

## Prime Front — extracted research

Historical source: `tfpt_1_architecture_e8.tex` at revision `ba2f0fc`. Original labels, results and caveats are retained; extraction changes ownership only. Complete original: `provenance/tfpt-source`. The publication assembly omits presentation boxes around excerpts.

## Contents

*Follow-up (2026-08-03, the Hecke tower):* the same  $\mathbb{Z}[i]$ -module structure carries a Hecke tower whose primitive degrees are *exactly* the Gaussian prime norms and whose  $\sigma$ -descended traces reproduce the atom layer of the Weil measure at deviation 0.00 ([verification/v714\_moonshot\_hecke\_groupoid.py], 19/19, MOONSHOT-GO; honestly fenced: the finite counting facts are  $E_8$ -unspecific on that slice — the specificity question belonged to stage 2 of PRIME.Z1.MOONSHOT.01 and has since been *measured*: the archimedean place of the same tower glues with *one* normalization ( $\kappa = (1, 1, 1)$  to  $< 10^{-12}$ ; [verification/v716\_moonshot\_arch\_glue.py], 18/18, MOONSHOT-STAGE2-GLUED), and among the dimension-8 unimodular lattices  $\{\mathbb{Z}^8, E_8\}$  (Mordell 1938) *only* the Gaussian  $E_8$  glues ([verification/v719\_moonshot\_traceformula.py], 24/24) —  $E_8$  becomes forced *at the gluing*, not at the finite places; the uniqueness *theorem* stays open, no marker moves).

*The null selector (2026-08-06):* a second, independent  $d = 4$  mechanism ([verification/v807\_lorentz\_nullselect.py], 14/14 on this part, NULLSELECTOR-ALGEBRAIC):  $|v_d|^2 = \det M_d = d(4 - d)$  vanishes iff  $d = 4$  on the RR/Serre functor vector, with anchor identities  $i+j = N_{\text{fam}}$ ,  $ij = |\mathbb{Z}_2|$ ,  $i^2+j^2 = g_{\text{car}}$ ,  $2ij = |\mu_4|$  and the bridge  $g^2 - N^2 = 4d = |H|^2$  tying null to glue index on the same invariant  $4d$  as the census above (the honest downgrade — no  $d$ -dependence in the seam family to vary — is typed) [E].

*The Reed–Muller cascade (2026-08-06):* the 15-label channel is a code ([verification/v819\_prime\_packet\_rm14.py], 21/21, RM14-EXACT; Lean PacketRM14.lean): the row hull of  $B$  is the punctured  $\text{RM}(1, 4)^* = [15, 5, 7]$  (enumerator  $1 + 15z^7 + 15z^8 + z^{15}$  exact),  $C_0$  the  $[15, 4, 8]$  simplex,  $\chi_{\text{NSR}}$  *one* codeword (the 7+8 split its two sides,  $C_2$  the affine bit), and the self-reproduction cycle  $\text{RM}(1, 3) \rightarrow E_8 \rightarrow V \rightarrow \text{RM}(1, 4)^* \rightarrow \text{RM}(1, 3)$  closes with exactly 1344 equivalences; the CSS code  $[[15, 1, 3]]$  with dual  $[15, 10, 4]$ , logical distances (7, 3) and triorthogonality  $8/4/2$ ;  $\zeta_8$  typed [H] [E].

*The canonical check rule (2026-08-06):* the Kraus protocol set meets the dual code ([verification/v820\_prime\_kraus\_rm24.py], 16/16+14/14, KRAUS-RM24-RULE-CANONICAL+PLANEFRAMES-EXACT): the literal leg-support claim is dead twice (weights  $1/2/3 < \text{distance } 4$ ;  $Sp(4, 2)$  orbits  $15+90$  vs  $45+60$ ), but the canonical  $\omega$ -built rule identifies the rule-A protocol set (45 reversal pairs + the  $15 \times 4$  fiber) with the 105 weight-4 checks of  $[15, 10, 4]$  exactly,  $Sp/\sigma/\text{deck}/\text{KMS}$ -covariant [E].

*The  $E_7$  vacuum completion (2026-08-06):* the check system completes ([verification/v821\_prime\_vacuum35.py], 22/22, VACUUM35-EXACT):  $140 = 105 + 35$ , Grams  $22I+6J / 6I+J / 28I+7J$  entrywise, and the spectral completion  $112 + 21 = 133 = \dim E_7$  is a *matrix identity* (the  $E_7 \times A_1$  corpus reading a [C] fingerprint, anchor v170); Weitzenböck  $H_{105}^T H_{105} = 2B^2 + 14I$  as the canonical label-side left factor (the v758 continuum wall untouched); the  $\pm B$  anisotropy cancels inside one Gram;  $\text{RM}(2, 4)$  dual-containing [E].

*The divisor lattice of 210, with its canonicity guard (2026-08-08):* the label register  $\mathbb{F}_2^4$  is the divisor lattice of  $210 = 2 \cdot 3 \cdot 5 \cdot 7$  — the Walsh–Hadamard transform carries the lattice Möbius function ( $4 \text{WH}[1111, v] = \mu(d(v)) = (-1)^{\text{wt}(v)}$  on all 16 labels), and the vacuum column is the rank-one Redheffer closure on the register’s own lattice, with the matrix-determinant lemma giving the Euler products *exactly* (half-weighted  $\det = \prod_p (1 - 1/p) = 8/35$ ; unweighted  $= \prod_p (1 - p^{-1/2})$ , exact radical maps) ([verification/v863\_redheffer\_colligation.py], sympy-exact). The hard canonicity guard then quantifies what selects the primes ([verification/v868\_divisor210\_audits.py], 36/36, DIVISOR210-GAUGE-FAMILY(2)): the Boolean/Walsh/ $\mu$  layer is *measured generic* (210/210 quadruples of primes  $< 30$  pass it — layer one carries no selection content, exactly as warned), the two deployed Euler determinants pin  $\{2, 3, 5, 7\}$  *uniquely* in the predeclared field (1 match each; non-matches separated by  $\geq 5.2 \times 10^{-3}$  at 50 digits; the tautological  $\prod p = 210$  test excluded from the verdict), the anchor prime 2 is *forced* by ramification (the unique  $\mathbb{Z}[i]$ -ramified prime onto the unique  $\sigma$ -fixed  $q^*=1$  class — the very prime at which  $V = L/(1+i)L$  is cut), the  $\mu_4$  weld grade cuts the gauge  $C_6 \rightarrow C_3$ , and after the ramification crossref exactly *two* gauge classes remain — the family-cycle chirality (3, 5, 7) vs (3, 7, 5), which is then *proven gauge* by two exact no-go wards (15/15, CHIRALITY-DEGENERATE: the register orientation functional vanishes identically — the Möbius complement  $d \mapsto 210/d$  pairs the orbits with reversed orientation — and quadratic readouts are provably chirality-blind by transposition). The Gram-only baseline is a simply transitive 720-torsor cut to 6 ordered bases by

the deployed pins — the semantics, not the lattice shape, carry the selection; the 6-vs-7 honesty note stands untouched (the clock alphabet  $\{2, 3, 5, 6\}$  and the quadruple  $\{2, 3, 5, 7\}$  are different objects:  $6 = 2 \cdot 3$  sits *inside* the lattice as the unique weight-2 divisor from two alphabet primes, 7 is prime and not in the alphabet); and the  $17 = 12 + 3 + 1 + 1$  moving-sector identification is honestly *buried at space level* (12/12, MOVING12-DIMENSION-ONLY: the count is exact, the principal angles against the only deployed bridge are  $\sim 90^\circ$ ) [E].

*The Redheffer/Mertens Smith echo (2026-08-08)*: the register's vacuum-completion pattern has an exact arithmetic instantiation ([verification/v861\_krein\_normalform\_lean.py], 37/37, REDHEFFER-ECHO-EXACT):  $\det R_n = M(n)$  (Bareiss integer determinants on 62 sizes; the matrix-determinant lemma extends the identity along all  $n \leq 2 \times 10^6$ ), the witness minor is 1 exactly, so  $\text{SNF}(R_n) = (1, \dots, 1, |M(n)|)$  — vacuum completion is a rank-one origin update whose *entire* Smith deviation is one new invariant factor; the mod-2 shadow is the squarefree-count parity; and the code-side instantiation is the vacuum transformation  $[[15, 5, 3]] \rightarrow [[16, 4, 4]]$  above, typed at parameter grade (pattern-grade cross-domain identification — no functor claimed). The same module types the register wiring: the  $\mu$ -sign and the deck are *two separate*  $C_2$ s (the deployed reading character separates them; the identification is one faithful- $\mu_4$  character away, three candidate constructions machine-verified — diagonal characters, the Galois bookkeeping  $\mu_K((p)) = \chi_4(p)$  on  $609 + 619$  primes, the Hall chain parity), and the 2-torsion weld law explains the refusal structurally ([verification/v862\_defect\_polar\_weld.py], 22/22, WELD-WITHOUT-CONTRACTOR: the deck's anticommutant is entirely faithful- $\mu_4$  — the deck enters a Clifford weld only as the quarter turn  $J = MD$ , never as a second involution; all four weld candidates close exactly on their forced domains, the unwelded control refuses exactly) [E].

**Hecke structure of the glue census is lattice-native (HECKE.GEOM.01).** The  $\mu_4$ -glue class thetas that refine the 240-root branching ( $240 = 52 + 64 + 60 + 64$  over  $\mathbb{Z}/4$ ) now carry a machine-checked Hecke package ([verification/v535\_hecke\_from\_geometry.py]): Kneser  $p$ -neighbours realise  $\#_{\text{iso\_lines}} = \sigma_3(p) \# \mathbb{P}^3(\mathbb{F}_p)$  at  $p = 2, 3, 5, 7$ ; the marked neighbour-sum is the affine element  $\nu_p = a \text{Id} + b T_p$  with cusp output  $a_3 = -4$ ,  $a_5 = -2$ ; and the seven census  $q$ -series span exactly the 2-adic oldform hull of  $E_4$  and  $f_8$  (levels  $\{1, 2, 4, 8, 16\}$  and  $\{8, 16\}$ ). Classical scaffolding named classical; weight-4  $\rightarrow \text{GL}(1)$  stays [O]; no RH claim. [E] (in-suite mechanics)

**Eichler trace layer at the  $E_8$  lattice (HECKE.GEOM.EICHLER.01).** The same census channels now carry a machine-checked Eichler/Witt package ([verification/v536\_eichler\_trace\_layer.py]; 23 checks,  $\sim 30$  s; builds on HECKE.GEOM.01 without duplicating Kneser/ $\nu_p$ /oldform checks). (A) The elementary Witt shadow  $\lambda_{\text{Eis}} = \sigma_3(\# \mathbb{P}^3 - \sigma_3) = L - \sigma_3^2$  is closed in  $p$  (identically and for all odd  $p \leq 100$ ). (B) The Eichler identity  $\lambda_{\text{geom}} = \lambda_{\text{Eis}} + a_p^2$  is two-sided at  $p = 3$  (live type-(i)/(ii) split:  $352 = 364 - 12$ ) and at  $p = 5$  via a T36-validated freeze ( $3784 = 3906 - 122$ ); the assembler  $R(p) = \sigma_3 - 1 - c(p^2)/8$  equals  $a_p^2$  with Kneser filter = identity. (C) Closed Type-A/B local densities  $N_A = \min(240(1 + p^3), p^7 + p^4 - p^3 - 1)$  (B empty iff  $p^4 < 241$ ;  $p = 7$  live 82560/743040; injectivity at  $p = 11, 13$ ). (D) Signed extraction  $a_p = -c(p)/8$  from geometric  $\text{Shell}(p)$  recovers  $(-4, -2, +24)$  and  $b \in \{24, 124, 368\}$ . (E)  $O(1)$  assembler for all odd  $p \leq 31$  against the eta product  $f_8$ . Classical Eichler/Siegel/Witt named classical; the claim is in-suite two-sidedness and the closed compiler densities. Honest fences:  $\text{Shell}(p^2)$  for  $p \geq 7$  out of scope; fibre refinement scoped to  $\{5, 7, 11\}$ ; weight-4  $\rightarrow \text{GL}(1)$  stays [O]; no RH claim; no marker moves. [E] (mechanics) / [O] (weight drop)

**Half-integral bridge (HECKE.GEOM.HALFINT.01).** The weight-4 cusp  $f_8$  now has a machine-checked half-integral companion in the compiler theta-monoid ([verification/v537\_halfintegral\_bridge.py]; 20 checks,  $\sim 90$  s; companion to HECKE.GEOM.01/HECKE.GEOM.EICHLER.01). (A) In the 70-element weight-5/2 theta-monoid (plain and  $q^2$ -scaled  $\vartheta_2, \vartheta_3, \vartheta_4$ ) there is exactly one object with the signed Shimura identity  $\text{Sh}_{t=2, \psi=1}(g) = -8 f_8$  (witness  $g = \vartheta_2(q^2)^2 \vartheta_3(q^2) \vartheta_4 \vartheta_4(q^2)$ , verified coefficientwise through  $n \leq 120$ ; three further monoid elements lift to scales  $+8, -16, +16$ ). (B)  $g$  is an exact  $T(p^2)$ -eigenform of weight 5/2, trivial nebentypus, with eigenvalues  $a_p(f_8)$  for  $p = 3, 5, 7, 11, 13$   $(-4, -2, 24, -44, 22)$ ; trial FE of the 5/2-side around centre  $5/4$  with  $N = 512$ ,  $\varepsilon = +1$ . (C)  $U_4(g) = 0$  exactly (mass mod 4 supported on  $\{1, 2\}$ ); level  $32 = 4 \cdot 8$  with  $M = 8$  even lies outside Kohnen 1982 — the plus-route is structurally closed (negative result part of the claim). (D)

With AFE-based twist  $L$ -values ( $\varepsilon_d = \chi_d(8)$ ;  $\varepsilon = -1 \Rightarrow L(2) = 0$ ), the Waldspurger quotient  $R(d) = b(|d|)^2 / (|d|^{3/2} L(f_8 \times \chi_d, 2))$  is constant on ten fundamental discriminants  $d \equiv 1 \pmod{8}$  ( $R = 23.1873585645\dots$ , relative spread  $\leq 10^{-12}$ , including  $d = 89$ ); the class  $d \equiv 5 \pmod{8}$  vanishes structurally. Classical Shimura/Kohnen/Waldspurger/Baruch–Mao named classical; the claim is the unique signed-scale monoid preimage, the in-suite chain, and the *measured* constant  $R$ . Honest fences:  $\mathrm{GL}(2)$  centre  $s = 2$ , not the  $\xi$ -line; no RH claim;  $\mathrm{ZETA.HP.CARRIER}$  untouched; no marker moves. [E] (Shimura/Hecke/Kohnen fence) / [C] (measured  $R$ )

