMPLING IDENTITY: the last term IS a prime exponential sum** (19 of 20 zones $h = 199$–1256, 1 dropped at the declared horizon); W^1 **does NOT oscillate**: $J = 3$ sign blocks, flat, CLOSED bound $J \le 130$ (trigonometric zero count) — one smooth three-lobe profile; **block/Leibniz device REFUTED**: block-Hölder $h^{+4.115}$, block-Abel $h^{+4.131}$ — 5.2–16.5× WORSE than $\ell^1 \times \sup$ at $h^{+3.595}$; head peel refuted ($K^* = M$ on 19/19; best overshoot 60.3–$4.80 \times 10^3 \sim h^{+2.428}$ = the h^2 itself); **NEW THEOREMS — the sign-definite MOMENT LAWS** $m_1 = -[S_0^2 + 2\sum_j P_j^2] \le 0$, $m_2 = -[2S_1 - (M-1)S_0]^2 \le 0$, m_p closed for every even p; the fixed witness (frac 0.2, degree 48) reproduces the arch half to worst slack 1.95×10^{-8} of the $O(1)$ total — **FLOOR-LIMITED at** 4.12–428× **the double-precision floor: SECOND HORIZON declared**, the arch half neither established nor refuted on any double-precision surface; sparsity refuted (support fills 0.32–0.64; support-localised ℓ^1 misses $h^{+3.505}$); **the SAMPLING IDENTITY** (1.0×10^{-16}–2.9×10^{-15}): $\sum_d c_d^{\mathrm{atom}} w_d = -\frac{1}{2}\sum_{n \le X}(2\Lambda(n)/\sqrt{n})\,\widehat{w}(\log n)$ ⇒ **the atom half IS a finite combination of** $\sum_{n \le X}\Lambda(n)n^{-1/2}\cos(t\log n)$ **at** 32 **EXPLICIT frequencies** $t = \pi(k \pm l)/\alpha$, needed to depth $h^{-2} = 2.19 \times 10^{-6}$–$1.12 \times 10^{-4}$, measured to cancel only to 0.00–0.37 of the trivial bound — **the h^2 cancellation is the INTRINSIC ARITHMETIC HARDNESS**, not an assembly artefact (finite von-Mangoldt table only; no zero read, no equivalence claimed); R1″: rank/support and corner (T2) refuted (the norm route in disguise, $\|M_{12}\|_2 \le 71.4$); **the surviving fact** $\rho = 1.0036$–$1.0140 > 1$ **on every window, FLAT** — arch dominates atom on every destructive direction; its inequality form refuted (floor $-7.93\ldots-1.09$; margin vs cross-coupling = factor 10^4); **U3 UPGRADED to THEOREM**: TV < 6, measured 3.9101–5.4153 vs closed 3.9105–5.4153 (FOUR digits); U1 PARTIAL: Z-law ⇔ ONE prime-free trigonometric inequality (pointwise form refuted — the weight is essential); end to end 3.79×10^{-10}–2.10×10^{-7}, with an exact atom half the SAME chain gives 1.0000 (deficit in ONE term); no-go breaks 6/6; balance 13/6/4/1 → 14 **THEOREM (6 new) / 4 CERT-UNIF / 3 CERT-WINDOW / 3 MEASURED / 8 refuted families**; T160-P1…P7 PENDING, no promotion; **rest: R-A the symbolic Chebyshev bound; R-B the prime-free inequality; R-C the exponential-sum bound at the** 32 **frequencies (not an exercise); R-D a fifth R1″ device**	PAIRING-RESISTS	46/46

Part	Probe	Contract / finding	Verdict	Checks
T161	`classical_closure`	the two classical rests plus the circularity triage, **and the triage returns its most consequential answer: THE CHAIN IS NOT CIRCULAR** (19 log-spaced zones $h = 50$–1445, a 28× lever arm; $X = e^{2\alpha} = n_{\text{zone}}^2$ exactly; $\psi(x)/x \le 1.038830$ verified at every jump to 400000); **R-A closes m-free**: $A = D\widehat{A}$ exactly (4.2×10^{-16}–2.0×10^{-13}), only the $1/s$ head binds ($i\pi$ margin 1.29–3.29) ⇒ **scale-free Bernstein rate** $\rho^* = (3+\sqrt{5})/2 = 2.618034$ (interval ratio 5 only), $M = 3.06$–5.93 uniform, envelope 0.055–$0.074 \le 1$; **the head needs NO peeling**: $c_d^{\text{arch}} = \Psi_d + D\widehat{G}(dD, D)$ with $\Psi_d = -\frac{1}{2}[(d{+}1)\log(d{+}1) - 2d\log d + (d{-}1)\log(d{-}1)]$ CLOSED, D-free, m-free (1.7–4.8×10^{-16} at every lag); full-interval rate $\rho_{\text{reg}} = 2.05$–3.38 NOT scale-free; "fixed degree" REFUTED: schedule $K(h) = O(\log h)$ both routes ($K = 12$–19 / 9–25); residual the prime-free LOG-MOMENT $\sum \Psi_d w_d = 2.15$–3.14 of the arch half, outside the polynomial ladder; **R-B REFUTED in all four readings** (per pair 3168/4320, worst +0.92; Abel tail 1229; aggregate by SIGN — arch negative, off-diagonal positive = the harmful direction; Abel condition false 3801/4320) — **survives: the FRACTION bound** $|\text{off}|/|Q^{\text{arch}}| = 0.1035$–$0.1713 \le \frac{1}{4}$**, flat** ($h^{-0.116}$), m-free version a 16×16 Gram question (Fejér 1915, Schur 1917); **NEW THEOREM:** $t(2\alpha) = 2\pi j$ **exactly — the** 32 **frequencies ARE the Fourier harmonics of the log-window**; the measured cancellation is FULLY the PNT main term ($(\sqrt{X}-1)/(\frac{1}{4}+t^2)$) to 0.014–0.232 of mass, improving $X^{-0.39}$ — a Mellin factor, no arithmetic saving); δ **arithmetic: needed** $\delta = 1.1482$–1.8809 **against RH strength** $\frac{1}{2}$ — gap $X^{0.648}$–$X^{1.381}$; absolute demand 0.012–0.31 of the LAST TERM, below the boundary term of every partial summation ⇒ no $\psi(x) - x$ input reaches it; frequency geometry worth $\delta = 0.29$–0.92 (suppression 7.0–19.8, matched control), already inside the numbers; **re-split moves the demand** (smooth prime term cancels the arch half to 0.50–0.76; $\delta_{\text{eff}} = 0.984$–$1.377$, still $> \frac{1}{2}$ on 18/18) ⇒ **the h^2 sits in the SPLIT, not in the primes**; balance 20/10/4/9/3; closed cores of T160+T161 promoted; **rest: R-A′ the log-moment; R-B′ the Gram fraction bound; R-C′ ONE split with $\delta_{\text{eff}} < \frac{1}{2}$; R-D a fifth device**	CLOSURE-RESISTS → v554	35/35

Part	Probe	Contract / finding	Verdict	Checks
T162	`third_split`	the search for the third split, **and it EXISTS while the exhaustion SATURATES** (the T161 surface rebuilt independently: 19 zones $h = 50$–1445, 18 windows inside the cond horizon $h = 50$–1218; two currencies declared before any candidate: δ_{val} the measured residual value, δ_{bnd} what a proof must beat, RH strength $\frac{1}{2}$ in both); C0/C1 REPRODUCE T161 to 0.01 (1.1482–1.8809 / 0.9839–1.3767); **C2 the MELLIN LADDER — the third genuine gain, an ASYMPTOTIC series**: closed cell moments $C_d(-(2k+\frac{1}{2}))$, optimum $K^* = 2$ on 18/18, gain 0.359–0.446 ($\delta_{\mathrm{val}} = 0.6247$–$0.9307$), past K^* the residual RISES $\times 15.5$–25.1 (the predicted log-divergence) — not iterable; **C5 the Abel ladder: optimal level EXACTLY ONE** ($32\pi/\alpha = 16.7$–$41.9 > 1$, closed), $\delta_{\mathrm{bnd}} = 1.3965$–$1.7867$, and after the step the demand is EXACTLY $\frac{1}{2} + \log(2\kappa\|\Delta w\|_1/|Q|)/\log X$ — **R-C′ is PRIME-FREE**, all arithmetic in $\kappa = 0.038821$; **C4 the FEJÉR split — the actual find:** $\delta_{\mathrm{bnd}} = 0.1332$–$0.4169$ **BELOW** $\frac{1}{2}$ **on** 18/18 (taper = smoothing of the Λ-mass by self-adjointness; lowering 1.15–1.51) — **at a price growing $h^{+2.86}$: RELOCATED, not closed**; frontier $P_{\min} = 7.2 \times 10^{1}$–$8.2 \times 10^{4} \sim h^{+2.0}$, 1.17–3.31 decades per δ unit; **R-A′ CLOSED**: the log-moment on three routes to machine precision (0.0–6.0×10^{-16}; Lerch/Frullani integral, $d = 1$ peel $= 2\log 2$ reproduces $\Psi_1 = -\log 2$; Wronskian boundary term completes T161); **R-B′ REFUTED**: the 16×16 Gram form INDEFINITE on 18/18 ($\lambda_{\min}/|\lambda_{\max}| = -13.34\ldots - 1.59$; Gershgorin inapplicable at 7.97–580.76) — survives: the a-weighted quarter bar 0.1435–0.2902 flat ($h^{-0.102}$), 14/18; end to end 6.1×10^{-17}–1.8×10^{-15} of the large scale; the factorising gap 0.77–$0.86 > 0.12$–0.43 to RH strength; surrogate breaks $\times 5.2 \times 10^{2}$–4.6×10^{7}, gauge break EXACTLY $c_0 \sum w_d$, under-resolution does NOT break (declared as INFO); **anti-fitting:** $\beta^* = 0.600$–$0.800 \neq \frac{1}{2}$ **would look** $100\times$ **better and is REFUSED** ($\frac{1}{2}$ forced by the $n^{-1/2}$ weighting); balance 10 THEOREM / 2 CERT-UNIF / 3 CERT-WINDOW / 9 MEASURED, zero FIT rows; T162-P1…P6 PENDING, no promotion; **rest: R-C″ the Pareto front; R-B″ a substitute for the Gram positivity; R-D a fifth device**	DELTA-REDUCED	30/30

Part	Probe	Contract / finding	Verdict	Checks
T163	`pareto_front`	the survey of the Pareto front, **and the front resists AS A THEOREM, not as a failed search** (28 zones $h = 142$–1445 with declared caps, 27 windows inside the cond horizon; the closed weight vector re-derived as a CORRELATION form, $O(h^2)$ and block-size-free, 1.6–2.0 $\times 10^{-15}$; $\psi(x)/x \le 1.038830$ verified at every jump to 400000, $\kappa = 0.038821$); **the price axis is CHAIN-DERIVED**: $Q(x^*) = 1/g_{16}$ (the T160 optimiser IS the T158 top rung), $P_{\text{aff}} = P_{\text{rung}}(1) = g_{16}B_{11} = 3.9 \times 10^3$–$1.0 \times 10^7 \sim h^{+3.049}$; the Fejér knob has BOTH endpoints in the chain ($\sigma = 1$ exactly the K=1 rung, $\sigma = \infty$ the K=16 rung) and is measured MONOTONE in both coordinates at all 27×50 grid points ⇒ **the knob curve IS the front**; flat caps 1.25/2/10 and every flat rung tier: 0/27 **below** $\frac{1}{2}$; only P_{aff} reaches 27/27 (-0.004–0.019); crossing $P_{\text{cross}} = 8.1 \times 10^2$–$1.0 \times 10^5 \sim h^{+1.906}$ at 0.0066–0.2047 of P_{aff} — 5–152× strictly INSIDE the accepted certificate, margin widening; **the price anatomy is an IDENTITY**: $\delta_{\text{bnd}} = \frac{1}{2} + \log(2\kappa g_{16}\text{TV}/P)/\log X$ exact at all 27×50 points ($\le 7.0 \times 10^{-14}$) ⇒ $\delta_{\text{bnd}} < \frac{1}{2} \iff P > 2\kappa g_{16}\text{TV}$, crossing exponent decomposes additively (TV +2.013, g_{16} −0.061); taper gain in TV bounded and h-flat (2.75–4.64, worth ≤ 0.276 of δ, decaying $1/\log X$; τ_{TV} median −0.0191); the taper DESTROYS the cancellation (CANC 2.2×10^{-6}–$3.6 \times 10^{-4} \to 0.449$–$0.934$) — the price is the cancellation depth read backwards; mode sweep $K = 16 \to 64$: price dividend, WORSE demand (+0.0135), TV exponent unmoved (+1.83/ + 1.82/ + 1.81); **NEW THEOREM (T1–T4):** $w_0 = \|a\|^2$**,** $\text{TV} \ge |w_0|$ **(telescoping),** $x_1 = 1 \Rightarrow \text{TV}(x) \ge 1/(4\sin^2(\pi/N)) \sim h^2$ **for EVERY admissible trial vector** — verified on all 1674 built, slack 5.0–23.2; hence $\delta_{\text{bnd}} < \frac{1}{2}$ forces $P > 2\kappa g_{16}/\mu_1^P = 5.5 \times 10^1$–$5.9 \times 10^3$, 5.5–590× above the largest flat cap — **R-C‴ CLOSED NEGATIVELY by an inequality**; **the h^2 IDENTIFIED: T162's $h^{2.86}$, the crossing's $h^{1.91}$ and T161's granularity are ALL the reciprocal smallest parity eigenvalue — the spectral gap of the parity Laplacian (KMS 1953) meeting $x_1 = 1$: NEVER IN THE PRIMES**; R-B″: the chain never needed the sign (duality + evaluated number, headroom $\ge 10^{14.9}$) — what remains is UNIFORMITY (R-B‴); R-D type mismatch (operator smoothing degrades $\times 2.3 \times 10^3$–$2.7 \times 10^5 \sim h^{+2.04}$ — the same h^2); no-go staircase MONOTONE $1.0 \to 9.1 \to 25.4 \to 129 \to 434$; out-of-sample $\sigma = 3$: $\delta_{\text{bnd}} = 0.176$–$0.380$ on 26/26 at 0.14–0.23 of P_{aff}; balance 11 THEOREM / 1 CERT-UNIF / 4 CERT-WINDOW / 10 MEASURED, zero FIT rows; **rest: R-E the successor (both arms prime-free); R-B‴; R-D**	FRONT-RESISTS → v555	32/32

Part	Probe	Contract / finding	Verdict	Checks
T164	`sector_change`	both arms of the successor R-E decided (the T163 surface rebuilt and re-verified: 28 zones $h = 142$–1445, 27 windows inside the cond horizon; $\kappa = 0.038821$ at every jump in $[100, 4 \times 10^5]$); **ARM A: the kernel spends the ceiling at power EXACTLY ONE** — $d\log(1-F)/d\log U = -1.0005\ldots - 1.0000$, $d\log r/d\log U = +1.0000$ exactly, an h^{ε} ceiling costs $h^{-\varepsilon}$ margin for every $\varepsilon > 0$; $\varepsilon^* = 0.50$ DISOWNED as the $\varepsilon = 0$ tailwind ($h^{+0.391}$ from the L-drift) divided by power one; **the single constant** $U_{\mathrm{ref}} = 4.9008 = \max_h 1/g_{16}$ **carries all** 9 **sub-surface windows out of sample**, $1/g_{16} = 1.7527$–$5.3286 \sim h^{+0.061}$ (9× lever arm, split halves $-0.010/+0.103$), every g_K strictly increasing (Schur 1917) ⇒ **the** $O(1)$ **gate is discharged window by window by a Cholesky identity — the rest is ONE QUANTIFIER:** $\sup_m 1/g_{16}(m) < \infty$; **ARM B — NEGATIVE BY A THEOREM:** Q **and TV homogeneous of degree two,** $x_1 = 1$ **fixes only the scale ⇒** Q/TV **and** δ_{bnd} **the same in EVERY sector — the entry normalisation is a GAUGE** (5670 combinations to 8.8×10^{-10}, horizon 2.6×10^{-8}); full space strictly worse ($\mu_0 = 0$ EXACTLY); weighted sector: floor flattens ($h^{+0.000}$ at $\kappa_s = 4$) but **floor × transfer** $= 1/\mu_1^P$ **IDENTICALLY at every shift (exponent sum** $+1.997$ **to** 10^{-9}**)**; Abel: $Q/\mathrm{TV} \le \|C\|_\infty = 30.3$–$930.6 \sim h^{+1.185}$ growing — forbids nothing; $\eta := (Q/\mathrm{TV})/\|C\|_\infty$; **the unconstrained ascent reaches** $Q/\mathrm{TV} = 2.38$–74.57 — 31–960× **over** 2κ **on** 9/9**,** $\eta = 7.9$–11.1% — **and T163's TV floor HOLDS on it** ($\mathrm{TV}\mu_1^P = 8.30$–$11.72$, a family T163 never searched) at price $h^{+3.299}$, above the crossing $h^{+1.91}$ and 3.5–7.6× above P_{aff}: relocated, not reduced; R-B‴ a NAMING problem (the T162 quarter bar 0.1001–0.1623 carries the -0.172, < 0.25 on 27/27; regression $-0.046\log h$ vs $-0.477\log\alpha$, α/h coupled — NARROWED); R-D settled in TYPE (P_K, r, F unchanged to 3.5×10^{-16} on 54 combinations — no fifth device from a sector change); stress: surrogate 1.5×10^3–4.9×10^7, T145 no-go MONOTONE $1.0 \to 9.1 \to 25.4 \to 129 \to 434$ (exact null control), gauge break 3.4×10^{-2}, random-v identity 3.8×10^{-16}; balance 11 THEOREM / 1 CERT-UNIF / 4 CERT-WINDOW / 6 MEASURED, zero load-bearing FIT rows; Q1…Q9 PENDING, no promotion; **rest: R-E-A the quantifier; R-E-B closed negatively; R-F the alignment question (**η **above** $2\kappa/\|C\|_\infty$ **at bounded price — no sector, no normalisation, no prime); R-B‴; R-D**	TOLERANCE-CARRIES / SECTOR-RESISTS	28/28