**Compiler relative-trace identity (HECKE.GEOM.RTF.01).** The three preceding packages are machine-checked as three *projections* of one finite relative-trace identity [ $E_8$ /glue orbital counting] = [Newform/Waldspurger spectral sum] ([verification/v538\_relative\_trace\_identity.py]; 18 checks,  $\sim 14$ s; promotion of discovery T53; companion to HECKE.GEOM.01/HECKE.GEOM.EICHLER.01/HECKE.GEOM.HALFINT.01). (R1)  $\mathrm{Tr}_V(\nu_p \circ \pi)$  is computed *both sides independently*: geometric  $a_p = -c(p)/8$  from the glue census (no modular input) equals spectral  $a_p$  from  $\eta(2\tau)^4 \eta(4\tau)^4$  (no geometry) for all  $(p, \pi) \in \{3, 5, 7\} \times \{\mathrm{Id}, \pi_{\mathrm{Eis}}, \pi_{\mathrm{cusp}}, \pi_{E_4}, \pi_{f_8}\}$  (anchors e.g.  $p=7$ : full trace 727680,  $f_8$ -channel 19840). (R2) The Eichler split  $\lambda_{\mathrm{geom}} = \lambda_{\mathrm{Eis}} + a_p^2$  is the RTF orbit decomposition (principal orbit  $\rightarrow \lambda_{\mathrm{Eis}} = L - \sigma_3^2$ ; elliptic  $\rightarrow a_p^2$ ; cross terms cancel; anchors  $\lambda_{\mathrm{Eis}} = 336/3780/19264$ ,  $a_p^2 = 16/4/576$ ). (R3) The period side is lattice counting:  $g$  is the quaternary form  $n = (x^2 + y^2)/2 + 2z^2 + u^2 + 2w^2$  ( $x, y$  odd; sign  $(-1)^{|u|+|w|}$ ), with  $b_{\mathrm{rep}} = b$  through  $n \leq 200$ , and  $[\text{signed lattice count}]^2 = R \cdot |d|^{3/2} \cdot L(f_8 \times \chi_d, 2)$  at  $R = 23.1873585645$  (rel  $\sim 10^{-12}$ ). Verdict **ONE-FORMULA**. Classical Jacquet-type RTF / Eichler / Witt / Waldspurger / theta correspondence named classical; NEW is the compiler-native realisation and bilateral independence. Honest fences: the identity is *finite* ( $p \in \{3, 5, 7\}$ , finite  $d$ -windows); the *infinite* RTF (sum over all  $p$  and  $d$  + archimedean term) remains the named open step; no RH claim; not  $\mathrm{ZETA.HP.CARRIER}$ -relevant; no marker moves. [E] (R1/R2 exact) / [C] (R3 measured  $R$ )

**Weil structure of the compiler family (RTF.GNS.WEIL.01).** The compiler family carries a fully identified Weil structure up to *two explicitly isolated obstructions* ([verification/v539\_weil\_structure\_family.py]; 25 checks,  $\sim 6$ s; promotion of discovery T55/T61/T62/T63; companion to HECKE.GEOM.RTF.01). (A) The canonical GNS pairing is the direct integral over the Gelfand spectrum of the character subalgebra: metric  $\ell^2(d, b^2/|d|)$  diagonal; fibres  $\sigma(d) = (\chi_d(3), \dots, \chi_d(13))$  partition the live family;  $\Lambda_{\mathrm{fam}}$  constant on each fibre; twist-mixing certificate per fibre  $\leq 5.037 \times 10^{-16} < 10^{-12}$ . (B) The trivial Sato–Tate isotype is algebraically the  $\mathrm{GL}(1)$  core  $G_0 = (1 + Y)/(1 - Y) = \zeta_p(w - 3)^2 / \zeta_p(2w - 6)$ , globally  $\zeta(u)^2 / \zeta(2u) = \sum 2^{\omega(n)} n^{-u}$  (Dirichlet coefficients exact through  $n \leq 2000$ ; character expansions  $\Phi_1 = \chi_0 + \chi_2$ ,  $\Phi_2 = \chi_4 - \chi_2$ ,  $\Phi_3 = 2\chi_0 - \chi_4 + \chi_6$ ,  $\Phi_4 = \chi_8 - \chi_6$ ; core series  $c_{k,0} = [1, 0, 2, 0, 2, 0]$ ). (C) The family form satisfies the exact linear relation  $Q_{\mathrm{fam}} = 2Q_\zeta(g) - 2Q_\zeta(g_b) + \mathrm{Arch} + \mathrm{Corr}$  ( $\mathrm{Prime}_F = 2P_\zeta(g) - 2P_\zeta(g_b)$  with max rel  $9 \times 10^{-16}$ ;  $Q_{\mathrm{fam}} = Q_F + \mathrm{Corr}$  with rel  $2 \times 10^{-16}$ ). **Obstruction 1:** the doubling term enters with a *minus* (family positivity does *not* imply  $Q_\zeta \geq 0$ ). **Obstruction 2:** the Plancherel correction is demonstrably non-automorphic (no polar/arch absorption; no finite Euler factor; identified as  $e^{-\sum p^{-u}}$ -type). Finite-class positivity  $Q_{\mathrm{fam}} \in [4.369, 11.486]$  is measured [C]; dense-class positivity stays open / RH-adjacent and is *not* claimed. Classical Weil 1952 / Gelfand /  $\mathrm{SU}(2)$ –Chebyshev / Sato–Tate /  $2^\omega$ -series / GNS named classical; NEW is the compiler-native identification and the machine-checked isolation of the two obstructions. Honest fences: *not* “almost RH” — the claim structurally shows why family positivity does not imply RH;  $\mathrm{ZETA.HP.CARRIER}$  untouched; no marker moves. [E] (A/B/C/obstructions exact) / [C] (finite-class  $Q_{\mathrm{fam}}$ )

**Amplitude route and positive linear carrier (RTF.GNS.AMP.01).** The route out of the square plane behind the two RTF.GNS.WEIL.01 obstructions is now machine-checked ([verification/v540\_amplitude\_linear\_carrier.py]; 34 checks,  $\sim 3$ s; promotion of discovery T67–T72; companion to HECKE.GEOM.RTF.01/RTF.GNS.WEIL.01). (A) The amplitude Dirac  $D = \begin{pmatrix} 0 & V \\ V^\top & 0 \end{pmatrix}$  with  $V[d, m] = \sqrt{w_d} b(d) \chi_d(m)$ ,  $w_d = |d|^{-1}$ , exists exactly and is Hecke-equivariant:  $D = D^\top$ ,  $D^2 = \mathrm{diag}(VV^\top, K)$  with  $V^\top V = K$  (rel  $2.7 \times 10^{-16}$ ), spectral  $\pm$ -symmetry;  $m$ -side intertwining

$VA_p^\top = \hat{A}_p V$  exact on the  $p$ -free locus ( $p = 3, 5, 7$ ) and  $d$ -side Shimura  $b(dp^2) = b(d)(a_p - \chi_d(p)p)$  integer-exact on 644 pairs. (B) The geometric polarisation  $b = N_+ - N_-$  is exact (parity-split lattice count of the quinary form, plus the  $\Theta$ -monomial identity coefficientwise);  $\Theta = N_+ + N_-$  is a *pure* Siegel–Weil Eisenstein  $T(p^2)$ -eigenform with eigenvalues  $\sigma_3(p) = 1 + p^3$  ( $p \leq 13$  exact; cusp component zero), carrying the Cohen seed identity  $\Theta(d) = -48 L(-1, \chi_d)$  exact-rational on 159 live  $d$  (Cohen 1975:  $H(2, d) = L(-1, \chi_d)$ ; generalised Bernoulli  $B_{2, \chi}$ ). (C) Every coefficient bilinear form inherits the even- $k$  deletion: the Cauchy–Littlewood window lemma with *free* Satake symbols has numerator  $1 - stuv X^2$ , hence  $1 - p^6 X^2$  pair-independently at  $st = uv = p^3$  (five channels checked); simultaneously the deletion *is* the square-class double counting of the towers, fam  $[1, 0, 2, 0] + 2b[0, 2, 0, 2] + \delta_{k1} = [2, 2, 2, 2]$  (generating-function exact; Möbius square sieve). (D) The positive linear carrier  $\ell^2(d, \mu)$  with  $\mu = 48 |L(-1, \chi_d)| |d|^{-a}$  carries *full* weights  $\lambda_k = 1 + p^{3k} - (\chi p)^k$  (layer  $[1, 1, 1, 1]$ ; no even- $k$  kill) and the plus balance  $Q = Q_\zeta(g_-) + Q_\zeta(g_+)$ ,  $g_\pm = e^{\pm 3u/2} g$  (rel  $2 \times 10^{-15}$ ); the  $b$ /doubling kernel enters *with plus* — the sign of the v539 minus inverted. (E) The completed functional equation  $\Lambda_\Theta(s) = 8^{1-s} \Lambda_{\Theta^\dagger}(5/2 - s)$  holds exactly (Fricke closed via the Jacobi inversions; rel  $\sim 10^{-40}$  modular and in the strip; poles  $5/2$  and  $0$  swapped; centre  $5/4$ , plus-balance locus off-centre by exactly  $1/4$ ); the guaranteed cone and the gap functional are FE-self-dual/-covariant. **Open boundary as content:** the positivity is *Euler-region* positivity (edge  $L$ -values, not the central line); the residual distance to the Weil cone is the FE-covariant gap functional  $\lambda^*$  on  $n \equiv 6 \pmod{8}$  — named, measured, Farkas-certified, and not removable by any finite signed theta library. Classical Cohen 1975 / Siegel–Weil / Jacobi–Fricke / Landen / Hecke split-Mellin / Cauchy–Littlewood / Shintani / Weil 1952 / Farkas named classical; NEW is the compiler-native consolidation. Honest fences: *not* “almost RH”; ZETA.HP.CARRIER untouched; no marker moves. [E] (A–E exact) / [C] (sampled cone facts;  $\lambda^*$  named open)

**Matching lemma and transport ledger package (RTF.GNS.LEDGER.01).** The discovery proof package T78–T85 is now load-bearing as one module in which every core check is *re-computed*, not cited ([verification/v541\_matching\_lemma\_ledger.py]; 33 checks,  $\sim 10$ s; companion to HECKE.GEOM.RTF.01/RTF.GNS.WEIL.01/RTF.GNS.AMP.01). (A) *Window proof.* The matching lemma of the hybrid certificate recipe is *proved exact-integer on*  $[4, 10^6]$ : the certificate  $7S(j) < 40A(j)$  (i.e.  $\rho_{\text{design}} F(j) < 1$  with  $\rho = 21/20$ ) holds at *every* atom  $j \leq 10^6$  (0 violations over 939 870 clash atoms; full enumeration, proven no-overflow, big-integer rechecks), with the exact Fraction margin  $X = 22242575693/270725400000 = 0.082159$  at the exact argmax  $j^* = 554400$  and  $\rho_{\text{crit}} = 1.1440 > 21/20$ ; the maximal-greedy identity makes the certificate *uniform* in the target set  $M$  and the greedy weights. Structure laws at 0 tolerance: the 8-law  $\Theta(4n) = 8\Theta(n)$  ( $n \leq 250000$ ); the seed-tower divisor formula  $\Theta(4^a s f^2) = 8^a \Theta(s) \alpha_s(f)$  and the  $\psi$   $w$ -table (all  $n \leq 50000$ ; Euler factor and  $w$ -recursion sympy-exact); the Cohen seeds  $\Theta(s) = c L(-1, \chi_\Delta)$  with *one* exact-rational constant per residue class  $\{-48, -80, -8, -8\}$  on all squarefree  $s \leq 2000$ ; the bracket  $\Theta \leq 2|\psi| \leq 3\Theta$  on the full window. (B) *Transport ledger.* The ledger  $Q_{\text{Weil}} = Q_{\text{cert}} + \Delta_{\text{arch}} + \Delta_2$  closes *exactly*:  $\Delta_{\text{pole}} \equiv 0$  and  $\Delta_{\text{conv}} \equiv 0$  are *proven identities* (sympy kernel collapse; quadrature  $2.8 \times 10^{-16}$ ; exact scale invariance), the residual is  $4.4 \times 10^{-16}$  on 24 exact-autocorrelation battery rows, and the odd-prime side of the full Weil functional agrees *exactly* with the certified plus combination  $P_\zeta^{\text{odd}}(h) = P_\zeta(g_-) + P_\zeta(g_+)$  (rel  $3.9 \times 10^{-16}$ ): the prime side was never the wall. (C) *Character-exact signed envelope.*  $-\psi = (\chi_{-4} + \frac{1}{4}\chi_8 + \frac{1}{4}\chi_{-8}) \cdot \Theta$  coefficient-exact on all odd  $n \leq 50000$  on both independent builds (character system solved sympy-exact); the signed certificate  $7S_{\text{net}} < 40A$  holds with 0 violations and margin factor  $8.86\times$ ; the confinement is a *set equality* on  $10^6$ :  $\{\text{zero-credit clash atoms}\} = \{\chi_{-4}\text{-coherent odd composites}\}$ . (D) *The archimedean term is internal.* Via Legendre duplication  $(2\pi)^{-s} \Gamma(s) = \frac{1}{2} \Gamma_{\mathbb{R}}(s) \Gamma_{\mathbb{R}}(s+1)$  (sympy-exact +  $1.8 \times 10^{-41}$ ) the ledger kernel obeys  $k_\zeta = K_{\text{fam}} - K_{\text{shift}}$  *pointwise* ( $7.9 \times 10^{-31}$  on 51 points; Gauss digamma anchor at  $t = 0$ ), and  $\Delta_{\text{arch}}(h) = A_{\text{fam}}(h) - A_{\text{shift}}(h)$  on the battery subset at rel  $9.9 \times 10^{-16}$  — I5 becomes the self-consistency of *one* heat family. (E)  *$\lambda$ -equivariant channel.* The CM carrier  $g_\lambda = \sum_{\mathbf{a}} \lambda_1(\mathbf{a}) q^{N_{\mathbf{a}}}$  exists exactly in two independent routes ( $\mathbb{Z}[i]$  lattice sum of  $z^4$  vs Cornacchia reconstruction, 0 mismatches  $n \leq 10^4$ ; CM laws exact;  $\text{supp}(c_1) = \{\mathbb{Z}[i]\text{-norms}\}$  as a set equality with  $c_1 \neq 0$  at every coherent atom;  $\vartheta_3^2 \equiv r_2$  on  $10^6$ ); the canonical phase mean  $\mu_1 = c_1/(c_0 d^2) \in [-1, 1]$  is exact,

and the  $\lambda$ -window certificate over all  $j \leq 10^6$  gives  $\max 7|S_\lambda|/(40A) = 0.0653$  strictly below the unlifted 0.2365 (factor  $3.6\times$ ; exact Fraction rechecks) — the sign-mixing of the Grossencharacter phases *is* the cancellation channel; coherent targets are reachable only by coherent rescalings (T81 reachability). (F) *Frontier facts* [C]: unlifted coherent-chain crossing  $k^* = 14$  (exact Fractions,  $\log_{10} N^* = 23.1$ ); absolute Robin tail factor 6.16 at  $J = 10^6$  and divergent — the constant route closes nothing beyond the window. **Named limits as content:** (i) *one* classically-shaped open lemma remains — the correlated-cancellation lemma on credit-rich non-coherent tail atoms (uniform form); provably-shaped  $\neq$  formal proof; (ii) I5 in one-family form ( $Q_{\text{cert}} + \Delta_2 + A_{\text{fam}} - A_{\text{shift}} \geq 0$ ) is the single remaining TFPT-specific object — by the closed ledger *equivalent* to Weil positivity  $\iff$  RH (Weil 1952): an equivalence *typing*, not a progress claim. Classical Gronwall 1913 / Robin 1983 *unconditional* / Cohen 1975 / Hecke 1918–1920 / Cornacchia / Fermat–Gauss / Dirichlet  $L(1, \chi)$  / Mertens-AP / Landau 1908 / Legendre duplication / Weil 1952–Guinand–Bombieri / Alaoglu–Erdős named classical; NEW is the compiler-native consolidation and the machine-checked window proofs. Honest fences: no RH-evidence language; *not* “almost RH”; ZETA.HP.CARRIER untouched; no marker moves. [E] (A–E exact) / [C] (frontier facts; two named limits open)

**The margin-chain identities of phase 2 (PRIME.MARGIN.IDENT.01).** The identity/theorem block of the discovery parts T128–T131 is now load-bearing as one *narrow* module ([verification/v542\_margin\_chain\_identities.py]; 44 checks,  $\sim 0.2$ s; companion to RTF.GNS.LEDGER.01), recomputed on small frame-A windows: 24 seam instances ( $n = 3 \dots 97$ ,  $\rho = 1.25 \dots 4.00$ , every inverted or diagonalised matrix  $\leq 300$ ), 43 graded/fine grid pairs and 400 randomised profiles, each identity a numerical residual against a preregistered tolerance *plus* a mutation control that must fail by  $\geq 10^{-3}$  relative. *Border geometry.*  $\sum_i (1 - g_i) = t_l + t_r$  exactly ( $5.5 \times 10^{-13}$ ), so the  $\kappa$  weights are a probability vector and the flat border vector gives  $\kappa_{\text{flat}} = 1$ ; the (T) four-term identity  $\tau_{\text{dn}} = \|y\|^2 - m_{\text{prot}} + m_{\text{fill}} - V_{\text{bord}}$  holds at  $2.2 \times 10^{-16}$ , with  $\tau$  recomputed independently from the full odd overlap matrix ( $1.1 \times 10^{-13}$ ). *Profile and curvature.*  $p_{N-1} = 2 - p_0 + 2E$  at  $2.3 \times 10^{-16}$ , hence the equivalence  $2E > p_0 \iff p_{N-1} > 2$  with 0 exceptions (both branches realised), and the Abel/Hölder chain  $\kappa \leq 2 - p_0 + \frac{N-2}{N} \sum_j |\Delta p_j - \overline{\Delta p}| + (p_{\text{max}} - p_{N-1})$  holds per instance with 0 violations — while the same chain *without* the curvature term is violated on 88/400 profiles, which is what makes the curvature term load-bearing. *Assembly and compression.* The matrix-free two-scale assembly equals  $J^T Q J$  ( $5.2 \times 10^{-16}$ ), the graded overlap operator equals  $J$ , and the C ea/Strang defect identity  $S_{\text{graded}} - S_{\text{uniform}} = R^T X^{-1} R$  holds as a *matrix* identity on all 43 grids ( $2.1 \times 10^{-14}$ ) with a PSD defect — the compression error is one-sided *by* the identity. *Sandwich, sign, counting.* The secular sandwich  $\varepsilon/(\|u\|^2 + \varepsilon/\mu_1) \leq \lambda_{\min}(Q) \leq \varepsilon/\|u\|^2$  holds on all 48 sections at which  $A > 0$  and  $\varepsilon > 0$  are *checked* (relative width  $\leq 1.4448$ ), with Albert’s sign equivalence verified in both directions;  $S^{-1} > 0$  entrywise (measured 48/48) implies a sign-constant and simple ground vector (Perron–Frobenius), with a control matrix where hypothesis and conclusion fail together; and  $n_{\text{run}} \leq n_{\text{cross}} + 1 \leq s_2 + 2$  together with  $\sum_j |\Delta p_j - \overline{\Delta p}| = \text{TV}(P)$  hold without exception on a non-vacuous battery. **Named limits as content:** every statement is *per instance on small windows* — nothing is uniform in the zone index; the  $\kappa$  law is *not* claimed (T129 broke it), only the chain underneath it; positivity of the pole-free section at depth stays open and carries no floor value here; the Perron step has a *measured* hypothesis; and the defect identity says nothing about the *size* of the defect. Abel / Hölder / Schur–Haynsworth 1968 / Albert 1969 / C ea 1964–Strang / Yserentant 1986 / Perron–Frobenius–Ostrowski 1937 named classical; Weil 1952 cited, never used as a criterion. [E] (per-instance identities; nothing uniform, no marker moves)

**The lumped  $M$ -matrix pair (PRIME.MMATRIX.IDENT.01).** The identity block of the discovery part T136 is load-bearing as a second narrow module ([verification/v543\_lumped\_pair\_identities.py]; 35 checks,  $\sim 1$ s; companion to PRIME.MARGIN.IDENT.01) on 20 frame-A seam windows ( $n = 4 \dots 59$ ,  $\rho = 1.25 \dots 4.00$ ,  $h = 25 \dots 299$ , every factorised matrix  $\leq 300$ ) giving 39 matrices with positive off-diagonals, 312 shifted ladder rungs and 8 whitened two-grid rungs. *The pair and the congruence.* With  $\Delta$  the positive off-diagonal part and  $L_\Delta = \text{diag}(\Delta \mathbf{1}) - \Delta$ , the lumped pair  $A_B = A + L_\Delta$  is Stieltjes and row-sum preserving ( $1.6 \times 10^{-15}$ ) with  $A_B \succeq A$  — an *upper* bound on  $\lambda_{\min}(A)$ , never a floor;  $A = A_B^{1/2}(I - W)A_B^{1/2}$  holds as a matrix identity ( $4.4 \times 10^{-15}$ ) with  $\lambda_{\max}(W) = \rho(L_\Delta A_B^{-1})$ ,

and the equivalence  $\lambda_{\max}(W) < 1 \iff A \succ 0$  is checked in *both* directions, so the Loewner sandwich is circular by itself. *Anchor, ladder, Varga.* With  $x = A_B^{-1}\mathbf{1} \geq 0$  and  $A_B^{-1} \geq 0$  entrywise (Ostrowski 1937, checked),  $\lambda_{\min}(A_B) \geq 1/\max_r x_r$  on all 39 rows (sharpest ratio 0.9070) from one solve and one sign check; lumping commutes with diagonal shifts and  $M$ -matrix inverses are entrywise monotone (Berman–Plemmons 1979), so the shifted ladder is *structurally* empty (0 exceptions of 312); and for the regular splitting  $A_B = D_0 - N_0$  Varga’s  $\rho(J) = \tau/(1 + \tau)$  is an identity ( $4.8 \times 10^{-15}$ ) whose Collatz–Wielandt bracket at the anchor vector gives an a-priori  $\rho(J) \leq 0.9861 < 1$  with no eigenvalue anywhere. *One negative result [X].* The  $k$ -scanned M18 majorant is *valid* at every  $k$  (39 pairs, 0 violations) and still stays above its preregistered bar  $1/2$  (best 3.080) while the measured bad mass is  $\leq 0.0120$ : the loss is the bound, the localisation term alone exceeds the bar, and the bar is not moved. **Named limits as content:** per instance on small windows, nothing uniform in the zone index, in  $D$  or in the gap; the T136  $D$ -exponents are fits and stay in the sandbox; the anchor and Varga rows are implications with hypotheses checked at the instance. Stieltjes / Ostrowski 1937 / Fan 1958 / Berman–Plemmons 1979 / Varga 1962 / Collatz 1942–Wielandt 1950 named classical; Weil 1952 cited, never used as a criterion. [E] (per-instance identities) + [X] (one closed route; no marker moves)

**The long-lag support structure (PRIME.LONGLAG.SUPP.01).** The structure block of the discovery part T137 is load-bearing as the companion module ([verification/v544\_long\_lag\_support.py]; 24 checks,  $\sim 0.2$  s) on the same 20 windows (3–25 prime-power atoms, 20555 positive off-diagonal edges, 455 loaded stripes, 39 bracketed lumped comparisons). *Support and amplitude.* The kernel splits exactly as  $c = c_{\text{arch}} - c_{\text{comb}}$  with  $c_{\text{comb}} \geq 0$ , and the archimedean section alone has *all* off-diagonals  $\leq 0$  on 20/20 windows, so every positive off-diagonal is comb-generated; all 20555 positive edges have their Hankel index  $M - 1 - r - t$  inside the atom-hat index set (per-window fraction exactly 1.000000), so the support is a union of anti-diagonal stripes at prime-power atom positions, each stripe a perfect matching with *exactly* orthogonal edge vectors; and  $\Delta_{rt} \leq c_{\text{comb}}[M - 1 - r - t]$  on all 20555 edges (sharpest ratio 0.99998532), with the Toeplitz lag in place of the Hankel index failing on 20415 of them. *The Green bracket.* For the lumped comparison,  $q_J = 1 - \min_r 1/(d_r x_r) < 1$  is an identity ( $8.2 \times 10^{-16}$ ) needing no measurement, and the Neumann partial sum brackets the Green function entrywise from below *because the discarded terms are nonnegative*, the anchor-weighted tail from above, on 39 rows. *A death certificate [X].* Gershgorin, every row-sum norm and every Collatz–Wielandt quotient at every *positive* weight are upper bounds for  $\rho(|E|)$ ,  $E = B^{-1}L_\Delta$ ; one rigorous lower bound therefore decides them all at once:  $\rho(|E|) \geq 1.3141 \geq 1$  on 20/20 rows, so no member of the absolute-value envelope family can *ever* certify  $\lambda_{\max}(W) < 1$  — while the *signed* radius  $\rho(E) = \lambda_{\max}(W) \leq 0.99995781$  on the same rows. The absolute value alone destroys the cancellation. **Named limits as content:** per instance on small windows, no decay law and no amplitude law; the support statement is one inclusion; the amplitude bound does *not* bound  $\lambda_{\max}(W)$ , and the structure bound assembled from it does not beat the margin (T137: 0 of 35 windows) and is not promoted; the bracket is for the lumped comparison, not the target. Gershgorin 1931 / Neumann series / Haynsworth 1968 named classical; Demko–Moss–Smith 1984 cited as a model and *not* applied (the comparison is dense); Weil 1952 cited, never used as a criterion. [E] (per-instance structure) + [X] (one family closed; no marker moves)

**The Hardy-core identities (PRIME.HARDY.IDENT.01).** The identity blocks of the discovery parts T140 and T141 are load-bearing as the third companion module (36 checks,  $\sim 1$  s) on 11 frame- $A$  windows ( $n = 8 \dots 139$ ,  $h = 58 \dots 285 \leq 300$ , 193–5102 positive off-diagonal edges,  $D$  spanning a factor 3.15). *Form lifting and finite core.* With  $C = \text{diag}(\sqrt{\Delta})M$  and  $M$  the edge–interval incidence,  $\text{Gram} = CHC^T$  ( $3.0 \times 10^{-15}$ ), so  $\text{rank}(\text{Gram}) \leq h - 1$ ; the covering kernel in closed geometric form  $K_{rs} = W([r \wedge s, r \vee s])$  equals  $M^T \Delta M$  ( $1.3 \times 10^{-15}$ ); and  $\rho(W) = \lambda_{\max}(K^{1/2} H K^{1/2})$  is an *exact* eigenvalue, agreeing with  $1 - \lambda_{\min}(A, A_B)$  ( $5.4 \times 10^{-14}$ ) and with  $\lambda_{\max}(\text{Gram})$  ( $7.8 \times 10^{-16}$ ) — three independent routes, no truncation — while  $H = \text{diag}(s) + L_N$  ( $4.7 \times 10^{-16}$ ) reads the sign law as mass plus a long-range Dirichlet form. *The four Hardy identities.*  $\rho(W) = u^T H u / u^T K^+ u$  at  $u = K^{1/2} v$  ( $6.6 \times 10^{-13}$ );  $L_N = D^T B D$  with the signed crossing kernel  $B_{kl} = \sum_{r \leq k \wedge l, k \vee l < s} N_{rs}$  ( $4.9 \times 10^{-16}$ );  $Q = B_+ \mathbf{1}$  ( $1.4 \times 10^{-15}$ ), so the Cauchy–Schwarz step is the Gershgorin diagonal

step; and  $DKD^\top = L_\Delta$  on the interior nodes ( $2.7 \times 10^{-14}$ ), whose diagonal is the endpoint edge mass. The crossing kernel of  $|N|$  does *not* reproduce  $L_N$ : the signs are load-bearing. The closed Muckenhoupt quantity  $A_{M0} = \max_k Q_k(\Delta \mathbf{1})_{k+1}$  follows from two routes agreeing to  $8.3 \times 10^{-15}$ ; its *size* is reported, not promoted. The checkerboard split  $\text{Band}_b = D + A_{\text{even}} + A_{\text{odd}}$  is entrywise exact on 9 (window,  $b$ ) pairs, with three Weyl steps dominating. *Two shapes, promoted as valid and not by size*. Every Loewner step is Cholesky-certified; the additive Hardy chain dominates  $\rho(W)$  on 11/11 ( $3.074\text{--}10.758\times$ ), its exact Weyl split dominates too ( $1.691\text{--}2.503\times$ ) and floors the additive family, and the joint shape  $\Lambda(H, Y) \cdot \Omega(Y)$  dominates on 11/11 for a positive definite Jacobi  $Y$  — the same  $Y$  without its mass term is singular and the certificate refuses. *A negative typing [X]*. Over the  $D$  range the exactly bounded object spreads  $1.36\times$  while every absolute variant spreads  $\geq 54.94\times$ ; the power laws ( $D^{-0.138 \pm 0.094}$  signed against  $D^{-0.776 \pm 0.352}$  and  $D^{-2.748}$ ,  $D^{-2.820}$ ,  $D^{-2.918}$ ) are fits and labelled as fits. **Named limits as content:** per instance on small windows, nothing uniform in the zone index or in  $D$ ; the bound rows promote *validity* and not size — neither shape beats the target — and the finite-core row is an identity, so the zone-uniform discrete Hardy inequality stays open. Abel summation / Cauchy–Schwarz / Weyl 1912 / Gershgorin 1931 / Muckenhoupt 1972 / Bradley 1978 / Opic–Kufner 1990 / Gantmacher–Krein 1950–1960 named classical; Weil 1952 cited, never used as a criterion. Machine-checked: [verification/v545\_hardy\_core\_identities.py]. [E] (per-instance identities) + [X] (one route typed; no marker moves)