Part	Probe	Contract / finding	Verdict	Checks
T165	`alignment_eta`	the alignment successor R-F closed BY CERTIFICATE, not by failed search (the T163/T164 surface rebuilt and re-verified, ladder to $h = 1445$, 27 windows inside the cond horizon; the dashboard R, η, P_{pr}, $\mathrm{TV}/(t_1 v)^2$, QB gauge-invariant under $v \to tv$, THEOREM); **the T165 exchange identity** $P_{\mathrm{pr}} = g_{16} \cdot R \cdot (\mathrm{TV}/(t_1 v)^2)/\mu_1^P$ **(**3.3×10^{-16}**) — four named objects, no inequality: demand and price are ONE equation**, so with T163's floor every crossing vector pays $\geq 2\kappa g_{16}/\mu_1^P =$ the old KMS h^2; anatomy: the free optimum is LOW-HARMONIC (93.7–96.8% in the first 32 parity modes vs 2.4–22.5% baseline, lift 4.2–39.7), orthogonal to the PNT main term ($-0.070 \ldots -0.031$), heavy edges on prime-power cells (lift 1.36–2.12), a CANCELLING sum (participation 8–18%); **the price decomposes EXACTLY (to** 0.0**):** $h^{+3.261} = +1.997$ **(KMS, the same** h^2 **since T152)** $+1.279$ **(demand overshoot)** $+0.093$ **(floor)** -0.108 **(**g_{16}**)**; bar-tight counterfactual $h^{+1.982}$; η(Cap) two-sided: the ceiling is a THEOREM, at Cap = 10 short by 5.5–392× on every window, 0/27 ladder points reach the bar; **EQUIVALENCE:** $R > 2\kappa$ **and T163's crossing predicate are THE SAME predicate (**105/105**) ⇒ R-F strictly STRONGER than the R2″ demand, the clauses collide — R-F is EMPTY, its** m**-uniform form IS R-E-A**; T2: the m-free budgets bound B_{11} sharply (21–26, $h^{+0.039}$) but it is the WRONG object ($1/g_1 \sim h^{+3.110}$, $1/g_{16} \sim h^{+0.061}$, closure exact $+3.110 - 3.049 = +0.061$) — the certification lives in the 15 ladder increments; T3, the DECOUPLED surface (25 windows / 6 zones, $\nu \in \{4, 5, 6, 8, 11, 16\}$, collinearity 0.87 → 0.68): **the quarter-bar drift is ZONE DEPTH, not window length (**19×**)**; honest costs: $\sup 1/g_{16} = 5.7327$ RETIRES $U_{\mathrm{ref}} = 4.90$ (+17%, frame-A-specific); union (52 windows): $P_{\mathrm{pr}} \geq 54.6$–$5865 > \Theta_{\mathrm{TOL}}$, closed for $h > 84.9$; stress: T145 monotone 1.0/2.8/12.1/27.5 (to 102×), surrogate 8.6×10^3–6.9×10^4, Dirichlet/KMS 10^{-15}; balance 11 THEOREM / 1 CERT-UNIF / 4 CERT-WINDOW / 6 MEASURED; **rest: EXACTLY ONE genuine open object** — $\inf_m g_{16}(m) > 0$**, i.e.** $g_{16}/g_1 \geq c\, h^{3-\varepsilon}$ **uniform in** m**: a quantifier over a certified flat list, a cancellation beyond absolute-value budgets**	ETA-RESISTS → `v556`	30/30

Part	Probe	Contract / finding	Verdict	Checks
T166	`schur_cascade`	the cascade lower bound dissected completely (a UNION surface: 27 frame-A + 36 decoupled-ν windows = 63, $h = 96$–1371, 14× lever arm, $\log h/\log \nu$ correlated only 0.33; T165's numbers reproduced on frame A, the union reading $h^{+2.759}/h^{+0.193}/h^{+2.566}$); **FOUR IDENTITIES, all THEOREM on** 63/63**: prefix (one Cholesky, all rungs), minor form (Schur 1917), the REGRESSION form** $g_K/g_1 = 1/(1 - R_K^2)$ **— the open object IS a near-collinearity — and DIAGONAL INVARIANCE under** $B \to DBD$**: the** h^3 **belongs to the arithmetic Gram block alone**; rung profile extreme (rung 2 alone median 59%, $K_{\text{half}} \in \{2..5\}$ on 63/63, flat first at $K = 6$); rung 2 closed: $1 - r_{12}^2 = 1.3 \times 10^{-7}$–$4.5 \times 10^{-4} \sim h^{-2.92}$ (frame A) against m-free budgets sharp to 21–26; **the CANCELLATION — a third mechanism: no entry cancels (**60–99% **kept), neither half is collinear alone (arch** 0.98**, atoms** 0.29**), yet the SUM degenerates to seven digits: it lives in the** 2×2 **GRAM DETERMINANT** (polarisation split 6.7×10^{-14}; pieces 2.6–630 vs det 1.0×10^{-3}–1.7×10^{-2}, ratio $\sim h^{-1.45}$): the T159/T160 mechanism ONE LEVEL UP, in a quadratic functional of the lag sums; best closed route $h^{+1.319}$ (required β precision 3.57×10^{-4}, all candidates miss by 12.4–2540); sharpest form: $1/g_K = \det G_{[1..K]}/(\mu_1^P \det G_{[2..K]})$**, first flat at** $K = 6$ **— exactly ONE uniform inequality missing**; $U_{\text{ref}}(\text{union}) = 7.45$ (supersedes 4.90/5.7327; zone-depth-driven; CERT-WINDOW, off-recipe); full chain $g_1 \leq$ best $\leq g_{16} \leq s$ on 63/63; $K = 64$: the surrogate's loss is TRUNCATION ($h^{+1.963} \to h^{+2.574}$), no certificate; **two honest corrections: F2 STRICTLY WEAKER than F1 (**$B_{11} \sim h^{+3.110}$**, F1 needs** $\varepsilon \leq -0.110$**); the confinement route a SUB-SURFACE ARTEFACT (**$h^{+2.714}$ **at** $h \leq 780$ **vs** $h^{+1.963}$ **on the union)**; stress: L_P no-go breaks COMPLETELY (gain $\equiv 1$ exactly), anti-fitting scramble factor 4569 — the mechanism hangs on WHERE the prime powers sit; balance +5 THEOREM / +3 CERT-WINDOW / +4 MEASURED, no new CERT-UNIF; P166.1…P166.9 PENDING; **verdict by exponent gap:** $h^{+1.319}$ **vs** $h^{+3.110}$**, gap** $h^{+1.791}$ **— the one missing inequality IS the cancellation, in three dresses**	CASCADE-RESISTS	30/30

Part	Probe	Contract / finding	Verdict	Checks
T167	`null_vector`	the most constructive dress attacked and closed AS A CONSTRUCTION (the 63-window union rebuilt bit for bit; T163–T166 reproduced as fits; 22 families preregistered); **the PIVOT IDENTITY** $u^{\mathsf{T}}Q_K u = 1/g_K + \delta^{\mathsf{T}}Q_K\delta$ **for** $\delta_1 = 0$ **(THEOREM,** 10206 **evaluations,** 3.2×10^{-9}**)** — **every candidate excess** $\rho = g_K\delta^{\mathsf{T}}Q\delta$ **is an EXACT number**; **gram2 EXACT at** $K = 2$ **(**$\rho = 0$**, THEOREM):** $(1, -Q_{21}/Q_{22})$ **attains** $1/g_2$ **on** 63/63 **(**8.7×10^{-10}**),** $B_{11}g_2 = 1/(1 - r_{12}^2)$ **an identity; certified gain exponent** $h^{+2.921}$ **frame A (**$= h^{3-0.079}$**),** $h^{+2.246}$ **union — dress (c) collapses onto dress (b)**; perturbation theory dies structurally: Jacobi diverges (radius 1.0–12.7) but **the KATO series CONVERGES (radius** 0.067**) — to the WRONG object** (the exact bottom eigenvector is itself useless, $\rho = 6.04$, overlap 0.083, relative gap 5.4×10^{-7}) — closed off in EVERY order; moment route $h^{+1.320}$ = T166's $h^{+1.319}$ (flagged); two thresholds separated (eps_{vec}, eps_{ent} = its square), envelopes on 189 perturbations each; **monotone SHARPER in** K **— mildest at** $K = 2$ **(**$1.90\times$**,** $h^{+0.353}$**), against the hypothesis and exactly where the vector is free**; at $K = 6$ no closed family hits (separation $h^{+1.138}$, ratio $264\times$ — the curves diverge); anchor $\text{eps}_{\text{ent}}(2) \to (1 - r_{12}^2)/4$ = T166's quarter; end-to-end gain 2207–7.84×10^6 certified; gram2's ρ at $K = 16$ in $[0.17, 6.16]$ — the deficit is the higher rungs, not the vector; **scramble TYPE-CHANGE:** 8/8 **windows lose positivity of the** 2×2 **diagonal ITSELF (**$Q_{11} = -1.35 \times 10^5 \ldots - 1.02 \times 10^4$**)**; anti-fitting cheat only $h^{-0.823}$; off-recipe $1 - r_{12}^2 \sim h^{-2.740}$ vs $h^{-2.921}$ (+0.181); **THE UNIFICATION AS AN IDENTITY:** $\text{eps}_{\text{ent}}(K) = \rho\,(1/g_K)/S_K$ — **dress (a) = det ratio, dress (b) =** $1 - r_{12}^2$ **(factor** $1 + 3r_{12}^2 \to 4$**), dress (c) = the threshold: ONE inequality, not three**; balance 6 THEOREM / 0 CERT-UNIF / 2 CERT-WINDOW / 7 MEASURED; ONE-TERM-MISSING deliberately NOT rendered (the one missing term IS dress (b)); **REST: R1, the only item — an** m**-free upper bound on** $1 - Q_{12}^2/(Q_{11}Q_{22}) \le C\,h^{-3+\varepsilon}$ **(required** 1.25×10^{-5} **median,** 3.19×10^{-8} **worst)**; theorem cores of T166+T167 promoted as v557	VECTOR-RESISTS → v557	39/39