**The capacity-chain identities (PRIME.CAPCHAIN.IDENT.01).** The identity spine of the capacity cycle T142–T145 is load-bearing as the fourth companion module (26 checks,  $\sim 0.5$ s) on 12 frame-A windows ( $n = 4 \dots 139$ ,  $h = 26 \dots 285 \leq 300$ ,  $D$  spanning  $7.6 \times 10^{-3}$ – $2.8 \times 10^{-2}$ ; the covering kernel positive definite on 11, where the first two items run). *The capacity decomposition.*  $K^{-1} = D^\top J^{-1} D + xx^\top / \text{cap}$  with  $J = DKD^\top$  ( $3.8 \times 10^{-15}$  entrywise),  $x = K^{-1} \mathbf{1}$ ,  $\text{cap} = \mathbf{1}^\top K^{-1} \mathbf{1}$ , as a matrix identity ( $1.4 \times 10^{-13}$ ); under the congruence the Dirichlet half is an *orthogonal projection* —  $\Omega = 1$  exactly ( $5.1 \times 10^{-12}$ ), where T141 had guessed 20.7–2724 — and its complement is the capacity rank-one. *The exact capacity–Rayleigh identity.* The assembled quotient for  $1 - \rho(W)$  equals  $v^\top (K^{-1} - H) v / v^\top K^{-1} v$  for every  $v$  ( $9.5 \times 10^{-14}$ ), reproduces  $1 - \rho(W)$  at the exact top eigenvector under the declared 1/gap conditioning, and satisfies the inf property; on the constant vector the whole denominator *is* the capacity term. *Cholesky nestedness.*  $\text{cap}_E([a, a+j])$  is a prefix sum of squared forward-substitution entries of one triangular solve (3404 interval capacities at  $2.0 \times 10^{-14}$ , exhaustive on the median window); the reversed ordering fails at  $\geq 4.6 \times 10^{-1}$ . *The whitening certificate.*  $\lambda_{\max}(W) \leq \kappa_{\text{up}} \leq$  Gershgorin ceiling ( $1.2245\text{--}1.4236$  against  $\leq 2.1213$ ), direction-correct, refusing at the shrunken shift. *The  $M_4$  split and the certified chain.* The mass half of Maz’ya’s Markov step is a pointwise theorem,  $\sigma_{\text{tot}} = 0.2197\text{--}0.4425 < 1$  on every window; licence 4 holds with  $|R|$  and *not*  $R^+$  (witness  $[[1, -\frac{1}{2}], [-\frac{1}{2}, 1]]$  built in); and S1’ is *certified per window on both routes*, better route  $c_0 = 2.5154\text{--}4.2265$  against the classical Dirichlet value 4, with Charikar’s cited density constant bracketed by exhaustive enumeration. **Named limits as content:** per instance on small windows, nothing uniform in the zone index or in  $D$ ; the  $c_0$  is *measured* at the minimiser’s own layer cake and the a-priori level lemma (T145’s L1) stays open — Maz’ya’s strong-type half is not used as a theorem anywhere. Maz’ya 1985 / Muckenhoupt 1972 / Fukushima–Ōshima–Takeda 1994 / Miclo 1999 / Charikar 2000 / Goldberg 1984 / Gershgorin 1931 named classical; Weil 1952 cited, never used as a criterion. Machine-checked: [verification/v546\_capacity\_chain\_identities.py]. [E] (per-instance identities and certified inequalities; no marker moves)

**The level-lemma identities (PRIME.LEVEL.LEMMA.01).** The a-priori core of T146 is load-bearing as the fifth companion module (20 checks,  $\sim 0.4$ s) on 12 frame-A windows ( $n = 4 \dots 139$ ,  $m = 26 \dots 285 \leq 300$ ), 9 random positive definite forms for the form-independent items, and a stress ladder  $m = 64 \dots 288$  — the companion to the capacity-chain module, whose certified chain carried a constant *measured* at the minimiser’s layer cake: here the same constant is a functional of the form alone. *The union identity for nested cakes.*  $\sum_{j,l} c_j c_l |S_j \cup S_l| = 2T \|\psi_t\|_1 - \|\psi_t\|^2$  — form-independent and  $O(m)$ , the reason the cake base is movable; exact (0.0) against the  $K \times K$  double sum and against union sizes computed with no nestedness used (controls  $3.2 \times 10^{-2}$  /  $4.8 \times 10^{-3}$ ). *The counting lemma at a general base.*  $G(b) \leq 2b^2 \|v\|_\infty \|v\|_1 / \|v\|^2 + \varepsilon$  for *any*

vector at all 6 preregistered bases; the classical dyadic 8 recovered at  $b = 2$  exactly, the base free (gain  $4/b_*^2 = 3.7612$ ); the one-level-down cake breaks the domination on every draw, so the  $+1$  is load-bearing. *The resolvent delocalisation bound.*  $\psi = \lambda R\psi$  ( $9.0 \times 10^{-14}$ , an identity — no Davis–Kahan step anywhere)  $\Rightarrow \|\psi\|_\infty \leq \lambda_{\text{up}} \max_j \|Re_j\|$  with Rayleigh–Ritz  $\lambda_{\text{up}}$  (1.0592–1.2883 of the gap) and residual-paid certified column norms, within 1.061–1.337 of the truth; the lever: the columns sit 2.85–10.17 below the trivial ceiling  $1/\lambda_{\text{min}}$ , giving  $\Gamma = 1.8356$ –2.2797 against the useless  $\sqrt{m} = 5.1$ –16.9. *The composite level lemma.*  $c_0^{\text{ap}} = 2b^2\Gamma \min(1, \Gamma_1) + \varepsilon = 3.9042$ –4.8488 (slack 1.986–2.418 over the measured constant, under the ceiling  $16 = 4 \times \text{Maz'ya's Dirichlet } 4$ ), the certified window inequality  $\text{cert\_lam\_max}(R) \leq c_0^{\text{ap}} \Psi_{\text{abs}}$  on 12/12, chain loss 0.0423–0.1272 of the exact gap; the built-in no-go discriminator ( $R = aa^T + \varepsilon I$ ,  $a_i = i^{-1/2}$ ;  $\Gamma$  grows  $3.676 \rightarrow 6.798$ , ratio  $1.85 \geq 1.5$ ,  $c_0^{\text{ap}} = 11.10$  exceeds every window value) fails exactly where it must, the theorem half surviving and the Dirichlet control flat (1.0063). **Named limits as content:** per instance on small windows — a finite list with an explicit maximum; nothing uniform in the zone index or in  $D$ , and explicitly *no* statement for all  $D$  (the asymptotic delocalisation of the Green columns stays open, typed open); the energy-route  $\sigma_{\text{tot}}$  stays measured. Maz'ya 1985 / Rayleigh 1877–Ritz 1909 / Cauchy–Schwarz / Miclo 1999 / Davis–Kahan 1970 (cited, deliberately not used) / Charikar 2000 / Goldberg 1984 / Gershgorin 1931 named classical; Weil 1952 cited, never used as a criterion. Machine-checked: `[verification/v547_level_lemma_identities.py]`. **[E]** (per-instance identities and certified inequalities; no marker moves)

**The Green/Szegő identities (PRIME.GREEN.SZEGO.IDENT.01).** The identity/certificate core of T147+T148 is load-bearing as the sixth companion module (21 checks,  $\sim 0.4$ s) on 12 frame-A windows ( $n = 4 \dots 139$ ,  $m = 26 \dots 285 \leq 300$ ), 7 random positive definite forms for the form-independent items, and a weight-class model ladder  $m = 64 \dots 288$  — the companion to the level-lemma module: here the two scalars its constant is made of are certified. *The factorisation identity.*  $\Gamma = \sqrt{m} \lambda_{\text{up}} \max_j \|Re_j\| = \sqrt{Q^*} \cdot S_w$  with  $S_w = \lambda_{\text{up}} \|R\|_F$  purely spectral and  $Q^* = m \max_j (R^2)_{jj} / \text{tr}(R^2)$  purely geometric — exact at  $2.2 \times 10^{-16}$ ; the certified factorisation dominates the true  $\Gamma$  everywhere ( $S_w = 1.1186$ –1.5777,  $Q^* = 2.0879$ –2.8634). *The bottom-block constant.*  $\sum_j (\Pi_B)_{jj} = |B|$  exactly ( $3.3 \times 10^{-16}$ ;  $B$  the preregistered  $\theta = 10$  block,  $|B| = 3$ –5), so  $Q_B \geq |B|$  is a theorem; measured  $Q_B/|B| = 1.5091$ –1.7980 inside the preregistered band  $[1, 2.2]$  against the Szegő/Widom sharp reference  $2|B|$  — the coordinate-projector control overshoots by  $\geq 3.9$ . *The second-difference  $\ell^1$  ceiling.*  $|a_k| \leq \min(1, \|s_k\|_\infty \|L_0^p \psi\|_1 / \mu_k^p)$  from the exact Dirichlet eigenpair, with the  $m$ -free closed form  $\|a\|_1 \leq 2\sqrt{\nu} + 1$  ( $\nu = (m+1)^{3/2} \|L_0 \psi\|_1 / (2\sqrt{2})$ ) and the chain  $m\|\psi\|_\infty^2 \leq 2\|a\|_1^2$ ; Dirichlet calibration  $\|a\|_1 = 1$  exact,  $\nu \rightarrow \pi$ . *The layer-cake  $S_w$  certificate.*  $\text{LDL}^T$  inertia counts equal the direct spectral counts on all 96 (window, rung) pairs, and the layer cake certifies  $S_w \leq 1.3288$ –1.8220 (true 1.1186–1.5777); dropping the bottom layer breaks it everywhere. *The Liouville transform.*  $m\|\psi\|_\infty^2 \leq 2\kappa_\Lambda \text{ceil}(\hat{\varphi})^2$  with  $\varphi = \Lambda^{-1/2}\psi$  and a-priori  $\kappa_\Lambda = \max \Lambda / \min \Lambda = 1.3868$ –2.5805, stepwise on every bottom-block mode; the weight-class discriminator at matched  $\kappa_\Lambda$  (mismatch 0.0013): the BV multiplier flat (1.003), the rough one (TV larger by 162.6) grows by 19.9 — the roughness, not the conditioning. **Named limits as content:** per instance on small windows — a finite list with an explicit maximum; nothing uniform in the zone index or in  $D$ , and explicitly *no* statement for all  $D$  (the one open lifting input,  $\nu_L \leq F(\text{form})$   $m$ -free / bounded  $\text{TV}(\log \Lambda)$ , stays open, typed open); the symbol-level order-2 hypothesis is refuted (T148:  $f(0) < 0$  on every surface window) and enters nowhere as a theorem;  $\sigma_{\text{tot}}$  stays retired as a route. KMS 1953 / Widom 1958–Basor–Ehrhardt 2009–Böttcher–Silbermann (the mechanism's address, never an authority) / Sylvester 1852–Bunch–Kaufman 1977 / Euler 1735 / Rayleigh 1877–Ritz 1909 / Davis–Kahan 1970 (cited, computed vacuous, not used) / Gershgorin 1931 named classical; Weil 1952 cited, never used as a criterion. Machine-checked: `[verification/v548_green_szego_identities.py]`. **[E]** (per-instance identities and certified inequalities; no marker moves)

**The gauge/parity identities (PRIME.GAUGE.PARITY.IDENT.01).** The identity/certificate core of T149+T150 is load-bearing as the seventh companion module (21 checks,  $\sim 0.3$ s) on 12 frame-A windows ( $n = 4 \dots 139$ ,  $m = 26 \dots 285 \leq 300$ ), 48 gauge assemblies (Jacobi, const, moving-geometric-

mean, and the arithmetic-free archimedean diagonal), the full-section parity items on the 6 windows with  $M \leq 300$ , and a parity control model at  $m = 2001$  — the companion to the Green/Szegő module: here the gauge *freedom* around that machinery and the parity *structure* underneath it are certified. *The gauge freedom.* For any positive diagonal  $\tilde{\Lambda}$ ,  $\lambda_{\min}(A) \geq \text{cert } \lambda_{\min}(\tilde{E}) \cdot \min_j \tilde{\Lambda}_j$  is a valid certified lower bound (an identity plus one Rayleigh step; family maximum valid);  $\Gamma = \sqrt{Q^*} S_w$  holds gauge-invariantly at  $2.4 \times 10^{-16}$ ; the const gauge has  $\text{TV}(\log \tilde{\Lambda}) = 0$  *exactly* with the certified entrywise sandwich  $\sigma = 1.3868\text{--}2.5805$ ; on the shifted indefinite form the completed-Cholesky floor refuses on 12/12. *The two-regime discriminator.* The flutter smoother removes TV by  $\geq 2.88\times$  while the Liouville-transported  $\nu_{\tilde{L}}$  moves  $\leq 0.0088$  — even the rough re-gauge moves it  $\leq 0.0046$  — while the untransported functional jumps  $0.41\text{--}9.76$  under the same rough gauge: the roughness is in the form, not in the multiplier (T149's withdrawal of the TV hypothesis, wired per instance). *The explicit zone diagonal.*  $\Lambda_r = c_0 - c_{M-1-2r} + \sum_{s \neq r} \max(c_{|r-s|} - c_{M-1-r-s}, 0)$  at residual  $5.9 \times 10^{-16}$ , the exact split  $c = c^{\text{arch}} + c^{\text{atom}}$  with  $c^{\text{atom}} \leq 0$ , the closed atom budget  $\|c^{\text{atom}}\|_1 \leq 4B\sqrt{N}$  ( $B = 1.038821$  verified on the table); the archimedean section alone is indefinite on 12/12 (the atoms are co-responsible for the positivity — the additive perturbation reading is structurally empty). *The parity compression.*  $U_-^\top T_M(c) U_- = T - H$  exactly (cross block  $1.4 \times 10^{-16}$ ): the reflection-odd section is the compression of the full symmetric Toeplitz section onto its antisymmetric parity sector, and the negative inertia of the sign-changing symbol sits entirely in the *even* sector (odd sector zero,  $\text{LDL}^\top$ ) —  $f(0) < 0$  explained rather than worked around. *The parity-sine calibration.* The parity sines are exact eigenpairs of the parity Laplacian (corner 3 forced by antisymmetry),  $\nu_k^P = \pi k^2$  on the exact model to  $3.0 \times 10^{-5}$ , and the per-window ladder  $\nu_k^P \leq Ck^2$  holds with certified  $C = 11.70\text{--}17.28$  and the derived per-mode ceiling  $m\|\psi_k\|_\infty^2 \leq 2(2\sqrt{C}k + 1)^2$ . **Named limits as content:** per instance on small windows — a finite list with an explicit maximum; nothing uniform in the zone index or in  $D$ , and explicitly *no* statement for all  $D$  (the one open input, an  $m$ -free  $C$  in the odd-sector ladder for a sign-changing symbol, stays open, typed open);  $\text{TV}(\log \Lambda)$  stays withdrawn as a hypothesis;  $\sigma_{\text{tot}}$  stays retired as a route. KMS 1953 (parity sector) / Widom 1958–Basor–Ehrhardt 2009–Böttcher–Silbermann (the parity sectors as the algebra's natural objects; the sign-changing symbol is exactly what Basor–Ehrhardt does not cover) / Sylvester 1852–Bunch–Kaufman 1977 / Chebyshev 1852 (verified on the table) / Abel / Weyl 1912–Bauer–Fike 1960–Davis–Kahan 1970 (cited, computed dead, not used) / Rayleigh 1877–Ritz 1909 / Gershgorin 1931 named classical; Weil 1952 cited, never used as a criterion. Machine-checked: [verification/v549\_gauge\_parity\_identities.py]. [E] (per-instance identities and certified inequalities; no marker moves)

**The odd-sector identities (PRIME.ODD.SECTOR.IDENT.01).** The identity/certificate core of T151 is load-bearing as the eighth companion module (19 checks,  $\sim 0.2$  s) on 12 frame-A windows ( $n = 4 \dots 139$ ,  $m = 26 \dots 285 \leq 300$ ), parity control models at  $m = 241$  and  $m = 2001$ , and a no-go size ladder  $m = 64 \dots 256$  — the companion to the gauge/parity module: here the *reroute* off the quadratic ladder is certified — through the pencil against the parity Laplacian and one elementary Sobolev step, to a per-mode bound *linear* in  $k$ . *The grid step-over.* The parity sines live on the shifted grid  $\theta_k = 2\pi k/N$  with  $\mu_k^P = 2 - 2\cos \theta_k$  exactly; the symbol has  $f(0) = -47.6 \dots -6.3 < 0$  with a nonempty negative window  $[0, \theta_c)$ , and  $\theta_c/\theta_1 = 0.371\text{--}0.485$  on every window: the odd grid steps over the negative window — odd-sector positivity is a grid fact. The honest negative beside it, promoted as content: the symbol-only pencil floor is vacuous ( $\rho \geq 1$  on 12/12) and the symbol moves by  $0.45\text{--}3.27$  across one mode spacing — the local model is a *matrix* statement, not a symbol statement. *The certified bottom ladder.*  $\lambda_k(A) \leq S\mu_k^P$  for  $k \leq 8$  by completed  $\text{LDL}^\top$  counts (Sylvester 1852),  $S = 1.1019\text{--}6.4725$  — order 2 in  $k$ , with  $f(0) < 0$  absorbed by the Rayleigh floor  $L_P \geq \mu_1^P I$ . *The Sobolev step.*  $\|\psi\|_\infty^2 \leq 2\|\psi\|_2 \|\nabla \psi\|_2$ , licensed by the odd sector's virtual node  $\psi_{-1} = 0$ ; on the exact parity model the pencil is  $(1, 1)$  (certified  $1.1 \times 10^{-12}$ ) and the per-mode price is exactly  $\pi$  ( $5.1 \times 10^{-8}$ ). *The closed archimedean minimum.*  $\min_r \Lambda_r^{\text{arch}} = c_0^{\text{arch}} - c_{M-1}^{\text{arch}}$  exactly (residual 0.0), attained at  $r = 0$ , from two per-window-checked shape properties of the smooth kernel. *The linear per-mode bound.* With the certified pencil floor  $\kappa = 0.1939\text{--}0.3599$  and  $K_{\text{bot}} = S$ ,  $b_k = 2m\sqrt{K_{\text{bot}}\mu_k^P/\kappa} \leq C_S k$  with  $C_S = 12.081\text{--}26.953 \leq 2\pi\sqrt{K_{\text{bot}}/\kappa}$  — linear where

the quadratic ladder grew, nothing additive anywhere; on the T145 no-go the bottom pencil ratio explodes ( $\times 15.8$  over  $m = 64 \dots 256$ ) while the parity model holds 1. **Named limits as content:** per instance on small windows — a finite list with an explicit maximum; nothing uniform in the zone index or in  $D$ , and explicitly *no* statement for all  $D$  (the one open input, the  $m$ -freeness of the bottom pencil ratio  $R = K_{\text{bot}}/\kappa = 3.7362\text{--}18.6607$ , stays open, typed open); the symbol route stays certified vacuous. KMS 1953 / Widom 1958–Basor–Ehrhardt 2009–Böttcher–Silbermann (the symbol question’s address — the computed answer: the pointwise symbol language does not extend to the bottom mode) / Weyl 1912 / Parlett / Hardy–Littlewood–Pólya 1934–Agmon 1965 / Sylvester 1852–Bunch–Kaufman 1977 / Rayleigh 1877–Ritz 1909 / Wilkinson 1968–Higham 2002 / Chebyshev 1852 named classical; Weil 1952 cited, never used as a criterion. Machine-checked: [\[verification/v550\\_odd\\_sector\\_identities.py\]](#). [E] (per-instance identities and certified inequalities; no marker moves)

**The fixed-size Ritz ceiling certificate (PRIME.RITZ.CEIL.01).** The certificate core of T154 is load-bearing as the ninth companion module (16 checks,  $\sim 0.4$ s) on the 12 *deepest* prime-power zones admitting a frame-A window inside the cap ( $n = 29 \dots 139$ ,  $m = 96 \dots 291 \leq 300$  — declared, not tuned: on the shallowest zones  $n \leq 11$  more than one bottom direction leaves the sine block and the fixed-depth certificate is not claimed there), a no-go size ladder  $m = 48 \dots 288$ , parity controls at  $m = 64, 128, 256$  and 3 Dirichlet control windows — the companion to the odd-sector module: here the *fixed-size* replacement for the ceiling step is certified. *The direction lemma.*  $\lambda_k(A) \leq \theta_k$  for every  $k \leq \dim$  — Ritz values are upper bounds for the eigenvalues of the *same index* (Courant–Fischer 1920 / Cauchy 1829), verified against exact spectra with zero one-sided excess — so the ceiling needs *no* residual argument; the reversed reading is refuted on 12/12 (overshoot  $\geq 18.6$ ). *The sixteen-column certificate.*  $\text{span}\{t_1..t_8\} + A^{-1}L_P \text{span}\{t_1..t_8\}$  (dimension 16 fixed, independent of  $m$ ; 8 completed Choleskys of matrices  $\leq 8 \times 8$ ) carries  $K^F = 1.1019\text{--}1.5845$  attaining the inertia-certified  $K_{\text{bot}}$  to  $\leq 9.9 \times 10^{-7}$  against the declared cap  $10^{-4}$  on 12/12 — the 8 size- $m$  LDL<sup>T</sup> counts retired from the ceiling step (controls: the sines alone overshoot 41.6–739; corrupted Green columns break the attainment on 12/12). *The one-direction anatomy.* Median of the seven aligned angles  $0.03\text{--}0.14^\circ$ , the largest  $83.79\text{--}89.70^\circ$  (measured): the single misaligned direction *is* the collapse price  $\lambda_1(A)/(t\mu_1^P) = 4.408\text{--}7.042$ , recovered in full per window by one Cholesky of  $A - \gamma I$  (size  $m$ , certified, not fixed size). *The no-go discriminator.* On the T145 reconstruction  $c(l) = 1/(1+l)$  the floor survives ( $t = 0.05$ ) but  $K_{\text{bot}}$  explodes  $\times 35.4$  and the ceiling explodes with it  $\times 57.5$  — no manufactured closure; the Dirichlet control (the Hankel half removed) is certified non-positive on 3/3: the reflection half is load-bearing for positivity. **Named limits as content:** per instance on small windows — a finite list with an explicit maximum; nothing uniform in the zone index or in  $D$ , and explicitly *no* statement for all  $D$ ; the attainment is not promoted to a theorem (the no-go overshoots — it is a property of the real section); the two open inputs after T154 (R1', an  $m$ -free floor for  $\lambda_{\min}(B_{HH})$  on the bulk parity block; R2', an  $m$ -free lower bound for the  $L_P$  floor on the Ritz complement, arithmetic-free and worth 91–100% of the collapse price) stay open, typed open. Courant–Fischer 1920–Cauchy 1829 / KMS 1953 / Schur 1917–Haynsworth 1968 / Sylvester 1852–Bunch–Kaufman 1977 / Weyl 1912 / Temple 1928–Kato 1949 (a floor device, not used here) / Kaniel 1966–Paige 1971 (quoted, never a bound) / Björck–Golub 1973 / Wilkinson 1968–Higham 2002 / Chebyshev 1852 named classical; Weil 1952 cited, never used as a criterion. Machine-checked: [\[verification/v551\\_ritz\\_ceiling\\_certificate.py\]](#). [E] (per-instance theorems and certified inequalities; no marker moves)

**The four fixed-size angle instruments (PRIME.ANGLE.INSTR.01).** The instrument cores of T155 and T157 are load-bearing as the tenth companion module (18 checks,  $\sim 0.4$ s) on the same declared surface as the Ritz module (the 12 deepest in-cap frame-A windows,  $n = 29 \dots 139$ ,  $m = 96 \dots 291 \leq 300$ ; a no-go size ladder  $m = 48 \dots 288$ ; parity and Dirichlet controls at  $m = 64, 128, 256$ ) — none of the four closes an angle, and none is promoted as anything but a per-instance instrument. *The complement-floor certificate (T155).*  $\min_{v \perp W} v^T L_P v \geq \mu_{13}^P - \lambda_{\max}(M^{1/2}(I - GG^T)M^{1/2})$  with  $G = T_{12}Q_W$  — an identity plus one certified Rayleigh bound, a theorem for every  $m$  and every subspace; never above the exact size- $m$  complement bottom, agreement  $\leq 1.01 \times 10^{-3}$  against the

declared cap  $2 \times 10^{-3}$  on this small surface, both closed-form controls to  $2.0 \times 10^{-11}$  (including the *worthless* configuration  $W = \text{span}\{t_2..t_{13}\} \rightarrow \mu_1^P$ : the instrument reports its own worthlessness); a corrupted overlap row moves it by  $\geq 0.549$ . *The sine-block confinement (T157)*. From the certified pencil floor and ladder ceiling alone,  $\|\gamma_H\|^2 \leq \lambda_1/(t \mu_{17}^P) \leq (S/t)/\rho_{17} = 0.0153\text{--}0.0245 < 1$  on 12/12 — the bottom eigenvector lives 97.5–98.5% inside the first 16 parity sines by a per-instance theorem (measured tail  $6.7 \times 10^{-6}\text{--}3.0 \times 10^{-5}$ ); with the pencil ceiling  $\kappa$  instead of the ladder  $S$  the bound is vacuous (17.6–477), and the inflated floor is refuted at  $e_1(A)$  by  $3.3 \times 10^3\text{--}1.0 \times 10^5$ . *The Schur-entry identity (T157)*.  $s = \mu_1^P t_1^T A^{-1} t_1 = (B^{-1})_{11}$  exactly ( $1.1 \times 10^{-11}$ ),  $(B^{-1})_{LL} S_L = I$  to  $5.8 \times 10^{-11}$  with  $S_L$  the fixed-size  $16 \times 16$  Schur complement the chain already forms:  $1/s = 4.0460\text{--}5.8962$  is an  $O(1)$  number carried by *one* entry, while the inversion-free Cauchy–Schwarz ceiling misses by  $250\text{--}7.27 \times 10^3$  (the cancellation is nearly complete); a one-index shift breaks the identity by 1.55–2.15. *The adaptive Lipschitz ceiling (T157)*. Interval bisection with the local Lipschitz constant certifies  $\inf f^{\text{arch}}/(4 \sin^2(\theta/2)) \geq 0.25$  on 12/12 windows in 519–1201 evaluations (a uniform grid at the global constant would need up to  $61 \times$  more); fed a target above the true grid minimum it *refuses* on 12/12. *The no-go discriminator*. On  $c(l) = 1/(1+l)$  the floor survives ( $t = 0.05$ ) while the confinement goes vacuous ( $\times 52.0$ ) and  $(S_L)_{11}$  explodes ( $\times 60.0$ ) against the flat 4.05–5.90 real band; parity control exact to  $4.9 \times 10^{-14}$ , the Dirichlet residual hits  $2/\sqrt{N}$  to  $7.4 \times 10^{-5}$ . **Named limits as content:** per instance on small windows — nothing uniform in the zone index or in  $D$ , and explicitly *no* statement for all  $D$ ; nothing about the angles is closed — the two open terms after T157 (an  $m$ -free lower bound on  $\hat{g}_1^2$ , equivalently an  $m$ -free upper bound on the one Schur diagonal entry; an  $m$ -free  $R1''$  domination with a non-shrinking margin) stay open, typed open. KMS 1953 / Schur 1917–Haynsworth 1968 / Courant–Fischer 1920–Cauchy 1829 / Cauchy–Schwarz (refuted as a ceiling, wired as a control) / Kantorovich 1948 / Temple 1928–Kato 1949 (floor devices, not used here) / Sylvester 1852–Bunch–Kaufman 1977 / Wilkinson 1968–Higham 2002 / Chebyshev 1852 / Szegő 1915–Widom 1958 (a named address, never a licence) named classical; Weil 1952 cited, never used as a criterion. Machine-checked: [verification/v552\_angle\_instruments.py]. [E] (per-instance identities and certified inequalities; no marker moves)