Part	Probe	Contract / finding	Verdict	Checks
T168	`lagrange_minors`	R1 attacked via the Lagrange identity — the determinant as a sum of squares (the same 63-window union as T163–T167, recipe untouched; T167's scalar reproduced bit-compatibly); anatomy EXACT: $Q = X^{\top}AX$ (3.9×10^{-15}), the mode vectors EUCLIDEAN-ORTHOGONAL ($r_{12} = 0$ to 6.2×10^{-17} — **the near-parallelism is created ENTIRELY by the arithmetic metric**), the Wronskian TELESCOPES ($W_r - W_{r-1} = (\mu_1 - \mu_2)u_r v_r$, 1.7×10^{-13}; one closed Dirichlet-partial-sum family), minor sup closed m-free $\sim N/\pi^2$ (attained 0.770), Cauchy–Binet verified as algebra; **FINDING: the $h \times h$ kernel A is POSITIVE DEFINITE on** 63/63 **($\lambda_{\min} = 5.0 \times 10^{-6}$–$8.5 \times 10^{-4}$, 8–12 orders above the solver floor) ⇒ the Lagrange identity is a GENUINE sum of squares,** $\rho_{\mathrm{LAG}} = 1.000$ **exactly — hard-fenced: a finite matrix per window, NO Weil test/assumption/reverse inference**; the key measurement SPLITS BY BASIS: closed basis — minors MAXIMAL ($\frac{1}{2}\sum M^2 = 1/(\mu_1\mu_2)$ exact $\sim h^4$: the Wronskians carry ZERO arithmetic, all smallness sits in the PSD kernel at one closed vector); eigenbasis — the sum is THIN (90% from 4–101 pairs $\leq 8 \times 10^{-4}$, top pair always $(\lambda_{\min}, \lambda_{\max})$ at 6.8–41.7% alone, raw $\kappa_w = 0.30$–0.94, typical minor 100:1); exponent ledger CLOSED, arithmetic-free: $1 - r_{12}^2 = \nu_1\nu_2/(\hat{a}_{11}\hat{a}_{22})$ with $\nu_1 \sim h^{+0.81}$, $\hat{a}_{11} \sim h^{+0.76}$, $\hat{a}_{22} \sim h^{+0.91}$ ($\nu_2 \sim h^{-1.38}$) — **every factor but one at the right power**; best m-free-shaped bound B_1 $h^{-0.548}$ union / $h^{-0.880}$ frame A — short by $h^{2.45}/h^{2.04}$; literal budget $h^{+2.049}$ (12 orders above); **the open factor EXACT: ν_2 with minimiser $t^* = Q_{12}/Q_{11}$ (factor** 1.0000**), needed to** 6.05×10^{-3} **median /** 2.82×10^{-4} **worst ($h^{-1.146}$) — by T168-TH7 that factor IS the target rewritten** (ONE-TERM-MISSING deliberately NOT rendered); no closed constant ($1/\sqrt{2}$ misses 0.194), no single-mode trial (overshoot 285–1.5×10^6), ground state only 0.982-confined (δ does not decay), Turán band neither sign-definite nor telescoping; chain $1/g_2 = Q_{11}(1 - r_{12}^2)$ (1.2×10^{-8}), certified gain ≥ 1.00–2.13 (weak, valid); **scramble breaks HARDER than T167:** 8/8 **lose $A \succ 0$ AND the Gram property (n_{neg} up to** 73**) — no window survives**; L_P: $Q_{12} = 0$, $1 - r_{12}^2 = 1$ exactly; **the hardness is SELF-SIMILAR under exact reformulation** (determinant → PSD compound at a closed vector → near-null Rayleigh → one ratio of two lag sums, same accuracy price); balance 8 THEOREM / 0 CERT-UNIF / 3 CERT-WINDOW / 11 MEASURED / 7 NO-GO	MINORS-RESIST	39/39

Part	Probe	Contract / finding	Verdict	Checks
T169	`tstar_ratio`	T168's one open factor attacked as a preregistered candidate table (same union; the block verified as a lag sum, 4.7×10^{-16}, with an exact arch/atom split, 5.6×10^{-16}); **every genuinely CLOSED candidate misses by** $O(1)$ **and DIVERGES from the threshold at** $h^{+1.99}$ (the block is ATOM-DOMINATED, $\lvert\text{atom}/\text{arch}\rvert = 3.5$–$43$ resp. 14–461; $\hat{a}_{11}^{\text{arch}} < 0$ on 63/63 — the arch half not even PSD; atom shares correlated 0.994 but NOT proportional, $s_{12}/s_{11} = 4.1$–10.7; budget interval vacuous BY SIGN); **the ONLY candidate meeting the threshold,** $K7 = \sqrt{\hat{a}_{22}/\hat{a}_{11}}$ **(**2.49×10^{-5} **median /** 2.27×10^{-4} **worst,** 63/63**), does so by T169-TH7:** $R(K7) = 2\sqrt{\hat{a}_{11}\hat{a}_{22}}\det\hat{A}/[(\hat{a}_{11}+\hat{a}_{22})(\sqrt{\hat{a}_{11}\hat{a}_{22}}+\hat{a}_{12})]$ **(**1.1×10^{-9}**) — it reintroduces** $\det\hat{A}$**: the** t^***-language is provably exactly as hard as T167's determinant, THE LOOP CLOSES AS AN IDENTITY** (T167 scalar → T168 factor → T169 candidate = the same object); gains: **R3 booked CERT-UNIF —** $\nu_1 \le \max(\hat{a}_{11}, \hat{a}_{22}) + \lvert\hat{a}_{12}\rvert$ **(Gershgorin, unconditional, uniform in** h**),** 63/63**, loses only** 1.10–1.19 **— the CERT-UNIF column moves off zero**; R2 refuted and downgraded to OPEN (the absolute-value budget gives $\hat{a}_{11} \ge -1.7 \times 10^2 \ldots -13$ — cannot even certify positivity); R4: Cholesky 63/63 CERT-WINDOW permanent, three uniform routes refuted (Gershgorin margin negative 63/63; Z-/M-matrix route cannot start; perturbation split off by 6.4×10^5); **Weil fence prominent: uniform-in-**h **positivity of** A_h **is RH-equivalent-shaped and never routed or claimed — the chain REBUILT R4-FREE (TH9)**, valid 63/63, loss 1.10–1.19, **frame-A trend** $h^{-2.948}$ **(**$\varepsilon = 0.052$ **in the** 0.5 **carry window)**; TH3: the $1/\sqrt{\mu}$ factors cancel — $\hat{a}_{ij} = t_i^{\mathsf{T}} A t_j$ are single-sine lag sums, the cond-10^8 ladder leaves the file; stress: L_P exact ($\hat{a}_{12} = 4 \times 10^{-17}$, overshoot exactly $4\cos^2(\pi/N)$ — the t^*-tightness MEASURES $\det\hat{A}$), scramble 8/8 (K7 $2.5 \times 10^{-5} \to 4.6$), flat-weight null 8/8, parity-pair only 6.7× (honestly PARTIAL); **REST: R1 in STANDARD SHAPE for the first time — a BILINEAR VON MANGOLDT SUM against closed Dirichlet weights** (diagonal ratio to $1.2 \times 10^{-2}/5.6 \times 10^{-4}$); R2 = R1, R3 closed, R4 not needed; balance 10 THEOREM / 2 CERT-UNIF / 1 CERT-WINDOW / 4 NO-GO	RATIO-RESISTS	41/41

Part	Probe	Contract / finding	Verdict	Checks
T170	`bilinear_sieve`	T169's standard shape cashed in: the bilinear form written out EXACTLY — $\hat{A} = B - \sum_n (\Lambda(n)/\sqrt{n}) X_n$ (TH1, 3.5×10^{-15}), $\det \hat{A} = \det B - D(B,S) + \det S$ (TH2, 1.3×10^{-10}), $\det S = \sum\sum \Lambda(n)\Lambda(m) K(n,m)/\sqrt{nm}$ with $K = \frac{1}{2} D(X_n, X_m)$ (TH3, 1.5×10^{-14}; Cauchy–Binet/Lagrange); X_n rank one (1.3×10^{-4}) ⇒ wedge square with mixed signs; **the reference window:** $\det B = 6.18$**,** $-D(B,S) = -198.8$**,** $\det S = 192.6$ **summing to** 4.22×10^{-3} **— a FIVE-ORDER cancellation between three** ∼200**-sized pieces** (absolute target 1.10×10^{-5}); the route table (target $\delta \geq 3.0$): truth +2.747 MEASURED, CS −0.000, large sieve +0.669, MV 1973 −0.384, exact split +0.996 (best), Vaughan +0.669 — all THEOREM; **the collapse is structural, by two theorems: TH4 RANK-3** — D **is the polarisation of** det**, matrix** $[[0,1,0],[1,0,0],[0,0,-2]]$**, rank** 3**, signature** $(1,2)$**, so** $\operatorname{rank} K \leq 3$ **for every** h**,** X **(**$\sigma_4/\sigma_1 = 2.0 \times 10^{-15}$**, eff. rank** 1.38–2.71 **vs** $N \sim 10^3$**): the form IS** $S_{11}S_{22} - S_{12}^2$ **in THREE linear** Λ**-sums; TH5 TYPE II — the weights see** n **only through** $\log n$**, so Vaughan Type II blocks have effective rank** $O(1)$ **for every** U, V **(**2 **SVs** = 99.9% **vs** 153 **generic,** U = 12–70 **scanned, identity coefficient-exact)**; honest T161 comparison: GAINED 32 → 3 functionals (theorem), NOT GAINED the precision (5.3×10^{-5} at $h = 285$, h^{-3}, RH yardstick 1.1×10^3 too coarse; ℓ^1 mass 41×); **TH6: the chain R4-free for free** — $1 - r_{12}^2 = \det \hat{A}/(\hat{a}_{11}\hat{a}_{22})$ an identity, sharper than T169-TH9 (−2.747 vs −2.771), Weil fence never approached; best unconditional carry $h^{-0.996}$ vs required $\leq h^{-2.50}$ — **missing** 1.50 **in the exponent, and the shortfall is a THEOREM**; scramble: rank-3 and bounds UNCHANGED, truth moves 2.76 — **the arithmetic sits in the JOINT VALUES of** (S_{11}, S_{22}, S_{12}) **against** B; flat-weight breaks; L_P exact; ν-decoupling honest (frames fall $-1.70/-1.27/-1.74$ — T169's $h^{-2.95}$ is frame-A, the shortfall is not); **R1 FINALLY CLASSIFIED: the row-angle sine falls** $h^{-2.711}$ **— a NEAR-DEGENERACY of two explicit finite** Λ**-sum vectors at** h^{-3}**, not a size; beyond-RH JOINT precision**; R2/R3 booked (T169), R4 off the path; balance 6 THEOREM / 1 CERT-UNIF / 5 NO-GO → v558	SIEVE-RESISTS	40/40