**The exact-form identities (PRIME.EXACT.FORM.IDENT.01).** The theorem cores of T158 and T159 are load-bearing as the eleventh companion module (25 checks,  $\sim 0.2$ s), recomputed on the same declared surface as the two modules above (12 deepest in-cap frame-A windows,  $m = 96 \dots 291 \leq 300$ ; no-go ladders  $m = 48 \dots 288$  on *both* T145 forms; parity and Dirichlet controls at  $m = 64, 128, 256$ ) — the algebra of the sixteen-form itself, and it closes neither open term. *The Thomson dual form and the positive Cholesky ladder (T158)*.  $s = (B^{-1})_{11} = \max_x (2x_1 - x^T B x)$  with the direction *executed* (8 random trials per window never exceed  $s$ ); the ladder  $g_K$  from one Cholesky of the leading  $16 \times 16$  block: terms strictly positive, partial sums monotone,  $1/s \leq 1/g_{16} \leq 1/g_1 = B_{11}$  literally, tight to 1.0642–1.1522; a 5% corruption of one off-diagonal entry makes the Cholesky *refuse* on 12/12. *The two-dimensionality (T158)*.  $\text{span}\{t_1, A^{-1} L p t_1\}$  attains  $s$  exactly ( $9.0 \times 10^{-13}$ ) because  $L p t_1 = \mu_1^P t_1$ ; without the Green column the two-sine span misses by 1.42–4.8, the corrupted column by  $255\text{--}7.7 \times 10^3$ . *The gauge identity and the signature (T159)*. The reflection-odd section annihilates constant lag vectors entrywise with *zero* tolerance, hence  $\sum_d w_d = 0$  to  $1.1 \times 10^{-14}$  of  $\sup |w|$ : the form is blind to the lag mass of  $c$  — exactly where the archimedean and atom halves are  $h^2$ -sized and cancel; the Toeplitz-minus-Hankel signature holds to  $1.4 \times 10^{-16}$ ; a one-index Hankel shift breaks the identity loudly, and the T145 form is not even a parity section (signature defect  $7.4 \times 10^{-2}$ ). *The exact lag sum and the two closed scalars (T159)*.  $x^T B_{LL} x = \sum_d c_d w_d$  to  $2.7 \times 10^{-16}$  of the absolute scale, with the honest price measured (relative to the cancelled total only  $1.4 \times 10^{-11}$  on this small surface);  $w_0 = \sum_k x_k^2 / \mu_k^P$  and  $2w_0 - w_1 = \|x\|^2$  exact. *The Z-matrix law and the Collatz–Wielandt floor (T159)*.  $B_{HH}^{\text{arch}}$  is a symmetric Z-matrix *raw* (no tolerance, 12/12); the closed sign-based floor  $\min_k (B_{HH}^{\text{arch}} \mathbf{1})_k = 0.9093\text{--}1.1284 \geq 0.25$  with the theorem direction verified against the exact  $\lambda_{\min} = 0.9138\text{--}1.1432$ ; the atom block obeys *no* sign law (a coin toss) and the full-block row-sum floor is negative — admissible and vacuous; the checkerboard similarity (an isometry) destroys the raw law: an entrywise fact, not a spectral one. *The no-go discriminator*. The T145 family breaks on three axes of this module’s structure, while on the

$c(l) = 1/(1+l)$  reconstruction the ladder theorem survives and  $1/g_{16}$  explodes ( $1187 \rightarrow 7.11 \times 10^4$ ) — the instrument reports the collapse. **Named limits as content:** per instance on small windows — nothing uniform in the zone index or in  $D$ , and explicitly *no* statement for all  $D$ ; the two open terms after T159 (the one explicit pairing inequality for  $R2''$ , needing correlation instead of sizes; a third device for the atom off-band block for  $R1''$ ) stay open, typed open. Maz'ya 1985 / Miclo 1999 / Schur 1917–Haynsworth 1968 / KMS 1953 / Dirichlet 1829 / Perron 1907–Frobenius 1912 / Collatz 1942–Wielandt 1950 / Gantmacher–Krein 1950 named classical; Weil 1952 cited, never used as a criterion. Machine-checked: [verification/v553\_exact\_form\_identities.py]. [E] (per-instance identities and certified inequalities; no marker moves)

**The sampling/harmonics identities (PRIME.SAMPLING.HARM.01).** The closed theorem cores of T160 and T161 are load-bearing as the twelfth companion module (21 checks,  $\sim 0.5$ s), recomputed on the same declared surface as the module above (12 deepest in-cap frame-A windows,  $m = 96 \dots 291 \leq 300$ ) plus a declared  $\Lambda$ -battery of six log-spaced cut-offs  $X = 10^3 \dots 3.2 \times 10^5$  (finite von-Mangoldt sums only, no matrix anywhere) — what the sixteen-form *pairs against*, and it closes no open term. *The sampling identity (T160).* The atom half of the pairing equals  $-\frac{1}{2} \sum_{n \leq X} (2\Lambda(n)/\sqrt{n}) \hat{w}(\log n)$  ( $\hat{w}$  the linear interpolant of the lag weights; exact to  $5.6 \times 10^{-15}$  of the absolute atom scale on 12/12): identically a finite combination of sums  $\sum_n \Lambda(n) n^{-1/2} \cos(t \log n)$  at 32 explicit frequencies; a one-index grid shift breaks the identity loudly. *The moment laws and the TV theorem (T160).*  $m_0 = 0$ ,  $m_1 = -[S_0^2 + 2 \sum_j P_j^2] \leq 0$  and  $m_2 = -[2S_1 - (M-1)S_0]^2 \leq 0$  (a perfect square), both strictly negative on every window; and U3 as a theorem with *checked* hypotheses:  $\text{TV}(c^{\text{arch}}) = 3.98\text{--}4.90 \leq |c_0| + 2 \sup < 6$ , closed and window-independent — the full atom-including kernel violates the monotone hypothesis on 12/12 (the wired mutation). *The harmonic-frequency theorem and the PNT main term (T161).*  $t_j(2\alpha) = 2\pi j$  *exactly*: the 32 frequencies are the Fourier harmonics of the log-window, and the closed parameter-free prediction  $(\sqrt{X} - 1)/(\frac{1}{4} + t^2)$  (de la Vallée Poussin 1896) matches the finite sums to 0.0007–0.097 of the trivial mass against the declared cap 0.12; a wrong Mellin pole and a mass-matched uniform fake measure both fail loudly — the agreement is a property of the von-Mangoldt measure. *The head split and the scale-free Bernstein rate (T161).*  $c_d^{\text{arch}} = \Psi_d + D\hat{G}(dD, D)$  with  $\Psi_d = -\frac{1}{2}[(d+1)\log(d+1) - 2d\log d + (d-1)\log(d-1)]$  closed,  $D$ -free and  $m$ -free, exact to  $3.2 \times 10^{-16}$  at every lag;  $\rho^* = (3 + \sqrt{5})/2 = 2.618034$  is the root of  $x^2 - 3x + 1$ , scale-free (interval ratio 5 only), with every window's  $\rho(D) = 2.588\text{--}2.608$  below it. *The fraction bound and the sign refutation (T161).*  $|\text{off-diagonal}| \leq \frac{1}{4}|Q^{\text{arch}}|$  certified per window (0.171–0.212 on 12/12), with the *refuted* sign law wired as the negative control (arch half negative, off-diagonal block positive, 176/240 pairs violate the per-pair form) — nothing single-signed is promoted. **Named limits as content:** per instance on small windows — nothing uniform in the zone index or in  $D$ ; finite  $\Lambda$ -sums allowed and used, *no* RH statement, and the  $\delta$  triage of T161 (required depth against RH strength) is a documentation item, not a module claim; the open terms after T161 (R-A' the prime-free log-moment; R-B' the  $m$ -free  $16 \times 16$  Gram fraction bound; R-C' one split of the pairing with  $\delta_{\text{eff}} < \frac{1}{2}$ ; R-D a fifth  $R1''$  device) stay open, typed open. Dirichlet 1829 / Abel 1826 / Fejér 1915–Schur 1917 / Bernstein 1912 / Chebyshev 1852–Mertens 1874 / de la Vallée Poussin 1896 / Clausen–Lerch / KMS 1953 named classical; Weil 1952 / Bombieri 2000 cited, never used as a criterion. Machine-checked: [verification/v554\_sampling\_harmonics\_identities.py]. [E] (per-instance identities and certified inequalities; no marker moves)

**The Pareto/total-variation identities (PRIME.PARETO.TV.01).** The closed theorem cores of T162 and T163 are load-bearing as the thirteenth companion module (23 checks,  $\sim 0.4$ s), recomputed on the same declared surface as the module above (12 deepest in-cap frame-A windows,  $m = 96 \dots 291 \leq 300$ ) with the one arithmetic input  $\kappa = 0.038821$  *verified* at every jump point of  $\psi(x)/x$  on the von-Mangoldt table — the *price structure* of the pairing, and it closes no open term beyond the negative closure that the floor inequality itself is. *The exchange law (T163).*  $\delta_{\text{bnd}}(x) = \frac{1}{2} + \log(2\kappa g_{16} \text{TV}(x)/P(x))/\log X$  is an identity at all 612 grid points of the declared Fejér knob ( $5.1 \times 10^{-16}$ ), with the crossing criterion  $\delta_{\text{bnd}} < \frac{1}{2} \iff P > 2\kappa g_{16} \text{TV}$  an exact equivalence at every point: price and demand are two coordinates of one object; the wrong constant breaks the law loudly. *The total-variation floor (T163).*  $w_0 = \|a\|^2$  ( $4.9 \times 10^{-16}$ ),  $\text{TV} \geq |w_0|$  by

telescoping, and  $x_1 = 1$  forces  $\mu_1^P \text{TV} = 5.00\text{--}28.84 \geq 1$  on all 708 trial vectors built — the closed floor  $1/\mu_1^P = 1/(4\sin^2(\pi/N)) \sim h^2$  — so every sub- $\frac{1}{2}$  point pays  $P > 2\kappa g_{16}/\mu_1^P$ : a flat-price sub- $\frac{1}{2}$  demand is impossible in the parity sector on this surface (closes R-C''' negatively); the inadmissible vector  $0.01x^*$  undercuts the floor and is recognised (the wired negative control). *The chain-derived price axis (T163)*.  $\sigma = 1$  is exactly the  $K = 1$  rung of the T158 Cholesky ladder ( $4.1 \times 10^{-16}$ ) and  $\sigma = \infty$  the  $K = 16$  top rung ( $1.2 \times 10^{-12}$ ); the knob is monotone in both coordinates (its curve is the Pareto front); a corrupted sixteen-block makes the Cholesky refuse. *The Mellin cell moments and the Abel step (T162)*. The  $k$ -th Mellin rung  $\sum_d w_d C_d(-(2k + \frac{1}{2}))$ , closed, agrees with an independent quadrature to  $2.8 \times 10^{-13}$  ( $k = 1 \dots 4$ ); the boundary-free Abel identities hold at levels 1–3 ( $8.3 \times 10^{-17}$ ), the optimal level is one for the closed reason  $32\pi/\alpha = 40.6\text{--}59.4 > 1$ , and a gauge violation breaks by its closed prediction ( $1.2 \times 10^{-15}$ ). *The Lerch/Frullani log-moment (T162)*.  $L = \sum_d \Psi_d w_d$  on three routes (Wronskian form  $2.8 \times 10^{-14}$ , integral with the  $d = 1$  peel  $1.8 \times 10^{-15}$ ,  $2\log 2$  to  $2.2 \times 10^{-16}$ ,  $\Psi_1 = -\log 2$  exact); plus the monotone no-go staircase ( $1.00 \rightarrow 3.49 \rightarrow 10.81 \rightarrow 24.98$ , exact null control). **Named limits as content:** per instance on small windows — nothing uniform in the zone index or in  $D$ ; the T163  $h$ -exponents are sandbox fits, *not* consumed; finite  $\Lambda$ -sums allowed and used, *no* RH statement; the open terms after T163 (R-E the successor, both arms prime-free; R-B''' the  $h$ -uniform positivity margin; R-D a fifth R1'' device) stay open, typed open. Fejér 1915 / Mellin 1896 / Abel 1826 / Chebyshev 1852–Rosser–Schoenfeld 1962 / Legendre / Dirichlet 1829 / KMS 1953 / Frullani 1828–Lerch 1887–Clausen 1832 / Schur 1917–Haynsworth 1968 named classical; Weil 1952 / Bombieri 2000 cited, never used as a criterion. Machine-checked: [verification/v555\_pareto\_tv\_identities.py]. [E] (per-instance identities and certified inequalities; no marker moves)

**The gauge/ $P_{\text{pr}}$  identities (PRIME.GAUGE.PPR.01).** The closed theorem cores of T164 and T165 are load-bearing as the fourteenth companion module (23 checks,  $\sim 0.3$ s), recomputed on the same declared surface as the module above (12 deepest in-cap frame-A windows,  $m = 96 \dots 291 \leq 300$ ) with the one arithmetic input  $\kappa = 0.038821$  *verified* at every jump point of  $\psi(x)/x$  on the von-Mangoldt table — the *gauge structure* of the pairing, and it does not close the one remaining quantifier. *The gauge theorem (T164)*.  $x_1 = 1$  fixes only the *scale* of  $a$ ;  $Q$  and TV are homogeneous of degree two; hence the demand ratio  $Q/\text{TV}$  of the same  $a$ -direction is the *same number in every sector* — unchanged to  $2.9 \times 10^{-11}$  on 2520 sector-by-trial-vector combinations (5 weight families  $\times$  7 shifts  $\times$  6 taper widths), with the pure shift scaling the certified value by exactly  $\theta = \mu_1^P/(\mu_1^P + \kappa_s)$ ; the full space is settled strictly worse ( $\mu_0 = 0$  exactly); the wrong transport map breaks loudly ( $2.0 \times 10^2\text{--}5.4 \times 10^3$ ). *The  $h^2$  conservation law (T164)*. floor  $\times$  transfer  $= 1/(\mu_1^P + \kappa_s) \cdot (1 + \kappa_s/\mu_1^P) = 1/\mu_1^P$  *identically* at every shift ( $2.2 \times 10^{-16}$ ): the  $h^2$  is conserved, never removed — it lives in the ratio, not in the floor; the wrong transfer misses by 0.9995 at the largest shift. *The power-one spend (T164)*. The  $r$ -ceiling  $r \leq U/L$  with  $L = \lambda_1(A)/\mu_1^P$  is linear in the entry ceiling, so  $d \log r / d \log U = +1$  exactly ( $1.1 \times 10^{-14}$ ); the quadratic consumption reads slope 2 and is recognised. *The  $P_{\text{pr}}$  identity and the predicate equivalence (T165)*.  $P_{\text{pr}} = g_{16} \cdot R \cdot (\text{TV}/(t_1 v)^2)/\mu_1^P$  exact to  $4.3 \times 10^{-16}$  on all 192 battery vectors, with the predicates  $R > 2\kappa$  and  $P_{\text{pr}} > 2\kappa g_{16} \text{tvf}/\mu_1^P$  agreeing as booleans everywhere and every crossing vector paying  $P_{\text{pr}} > 2\kappa g_{16}/\mu_1^P = 15.4\text{--}142.8$  above the preregistered tolerance 10 on 12/12 — the successor question R-F closes negatively by an identity plus a floor; the price with  $(t_1 v)$  unsquared breaks the gauge invariance loudly. *The Schur-cascade direction and the exponent closure (T164/T165)*.  $s = \mu_1^P t_1^T A^{-1} t_1 \geq g_{16} \geq g_1$  window by window with  $P_K = \hat{a} s \geq 1$  (Kantorovich 1948), and the exponent closure  $\log(1/g_1) = \log(g_{16}/g_1) + \log(1/g_{16})$  is exact per instance and on the fitted lists ( $1.3 \times 10^{-15}$ ); plus the monotone no-go staircase and exact Dirichlet/parity controls. **Named limits as content:** per instance on small windows — nothing uniform in the zone index or in  $D$ ; the T164/T165  $h$ -exponents and the measured anatomy are sandbox, *not* consumed; finite  $\Lambda$ -sums allowed and used, *no* RH statement; **the one open object after T165 stays open, typed open:**  $\inf_m g_{16}(m) > 0$ , equivalently a lower bound on the sixteen-step Schur-cascade gain  $g_{16}/g_1$  uniform in  $m$  — a quantifier over a certified flat list. Schur 1917–Haynsworth 1968 / KMS 1953 / Abel 1826 / Chebyshev 1852–Rosser–Schoenfeld 1962 / Kantorovich 1948 / Fejér 1915 / Dirichlet 1829 named classical; Weil 1952 / Bombieri 2000

cited, never used as a criterion. Machine-checked: [verification/v556\_gauge\_ppr\_identities.py]. [E] (per-instance identities and certified inequalities; no marker moves)

**The cascade/vector identities (PRIME.CASCADE.VECT.01).** The closed theorem cores of T166 and T167 are load-bearing as the fifteenth companion module (23 checks,  $\sim 0.1$  s), recomputed on the same declared surface as the module above (12 deepest in-cap frame-A windows,  $m = 96 \dots 291 \leq 300$ ) with the one arithmetic input  $\kappa = 0.038821$  *verified* at every jump point of  $\psi(x)/x$  on the von-Mangoldt table — the *anatomy of the cascade* and the *exactness of the closed vector at  $K = 2$* , and it does not close the one remaining scalar. *The cascade identities (T166).* The prefix property (one Cholesky of the sixteen-block yields all sixteen rungs; 192 comparisons,  $1.1 \times 10^{-12}$ ), the minor form  $1/g_K = \det G_{[1..K]}/(\mu_1^P \det G_{[2..K]})$  on the raw arithmetic Gram block ( $2.7 \times 10^{-12}$ ; Schur 1917–Haynsworth 1968), and the regression form  $g_K/g_1 = 1/(1 - R_K^2)$  ( $8.8 \times 10^{-13}$ ) — the open object *is* a near-collinearity statement; the wrong minor misses by 0.99–1.45. *The diagonal invariance (T166).* The gain  $g_K/g_1 = B_{11}(Q_K^{-1})_{11}$  is unchanged under  $B \rightarrow DBD$  on 144 random positive diagonals ( $2.7 \times 10^{-12}$ ): the cascade does not see the KMS normalisation — the gain belongs to the arithmetic Gram block alone; a non-diagonal congruence moves it by 2.7–3.8. *The exact  $K = 2$  identity (T167).* The closed vector  $u = (1, -Q_{21}/Q_{22})$  attains  $1/g_2 = Q_{11}(1 - r_{12}^2)$  exactly ( $1.1 \times 10^{-12}$ ) and  $B_{11}g_2 = 1/(1 - r_{12}^2)$  identically ( $1.2 \times 10^{-12}$ ): at  $K = 2$  the fast-null-vector is free — the third dress collapses onto the second, one scalar built from three closed lag sums; the wrong-sign vector pays  $4r_{12}^2/(1 - r_{12}^2) = 6.7 \times 10^2$ – $1.5 \times 10^4$ . *The pivot identity and the unification (T167).*  $u^T Q_K u = 1/g_K + \delta^T Q_K \delta$  for  $\delta_1 = 0$  ( $2.0 \times 10^{-12}$  on 252 evaluations) — every candidate excess is an exact number; the sign-aligned entry attainment  $|x^{*\top} E x^*| = \varepsilon S_K$  ( $3.8 \times 10^{-16}$ ) makes the entry threshold exact, and the unification  $(1/g_2)/S_2 = (1 - r_{12}^2)/(1 + 3r_{12}^2)$  ( $1.2 \times 10^{-12}$ ) makes the three dresses *one* inequality; the  $\delta_1 = 0.1$  mutation breaks by exactly the closed cross term  $2\delta_1$ . *The trial theorem and the two must-breaks (T166/T167).* All 288 random trials with  $u_1 = 1$  bound  $g_K$  from below (min ratio 15.14; Schur 1917); the  $L_P$  no-go gives cascade gain exactly 1 (deviation 0.0); the scramble type-change makes  $B_{11}$  itself lose positivity on a majority of draws on 12/12 windows — at  $K = 2$  the collapse under scrambling is a change of type, not a factor; plus exact Dirichlet/parity controls. **Named limits as content:** per instance on small windows — nothing uniform in the zone index or in  $D$ ; the T166/T167  $h$ -exponents and the measured anatomy are sandbox, *not* consumed; finite  $\Lambda$ -sums allowed and used, *no* RH statement; **the one open object after T167 stays open, typed open:** R1, the single scalar — an  $m$ -free upper bound on  $1 - r_{12}^2 = 1 - Q_{12}^2/(Q_{11}Q_{22}) \leq C h^{-3+\varepsilon}$ , three closed lag sums and nothing else. Schur 1917–Haynsworth 1968 / KMS 1953 / Rayleigh 1877–Courant–Fischer 1920 / Cauchy 1829 / Kato 1949 (cited as the closed-off route, never used) / Chebyshev 1852–Rosser–Schoenfeld 1962 / Dirichlet 1829 named classical; Weil 1952 / Bombieri 2000 cited, never used as a criterion. Machine-checked: [verification/v557\_cascade\_vector\_identities.py]. [E] (per-instance identities and certified inequalities; no marker moves)

**The bilinear/rank identities (PRIME.BILINEAR.RANK.01).** The closed theorem cores of T168, T169 and T170 are load-bearing as the sixteenth companion module (27 checks,  $\sim 0.2$  s), recomputed on the same declared surface as the module above (12 deepest in-cap frame-A windows,  $m = 96 \dots 291 \leq 300$ ) with the one arithmetic input  $\kappa = 0.038821$  *verified* at every jump point of  $\psi(x)/x$  on the von-Mangoldt table — the *exact shape of the one remaining scalar*, and it does not close that scalar. *The Lagrange/Wronskian anatomy (T168).* The lag-sum identity  $\hat{a}_{ij} = t_i^T A_h t_j = \sum_d c_d W_d^{ij}$  with the closed correlation weights ( $1.6 \times 10^{-15}$ ; the  $1/\sqrt{\mu}$  factors cancel, T169-TH3); Euclidean orthogonality  $t_1 \cdot t_2 = 0$  ( $4.4 \times 10^{-16}$ ) — the near-parallelism is created *entirely* by the arithmetic metric; the Wronskian telescope  $(\mu_1 - \mu_2)u_r v_r = W_{r-1} - W_r$  with  $W_{-1} = W_{h-1} = 0$  ( $1.9 \times 10^{-17}$ ) and  $\frac{1}{2} \sum M^2 = 1/(\mu_1 \mu_2)$  exactly ( $4.6 \times 10^{-16}$ ; Lagrange 1773); the wrong eigenvalue pair misses by 1.66–1.67 relative. *The exponent account and the Gershgorin certificate (T168/T169).*  $1 - r_{12}^2 = \nu_1 \nu_2 / (\hat{a}_{11} \hat{a}_{22})$  identically ( $2.5 \times 10^{-13}$ );  $\nu_1 \leq \max(\hat{a}_{11}, \hat{a}_{22}) + |\hat{a}_{12}|$  (Gershgorin 1931, unconditional) on 12/12 with loss 1.1885–1.1947; dropping  $|\hat{a}_{12}|$  fails on every window. *The det-collapse theorem (T169-TH7).*  $R(K7) = 2\sqrt{\hat{a}_{11}\hat{a}_{22}} \det \hat{A} / [(\hat{a}_{11} + \hat{a}_{22})(\sqrt{\hat{a}_{11}\hat{a}_{22}} + \hat{a}_{12})]$  in closed form ( $8.7 \times 10^{-13}$ ) — the loop T167 scalar  $\rightarrow$  T168 factor  $\rightarrow$  T169 candidate closes as an identity;

usable form  $\nu_2 \leq 2 \det \hat{A}/(\hat{a}_{11} + \hat{a}_{22})$ , tight to  $< 2$  exactly; the closed constant  $1/\sqrt{2}$  overshoots by  $17\text{--}2.3 \times 10^2$ . *The exact bilinear von Mangoldt form (T170)*.  $\hat{A} = B - S$  against the direct block ( $1.7 \times 10^{-14}$ );  $\det \hat{A} = \det B - D(B, S) + \det S$  ( $1.0 \times 10^{-10}$ );  $\det S = \sum_{n,m} \Lambda(n)\Lambda(m)K(n,m)/\sqrt{nm}$  with  $K = \frac{1}{2}D(X_n, X_m)$  through the *explicit* kernel matrix ( $7.5 \times 10^{-16}$ ; Cauchy–Binet); the wedge reading to the measured rank-one defect  $4.6 \times 10^{-3}\text{--}3.1 \times 10^{-2}$ . *The rank-3 theorem and the Type II collapse (T170)*. The polarisation matrix has eigenvalues  $\{-2, -1, +1\}$  exactly — rank 3, signature  $(1, 2)$  — so  $\sigma_4/\sigma_1 \leq 1.6 \times 10^{-16}$  on every window: the bilinear form *is* the rank-3 polynomial  $S_{11}S_{22} - S_{12}^2$  in three linear  $\Lambda$ -sums; Vaughan’s identity is coefficient-exact for every  $U = V \in \{3, 4, 5, 6\}$  and the Type II kernel has 99.9%-energy rank  $[2, 2, 2, 2]$  against generic  $[20, 15, 11, 9]$ . *The R4-free chain and the discriminators (T170/T169)*.  $1 - r_{12}^2 = \det \hat{A}/(\hat{a}_{11}\hat{a}_{22})$  is an identity — no  $A$ -positivity, the Weil fence never approached; the  $L_P$  no-go gives  $\hat{a}_{12} = 1.1 \times 10^{-17}$  identically (no decay); the scramble leaves the rank-3 collapse *untouched* on all 60 draws while the value moves  $\times 834.8$  median — the arithmetic sits in the joint values of  $(S_{11}, S_{22}, S_{12})$  against  $B$ ; plus exact Dirichlet/parity controls. **Named limits as content:** per instance on small windows — nothing uniform in the zone index or in  $D$ ; the T168–T170  $h$ -exponents and route-table deltas are sandbox, *not* consumed; finite  $\Lambda$ -sums allowed and used, *no* RH statement — the one open object after T170 (R1, now finally *classified* but not closed: an  $m$ -free unconditional certificate that the two rows of  $\hat{A}$  become collinear at rate  $h^{-3+\varepsilon}$  — a joint near-degeneracy of three explicit linear  $\Lambda$ -sums, not a size) stays open, typed open. Lagrange 1773 / Cauchy–Binet 1812–1815 / Gershgorin 1931 / Rayleigh 1877 / Montgomery–Vaughan 1973–Vaughan 1977 (the priced route) / KMS 1953 / Chebyshev 1852–Rosser–Schoenfeld 1962 / Dirichlet 1829 named classical. Machine-checked: `[verification/v558_bilinear_rank_identities.py]`. [E] (per-instance identities and certified inequalities; no marker moves)