Part	Probe	Contract / finding	Verdict	Checks
T171	`final_map`	**the capstone: the map is complete** — all sixteen links of the reduction chain, from the I5 floor to the final R1 shape, reproduce in ONE connected run on 12 windows over two frames ($\nu = 4/6$, $h = 142$–1445, lever arm $10.2\times$): 13 THEOREM ($T152$ Schur two-block sharp at $0.5\lambda/1.02\lambda$; T151 Sobolev 0.483; T153 Ψ-collapse 1.19×10^{-15} with $\sum w_d = 0$; T154 Ritz from below $g_{32}/s = 0.923$–0.986; T158 Thomson = ladder 1.45×10^{-9}, 33/33 trials below; T160 sampling 1.05×10^{-15}; T163 swap/TV overshoot $\times 7.3 \times 10^{5}$–6.2×10^{8}; T164 gauge/quantifier $\Psi(x^*) = 1/g_{16}$; T165 $P_{\mathrm{pr}} = 1$; T166 cascade diagonal-invariant; T167 $K = 2$ mildest; T169 det collapse R4-free on ALL 12 incl. the indefinite window; T170 rank-3 $\{-2, -1, +1\}$) $+3$ CERT (T155 12×12 floor sharp; T156 $F(P, r)$ $L_{16} = 5.0$–19.6; T170 R1 shape $\sin\angle = 1.09 \times 10^{-7}$–$1.15 \times 10^{-4}$, rate frame-dependent $\nu = 4$: $h^{-1.86}$ / $\nu = 6$: $h^{-2.87}$, COLLAPSE not); honest exclusion: $\nu = 4$, $h = 1445$ has $\lambda_{\min}(B_{LL}) = -343$ — ladder links 11/12, identity links 12/12; **all eight classified no-gos fail as classified** (budget $h^{+1.32}$; symbol < 0 on 12/12; sector invariant 2.1×10^{-9}; Kato 2.0×10^{4}–1.9×10^{7} off scale; atom band $+0.79$ faster; sieve $\delta = +1.044$ vs 3.0; scramble type-flip $h^{-2.57} \to h^{+0.21}$, $\times 11421$; L_P gain = 1 exactly); **the precision ledger closes**: needed 2.284×10^{-7} at $h = 1444$ (rate $h^{-3.0}$) vs RH yardstick $h^{-0.5} = 2.63 \times 10^{-2}$ (short $1.2 \times 10^{5}\times$) and best unconditional $h^{-0.996} = 7.13 \times 10^{-4}$ (short $3.1 \times 10^{3}\times$); both self-similarity identities check (4.2×10^{-10} / 9.9×10^{-10}, sign-agnostic); balance of the quotable parts 40 THEOREM / 5 CERT-UNIF / 9 CERT-WINDOW / 16 MEASURED / 9 NO-GO (T126–T161 unaggregated); **adds ZERO new uniform-in-m statements — exactly why it is promotable**; R1 the only open phase-2 object, classified → v559	MAP-COMPLETE	43/43

Part	Probe	Contract / finding	Verdict	Checks
T172	`frame_beyond`	**the map is portable, the numbers are not** — the recipe inventory has FOUR degrees of freedom (frame rule D; ν; zone family; the atom set NOT free — Λ lives on prime powers, "beyond" means different ANCHOR arithmetic), measured on four surfaces (54 windows: frame-A reference; gap-blind frame B $D = \alpha/h +$ ν-extremes 3/8; non-prime-power anchors; congruence classes $p \equiv 1$ vs 3 mod 4) + 3 truncated-comb stress windows; **IDENTITY TRANSFER:** 13/16 **links UNIVERSAL,** 0 **broken** (the whole identity tower + both direction lemmas; the sampling identity — the most zone-sensitive link — exact on every window of every surface); the 3 that shift are exactly the number-carrying ones (links 5/6 = window certificates, link 16 = the rate); **indefiniteness LOCALISED at the sieve horizon** ($\lambda_{\min} = -3.7 \times 10^5 - -6.2 \times 10^6$ on 3/3 cut-comb stress windows against 0/54 complete-comb; identity links hold there $\sim 10^{-15}$ — no positivity used), NOT at a frame or zone — T171's worry retired; constants map: 8/11 inside frame A's band, 3 wander on S3 (K^F $\times 2.3$, U_{ref} $\times 1.2$, s $\times 1.5$) — a certificate on a new surface is a NEW certificate (procedure + directions transfer, 54/54); **R1's near-degeneracy PERSISTS everywhere** (angle $< 10^{-2}$ on 54/54, decay on 9/9 legs, $1 - r_{12}^2 = 1.4 \times 10^{-7}$–$6.2 \times 10^{-4}$), the RATE frame-bound ($\nu = 3$: $h^{-2.19}$ / $\nu = 4$: $h^{-2.46}$ / $\nu = 6$: $h^{-2.55}$ / $\nu = 8$: $h^{-1.83}$ / B: $h^{-2.76}$; zone spread 0.40 vs frame spread 1.03); **THE PLACEMENT FINDING: scramble at fixed Λ-value multiset raises the angle** $\times 6.6 \times 10^4$ **median and REMOVES the decay (**$h^{+0.11}$ **vs** $h^{-1.93}$**) — the collapse belongs to the actual prime-power placement**; candidates P1–P4 PENDING (bundling with T174)	PARTIALLY-PORTABLE	44/44

Part	Probe	Contract / finding	Verdict	Checks
T173	`frame_rate`	**the demanded rate is itself a frame datum, and the deficit is the number of the phase** — 284 windows + 109 scramble twins over ELEVEN preregistered frame data (A at $\nu \in \{3,4,5,6,8,10,12,16\}$, gap-blind B, smoothed C at $\nu = 4/6$); $q = 1 - s$ **EXACT** (1.9×10^{-15}) **and** $q = 0.667$–$0.777 < 1$ **on every frame datum — the** h^{-3} **target was the** $q = 1$ **idealisation of a** $\Theta(D^3)$ **law**; gap functional CALIBRATED, not guessed (6 preregistered candidates; the relative I5 gap $t/\lambda_{\max}(B_{LL})$ hits the chain's 3 on frame A — median 3.033 — and is the only dimensionless one; $(D/\alpha)^3$ named a tautology); REQ $= 2.831$–3.438 (spread 0.607); deficit as ONE slope of $R = \mathrm{GAP}/(1 - r_{12}^2)$: **pooled** $+0.155 \pm 0.102$ ($\times 1.54$ over-scatter inflation; 3.0σ above zero), $1.8\times$ flatter than either side (χ^2/dof: REQ 4.87 / DEL 4.17 / difference $2.38 = 2.4\sigma$ — constancy REJECTED), split-stable to 0.214 where both sides drift $\sim$0.5; exact supports: μ_1^P-convention shifts by exactly 2 (1.9974), fixed-h^{-3} χ^2 = delivery χ^2 to 8×10^{-15} (a frame-independent demand cancels nothing); **invariant under the D-rule at fixed** ν **(**$\chi^2/\mathrm{dof} \leq 0.64$ **— theorem candidate for that half), NOT under** ν **(**$r = +0.76$**, 2.8**σ**, confounded with arithmetic content under the cap;** q **the weakest driver,** 0.2σ**);** **CORRECTION TO T172 (P7): fixed-**h^{-3} **gives** $+0.015 \pm 0.084$ **— consistent with zero; T172's** C**-growth neither sign- nor split-stable (verdict untouched)**; h-vs-D not resolved (declared; fixed-α diagnostic legs do not settle it); **R1 frame-free: the delivered near-degeneracy closes** $h^{+0.155 \pm 0.102}$ **SLOWER than the relative I5 gap it must pay — no frame closes it (**0/11 **upper edges** ≤ 0**, best** $+0.170$ **at B;** 8/11 **individually consistent with zero — pooled, not claimed per leg); frame shopping over by numbers** (B delivers 3.449, demands 3.438); stress: scramble removes the decay (4/4 legs), lag identity $\leq 1.9 \times 10^{-16}$, jackknife ≤ 0.026, shuffle null 3.7σ away; candidates P5–P8 PENDING (bundling with T174)	DEFICIT-VARIES	40/40

Part	Probe	Contract / finding	Verdict	Checks
T174	`cancellation_identity`	**the gauge route is exhausted, by theorems, and the deficit is measured frame-free at** 5.0σ — both sides of the T173 deficit are scale-free functionals of ONE raw Gram $\hat{a}_{ij} = t_i^{\mathsf{T}} A_h t_j$ (demand = relative gap of the 16×16 μ-congruence; delivery = the 2×2 corner of the SAME matrix), so "do the frame factors cancel" becomes checkable blindness algebra; THE LEDGER: amplitude gauge THEOREM (≤ 0.028 of the per-cell conditioned bar over 12 decades), **the COMPLETE** $\mu_1^P \approx h^{-2}$ **channel THEOREM (**≤ 0.009 **— exactly absent from the deficit)**, full positive diagonal group THEOREM (delivery exact; the demand keeps the SHAPE — the asymmetry is the anatomy), KMS shape vs k^2 CERTIFIED (unconditional band $2\log((1+\varepsilon)/(1-\varepsilon))$, $\varepsilon = (K\pi/(2h+1))^2/6$; ≤ 0.0075 in deficit units = 4.8% of T173's number), **arch/comb mixture DOES NOT CANCEL** (additive and exact; NO factorisation; neither term alone has a positive Schur floor on one of 84 cells), frame rule THEOREM ($R = R(\alpha, M)$ identically — a SAMPLING map); explained: G2 the UNIQUE doubly gauge-invariant arithmetic member (integer degrees); the G3 offset exactly 2 ($G2$ is μ_1^P-blind; $h^{-1.9975}$); **the rectangle measurement**: $a = -0.281 \pm 0.066$ (4.3σ) with the anchor direction flat; the exact between/within split puts the deficit in the GRID response (3.3× selection, whose sign is mostly wrong — selection reading rejected); the driver is COMB DENSITY per lag cell (monotone $+0.281 \to +0.062$ across the cuts; on legs $\log \nu$ collinear with $\log$ dens at $r = -0.921$ — T173's ν-driver discharged); **dense corner exhausted (**1069 **cells,** 151 **anchors, cluster-robust): DEFICIT(frame-free, dens** ≥ 3**)** $= +0.1111 \pm 0.0222$ — 5.0σ **from zero,** 0.4σ **from T173's** $+0.155 \pm 0.102$; P6 NOT a theorem (multiplicative half only) but less load-bearing — the deficit is measured D-rule-free; candidates P9–P15; rest: per-anchor scatter $\chi^2/\mathrm{dof} = 3.02$ (candidate: placement phases $\mathrm{frac}(\log p/D)$), the admissibility question (0.11 vs ≥ 3 atoms/cell: factor 2.5), the sparse corner → v560	PARTIAL-CANCEL	37/37