**The phase-2 capstone (PRIME.PHASE2.CAPSTONE.01).** The capstone *verifies the map and claims nothing new*: T126–T170 reduced the I5 uniformity question through sixteen exact reformulations to one object (R1), T171 (contract FINAL.MAP, verdict MAP-COMPLETE) reproduced every link of that reduction in one connected sandbox run, and the seventeenth companion module (35 checks,  $\sim 0.3$ s) is the load-bearing version of that run on the same declared surface as the module above (12 deepest in-cap frame-A windows,  $m = 96 \dots 291 \leq 300$ ;  $\kappa = 0.038821$  *verified* at every jump point of  $\psi(x)/x$ ; the  $16 \times 16$  low block positive definite on  $12/12$  — a numerical fact, never routed through the Weil criterion). *The chain, sixteen links in one pass* (13 THEOREM / 3 CERT): the T152 Schur two-block criterion sharp on both sides (holds at  $0.5\lambda_{\min}$ , fails at  $1.02\lambda_{\min}$ ; KMS  $1.3 \times 10^{-15}$ , Haynsworth  $3.8 \times 10^{-15}$ ); the T151 Sobolev bound  $b_k \leq C_S k$  (worst ratio 0.4816); the T153  $\Psi$ -collapse  $\Psi = \sum_d c_d w_d = x^T G x$  ( $6.3 \times 10^{-16}$ ;  $\sum_d w_d = 0$  to  $9.0 \times 10^{-17}$ ); the T154 Ritz ceiling from *below* ( $g_{32}/s = 0.9520\text{--}0.9849$ ); the sharp T155  $12 \times 12$  floor ( $t = 0.2295\text{--}0.3719$ ); the T156  $F(P, r)$  chain ( $L(16) = 6.6\text{--}15.6$ ); the T158 Thomson/ladder duality ( $3.1 \times 10^{-12}$ ; 36/36 trials below); the T160 sampling identity ( $1.1 \times 10^{-15}$  /  $9.9 \times 10^{-15}$ ); the T163 Abel swap with the TV floor (overshoot  $\times 5.2 \times 10^4\text{--}1.1 \times 10^6$ ); the T164 gauge/quantifier collapse ( $\Psi(x^*) = 1/g_{16}$  to  $7.0 \times 10^{-12}$ ; 48/48 perturbed vectors larger); the T165  $P_{\text{pr}}$  identity ( $= 1$  at  $x^*$ ); the T166 cascade  $1/(1 - R_{16}^2)$  ( $1.8 \times 10^{-12}$ , diagonal-invariant); the T167  $K = 2$  exactness (milder  $\times 1.62\text{--}5.20$ ); the T169 det collapse, R4-free ( $\leq 4.2 \times 10^{-12}$ , no positivity of  $A_h$  consumed — the Weil fence never approached); the T170 rank-3 theorem ( $J$ -eigenvalues  $\{-2, -1, +1\}$  exact;  $\sigma_4/\sigma_1 \leq 1.6 \times 10^{-16}$ ); and the T170 R1 shape via the  $2 \times 2$  Lagrange identity ( $\sin \angle = 1.169 \times 10^{-4}\text{--}2.467 \times 10^{-3} \leq 10^{-2}$  per window — *rate and quantifier typed OPEN*). *The no-go battery*, all eight failing as classified: size budget  $\times 2.7 \times 10^3\text{--}1.6 \times 10^4$ ; symbol infimum  $< 0$  on  $12/12$ ; sector change an invariance ( $\leq 4.2 \times 10^{-12}$ ); perturbation  $\times 9.8 \times 10^2\text{--}2.0 \times 10^4$  off scale (Kato 1949); atom band dominates  $\times 1.71\text{--}2.83$ ; sieve operator-norm  $\times 8.8 \times 10^3\text{--}7.5 \times 10^4$  over (Montgomery–Vaughan 1973 / Vaughan 1977 the priced route); scramble  $\times 679$  median with rank-3 *untouched* on 60 draws;  $L_P$  gain = 1 exactly. *The precision ledger*: needed  $2.752 \times 10^{-4}$  at the deepest window — finer than the RH yardstick  $h^{-0.5}$  by  $215\times$  and the best unconditional  $h^{-0.996}$  by  $13\times$  on this surface; both self-similarity identities  $\leq 6.8 \times 10^{-13}$ ; mutations fail loudly. **Named limits as content:** per instance on small windows — nothing uniform in the zone index or in  $D$ ; the T171 trend fits (pooled

$h^{-2.33}$ , per-frame  $h^{-1.86}/h^{-2.87}$ ,  $\delta = +1.044$  against 3.0, the full-surface numbers  $2.284 \times 10^{-7} / 1.2 \times 10^5 \times / 3.1 \times 10^3 \times$ ) are sandbox, *not* consumed; *zero* new uniform-in- $m$  statements — which is exactly why the capstone is promotable; the one open phase-2 object (R1, an  $m$ -free unconditional certificate that the two rows of  $\hat{A}$  become collinear at rate  $h^{-3+\varepsilon}$ ) stays open, typed open; *no* RH statement. Schur 1917–Haynsworth 1968 / KMS 1953 / Cauchy 1829–Courant–Fischer 1920 / Rayleigh 1877 / Abel 1826 / Dirichlet 1829 / Lagrange 1773 / Cauchy–Binet 1812–1815 / Kato 1949 (the closed-off route) / Montgomery–Vaughan 1973–Vaughan 1977 / Chebyshev 1852–Rosser–Schoenfeld 1962 / Fejér 1915 named classical; Weil 1952 / Bombieri 2000 cited, never used as a criterion. Machine-checked: [verification/v559\_phase2\_capstone.py]. [E] (per-instance identities and certified inequalities; no marker moves)

**The frame/deficit identities (PRIME.FRAME.DEFICIT.01).** The gauge route of phase 2, closed as algebra: the eighteenth companion module (24 checks,  $\sim 0.5$  s) promotes the theorem cores of T172 (contract FRAME.BEYOND), T173 (FRAME.RATE) and T174 (CANCEL.IDENTITY) — the companion to the capstone above, certifying what those three parts established *about* the map’s frame dependence, on a small declared surface (a  $4 \times 3$  (anchor,  $h$ ) rectangle of gap-blind frame-B cells,  $n = 41 \dots 557 \times h = (160, 226, 320)$ , comb complete, positive definite on 12/12; one non-prime-power instance  $n = 210$ ; one truncated-comb stress instance  $n = 1009$ ;  $\kappa = 0.038821$  *verified* at every jump point of  $\psi(x)/x$ ). *The identity transfer + sieve-horizon discriminator* (T172): four chain links — the KMS/Schur pair ( $3.2 \times 10^{-16}$ ), the lag-sum identity ( $1.4 \times 10^{-16}$  of the larger half; Abel 1826), the R4-free det collapse (at 0.020 of the conditioned bar) and the additive split ( $7.6 \times 10^{-16}$ ) — re-derived *exactly* on a frame-B and a non-prime-power instance; the truncated-comb instance turns indefinite ( $\lambda_{\min} = -9.6 \times 10^4$ ) against 12/12 positive definite complete-comb cells, with the identity links holding either way (none uses positivity): indefiniteness sits at the declared sieve horizon, not at a frame or zone. *The  $q = 1 - s$  identity + calibration* (T173):  $hD = \alpha$  per cell forces the OLS slopes to obey  $q = 1 - s$  exactly ( $2.9 \times 10^{-14}$ ); the gauge degrees come out as integers —  $G2 = t/\lambda_{\max}(B_{LL})$  and the delivery  $1 - r_{12}^2$  both (0, 0), so  $G2$  is the unique doubly gauge-invariant arithmetic member of the family;  $G3 = \mu_1^P G2$  identically (the offset-2 explained);  $(D/\alpha)^3 h^3 = 1$  a tautology. *The blindness theorems + the additive limit* (T174): the amplitude  $c \rightarrow \kappa c$  is a gauge of both sides and of  $R$  over twelve decades (worst 0.026 of the per-cell conditioned bar); the scalar  $\mu_1^P$  — *the entire  $h^{-2}$  channel of the demand* — cancels exactly on both sides; the delivery is invariant under the full positive diagonal group while a non-scalar diagonal moves the demand by 0.23–1.40 (the asymmetry is the anatomy); and neither the arch term nor the comb term alone has a positive Schur floor on a single cell (0/12 + 0/12 against 12/12 for the sum): no factorisation  $\hat{A} = (\text{frame}) \times (\text{arithmetic})$  exists — P6 is *not* a theorem, only its multiplicative half. *The KMS shape band* (unconditional, declared  $\varepsilon = (K\pi/(2h+1))^2/6$ ): the  $k^2$ -swap moves log GAP by  $\leq 0.312$  of the certified band while the delivery does not move at all. *The placement discriminator*: the value-multiset scramble destroys the floor on 24/24 draws. **Named limits as content:** per instance on a small declared surface — nothing uniform in the zone index or in  $D$ ; the frame-free deficit  $+0.1111 \pm 0.0222$  ( $5.0\sigma$ , 151 anchor clusters) is a sandbox MEASURED number, *not* consumed; the one open phase-2 object (R1, now stated frame-free) stays open, typed open; *no* RH statement. Schur 1917 / KMS 1953 / Dirichlet 1829 / Rayleigh 1877 / Abel 1826 / Chebyshev 1852–Rosser–Schoenfeld 1962 / Wilkinson 1968–Higham 2002 named classical; Weil 1952 / Bombieri 2000 cited, never used as a criterion. Machine-checked: [verification/v560\_frame\_deficit\_identities.py]. [E] (per-instance identities and certified inequalities; no marker moves)

**The dense-limit identities (PRIME.DENSE.LIMIT.01).** The exact cores of the phase-2 endgame (T176, contract DENSE.LIMIT, verdict SITS-AT-ZERO — the probe that raised the sieve twentyfold, extended the deficit(dens) curve to a densest bin consistent with zero, and *closed the phase-2 measurement programme as planned*), as one machine-checked module (16 checks,  $\sim 0.5$  s, deterministic) on a small declared surface (18 prime-power anchors in  $[40, 565] \times$  the declared  $2^{1/4}$  ladder  $h = (128 \dots 304)$ , comb complete, positive definite on 108/108;  $\kappa = 0.038821$  verified at every jump point; the big sieve is *not* rebuilt — the identities are sieve-size-free). *R1 closed* (T176 P2): the Feynman–Hellmann identity  $d \log R/d\delta = v^T d\tilde{A}v/\lambda_{\min} - w^T d\tilde{A}w/\lambda_{\max} - d\text{DEL}/\text{DEL}$  with the

exact lag derivative reproduces the full nonlinear rebuild on 6/6 instances at the minimum of a declared step ladder whose U-shape is exhibited ( $2.3 \times 10^{-7}$  at  $\delta = 10^{-6}$ ; a wrong formula has a floor, not a minimum) — T175’s measured intervention slope is retired as a separate claim, and the near-degeneracy channel  $d\text{DEL}/\text{DEL}$  carries the largest share (0.54–0.98). *The phase pole* (T176 P3): positive definiteness survives only 2% of the phase period,  $\delta_{\text{PD}} \cdot |d \log R/d\delta| = O(1)$ , and the two lowest modes are already near-degenerate at  $\delta = 0$  — the  $\sim 300\times$  harmonic anomaly is retired (a first-harmonic regression of a function with a pole in the interval). *Sieve consistency at construction level* (T176 P4): complete-comb lag vectors agree bit for bit under a bigger table,  $R$  exactly, the bin deficit identically — what lets any later probe raise the cap without re-deriving the chain; the truncated must-break moves as it must. *The estimator + the ladder caveat as a check* (T176 P1): the cluster-robust bin statistic is deterministic and exactly reproducible — and *ladder-dependent* (fine vs coarse rung ladder shift 0.036 here, up to 0.20 on T176’s surface): any quoted bin deficit must cite its rung ladder, and the ledger row does. **Named limits as content:** T176’s SITS-AT-ZERO (pooled  $+0.0496 \pm 0.0372$  over the two new bins; plateau window  $3.6\times$  narrower) is a sandbox MEASURED number with its rung ladder, *not* consumed; the limit — true zero, power law, plateau — stays open, typed open, under *every* resource-reachable sieve; the closure is of the measurement programme, not of the question; *no* RH statement. Feynman–Hellmann (Hellmann 1937, Feynman 1939) / Schur 1917 / KMS 1953 / Dirichlet 1829 / Chebyshev 1852–Rosser–Schoenfeld 1962 named classical; Weil 1952 cited, never used as a criterion. Machine-checked: [verification/v562\_dense\_limit\_identities.py]. [E] (per-instance identities and certified inequalities; no marker moves)

**The Paper-II readout closure (PRIME.PAPER2.READOUT.01).** The check closure of the classification paper (Paper II, `parity_toeplitz_classification`): every number that paper quoted from an exploration probe without a recomputing module is recomputed on the identical declared surface (24 checks,  $\sim 0.5$  s) — the T170 reference window bit for bit ( $h = 540$ ,  $X = 157^2$ ;  $\det \hat{A} = 4.22 \times 10^{-3}$ , a 4.7-order cancellation), the  $\ell^1$ -to-signed ratio  $40.6\times$  (Cauchy–Binet at round-off), the tightness envelope  $\mu_1^P \text{TV} \in [5.00, 23.20]$  with both ends attained, the density-curve endpoints on identical bin edges — and the one caveat Paper II declared without exhibiting it is exhibited as a structure fact: the rung-ladder shift of the binned exponent is *anchor selection* (fine vs coarse ladders select different anchor sets in the same bin, 13 vs 12; removing the extra anchors reproduces the whole shift, curvature transport moves it by only  $-0.0111$ ). Pure readout/identity checks — no new claim, no rate, no uniformity; every measured quantity stays MEASURED with its rung ladder; no RH statement. Machine-checked: [verification/v563\_paper2\_readouts.py]. [E] (readout/identity checks with mutation controls; no marker moves)

v798/v819

and the two off-origin plane classes, i.e. the v821 and v834 counts as rows of *one* table. The *local* Hecke theorem holds at every one of the 15 points: 3 isotropic + 4 non-isotropic directions through  $x$ , the isotropic ones in bijection with the points of  $x^\perp/\langle x \rangle$ , and the 28 off-origin flats through  $x$  split  $12 + 16$  with  $A_3 + B_3 = 28 = \sigma_3(3)$  and  $A_3 - B_3 = -4 = a_3$  (the  $\eta$ -product Hecke eigenvalue, now a *local* statement;  $28 - (-4) = 32 = 2^5$ , the  $\pi_{\text{cusp}}$  denominator).

*The  $-4$  triple point:* the local Hecke count  $12 - 16$ , the bent Gauss sum  $-2^{4/2}$  and the Walsh eigenvalue are the *same machine-checked integer*  $= a_3$ ; and

and the cross-machinery identity lands: the v817 prime-packet numerator *equals* the code Walsh transform at every level  $\leq 16$  (Eisenstein normal form  $15 \Theta_{D8} = 7 \Theta_L + 8 \Theta_0$ ), so  $\hat{m}_2(n) = -1/15$  holds as an *integer identity at every odd  $n \leq 16500$*  (all 1911 odd primes covered; upgraded to *all* odd shells in round 36 — see the global shell theorem below) — the per-prime measurement was the prime restriction of a character theorem of this code.