Part	Probe	Contract / finding	Verdict	Checks
T175	`phase_placement`	**the placement phases are real and causal, but non-smooth — and the heterogeneity dissolves into the error bar** — 1504 cells on 188 anchors (EVERY prime power in $[40, 1090]$; no frame rule anywhere) × the T174 ladder + declared floor $h = 128$; the deficit REPRODUCES ($+0.1069 \pm 0.0222$, 0.13σ from T174); **the trap said out loud: the preregistered anchor invariant** $\theta_p = \mathrm{frac}(\log p/\alpha)$ **is a smooth monotone function of** α **(Rayleigh** $Z = 1296$**) — a regression on it would be an** α**-regression in disguise**; only the cell phase $\varphi_p(h) = \mathrm{frac}(h\theta_p)$ is free and equidistributed (max $Z = 1.41$; generating identity certified 1.1×10^{-13}, Weyl 1916); **the phases are REAL in** $\log R$ ($F = 10.80$ vs within-rung null 95% point 1.69; +10.3% held out; composite placebo $0.81/-0.0275$; amplitudes monotone $0.018 \to 0.141$) **but NOT the heterogeneity channel** (χ^2/dof $3.14 \to 2.52$, target < 2 missed; held out -15.4%; deficit untouched, 0.18σ); the exploratory scan excludes every anchor statistic; **the resolution is the ERROR BAR: jackknife slope errors take the excess scatter** $0.1224 \to 0.0000$ **and the robust pull** $1.41 \to 1.22$ **(half-split** 0.2722 **vs sd** 0.2789**) — T174's** χ^2 **was substantially an OLS independence assumption, the** 5σ **deficit unaffected**; **the dense corner EXHAUSTED** (dens ≤ 361 by the caps; 359 reached): the curve falls $+0.982 \pm 0.213 \to -0.057 \pm 0.132$ over 3 decades, monotone, slope 6.1σ, **the densest bin CONSISTENT WITH ZERO — true zero / power law / plateau** < 0.132 **undecidable under this sieve**; both channels significant ($\log h$ +0.2435, $\log \mathrm{dens}$ −0.0631; curvature −0.137) — not a single exponent; alternative assemblies INADMISSIBLE (lump 0%/wide 0% PD dense), deformation sensitivity ratio 0.4 — the sparse corner is no assembly artefact; rule proposal dens ≥ 1 (6.5σ, not retroactive); scramble 12/12; **the INTERVENTION (exact rebuild control): displacing the six family atoms gives** $d \log R/d\delta = 879$**, linear over four declared decades — the placement phase is CAUSAL for** R**, the first interventional statement of the series; the first harmonic predicts** 2.89 **(factor** ~ 300 **too small), PD dies at** $\delta \geq 10^{-3}$**, GAP** $= 1.6 \times 10^{-9}$–4.8×10^{-6}**: the floor is a near-degeneracy,** $\log R$ **non-smooth in the phases**; candidates P175-1–P175-8 PENDING	PHASES-RESIST	38/38

Part	Probe	Contract / finding	Verdict	Checks
T176	`dense_limit`	**the density curve sits at zero, and the phase-2 measurement programme closes as planned** (landed after T177; numbered by contract) — the bigger sieve and ONE change only: `ATOM_MAX` $1.2 \times 10^6 \to 2.5 \times 10^7$ (factor 20.8; cost 0.08 s / 59.7 MB peak — **the sieve was never the binding constraint**, the grid costs the 110 s), 1 566 727 prime-power atoms, density ceiling 361 → 6120 (6118 reached), 14 532 cells on 692 anchors, 0 indefinite; **the extended curve** (eight ratio-4 bins, cluster-robust): $+0.8886 \pm 0.1027 \to +0.5148 \to +0.1948 \to +0.3922$ (declared violation, curvature) $\to +0.1520 \to +0.0579 \to +0.0376 \pm 0.0410$ (dens $\sim$779, 0.9σ, NEW) $\to +0.1046 \pm 0.0881$ (dens $\sim$2512, 1.2σ, NEW); **pooled over the new bins** $+0.0496 \pm 0.0372$ **(**1.3σ**) — no sign change, no stabilisation above zero**; density slope 8.0σ over 4.0 decades; **plateau window** $0.264 \to 0.074$ **at** 2σ **— a** $3.6\times$ **narrowing, the whole content of the bigger sieve**; sieve consistency: all six T175 bins at 0.0σ under T175's exact ladder + anchors; both curvature channels survive with T175's coefficients ($\log h$ $+0.1529 \pm 0.0258$, 5.9σ; $\log \text{dens}$ -0.0434 ± 0.0060, 7.3σ; $a_2 = -0.0842 \pm 0.0085$; $\leq 1.7\sigma$), none of six preregistered reparametrisations is one exponent (the data wants $\text{dens}^{+0.284} h$); **R1 CLOSED: the Feynman–Hellmann form reproduces the rebuild to** 3.37×10^{-6} at the U-minimum of a declared step ladder (dDEL/DEL carries 84%, $\lambda_{\min}$ 7%); **the** $300\times$ **harmonic anomaly is a POLE** (PD survives 2% of the period, $\delta_{\text{PD}} \cdot |d \log R / d\delta| = 5.8$; the two lowest modes 13.4% apart at $\delta = 0$ — the intervention runs along a crossing); scramble 194/194; **anti-promotion: the estimator is LADDER-DEPENDENT** (0.20) **— every quoted bin deficit must cite its rung ladder**; resource fence: factor 5 more sieve fits, factor 100 ($\sim 2.5 \times 10^9$, one more decade of window) does not; true zero / power law / plateau stay OPEN → v562	SITS-AT-ZERO	24/24

Part	Probe	Contract / finding	Verdict	Checks
T177	`cp_invariant`	**the reverse flow resumed at the CP ledger: two invariants carry $\pi/3$, four rows narrowed, none upgraded** — the gauge-degree toolkit (T173/T174) turned on the four CP ledger rows; all four constructions rebuilt exactly from the compiler primitives ($\delta = \pi/3 + 3\lambda_C^2 = 1.198231638$ certified vs v88/v220; $\det(\mathbf{1}, d, n) = 21$; $J = 3.327 \times 10^{-5}$, CKM unitary to 10^{-41}); **new exact identity**: on the hexagonal unit the phase channel carries $\sum_k \rho^{d_k} = 4\rho$ (arg $= \pi/3$, sheet-EVEN — the E_8 degrees are even) against the magnitude channel $\sum_k \rho^{m_k} = 4$ (arg $= 0$, sheet-ODD — the exponents are odd): the E8.CHAN.01 typing has an exact fiber realisation (mutation controls: $e^{i\pi/5}$ and mutated lists break; caveat: exponent-sum reality alone is not hexagon-specific); THE TABLE (6 functionals × 4 groups, character bookkeeping a homomorphism in every non-trivial cell): the magnitude data carry NO phase (arg $\prod c = 0$ on both triples), the full δ has NO character (CP-even correction on a CP-odd angle), the raw orientation is HOMOGENEITY DEGREE +1 in the frame normalisation ($21\sin(\pi/3)$ is frame data) while $\mathrm{Im}(\rho\det)/\det = \mathrm{Im}\,\rho$ removes the frame exactly, and Jarlskog passes (a)–(c), failing only the $\pi/3$-carrier test (its content is δ) — the benchmark validates the criterion; **Tier-1 is EMPTY BY ALGEBRA** (kernels commute with conjugation ⇒ every $\pi/3$-carrier is κ-odd); **Tier-2: TWO passers, P2 and V5, in two inequivalent character classes** (sheet-even vs sheet-odd), NEITHER frame-coupled; four ledger rows narrowed (REDTEAM.D.01 residual = a fiber datum; E8.CHAN.01 phase channel sheet-blind; CP.MOD.01 scoped to the leading term; DUAL.FRAME.01 orientation reading degree +1, invariant core $\sin(\pi/3)$, sign a declared orientation), NO marker moves; rest: which Tier-2 character is physical (the sheet-odd reproduces CP.MU6.01 — a preference, not a derivation), the deck column a declared model → `v561`	DEGENERATE	49/49

Part	Probe	Contract / finding	Verdict	Checks
T178	`cp_frame_carrier`	**the frame door of the CP mirror, decided: R1 by theorem, R2 by anchor, the degeneracy explained** (post-closure backflow part) — **R1, natural half, NEGATIVE BY THEOREM**: the fiber acts by a scalar on the n-row, so every homogeneous frame block factorises $P(\rho n) = \rho^m P(n)$ (fully generic degree-1..4 polynomials, symbolic coefficients) and every scale-free functional is $\sum_k A_k \operatorname{Im}(\rho^k)$ with real degree-0 frame ratios; the metric-free invariants of $(\mathbf{1}, d, n)$ have ONE generator (exact bracket enumeration: six permutations $= \pm\Delta$, 21 vanish identically; Weyl 1946 FFT the cited class boundary) $\Rightarrow$ every scale-free ratio is constant — no frame-coupled scale-free $\pi/3$-carrier without a metric; **R1, metric half, NON-CANONICAL**: frame-coupled Tier-2 carriers exist but split into two character classes ($\Delta/\langle d, n\rangle$, $\Delta/\langle \mathbf{1}, n\rangle$ at $(-1,-1,+1)$ vs $\Delta^2/\langle n, n\rangle$ at $(-1,-1,-1)$) and the normalisers contain the continuum $q_t = tn_1^2 + n_2^2 + n_3^2$ (values $441\sin(\pi/3)/(25t+117)$) — no canonical frame factor (21/8, 21/2, 441/142 normaliser-relative, anti-numerology fence as a check); **the SINGLE-SINE LEMMA**: $\{|\operatorname{Im}\rho_n^k|\}$ has one nonzero element iff $n \in \{1,2,3,4,6\}$ (crystallographic, $\varphi(n) \le 2$; $\sin(4\pi/n) - \sin(2\pi/n) = 2\sin(\pi/n)\cos(3\pi/n)$, zero only at $n = 6$ for $n \ge 5$) — the compiler clocks μ_4/μ_6 are the two maximal single-sine fibers, so T177's DEGENERATE was FORCED (on μ_5 values separate carriers); **R2 SELECTED by the anchor**: Jarlskog carries the full tuple $(-1,-1,-1)$ at 40 dps, matching the sheet-odd orientation class only — the unique anchor-matching canonical carrier is the fiber-pure $\operatorname{Im}\rho = \sin(\pi/3)$ (conditional on the anchor, not derived); no marker moves, REG.FREEZE.01 untouched → v564	FIBER-PINNED	31/31
T179	`lambda_pencil`	**the relative pencil on the declared Paper-II surface: the one-mode question, answered** (post-closure backflow part) — the pencil identities $\det(B - S) = \det B\,(1-\lambda_1)(1-\lambda_2)$, $D(B,S) = \det B\,(\lambda_1 + \lambda_2)$, $\det S = \det B\,\lambda_1\lambda_2$ machine-tight on all 70 declared T170 frame-A windows (worst residual 2×10^{-14}; the v563 surface bit for bit); the LOCKING mode is λ_2 hugging 1 (median 1.000021) while the spectator λ_1 is uniformly separated — $\min|1-\lambda_1| = 3.53$, $\lambda_1 \in [4.53, 180.64]$, always ABOVE 1 (the review guess, reversed): the two-dimensional cancellation reduces to the SINGLE scalar $1 - \lambda_2$ on the reachable surface; $h = 540$ reads $1 - \lambda_2 = -2.27 \times 10^{-5}$ (reproducing $\det \hat{A} = 4.22 \times 10^{-3}$); position scramble explodes $|1-\lambda_2|$ by $\times 1.9 \times 10^6$ while the identities keep holding — arithmetic placement, not an identity artefact; declared surface only, Problem 7.1 typed not bounded → v569	ONE-MODE	4/4

Part	Probe	Contract / finding	Verdict	Checks
T180	`separation_floor`	**the T179 open question closed on the surface: the spectator separation certified from closed scalars** (post-closure backflow part) — two one-line theorems ($P = \det S/\det B > 0$, $T = \operatorname{tr} Z > 0 \Rightarrow \lambda_1 \geq \sqrt{P}$; $P \leq 0 \Rightarrow \lambda_1 \geq T$) turn the separation into a closed per-window floor: $\min F = 2.130 \geq 1.05$ on ALL 70 declared windows (measured $\min \lambda_1 = 4.534$) — the separation follows from $\det S/\det B$ and $\operatorname{tr} Z$ alone; the dominance $P \in [4.5, 120.6]$ (median 30.1) grows $h^{0.90}$ (surface ladder), $1 - T + P = \det(B - S)/\det B$ links the scalars exactly; HONEST NEGATIVE: the cheap witness route is dead — $\det S$ is itself a pair-level cancellation (Cauchy–Binet exact; negative pair mass up to $13.7 \times \det S$; best single pair 0.13–$0.34 \times \det B$ vs ≥ 1.10 needed): the h-uniform dominance stays open, typed as pair-level size-vs-cancellation; scramble flips $\operatorname{tr} Z$ to -78.3; declared surface only, Problem 7.1 untouched → v570	FLOOR-CERTIFIED	9/9
T181	`pair_band_structure`	**where the dominance lives: LONG-RANGE** (post-closure backflow part) — the Cauchy–Binet band decomposition of $\det S$ is exact per window; EVERY near band (< 16 lag cells) and the diagonal drag NEGATIVE on every selected window (near-diagonal budgets aim at the WRONG SIGN); the far band (≥ 16 cells) carries the WHOLE excess with stable net ($\text{far}/\det S \in [1.55, 1.91]$) while the gross far mass grows $4.1 \to 13.0 \times \det S$ — long-range coherence, not local mass; must-break both directions: the scramble EXPLODES $\det S$ $\times 49$ (the real placement holds it $\sim 50\times$ below random) and collapses the far share below 1; sharper: scale-locked ($\geq h/2$), weight-concentrated (top decile nets 0.85–$1.00 \times \det S$), phase-flat — the target is heavy-pair window-scale coherence; per-atom share oscillating (standing-wave-like); typing of the v570 dominance question; declared surface/subset only → v573	LONG-RANGE	10/10
T182	`cheb_loewner`	**the second review's Toeplitz strand, audited: everything EXACT** (post-closure backflow part) — the polarized cross-weight formula closes the whole $K = 2$ read block (symbolic $N \leq 11 + 10^{-11}$ surface); the edge read is a scaled LOEWNER matrix of U_{s-1} ($\sin((N-s)\omega) = -\sin(s\omega)$, exact; rank $\leq s - 1$, $s = 2$ rank ONE by theorem); the leading profile's Lorentz image is the null triple $(5, -3, 4) = (g_{\text{car}}, N_{\text{fam}}, \lvert\mu_4\rvert)$ via Euclid (typed COMPRESSION); the edge defect is the exact coefficient $-\frac{3\pi^4}{175}(s^2 - 4)(11s^2 + 6)N^{-4}$ (locally better than the h^{-3} demand); the pair polynomial $P(s, z)$ ($P(2, 3) = 0$ exact, transition $s/z \approx 0.503/1.987$) is the analytic prototype of T181's pattern; macro kernels $W^{\infty}(\sigma)$ verified; three-regime split: edge CLOSED, macro/transition OPEN → v576	FORMULAS-EXACT	9/9

Part	Probe	Contract / finding	Verdict	Checks
T183	`nullray_census`	**the null-ray census: correlated, partial, honest** (post-closure backflow part) — the locking mode aligns with $(2,-1)$ at 23.5–33.8° on ALL 70 windows (below the 45° isotropic mean), declines slowly ($h^{-0.10}$ — the clean success bar NOT met), and the scramble destroys the alignment (+21°): a real PARTIAL carrier; the macro regime bends the direction — follow-up is $K_\infty(\sigma,\tau)$ → v577	NULL-RAY-CORRELATED	5/5
T184	`macro_kernel`	**the two-scale kernel signs match everywhere** (post-closure backflow part) — sign $K_\infty(\sigma,\tau)$ matches the real two-lag kernel on EVERY resolved grid cell at $h=300$ (332/332; 330/332 at $h=150$, finite-size); the diagonal $\det W^\infty(\sigma)$ is negative on ∼72% of $(0,1)$ (same-scale pairs couple NEGATIVE analytically — the macro origin of T181's near drag; positive islands only at $\sigma \geq 0.66$); near/far asymmetry geometric; wrong polarisation breaks the match — the GEOMETRIC half of Regime B closes, the Λ-weighted arithmetic occupation stays open → v579	SIGNS-MATCH	6/6
T185	`occupation_map`	**the primes populate the sign map** (post-closure backflow part) — the lag-side polarization identity $\det S = \frac{1}{2}\sum c_{\mathrm{at}} c_{\mathrm{at}} D(W,W)$ EXACT (10^{-9}, binned map resums exactly); EVERY significant occupation cell sign-matches K_∞ on every tested window (50/50, 67/67, 65/65, 65/65); mid-diagonal negative, cross-scale positive; scramble: identity holds while $\det S$ explodes ×49 — the open Regime-B content is QUANTITATIVE (the amounts), not sign-level → v580	OCCUPATION-FOLLOWS-KERNEL	5/5
T186	`locking_transport`	**the multilevel transport lemma, measured and killed** (post-closure backflow part) — 40 doubling pairs; drift 0.3–6.7° per octave (median 2.35°) but GROWING ($h^{+0.36}$ vs the bar $h^{-1-\delta}$): not summable, the review's own kill fires — no scalar-defect transport along a fixed direction; future transports must carry the window-dependent macro direction → v581	TRANSPORT-KILLED	4/4
T187	`density_dominance`	**the dominance is DENSITY-DRIVEN** (post-closure backflow part) — the smoothed comb reproduces $\det S$ to 99.8–99.9% (2% window; 98.7–99.3% at 5%; smoothly degrading to 81–89% at 20%) on every regular window via the exact lag-side identity; scrambled control captures only 70% — the real comb is ANOMALOUSLY SMOOTH at these scales: the placement content is the deterministic profile (which suppresses $\det S$ ∼50× below random); boundary $h=1292$ named; the v570 h-uniform question relocates to PNT-level regularity of the smooth density functional → v582	DENSITY-DRIVEN	5/5

Part	Probe	Contract / finding	Verdict	Checks
T188	`pnt_model`	**the prime-free closed form** (post-closure backflow part) — the two-term classical law $\text{mass}(u) = 4e^{u/2} - 2(\zeta'/\zeta)(1/2)$ (pole + $s = 1/2$ constant, NO zeros) reproduces $\det S$ on all 69 regular windows PARAMETER-FREE to 1.016–1.143 (entries 0.995–1.001, dominance correlation 0.998); the $h = 1292$ sign flip is genuine prime-fluctuation content (prime-free $+1844$ vs real -1465); the dominance splits into a prime-free two-integral inequality + a zero-oscillation bound → v583	PRIME-FREE-CLOSED-FORM	8/8
T189	`pnt_locking_split`	**the two-layer locking split** (post-closure backflow part) — the v583 prime-free model through the Paper-II defect: misses the real lock by 2–4 orders (median 1350×, growing $h^{1.08}$) while reproducing $\det S$ — THE DOMINANCE IS DENSITY, THE LOCKING IS ARITHMETIC; layers $h^{-1.43}$ (density, positive, coarse lock 10^{-2}–10^{-4}) × $h^{-1.08}$ (primes, sign overshoots at $h = 1219/1445$) = $h^{-2.51}$ real; the 7.1 budget splits -1.4 classical + -1.1 zero-oscillation → v585	TWO-LAYER-SPLIT	6/6
T190	`pnt_lock_direction`	**the locking direction is density-fixed** (post-closure backflow part) — real vs prime-free locking eigenvector: max 0.331°, median 0.030°, decay $h^{-1.16}$ on all 69 regular windows (depth stays 1350× apart: THE PRIMES ARE A PURE DEPTH AMPLIFIER); the deterministic direction certifies the real defect within $[0.98, 2.9]$; the v577 slow decay was a comparator artifact (the deterministic direction drifts, limit $-0.551 \sim -0.5$ under $1/\log h$, not settled); scramble breaks to 6–25° → v586	DIRECTION-DENSITY-FIXED	7/7
T191	`w_closed_form`	**the exact diagonal weight formula** (post-closure backflow part) — NEW EXACT FORMULA $W_{kk}(d) = (2/N)[(N-1-d)\cos(\omega_k d) + \sin(\omega_k(d+1))/\sin\omega_k]$, machine-exact ($10^{-15}$) for all lags, $h = 7$–1000, $k = 1$–3; the piecewise break cancels, the edge suppression is a two-term cancellation; the exact cell-sum reproduces the prime-free entries (1.001–1.004) and det (0.9999–1.0003); macro limit $g_k(\sigma)$ + v579 dictionary ($W^{\infty}(\sigma) = g(1-\sigma)$) → v587	CLOSED-FORM-EXACT	6/6
T192	`closed_delta`	**the closed deterministic defect** (post-closure backflow part) — one exact function $G_{ij}(\beta)$ carries both sides: pole exponent (S) vs the TRIVIAL-ZERO LADDER (B: $c_{\text{ar}}(d) = -Df(dD)$, $f = \sum_n e^{-(2n+1/2)w}$); closed B to 2×10^{-5}, closed $\tilde{\delta}$ to 0.2% on ALL 70 windows, census decay $h^{-1.40}$ (= the v585 layer); the deterministic half of Problem 7.1 = the explicit-formula skeleton in closed form → v588	CLOSED-DELTA-EXACT	6/6

Part	Probe	Contract / finding	Verdict	Checks
T193	`zero_comb`	**the zeta zeros enter the instrument** (post-closure backflow part) — the atom table's residual oscillation IS the explicit-formula comb $-4\sum \sin(\gamma u)/\gamma$ (correlation 0.82, slope 1.28 with 200 damped zeros; pointwise at low u); the comb recovers a factor 5.7 of the lock depth at $h = 184$ ($176 \to 31$, non-monotone in truncation — honest); deep windows out of reach ($h = 540$: 2646×); the zero-oscillation bound = a quantified target (resolution π/D, amplitude = lock depth) → v589	ZERO-COMB-IDENTIFIED	4/4
T194	`pole_rank_one`	**the rank-one pole term** (post-closure backflow part) — the Laplace denominators factor completely and the pole part is EXACTLY separable ($S^{\text{pole}} = -32\pi^2 a e^a\, gg^T$, $g_k = k/(a^2 + 4\pi^2 k^2)$, det ≡ 0 symbolically); the closed direction law $v_2/v_1 = -(a^2 + 16\pi^2)/(2(a^2 + 4\pi^2))$ with limit exactly $-1/2$ DERIVES the v577 null-ray conjecture and explains the v586 drift ($-6\pi^2/(a^2 + 4\pi^2)$); ladder match 0.04–1.4% → v591	POLE-RANK-ONE	5/5
T195	`continuum_det_law`	**the continuum determinant law** (post-closure backflow part) — $\det S(a)$ closed, e^{2a} coefficient identically zero, exact leading law $640\pi^2 e^a/a^3$ (one full order suppressed); the real dominance tracks the closed formula at correlation 0.9985 (the v570 growth derived); finite-N gap typed ($0.19 \to 0.50$); entry-level arithmetic certificate $\sup|\text{Osc}| \times \text{TV}$ with margin 47.6× → v592	DET-LAW-DERIVED	6/6
T196	`cutoff_completion`	**the cutoff completes the closed entries** (post-closure backflow part) — the v592 finite-N det gap (0.19–0.50) was the integration cutoff; σ_0-corrected closed integrals match entries to 0.7–1% and det to 0.2–1%; residual $O(1/N^2)$, shrinking — THE DENSITY LAYER IS CLOSED END-TO-END → v593	CUTOFF-CLOSES	4/4
T197	`unconditional_cert`	**the first unconditional statement** (post-closure backflow part) — partial summation (verified) + Büthe 2018 ($|\psi(x) - x| < 0.94\sqrt{x}$ unconditional to 10^{19}) give $\sup|\text{Osc}| \le 17.4$ on the surface (measured 1.474); the entry certificate holds with margin 565×; the determinant level stays with the zero-oscillation theorem → v594	UNCONDITIONAL-ENTRY-CERT	5/5
T198	`mapping_completion`	**the dictionary closes the books** (post-closure backflow part) — the v593 residual was the σ mapping ($\sigma = d/N$, $a_{\text{eff}} = ND/2$, limits $[u_0/(ND), 2\alpha/(ND)]$); with the exact dictionary the closed entries AND determinant match at identity level (1.00000–1.00017; 1.00000 at $h = 1445$); kernel piece 9×10^{-6} = the genuine $O(1/N^2)$; honest amendment of the v593 attribution; det law in Lean (`det_S_closed`) → v595	MAPPING-CLOSES	4/4

Part	Probe	Contract / finding	Verdict	Checks
T199	`lock_projection`	**the 1D lock projection** (post-closure backflow part) — along the CLOSED v591 direction the zero functional cancels the closed density value in a NEAR-IDENTITY (median $\lvert q_{\text{real}}/q_{\text{model}}\rvert = 7.7 \times 10^{-4}$, decay $h^{-1.01}$); coupling spectrum COLLECTIVE (max share 5.9%, $N_{50} = 56$; heavy zeros 14.1/25.0/30.4/40.9); 400-zero truncation fails (v589 plateau); scramble breaks by 10^6; the arithmetic content of Problem 7.1 = one explicit identity family → v596	NEAR-IDENTITY-1D	6/6

B Symbols used in Sections 5–7

All of these are sandbox objects; they are listed so the figures and formulas above can be read without the diary.

Symbol	Meaning
Q_{Weil}, Q_{cert}, Δ_2, Δ_{arch}	the four terms of the transport ledger; $Q_{\text{Weil}} = Q_{\text{cert}} + \Delta_{\text{arch}} + \Delta_2$ closes at 4.4×10^{-16} (v541)
A_{fam}, A_{shift}	the two archimedean halves of the heat family; $\Delta_{\text{arch}} = A_{\text{fam}} - A_{\text{shift}}$ (T82)
I5	the one remaining coupled inequality, $Q_{\text{cert}} + \Delta_2 + A_{\text{fam}} - A_{\text{shift}} \geq 0$ (T79)
μ_k, w_k, g_k	atom strength, handover window width, atom spacing at zone k
$D_k(\alpha)$	the Schur-complement defect of the induction step; target $D_k \leq \mu_k/2$ (T98)
$\sigma_k(\delta)$	the E_- Schur profile of the genuine Weil form just above atom entry (T102)
E_-, E_0, E_+	the three atom eigenspaces; the k-th atom acts as $\text{diag}(-\frac{1}{2}, 0, +\frac{1}{2})$
bare_k, b_0	$\lambda_{\min}(Q_{\text{full}}\lvert_{E_-})$ and its certified excess over $\mu_k/2$ (T105)
L, Λ_-	the soft dressing scalar and its free cap (5.63–7.52, divergent) (T105)
J	the mirror-parity involution; $JQJ = Q$, $JB = -BR$ (T105)
$\rho_\pm$	the Friedrichs-angle cosines of the even / odd channel (T106)
(R)	$Q_{\text{full}}(\alpha_k)\lvert_{J=-1} \succeq (\mu_k/2)P_-\lvert_{J=-1}$, the remaining Loewner statement (T106)
s^*, τ, κ, ε	Sherman–Morrison scalar, its Woodbury split $s^* = \tau + \kappa$, and $\varepsilon = 1 - \tau$ (T107)
r	the residual ratio κ/ε; (R) $\iff$ $r \leq 1$; measured 0.0053–0.1814 (T107)
x, h, x_0	the pole response $x = T_{\text{odd}}^{-1}\tilde{t}$, its demand compression $h = V^{\mathsf{T}}x$, and the single edge entry that carries $\lVert h\rVert$ on 8 zones (T108)
ω, ω_{cert}	the wing scalar $(\mu/2)\lambda_{\max}(\Gamma)$ with $\Gamma = V^{\mathsf{T}}T_{\text{odd}}^{-1}V$, and its graded-matrix-cap certificate (T108/T109)
m_k, need109	the strict odd-channel margin $\lambda_{\min}(Q\lvert_{\text{odd}})$ at zone k, and the explicit demand $(\mu/2)H_{\text{cert}}^2/((1-\omega_{\text{cert}})\kappa)$ of the T109 chain; $m_k \geq$ need109 on 16/16 measured zones (T110)
(Σ, r, σ); θ, φ	the boundary state of the Riccati recursion (partial minimisation of the pole-free form over the interior; carries the pole exactly), and the two exponents of the measured law $\varepsilon \approx 8.34\, D^\theta \alpha^\varphi$, $\theta = 1.790$, $\varphi = -6.04$ (T116)
δ_ℓ, $\hat{s}_\ell$, U	the rung contribution of the level telescope ($\varepsilon_\ell = \varepsilon_{\ell+1} + \delta_\ell$), its oscillation-space residual data ($r_\ell = W\hat{s}_\ell$), and the certified Parseval majorant $U = Z^{\mathsf{T}}T_M(\text{up})Z$ of the per-rung bound (8R) (T124)